

システム計画論 第 10 回 課題

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$a = \mu_A(x)$, $b = \mu_B(y)$ とおく.

(1)

$$\begin{aligned}
 \mu_{B'}(y) &= \sup_x \min(\sqrt{a}, F_1(a, b)) \\
 &= \sup_{a \in [0, 1]} \min(\sqrt{a}, \max(\min(a, b), 1 - a)) \\
 &= \max \left(\sup_{\substack{a \\ \min(a, b) \leq 1 - a}} \min(\sqrt{a}, 1 - a), \sup_{\substack{a \\ \min(a, b) > 1 - a}} \min(\sqrt{a}, a, b) \right) \\
 &= \max \left(\sup_{\substack{a \\ \min(a, b) \leq 1 - a}} \min(\sqrt{a}, 1 - a), \sup_{\substack{a \\ \min(a, b) > 1 - a}} \min(a, b) \right) \\
 &= \max \left(\sup_{\substack{a \\ \min(a, b) \leq 1 - a}} \min(\sqrt{a}, 1 - a), b \right) \quad (\because a \in [0, 1])
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 \sqrt{a} &= 1 - a \\
 a &= (1 - a)^2 \\
 a &= \frac{-1 + \sqrt{5}}{2} \quad (\because a \geq 0)
 \end{aligned} \tag{2}$$

式 (1), (2) より, 次が成り立つ.

$$\mu_{B'}(y) = \max \left(\frac{-1 + \sqrt{5}}{2}, \mu_B(y) \right) \tag{3}$$

(2)

$$\begin{aligned}
 \mu_{B'}(y) &= \sup_x \min(a^2, F_4(a, b)) \\
 &= \sup_{a \in [0, 1]} \min(a^2, \min(1, 1 - a + b)) \\
 &= \sup_{a \in [0, 1]} \min(a^2, 1 - a + b)
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 a^2 &= 1 - a + b \\
 a &= \frac{-1 + \sqrt{4b + 5}}{2} \quad (\because a \geq 0) \\
 a^2 &= \frac{2b + 3 - \sqrt{4b + 5}}{2}
 \end{aligned} \tag{5}$$

式 (4), (5) より, 次が成り立つ.

$$\mu_{B'}(y) = \frac{2\mu_B(y) + 3 - \sqrt{4\mu_B(y) + 5}}{2} \tag{6}$$

(3)

$$\begin{aligned}\mu_{B'}(y) &= \sup_x \min(\sqrt{a}, F_4(a, b)) \\ &= \sup_{a \in [0, 1]} \min(\sqrt{a}, \min(a, b)) \\ &= \sup_{a \in [0, 1]} \min(a, b) \quad (\because a \in [0, 1]) \\ &= b = \mu_B(y)\end{aligned}\tag{7}$$