HW 2

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Problem 1

$$C(X,Y,Z) = \alpha_1 P_1(X_1,Y_1,Z_1) + \alpha_2 P_2(X_2,Y_2,Z_2) + \alpha_3 P_3(X_3,Y_3,Z_3)$$

(i) Normalized coordinates (chromaticity) of primaries

P_1

$$x_1 = \frac{X_1}{X_1+Y_1+Z_1} \qquad y_1 = \frac{Y_1}{X_1+Y_1+Z_1} \qquad z_1 = \frac{Z_1}{X_1+Y_1+Z_1}$$

P_2

$$x_2 = \frac{X_2}{X_2+Y_2+Z_2} \qquad y_2 = \frac{Y_2}{X_2+Y_2+Z_2} \qquad z_2 = \frac{Z_2}{X_2+Y_2+Z_2}$$

P_3

$$x_3 = \frac{X_3}{X_3+Y_3+Z_3} \qquad y_3 = \frac{Y_3}{X_3+Y_3+Z_3} \qquad z_3 = \frac{Z_3}{X_3+Y_3+Z_3}$$

(ii) Normalized chromaticity coordinates of colour C

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z} \qquad z = \frac{Z}{X+Y+Z} \quad — (1)$$

We know that

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$$
$$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3 \quad — (2)$$
$$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$$

So, normalized chromaticity coordinates of color C in terms of chromaticity coordinates of P_1, P_2, P_3

From (1) & (2)

$$x = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$y = \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$z = \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$\rightarrow$ (3)

(iii) To show normalized values of colour C as linear combination of normalized primaries, we have to show

$$x = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$
$$y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$$
$$z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3$$

$$\hat{C}(x, y, z) = \beta_1 \hat{P}_1 (x_1, y_1, z_1) + \beta_2 \hat{P}_2 (x_2, y_2 + z_2) + \beta_3 \hat{P}_3 (x_3 + y_3, z_3)$$

We know that

$$x_1 = \frac{X_1}{X_1+Y_1+Z_1}$$

So $\quad X_1 = x_1 (X_1 + Y_1 + Z_1)$

$$\alpha_1 X_1 = \alpha_1 x_1 (X_1 + Y_1 + Z_1)$$

Similarly

$$\alpha_2 X_2 = \alpha_2 x_2 (X_2 + Y_2 + Z_2) \quad - (4)$$

$$\alpha_3 X_3 = \alpha_3 x_3 (X_3 + Y_3 + Z_3)$$

Sub (4) in (3)

$$x = \frac{\alpha_1 (X_1 + Y_1 + Z_1) x_1 + \alpha_2 (X_2 + Y_2 + Z_2) x_2 + \alpha_3 (X_3 + Y_3 + Z_3) x_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$\hookrightarrow$ (5)

From (5), we get

$$\beta_1 = \frac{\alpha_1 (X_1 + Y_1 + Z_1)}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$\beta_2 = \frac{\alpha_2 (X_2 + Y_2 + Z_2)}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$\beta_3 = \frac{\alpha_3 (X_3 + Y_3 + Z_3)}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

Hence $x = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

Similarly, $y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$

$z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3$

Problem 2.

(i) Quantized sequence.

23, 25, 25, 29, 29, 29, 26, 27,
27, 27, 22, 20, 21, 21, 23, 25,
25, 25, 24, 25, 21, 17, 11, 11,
9, 12, 7, 10, 10, 13, 16, 20

(ii) Range $[0, 32]$

So, 5 bits / sample $\times$ 32 sample $= \boxed{160 \text{ bits}}$

(iii) Differences

2, 0, 4, 0, 0, −3, 1
0, 0, −5, −2, 1, 0, 2, 2
0, 0, −1, 1, −4, −4, −6, 0
−2, 3, −5, 3, 0, 3, 3, 4

