CSCE 5400 Formal Languages, Automata, and Computability - Fall 2024

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Assignment-2

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1. Find an inductive definition for each set S. In this question, N is the set of Natural numbers and includes 0. (4 points)

a) S: {1, 5, 13, 29, 61, …}

1) a) Basis: $1 \in S$

Induction: If $x \in S$ then $x + 2^{n+1} \in S$

where n is a natural number.

Or

1a) Basis: $1 \in S$

Induction: If $x \in S$ then $2x + 3 \in S$

b) S: {a2n | n ϵ N } U { a2n+1 | n ϵ N }

1b) Basis: $\varepsilon \in S$

Induction: If $x \in S$ then $ax \in S$.

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2. Define a grammar for each of the following languages: (6 points)

a) L = {bb, bab, baab, baaab, ... }

2) a) $L = \{bb, bab, baab, baaab, \ldots\}$

$$\begin{cases} S \rightarrow bAb \\ A \rightarrow \varepsilon \mid aA \end{cases}$$

b) $\{an \mid n \in N\} \cup \{bcn \mid n \in N\}$ * N is the set of Natural numbers and includes

2 b) $\{a^n \mid n \in N\} \cup \{bc^n \mid n \in N\}$

$$\begin{cases} S \rightarrow A \mid B \\ A \rightarrow aA \mid \varepsilon \\ B \rightarrow bC \\ C \rightarrow cC \mid \varepsilon \end{cases}$$

c) aa*cbb*d ∑= {a,b,c,d}

2c) $aa^*cbb^*d \quad \Sigma = \{a,b,c,d\}$

$$\begin{cases} S \rightarrow aAcbBd \\ A \rightarrow \varepsilon \mid aA \\ B \rightarrow \varepsilon \mid bB \end{cases} \quad \text{or} \quad \begin{cases} S \rightarrow aAC \\ A \rightarrow aA \mid \varepsilon \\ C \rightarrow cbB \\ B \rightarrow bB \mid D \\ D \rightarrow d \end{cases}$$

