A Project Report

On

Laplace Transformation of Special Functions

Submitted by

Krishankant Sarswat, Roll No.2215500087

Sachin Raghav ,Roll No.2215500126

Lucky Goyal, Roll No.2215500094

Parth Mittal, Roll No.2215500105

Vivek Kumar Sharma, Roll No.2215500175

Prince Bazad, Roll No.2215500111

Supervisor

Dr.Ambuj Kumar Mishra

Associate Professor

Department of Mathematics

GLA UNIVERSITY

GLA University, Mathura - 281406

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DECLARATION

We **Krishankant Saraswat, B.Tech 1st year, Roll No.2215500087 , Sachin Raghav, B.Tech 1st year, Roll No.2215500126 , Lucky Goyal, B.Tech 1st year, Roll No.2215500094, Vivek kumar Sharma, B.Tech 1st year, Roll No.2215500175 , Parth Mittal, B.Tech 1st year, Roll No.2215500105 , Prince Bazad, B.Tech 1st year, Roll No.2215500111** hereby declare that the work presented in this project report entitled Laplace Transformation of Special Functions is an authentic record of our own work carried out under supervision of Dr.Ambuj Kumar Mishra.

Krishankant Saraswat, Roll No.2215500087 Sachin Raghav, Roll No.2215500126

Prince Bazad, Roll No.2215500111 Vivek Kumar Sharma, RollNo.2215500175

Parth Mittal, Roll No.2215500105 Lucky Goyal, RollNo.2215500094

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CERTIFICATE

This is to certify that the above statement made by the students are correct to the best of my knowledge

Date: 16/05/2023

Place: Mathura

Dr.Ambuj Kumar Sharma
Associate Professor
Department of Mathematics

Contents

Chapter - 1

Introduction, Motivation and Object

PIERRE-SIMON LAPLACE

Developed mathematics in astronomy, physics and statistics

Began work in calculus which led to the laplace transform.

Focused later on celestial mechanics

One of the first scientists to suggest the existence of black holes

LAPLACE TRANSFORM

Definition. Let $f(t)$ be function defined for all positive values of t, then

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

provided the integral exists, is called the **Laplace Transform** of $f(t)$. It is denoted as

$$\mathrm{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

SPECIAL FUNCTIONS IN LAPLACE TRANSFORM

Two types of special functions in laplace transform are :-

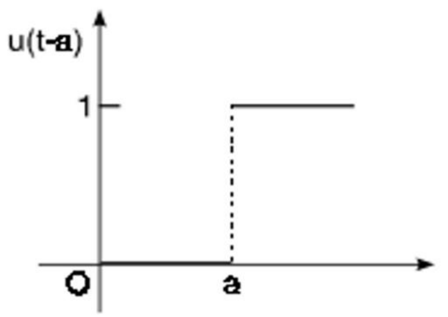
1. Unit Step Function.

2. Dirac Delta Function.

UNIT –STEP FUNCTION

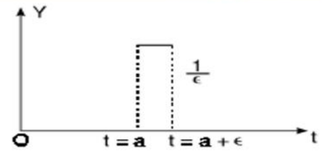
The unit step functions $u\,(t-a)$ is defined as follows:

$$u\,(t-a) = \begin{cases} 0 & \text{when } t < a \\ 1 & \text{when } t \geq a \end{cases} \quad \text{where } a \geq 0.$$



DIRAC DELTA FUNCTION

When a large force acts for a short time, then the product of the force and the time is called impulse in applied mechanics. The unit impulse function is the limiting function.



$$\delta(t-1) = \frac{1}{\varepsilon},\ a < t < a+\varepsilon$$
$$= 0, \quad \text{otherwise}$$

The value of the function (height of the strip in the figure) becomes infinite as $\varepsilon \to 0$ and the area of the rectangle is unity.