Range $= [-6, 4] = \underline{\underline{11}}$ values

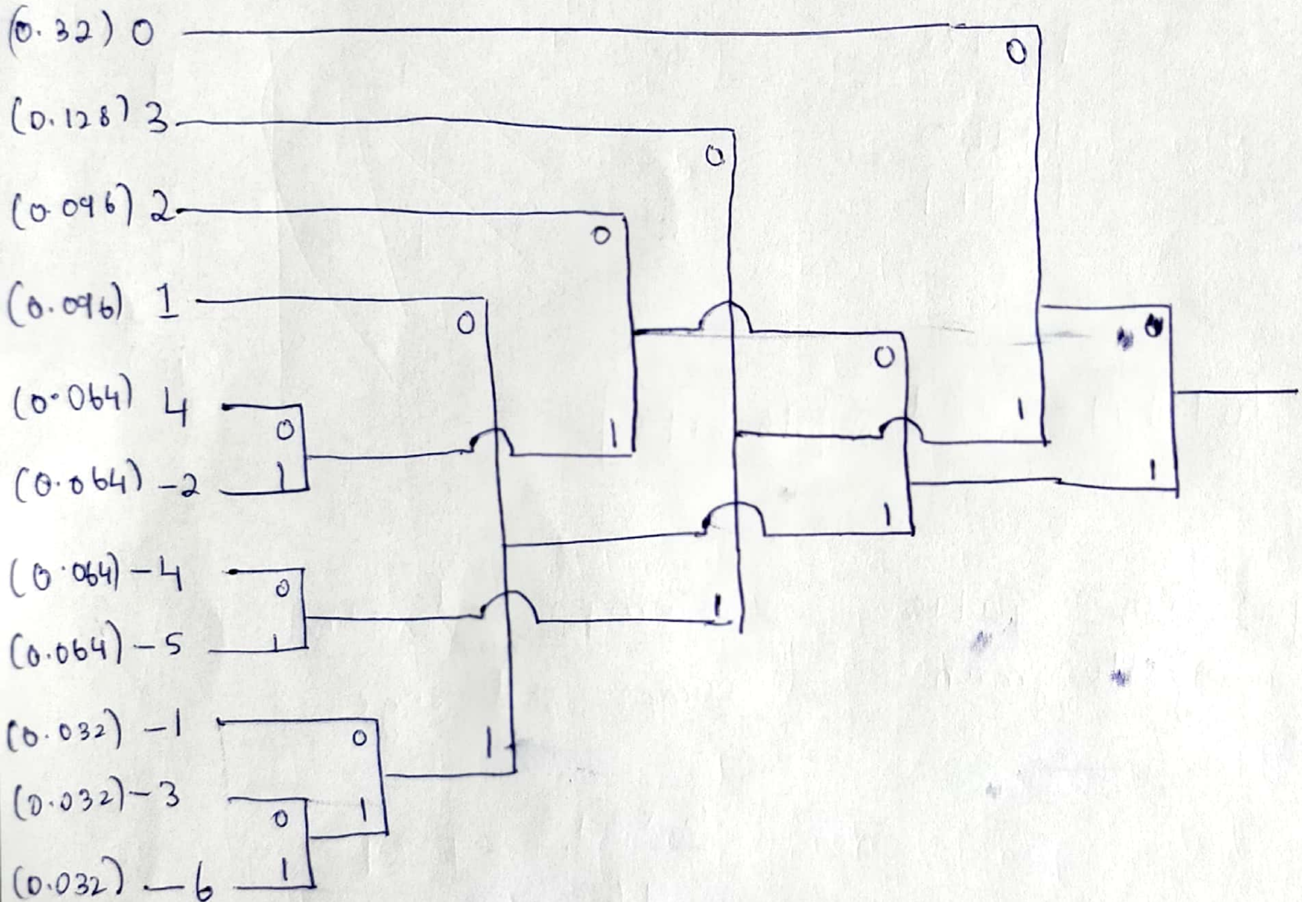
So, Requires 4 bits / sample.

Total bits $= 4 \times 31 = \boxed{124 \text{ bits}}$

(iv) Compression ratio $= \dfrac{\text{uncompressed size}}{\text{compressed size}}$

$= \dfrac{155}{124} = 1.25$

(v) Huffman coding



Diffrence	Huffman code	Li	Frequency	No of bits
0	00	2	10	20
3	010	3	4	12
2	100	3	3	9
1	110	3	3	9
4	1010	4	2	8
-2	1011	4	2	8
-4	0110	4	2	8
-5	0111	4	2	8
-1	1110	4	1	4
-3	11110	5	1	5
-6	11111	5	1	5
				(96)

Totalbits used = $\boxed{96 \text{ bits}}$

(v)

