CSCE 5400.001 Formal Languages, Automata and Computability

Assignment-3 Key

1. Find grammar for the following languages:
   a. $L = \{a^n b^m c^k \mid n = k + m \text{ and } n \geq 0, m \geq 0, k \geq 0\}$
      We can rewrite it as $a^k a^m b^m c^k$
      S -> aSc | T
      T -> aTb | ε

   b. $L = \{(ab)^n (cd)^n : n \geq 1\}$
      S -> aTd
      T -> bSc | bc
      or
      S -> abScd | abcd

   c. Language whose 3rd symbol from last is b.
      CFG for (a+b)* b (a+b) (a+b)
      S -> AbBB
      A -> aA | bA | ε
      B -> a | b
      or
      S -> AbB
      A -> aA | bA | ε
      B -> aa | ab | ba | bb

2. What is the language of the following CFG?
   S -> AB | ε
   A -> 1A | S
   B -> 0B | S

   Answer:
   (1*0*)* or **(0+1)***

3. Remove the ε-Productions:
   S -> AaB | aaB
   A -> ε
   B -> aaA | ε

Answer:

Step-1: Remove A -> ε

S -> AaB | aaB | aB
B -> aaA | ε | aa

Step-2: Remove productions with A, since all productions for A are eliminated.

S -> aaB | aB
B -> aa | ε

Step-3: Remove B -> ε

S -> aaB | aB | aa | a
B -> aa

4. Convert the following CFG into an equivalent CFG in Chomsky Normal Form:

A -> BAB | B | ε
B -> 00 | ε

Answer:

Step-1: Eliminate ε productions

A -> BAB | B | **BB | AB | BA** | ε
B -> 00

Step-2: Eliminate unit productions

A -> BAB | **00** | BB | AB | BA | ε
B -> 00

Step-3: Remove useless productions

No useless productions in this grammar.

Step-4: CFG in Chomsky Normal Form (CNF: **A -> BC** or **A -> a**)

A -> BAB | **XX** | BB | AB | BA | ε
X -> 0
B -> **XX**

Final CFG in CNF:

A -> B**Y** | XX | BB | AB | BA | ε
Y -> AB
X -> 0
B -> XX

5. i) Find the grammar for the following language.

$L = \{a^n b^n c^m d^m \mid n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$

Grammar:

S -> AB | C
A -> aAb | ab
B -> cBd | cd
C -> aCd | aDd
D -> bDc | bc

ii) Convert the generated Context Free Grammar to Greibach Normal Form.

Step-1: Eliminate ε productions

No ε productions in this grammar.

Step-2: Eliminate unit productions

S -> AB | **aCd | aDd**
A -> aAb | ab
B -> cBd | cd
C -> aCd | aDd
D -> bDc | bc

Step-3: CFG in Greibach Normal Form (GNF: **A -> aα** or **A -> a**)

S -> **aAbB | abB** | aCd | aDd
A -> aAb | ab
B -> cBd | cd
C -> aCd | aDd
D -> bDc | bc

Final CFG in GNF:

S -> aAPB | aPB | aCR | aDR
A -> aAP | aP
B -> cBR | cR
C -> aCR | aDR
D -> bDQ | bQ
P -> b
Q -> c
R -> d

iii) Is the grammar for language L ambiguous? Why?

Yes, the grammar for language L is ambiguous.

Example:

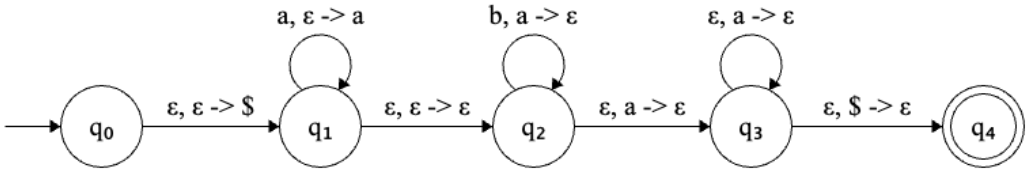
String “aabbccdd” can be generated using two different Left-most derivations

1. $S \underset{lm}{\Rightarrow} AB \underset{lm}{\Rightarrow} aAbB \underset{lm}{\Rightarrow} aabbB \underset{lm}{\Rightarrow} aabbcBd \underset{lm}{\Rightarrow} aabbccdd$
2. $S \underset{lm}{\Rightarrow} C \underset{lm}{\Rightarrow} aCd \underset{lm}{\Rightarrow} aaDdd \underset{lm}{\Rightarrow} aabDcdd \underset{lm}{\Rightarrow} aabbccdd$

