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Group multi-attribute decision model to partner selection in the formation of virtual enterprise under incomplete information

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ABSTRACT

Partner selection is very important in the formation of a virtual enterprise. However, in many practical situations, during the partner selection process, the decision makers are generally unsure of their preferences because the information about the candidates and their performances are incomplete and uncertain. Therefore, two MADM (multi-attribute decision model) methods for group decision making with interval values are proposed to solve partner selection problem under incomplete information in this paper. The first method is a technique for order preference by similarity to ideal solution (TOPSIS) for group decision making based on deviation degree. The second method is a TOPSIS for group decision making based on risk factor. Finally, an illustrative example is used to demonstrate the feasibility and practicability of these two extended TOPSIS methods for group decision making with interval values. Results show that these two extended TOPSIS methods for group decision making with interval values can effectively deal with the partner selection problem under incomplete information.

1. Introduction

The virtual enterprise (VE) is a kind of new organizational mode in the manufacturing world. A VE is a temporary alliance of autonomous, diverse, and possibly geographically dispersed organizations that share skills or core competencies to meet a particular mission and exploit fast changing market opportunity (Park & Favrel, 1999; Lau, Chin, Pun, & Ning, 2000; Wu & Su, 2005). Hence, A VE is a kind of dynamic alliance of member companies, which join to take advantage of a market opportunity. When the market opportunity passes, the VE will be dissolved (Ip, Huang, Yung, & Wang, 2003).

The life cycle of a VE has four stages: creation, operation, evolution and dissolution. A major issue in the formation of the VE is to select suitable partners in the process of the dynamic alliance establishing, and it has attracted much research attention recently. For example, Talluri and Baker (1996) proposed a two-phase mathematical programming approach to solve the partner selection problem in the formation of a VE, where the factors of cost, time and distance were considered. Wang, Ip, and Yung (2001) developed a fuzzy decision embedded genetic algorithm to get the partner selection solution with due date constraint in a VE where the sub-projects form a precedence network. Ip et al. (2003) described and modeled a risk-based partner selection problem, and a rule-based genetic algorithm was developed to solve the partner selection problem. Zeng, Li, and Zhu (2006) proved that the partner selection problem with a due date constraint in a VE is a NP-complete problem, and a nonlinear integer programming model for this partner selection problem was established.

However, these traditional methods are problematic in that they use crisp values to express the decision makers' judgment on a comparison of candidates. In reality, during the partner selection process, the decision makers are generally unsure of their preferences because the information about the candidates and their performances are incomplete and uncertain. Furthermore, some of the decision attributes are subjective and qualitative, so the decision makers cannot easily express their judgment on the candidates (Mikhailov, 2002; Wang & Chen, 2007). Nevertheless, the decision makers can utilize interval values to express the information about the candidates and their performances under incomplete and uncertain information. It means that some new methods are needed to be proposed to solve partner selection problem with interval values.

On the other hand, when selecting partners for a market opportunity in a VE, many factors will be taken into consideration, such as cost, quality, trust, credit, delivery time, reliability and so on Wu and Su (2005). And several decision makers will take part in the evaluation of candidates. Obviously, partner selection in the formation of a VE is a kind of group multi-attribute decision making (MADM) problem, which refers to making preference decisions over the available candidates that are characterized by multiple, usually conflicting, attributes for each decision maker. So group MADM methods can be used to solve partner selection problem of a VE.

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According to the above mentioned analysis, two group MADM methods with interval values in solving partner selection problem under incomplete information are proposed in this paper. The first method is a technique for order preference by similarity to ideal solution (TOPSIS) for group decision making based on deviation degree. The second method is a TOPSIS group decision making based on risk factor.

This paper is organized as follows. Section 2 presents the group MADM problem with interval values. In Section 3 a TOPSIS for group decision making based on deviation degree and a TOPSIS for group decision making based on risk factor are proposed to solve the group MADM problem with interval values. Section 4 presents an illustrative example and Section 5 summarizes the work of this paper.

2. Presentation of the problem

Assuming an enterprise gets a bid for a large project consisting of several sub-projects. The enterprise is not able to complete the whole project only by using its own capacity. Therefore, the enterprise has to select partners for the sub-projects. The partner selection problem in the formation of a VE is described as follows:

Let $X = \{X_i | i = 1, \ldots, n\}$ be a finite set of possible candidates and $A = \{A_j | j = 1, \ldots, m\}$ be a finite set of attributes according to which the desirability of a candidate is to be judged. Because the information about the candidates is incomplete and uncertain in the partner selection process, the decision maker $k(k = 1, 2, \ldots, K)$ cannot easily express a crisp value to candidate X_i with respect to attribute A_j. But the decision maker k can utilize an interval value $\tilde{a}_{ij}^k$ to candidate X_i with respect to attribute A_j, where $\tilde{a}_{ij}^k = [a_{ij}^{k^L}, a_{ij}^{k^U}]$. And let $\tilde{A}^k = [\tilde{a}_{ij}^k]_{n\times m}$ be the decision matrix in the form of interval values. The group MADM problem with interval values can be expressed in matrix format as follows:

$$\tilde{A}^k = \begin{bmatrix} \tilde{a}_{11}^k & \tilde{a}_{12}^k & \cdots & \tilde{a}_{1m}^k \\ \tilde{a}_{21}^k & \tilde{a}_{22}^k & \cdots & \tilde{a}_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1}^k & \tilde{a}_{n2}^k & \cdots & \tilde{a}_{nm}^k \end{bmatrix} \tag{1}$$

