MA203 Project Presentation

MATHEMATICALLY MODELING THE CHARGE DISTRIBUTION ON THE SURFACE OF A SPACECRAFT IN OUTER SPACE ORBITS

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ABSTRACT

Understanding the interaction of a spacecraft with the outer space environment forms a crucial part of its functioning. Various phenomena including solar radiation, plasma interaction, dielectric charging, etc., cause a spacecraft to acquire a net charge, which is non-uniformly distributed. This charge distribution poses a risk to the functioning of the spacecraft as they lead to electrical discharges (such as Corona discharge or arcing), radio frequency interference, thermal effects leading to uneven temperature distribution etc.. Consequently, understanding and analyzing the spacecraft's charge distribution is a critical aspect of space exploration and satellite technology.

PROBLEM STATEMENT

The objective of this project is to mathematically model the charge distribution on the surface of the spacecraft in outer space orbits. While all spacecraft surfaces share the same potential due to their interconnected nature, the challenge lies in accurately capturing the dynamic charge distribution resulting from electrostatic interactions among these surface charges.

APPROACH

Two significant numerical methods were employed to solve this problem:

1. **Method of Moments** to solve for an unknown function
2. **Gauss-Jordan elimination** to solve a system of linear equations

We are modeling the charge distribution for a symmetric, cubical conducting body. In our initial step, we concentrate on an individual isolated surface of the body, which we subdivide into smaller segments (of equal area) systematically arranged in an m*n grid. Each of these segments possesses its electric potential, stemming from a combination of contributions from other adjacent segments and its inherent self-potential. This approach enables us to formulate an equation for every segment that describes the potential of each segment. In this equation, the charge density of the segment is unknown while the potential is known. This involves the use of the method of moments.

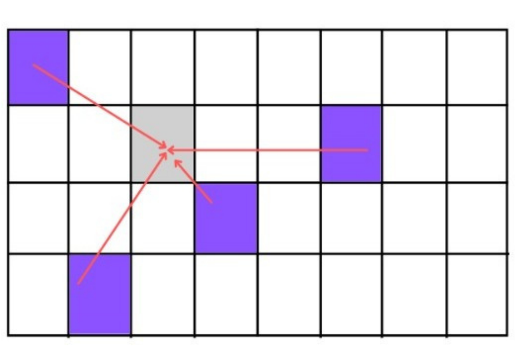

Figure 1: How the potential on each segment is affected by its own charge distribution and the charge distribution on all other segments

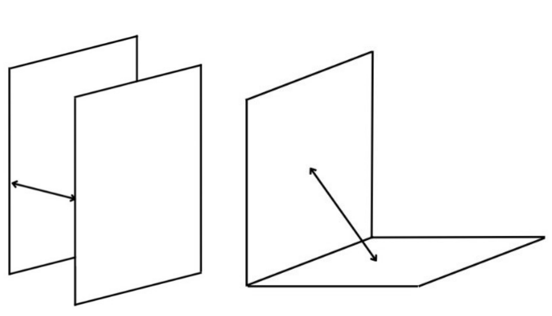

Figure 2: Electrostatic interaction between two parallel and perpendicular surfaces

The general form of an equation to be solved using the method of moments is given as:

$$g = L(f)$$

The equation that we use here is given as:

$$\phi = \frac{1}{4\pi\epsilon}\int \frac{\sigma \rho ds}{|r - r'|}$$

where, the operator L corresponds to integration, the known function g is potential, while the unknown function f is a function of ϱ, which is the charge density. The above equation is for expressing the electric potential generated on one segment because of the charge on another segment. But for calculating the self- electric potential of a segment, we use the following expression:

$$V(self) = \alpha \frac{\sqrt{cd}}{\pi\epsilon_0}\ln(1+\sqrt{2})$$

Numerical integration has been used to convert the integral operator into a summation which is further evaluated using matrix multiplication. The equations involved are as follows:

$$V[m] = \sum_{n=1}^{M} N(\alpha[n] \cdot \frac{\Delta S}{\sqrt{(x[m]-x[n])^2 + (y[m]-y[n])^2}}$$

$$L[m][n] = \frac{\Delta S}{\sqrt{(x[m]-x[n])^2 + (y[m]-y[n])^2}}$$

$$V[m] = \sum_{n=1}^{M} N(\alpha[n] \cdot L[m][n])$$

Here, the area of each segment is represented using ΔS

This L matrix is multiplied with a vector α containing unknown elements that correspond to the charge density of every segment in a single plate (or in both the plates, depending upon the case). This gives the potential vector g (known) that contains values for potential of every segment. This system of equations is solved using Gauss Jordan Elimination.

