Instructions: Try to solve all problems on your own. If you have difficulties, ask the instructor or TAs.

Bindu Metal Works cuts large Aluminium rolls of length 1030cm and width 150cm into smaller sheets of different lengths and width 150cm. The smaller sheets are then used to make parts of transformers. The demand (d_i) for smaller sheets is known and we would like to cut the sheets in a way that the number of rolls used is minimized (i.e. waste is minimized). Note that in these problems, we are only concerned about the length, and width does not play any role in decision making. A roll can be cut in various patterns. For instance, if three types of smaller sheets are required (of lengths, say, 650, 50, 40) then one possible pattern is $1 \times 650 + 7 \times 50 + 0 \times 40$ which will lead to a wastage of 30cm of the roll. Given the demands of smaller sheets, one can cut in many patterns. The decision variable is how many times we use each pattern.

We will work with two data sets of varying difficulty level.

Exercise 1: A simple formulation

Let us suppose there are n demands of small sheets, each of length b_i and demand $d_i, i = 1, \ldots, n$. Let pattern j be used t_j times. Let x_{ij} denote the number of small sheets of size i generated by pattern j. For instance, in the above small example, if pattern 1 is $1 \times 650 + 7 \times 50 + 0 \times 40$, then $x_{1,1} = 1, x_{2,1} = 7, x_{3,1} = 0$. Write a small optimization problem formulation using x and t variables that can be solved to minimize the number of rolls used to meet all demand. This formulation could be nonlinear. Is it convex? Use BARON solver available through AMPL or Pyomo to solve the problem for two data sets.

Data set 1 (10 items)

Width of Sheet	1000	980	940	640	630	60	55	50	45	40
Demand	76	50	1189	3876	4187	945	817	138	1380	140

Data set 2 (15 items)

Width of Sheet	1000	980	940	920	900	710	700	650	640	630
Demand	76	50	1189	2210	987	894	2850	6603	3876	4187
Width of Sheet	60	55	50	45	40					
Demand	945	817	138	1380	140					

Write formulation clearly and report the solution that you obtain.

Exercise 2: How many patterns?

One may be interested in finding how many 'maximal' patterns are possible. A 'maximal' pattern is one in which no more smaller sheets can be cut. The example given in the beginning is maximal, but the pattern $1 \times 650 + 6 \times 50 + 1 \times 40$ is not. Develop an algorithm to find all possible maximal patterns for a given set of smaller sheets (here the value of demands is not important). Clearly write your algorithm. Comment on its running time. How many maximal patterns can you find for the above two data sets?