Compression ratio of Huffman $= \dfrac{\text{uncompressed size}}{\text{compressed size}}$

$$= \frac{5 \times 31}{96} \approx 1.61$$

Huffman coding has better compression ratio than DPCM

Problem 3

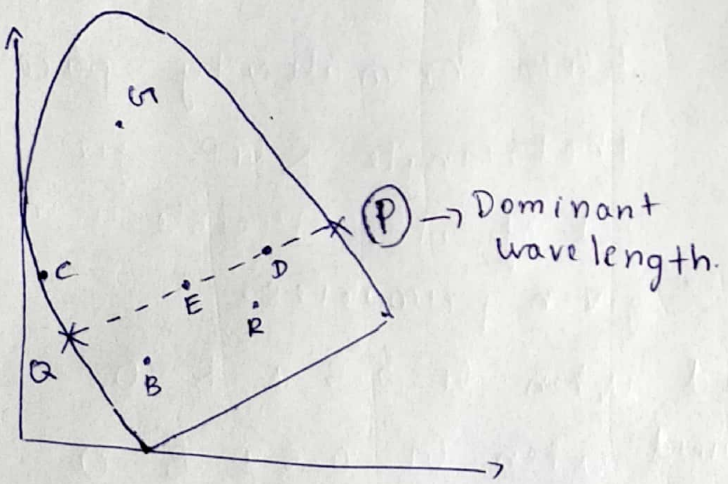
For color C

Dominant + white = color C

color C + complementary = white

(1)

When the straight line drawn from a color to the equiluminous point is extrapolated to intersect the gamut, the intersection with the gamut gives its dominant color.

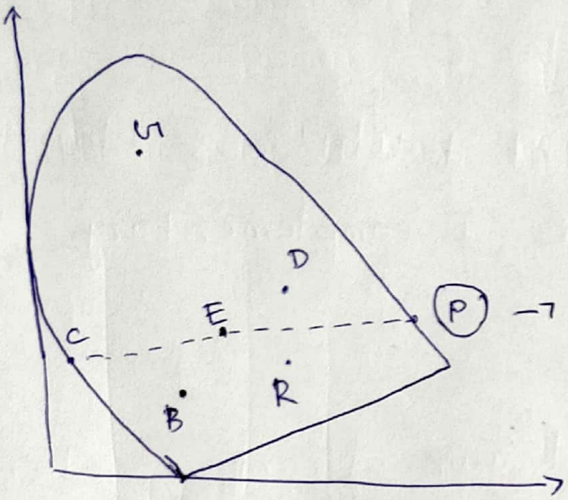


When line from D to E is extrapolated to meet the gamut, we get two colors P & Q. The one closest to D, which is P in this case, is the dominant wavelength.

(ii) No, all colors do not have a dominant wavelength.

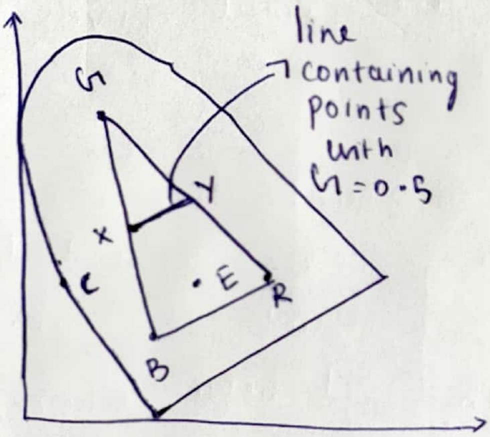
(Eg) Some colors like purple and magenta, since their extrapolated line hits the boundary in the flat part rather than the horse shoe part. But, they have a "Complimentary Dominant wavelength", lying on the opposite side.

(iii)



→ Complementary wavelength of C

(iv)



G = 0.5 will give you a ~~set~~ line in the normal RGB chromaticity space. The triangle RGB gives the set of colors visible using the given primaries.

X is the point obtained when G = 0.5 & R = 0

Y is the point obtained when G = 0.5 & B = 0

And line XY is the set of points, where G = 0.5

Since value of G is fixed, this ~~RGB~~ ~~XXX~~ will represent a plane in RGB space, which when projected to chromaticity space will be a line.

(v) The locus line $G = 0.5$ in chromaticity space will map as a plane in the RGB space.

This is because the value of G is fixed at 0.5.

And R, B values will have the range [0,1]