RESULTS

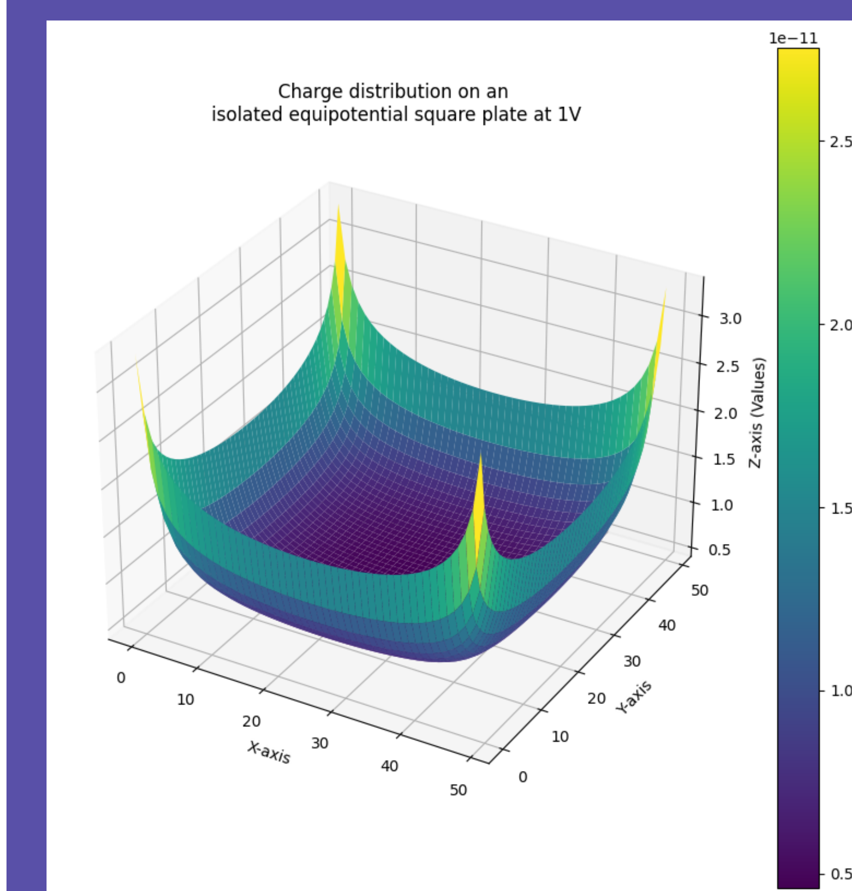

Figure 3 : Charge distribution on a single isolated equipotential plate

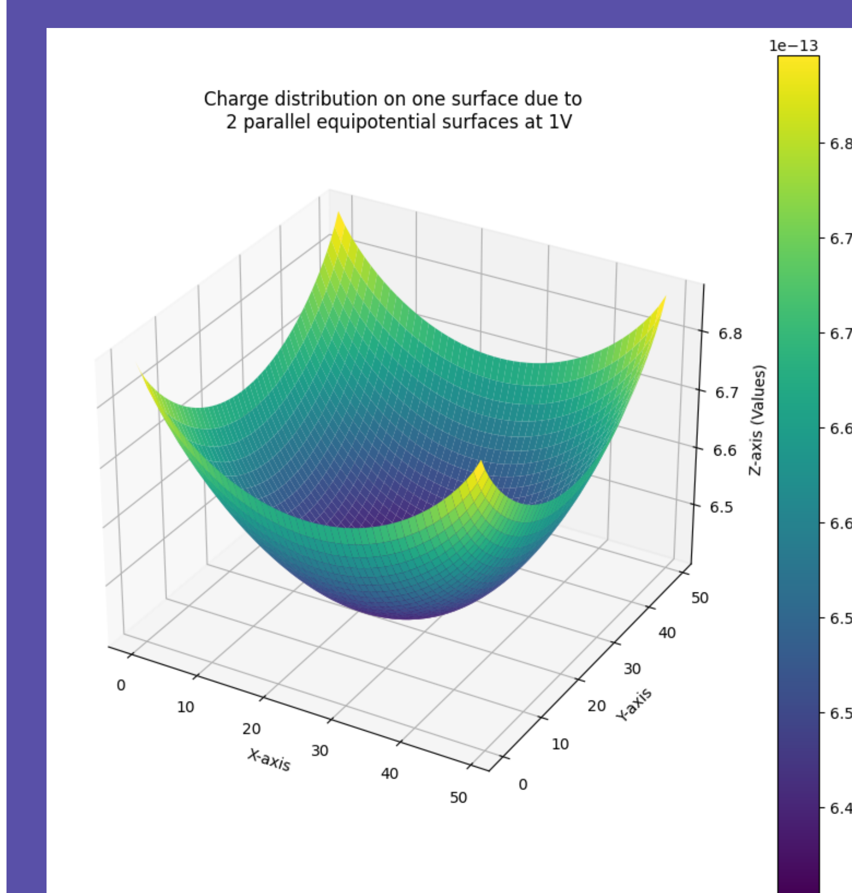

Figure 4 : Charge distribution due to plate's own influence and another parallel plate