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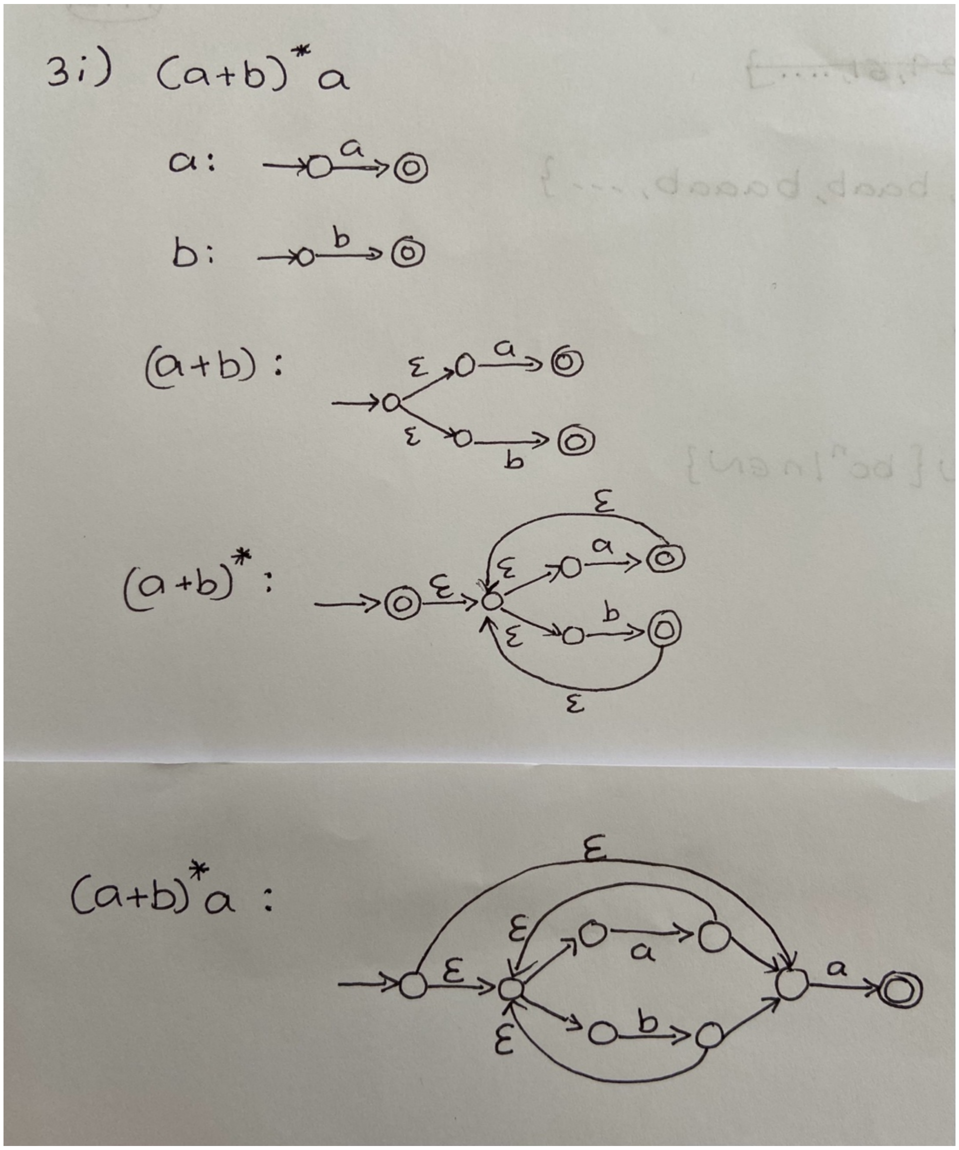
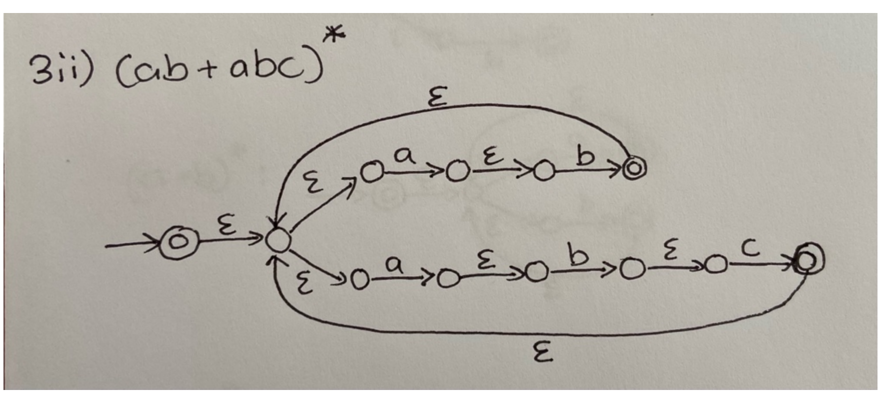
3. Construct an NFA for the following languages. (6 points)

i. (a + b)*a

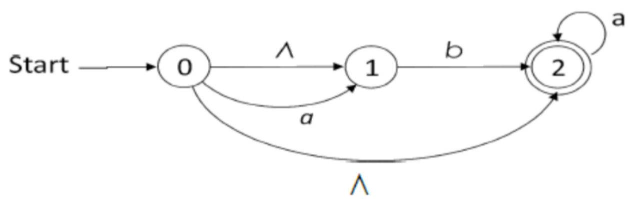


ii. (ab + abc)*



4. **(A)** Convert the following NFA to equivalent DFA. (**B**) Show the steps for the conversion. (10 points)

Over alphabet {a , b}. Hint: Symbol □ is another notation for empty string (∈)



4) NFA to DFA

Step 1: λ-closure

$\lambda(0) : \{0, 1, 2\}$

$\lambda(1) : \{1\}$

$\lambda(2) : \{2\}$

Step2:

T_N

		a	b	ε
	0	{1}	φ	{1, 2}
	1	φ	{2}	φ
F	2	{2}	φ	φ

T_D

	a	b
F $\lambda(0) = \{0, 1, 2\}$	{1, 2}	{2}
F {1, 2}	{2}	{2}
F {2}	{2}	φ
φ	φ	φ

$$T_D(\{0,1,2\}, a) = \lambda(T_N\{0,1,2\}, a)$$
$$= \lambda(T_N\{0\}, a) \cup \lambda(T_N\{1\}, a) \cup \lambda(T_N\{2\}, a)$$
$$= \lambda(1) \cup \lambda(\phi) \cup \lambda(2)$$
$$= \{1, 2\}$$