Assignment-4 Key

1. Find Push Down Automata and Context Free Grammars for each of the following languages. To find the PDA do not convert CFG to PDA.
   a. **$L = \{a^n b^m \mid n > m\}$**

      PDA:

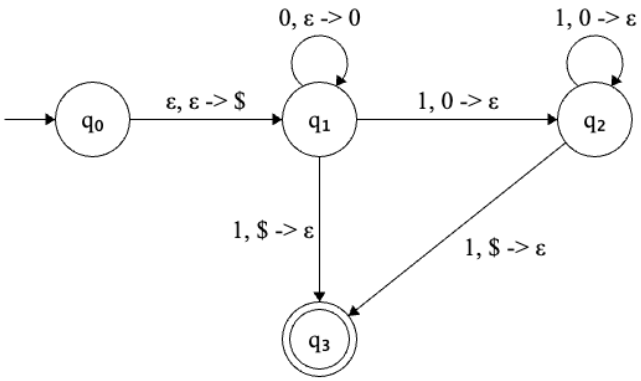


      CFG:

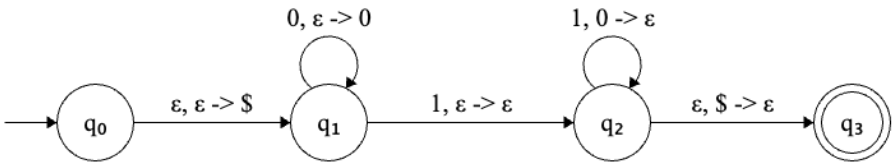
$$\begin{cases} S \rightarrow aSb \mid X \\ X \rightarrow Xa \mid a \end{cases}$$

   b. **$A = \{0^i 1^{i+1} \mid i \geq 0\}$**

      PDA:



      or



      CFG:

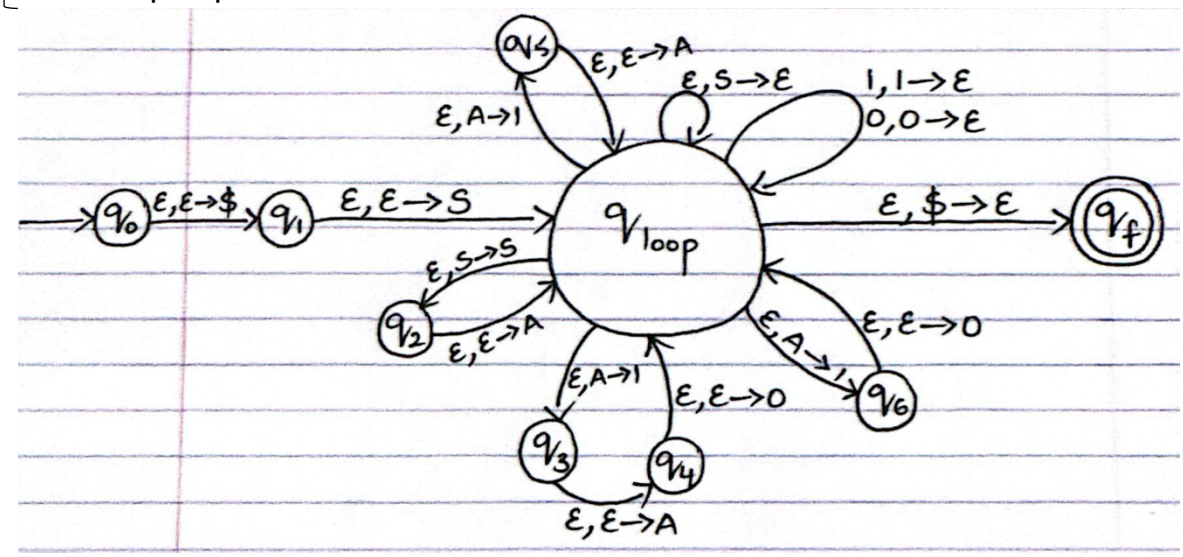
$$\begin{cases} S \rightarrow 0S1 \mid 1 \end{cases}$$

      or

$$\begin{cases} S \rightarrow 0S1 \mid X \\ X \rightarrow 1 \end{cases}$$

2. Convert the following Context Free Grammars to equivalent Push Down Automata.

S -> AS | ε
A -> 0A1 | A1 | 01



E -> E + T | T
T -> T x F | F
F -> (E) | a