

CSCE 5400 Formal Languages, Automata, and Computability - Fall 2024

Name: Kishan Kumar Zalavadia

EUID: 11685261

Assignment-6

1. Show that NP is closed under the star operation. Provide a detailed justification for your answer.

Ans:

If L is the language that belongs to NP, then we need to prove that L^* also belongs to NP.

Given $L \in \text{NP}$, to prove $L^* \in \text{NP}$

If L is an NP language, then a Turing Machine M should decide on language L in polynomial time.

Let us consider an input string w which belongs to L^* .

Now to construct a nondeterministic Turing machine M^* for L^* , we use the following steps:

- On input w , machine M^* nondeterministically breaks the input string w into parts such that $w = w_1 w_2 w_3 \dots w_n$
- Now, check for every w_i in w_n nondeterministically determines that it belongs to L .
- If all the w_i are in L , then M^* accepts the string w .
- If M accepts all the substrings, then M^* accepts w , otherwise, reject the input.

Algorithm:

Language L belongs to NP.

There is a nondeterministic Turing Machine M such that the language of M , $L(M) = L$

$M =$ "On input w

1. If $w = \epsilon$ then accept.
2. Nondeterministically select a number n such that $1 \leq n \leq |w|$.
3. Nondeterministically split w into n pieces such that $w = w_1 w_2 \dots w_n$.
4. For all i , $1 \leq i \leq n$: run M on w_i . If M is rejected, then reject.
5. Else (M accepted all w_i , $1 \leq i \leq n$), accept."