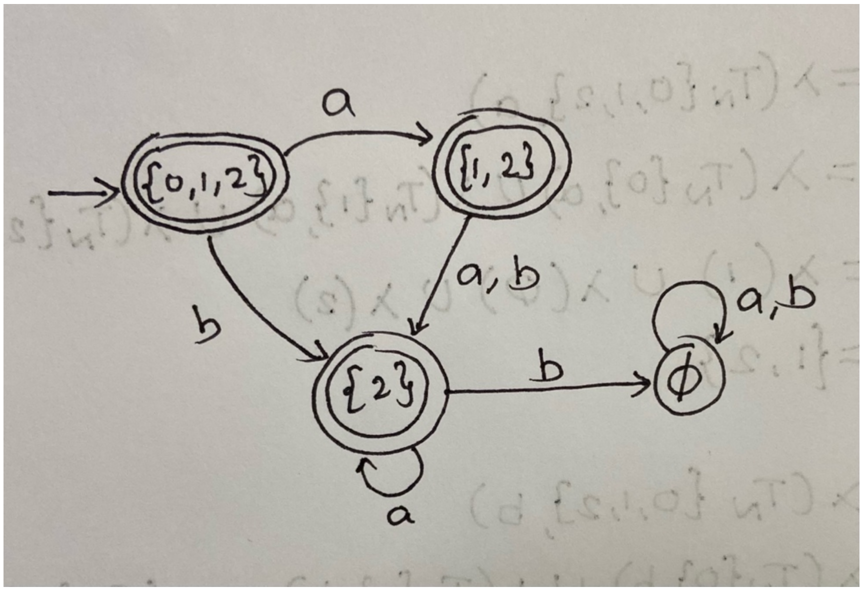
$$T_D(\{0,1,2\}, b) = \lambda(T_N\{0,1,2\}, b)$$
$$= \lambda(T_N\{0\}, b) \cup \lambda(T_N\{1\}, b) \cup \lambda(T_N\{2\}, b)$$
$$= \lambda(\phi) \cup \lambda(2) \cup \lambda(\phi)$$
$$= \{2\}$$

$$T_D(\{1,2\}, a) = \lambda(T_N\{1,2\}, a)$$
$$= \cancel{\lambda(T_N, a} \; \lambda(T_N\{1\}, a) \cup \lambda(T_N\{2\}, a)$$
$$= \lambda(\phi) \cup \lambda(2)$$
$$= \{2\}$$

$$T_D(\{1,2\}, b) = \lambda(T_N\{1,2\}, b)$$
$$= \lambda(T_N\{1\}, b) \cup \lambda(T_N\{2\}, b)$$
$$= \lambda(2) \cup \lambda(\phi)$$
$$= \{2\}$$

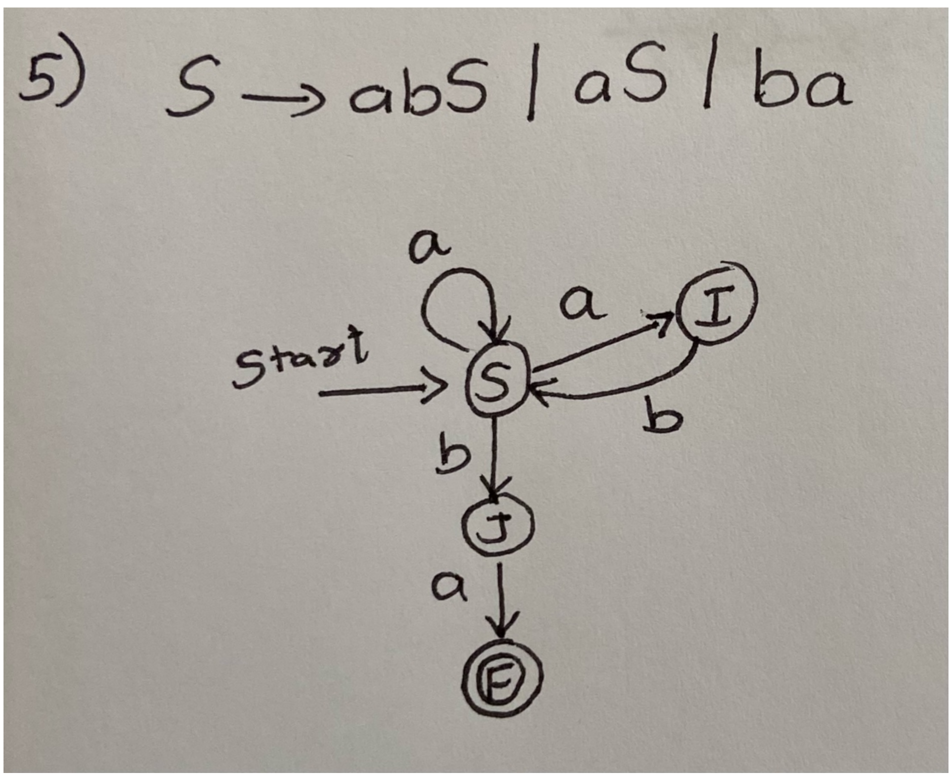
$$T_D(\{2\}, a) = \lambda(T_N\{2\}, a)$$
$$= \lambda(2)$$
$$= \{2\}$$