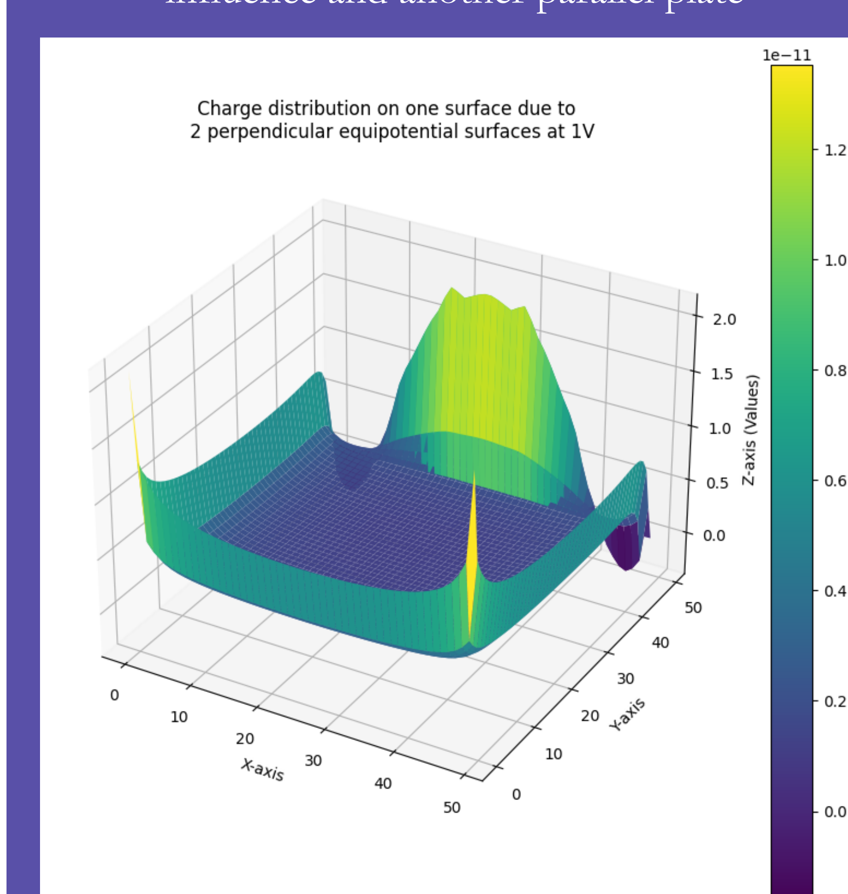

Figure 5 : Charge distribution due to plate's own influence and another perpendicular plate

The reason for such a peculiar charge distribution is the fact that the entire body of the spacecraft has to bear the same electric potential at all times, as governed by the laws of electrostatics. Hence, the charge distribution on each surface of the body is non-uniform, and the pattern is evident in the graphs so obtained after numerically utilising the expressions of the known electric potential for each surface by discretization.

The first plot represents the charge distribution on a single isolated plate, modelled as one of the surfaces of the spacecraft. We only consider the influence of the plate's own charges to express its electric potential.

Next, we consider two parallel, identical surfaces and solve for their charge distribution by considering their own as well as each others' electrostatic influence to express their electric potential.

Finally, we do the same for two perpendicular surfaces, having an edge in common.

The concave-shaped graphs suggest that the majority of the charge accumulates along the periphery of the surface due to the repulsion of like charges. These depictions of charge distribution will play a pivotal role in shaping the blueprint and construction of the spacecraft.

CONCLUSION

The results obtained after implementing our method of mathematically modeling and visualising the charge distribution on the surface of a spacecraft in outer space orbits, clearly align with how it actually should be. The bowl-shaped plots indicate that most of the charge is concentrated on the edges of the surface, since like charges repel. These charge distribution plots will be instrumental in determining the design and assembly of the spacecraft. This can be extended to all 6 surfaces similarly, using the concept of parallel and perpendicular surfaces.