(2) The Unit Impulse function is defined as follows:

$$\delta(t-a) = \begin{cases} \infty & \text{for } t = a \\ 0 & \text{for } t \neq a. \end{cases}$$

and $$\int_0^\infty \delta(t-a)\,.\,dt = 1. \qquad \text{[Area of strip = 1]}$$

(3) Laplace Transform of unit Impulse function

$$\int_0^\infty f(t)\,\delta(t-a)\,dt = \int_a^{a+\varepsilon} f(t)\cdot\frac{1}{\varepsilon}\,dt \qquad \left\{ \begin{array}{l} \text{Mean value Theorem} \\ \int_a^b f(t)\,dt = (b-a)\,f(\eta) \end{array} \right.$$

$$= (a+\varepsilon-a)\,f(\eta)\,.\,\frac{1}{\varepsilon} \qquad \text{where } a < \eta < a+\varepsilon$$

$$= f(\eta)$$

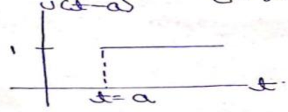
Chapter - 2

Description and Work done Chapter

Derivation of Unit Step function-

It is denoted by $u(t-a)$ and is defined as

$$u(t-a) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$$



$$F(t)\, u(t-a) = \begin{cases} F(t), & t \geq a \\ 0, & t < a. \end{cases}$$

$L[u(t-a)]$

$$L[F(t)] = \int_0^\infty e^{-pt} F(t)\, dt$$

$$L[u(t-a)] = \int_0^\infty e^{-pt} u(t-a)\, dt$$

$$\Rightarrow \int_0^a e^{-pt} \cdot 0\, dt + \int_a^\infty e^{-pt} \cdot 1\, dt$$

$$\Rightarrow \left\{ \frac{e^{-pt}}{-p} \right\}_a^\infty$$

$$\Rightarrow 0 + \frac{e^{-ap}}{p}$$

$$\Rightarrow \boxed{L[u(t-a)] = \frac{e^{-ap}}{p}}$$

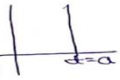
$$\boxed{\begin{gathered} F(t-a)\, u(t-a) = \begin{cases} F(t-a), & t \geq a \\ 0, & t < a \end{cases} \\ \therefore L[F(t-a)\, u(t-a)] = e^{-ap} \cdot F(p) \end{gathered}}$$

eg- find $L\left[\sin\left(t-\frac{\pi}{3}\right) u\left(t-\frac{\pi}{3}\right)\right]$

$$\Rightarrow e^{-\pi/3\, p} \frac{1}{p^2+1}$$

Derivation of Dirac Delta function-

$$\delta(t-a) = \begin{cases} \infty, & t=a \\ 0, & t\neq a \end{cases}$$



we define this by $\delta_\epsilon(t-a)$

$$\left[\lim_{\epsilon\to 0} \delta_\epsilon(t-a) = \delta(t-a)\right] \quad = \begin{cases} \frac{1}{\epsilon}, & a<t<a+\epsilon \\ 0, & \text{otherwise} \end{cases}$$

Evaluation of $\int_0^\infty \delta(t-a)\,dt = \lim_{\epsilon\to 0}\int_0^\infty \delta_\epsilon(t-a)\,dt$

$$\Rightarrow \int_0^a \cancel{0}\,dt + \int_a^{a+\epsilon} \frac{1}{\epsilon}\,dt + \int_{a+\epsilon}^\infty \cancel{0}\,dt$$

$$\Rightarrow \frac{1}{\epsilon}[t]_a^{a+\epsilon}$$

$$\Rightarrow \frac{1}{\epsilon}[a+\epsilon-a]$$

$$\Rightarrow 1$$

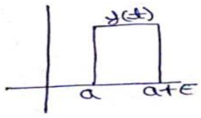
Evaluation of $\int_0^\infty f(t)\,\delta(t-a)\,dt$.

$$= \lim_{\epsilon\to 0}\int_0^\infty f(t)\,\delta_\epsilon(t-a)\,dt.$$

$$\Rightarrow \int_0^a f(t)\,0\,dt + \int_a^{a+\epsilon} f(t)\frac{1}{\epsilon}\,dt + \int_{a+\epsilon}^\infty 0\,dt$$

$$\Rightarrow \frac{1}{\epsilon}\int_0^{a+\epsilon} f(t)\,dt$$

$$\Rightarrow \frac{1}{\epsilon}(a+\epsilon-\cancel{a})f(a) = f(a)$$



$$L[\delta(t-a)]$$

$$= \int_0^\infty e^{-pt}\,\delta(t-a)\,dt$$

$$= e^{-pa}.$$

eg- $\int_{-\infty}^\infty e^{2t}(t-3)\,dt = e^6.$

$$\boxed{L[\delta(t)] = e^{-p\cdot 0} = 1} \quad \boxed{L^{-1}\{1\} = \delta(t)}$$

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