システム計画論第2回課題

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$$\chi_{(\tilde{A})_h}(x) = \begin{cases} 1 & \text{if } x \in (\tilde{A})_h, \\ 0 & \text{otherwise.} \end{cases}$$
 (1)

$$\sup_{h \in [0,1]} \min \left(h, \chi_{(\tilde{A})_h}(x) \right) \tag{2}$$

$$= \max \left(\sup_{h \in \left[0, \mu_{\tilde{A}}(x)\right)} \min \left(h, \chi_{\left(\tilde{A}_{h}\right)}(x)\right), \quad \sup_{h \in \left[\mu_{\tilde{A}}(x), 1\right]} \min \left(h, \chi_{\left(\tilde{A}_{h}\right)}(x)\right) \right)$$
 (3)

$$= \max \left(\sup_{h \in \left[0, \mu_{\tilde{A}}(x)\right)} \min\left(h, 1\right), \quad \sup_{h \in \left[\mu_{\tilde{A}}(x), 1\right]} \min\left(h, \chi_{\left(\tilde{A}_{h}\right)}(x)\right) \right) \quad \left(\because x \in (\tilde{A})_{h}, \ \forall h \in \left[0, \mu_{\tilde{A}}(x)\right)\right)$$

$$(4)$$

$$= \max \left(\sup_{h \in \left[0, \mu_{\tilde{A}}(x)\right)} \min\left(h, 1\right), \quad \sup_{h \in \left[\mu_{\tilde{A}}(x), 1\right]} \min\left(h, 0\right) \right) \quad \left(\because x \notin (\tilde{A})_h, \ \forall h \in \left[\mu_{\tilde{A}}(x), 1\right] \right)$$
 (5)

$$= \max \left(\sup_{h \in [0, \mu_{\tilde{A}}(x))} h, \quad 0 \right) \tag{6}$$

$$= \max\left(\mu_{\tilde{A}}(x), \quad 0\right) \tag{7}$$

$$=\mu_{\tilde{A}}(x) \tag{8}$$