システム計画論 第 8 回 課題

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\begin{aligned}
\text{maximize} \quad & \min\left(\mathrm{Nes}\left(c_1x_1+c_2x_2\geq 450\right),\ \ \mathrm{Pos}\left(c_1x_1+c_2x_2\geq 530\right)\right),\\
\text{subject to} \quad & \mathrm{Nes}\left(a_1x_1+b_1x_2\leq 240\right)\geq 0.8,\\
& \mathrm{Nes}\left(a_2x_1+b_2x_2\leq 400\right)\geq 0.8,\\
& \mathrm{Nes}\left(a_3x_1+b_3x_2\leq 210\right)\geq 0.8,\\
& x_1,x_2\geq 0.
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\tilde{A}_1=\langle 2,0.7\rangle,\quad \tilde{A}_2=\langle 4,1.5\rangle,\quad \tilde{A}_3=\langle 1,0.5\rangle,\quad \tilde{B}_1=\langle 3,0.5\rangle,\quad \tilde{B}_2=\langle 2,0.3\rangle,\quad \tilde{B}_3=\langle 3,0.3\rangle,\\
&\tilde{C}_1=\langle 5,1\rangle,\qquad \tilde{C}_1=\langle 7,0.7\rangle.
\end{aligned}
\tag{2}
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$$
\begin{aligned}
&\mathrm{Nes}\left(a_1x_1+b_1x_2\leq 240\right)\geq 0.8\\
&(2x_1+3x_2)+0.8(0.7x_1+0.5x_2)\leq 240 \qquad (\because (2))\\
&2.56x_1+3.4x_2\leq 240
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&\mathrm{Nes}\left(a_2x_1+b_2x_2\leq 400\right)\geq 0.8\\
&(4x_1+2x_2)+0.8(1.5x_1+0.3x_2)\leq 400 \qquad (\because (2))\\
&5.2x_1+2.24x_2\leq 400
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&\mathrm{Nes}\left(a_3x_1+b_3x_2\leq 210\right)\geq 0.8\\
&(x_1+3x_2)+0.8(0.5x_1+0.3x_2)\leq 210 \qquad (\because (2))\\
&1.4x_1+3.24x_2\leq 210
\end{aligned}
\tag{5}
$$

$h_N\leq \mathrm{Nes}\left(c_1x_1+c_2x_2\geq 450\right),\ \ h_P\leq \mathrm{Pos}\left(c_1x_1+c_2x_2\geq 530\right),\ \ h=\min(h_N,h_P)$ とする.

$$
\begin{aligned}
&\mathrm{Nes}\left(c_1x_1+c_2x_2\geq 450\right)\geq h_N\\
&(5x_1+7x_2)-h_N(x_1+0.7x_2)\geq 450 \qquad (\because (2))\\
&h_N\leq \frac{5x_1+7x_2-450}{x_1+0.7x_2}
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&\mathrm{Pos}\left(c_1x_1+c_2x_2\geq 530\right)\geq h_P\\
&(6x_1+7.7x_2)-h_P(x_1+0.7x_2)\geq 530 \qquad (\because (2))\\
&h_P\leq \frac{6x_1+7.7x_2-530}{x_1+0.7x_2}
\end{aligned}
\tag{7}
$$

$h \leq h_N,\ \ h \leq h_P$ と (3) $\sim$ (7) より, (1) は以下のように書き換えられる.

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\begin{aligned}
\text{maximize}\ \ & h, \\
\text{subject to}\ \ & \frac{5x_1 + 7x_2 - 450}{x_1 + 0.7x_2} \geq h, \\
& \frac{6x_1 + 7.7x_2 - 530}{x_1 + 0.7x_2} \geq h, \\
& 2.56x_1 + 3.4x_2 \leq 240, \\
& 5.2x_1 + 2.24x_2 \leq 400, \\
& 1.4x_1 + 3.24x_2 \leq 210, \\
& x_1, x_2 \geq 0.
\end{aligned}
\tag{8}
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(8) において $t = \frac{1}{x_1 + 0.7x_2},\ \ z_i = x_i t, (i = 1, 2)$ とすると次の式が得られる.

$$
\begin{aligned}
\text{maximize}\ \ & h, \\
\text{subject to}\ \ & z_1 + 0.7z_2 = 1 \\
& 5z_1 + 7z_2 - 450t \geq h, \\
& 6z_1 + 7.7z_2 - 530t \geq h, \\
& 2.56z_1 + 3.4z_2 \leq 240t, \\
& 5.2z_1 + 2.24z_2 \leq 400t, \\
& 1.4z_1 + 3.24z_2 \leq 210t, \\
& z_1, z_2, t, h \geq 0.
\end{aligned}
\tag{9}
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$5z_1 + 7z_2 - 450t \geq h,\ \ 6z_1 + 7.7z_2 - 530t \geq h$ は単に変数変換をしただけである.

非負制約を除く残りの制約式は両辺に t をかけて z_1, z_2 についての式に変形している. これらは $t = z_1 + 0.7z_2 = 1$ という制約のもとで, (8) 中の元の制約と同値である.