$$T_D(\{2\}, b) = \lambda(T_N\{2\}, b)$$
$$= \lambda(\phi)$$
$$= \phi$$

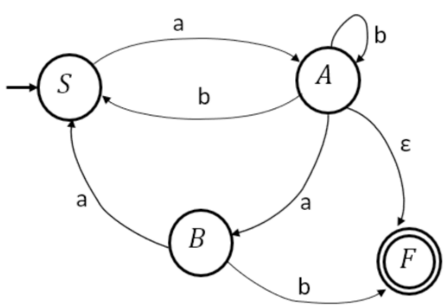


5. What is the NFA of the following regular grammar? (4 points)

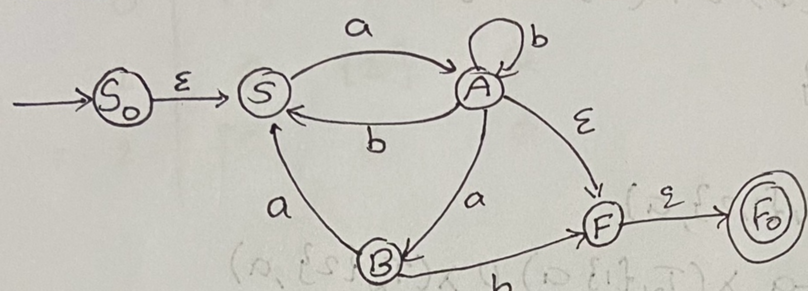
S → abS | aS | ba



6. Convert the following NFA to an equivalent Regular Expression using **GNFA method**. Please first delete state B, then state S, and finally state A. (10 points)



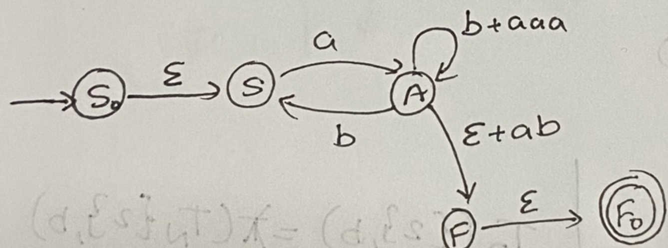
6) GNFA:



Step 1: Delete B

in	out
1	2

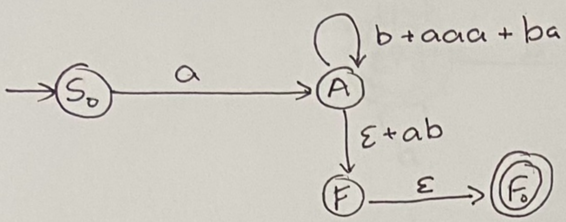
$\Rightarrow 1 \times 2 = 2$ new transactions



Step 2: Delete S

in	out
2	1

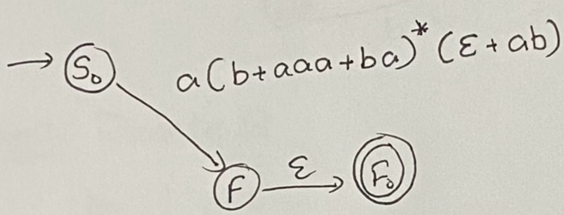
2×1 = 2 new transactions



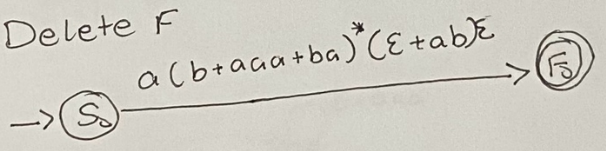
Step 3: Delete A

in	out
1	1

1×1 = 1 new transaction



Step 4: Delete F



So, the Regular Expression is

$$a(b+aaa+ba)^*(\varepsilon+ab).$$