Each DM will elicit weights for the attribute A_j as w_j^k, where $j = 1, 2, \ldots, m$. All $w_j^k (j = 1, 2, \ldots, m)$ belong to Park and Favrel (1999) and sum to one, i.e.,

$$1 \geqslant w_j^k \geqslant 0, \quad j = 1, 2, \ldots, m, \quad \sum_{j=1}^{m} w_j^k = 1 \tag{2}$$

The values of different attributes have different dimensions. Thus, the decision matrix in the form of interval values $\tilde{A}^k (k = 1, 2, \ldots, K)$ needs to be standardized with matrix $\tilde{R}^k$ in order to eliminate disturbance in the final results. And let $\tilde{R}^k = [\tilde{r}_{ij}^k]_{n\times m}$ be the standardized decision matrix in the form of interval values, where $\tilde{r}_{ij}^k = [r_{ij}^{k^L}, r_{ij}^{k^U}]$.

Generally there are two kinds of attributes, the benefit type and the cost type. The higher the benefit type value is, the better it will be. While for the cost type, it is the opposite.

For the benefit type, the normalized formulas for the interval value are described as follows (Zhu, Liu, & Fang, 2007):

$$\begin{cases} r_{ij}^{k^L} = \dfrac{a_{ij}^{k^L}}{\sqrt{\sum_{i=1}^{n} (a_{ij}^{k^U})^2}} \\ r_{ij}^{k^U} = \dfrac{a_{ij}^{U}}{\sqrt{\sum_{i=1}^{n} (a_{ij}^{k^L})^2}} \end{cases} \quad \text{if } A_j \text{ is the benefit type} \tag{3}$$

Similarly, the formulas for interval value of the cost type are described as follows:

$$\begin{cases} r_{ij}^{k^L} = \dfrac{1/a_{ij}^{k^U}}{\sqrt{\sum_{i=1}^{n} (1/a_{ij}^{k^L})^2}} \\ r_{ij}^{k^U} = \dfrac{1/a_{ij}^{k^L}}{\sqrt{\sum_{i=1}^{n} (1/a_{ij}^{k^U})^2}} \end{cases} \quad \text{if } A_j \text{ is the cost type} \tag{4}$$

Now interval value $[r_{ij}^{k^L}, r_{ij}^{k^U}]$ is normalized of interval value $[a_{ij}^{k^L}, a_{ij}^{k^U}]$. The normalization method mentioned above is to preserve the characteristic that the ranges of normalized interval values belong to [0, 1].

Considering different importance of each attribute, we can build the weighted normalized interval decision matrix as Jahanshahloo, Lotfi, and Izadikhah (2006)

$$\overline{r_{ij}^{k}}^L = w_j^k r_{ij}^{k^L}, \quad j = 1, \ldots, m, \quad i = 1, \ldots, n, \quad \text{and} \quad k = 1, \ldots, K \tag{5}$$

$$\overline{r_{ij}^{k}}^U = w_j^k r_{ij}^{k^U}, \quad j = 1, \ldots, m, \quad i = 1, \ldots, n, \quad \text{and} \quad k = 1, \ldots, K \tag{6}$$

Let $\tilde{\overline{R^k}} = [\tilde{\overline{r_{ij}^k}}]_{n\times m}$ be the weighted normalized decision matrix in the form of interval values, where $\tilde{\overline{r_{ij}^k}} = [\overline{r_{ij}^k}^L, \overline{r_{ij}^k}^U]$.

3. Extended TOPSIS for group decision making applied into partner selection

Hwang and Yoon (1981, 1985) developed a TOPSIS for multiple attribute decision making. TOPSIS is based on the concept that the chosen candidate should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS) for solving a multi-attribute decision-making problem. In traditional TOPSIS, most of the input variables are assumed to be crisp numerical data (Saremi, Mousavi, & Sanayei, in press). However, in reality, the information about the candidates is incomplete and uncertain during the partner selection process. So traditional TOPSIS can not be utilized to solve partner selection problem under incomplete information situation. In this paper we propose two extended TOPSIS methods for group decision making to solve partner selection problem with interval values.

3.1. TOPSIS for group decision making based on deviation degree

First, we should identify positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) for each decision maker as

$$\tilde{\overline{R^{k^+}}} = (\tilde{\overline{r_1^{k^+}}}, \tilde{\overline{r_2^{k^+}}}, \ldots, \tilde{\overline{r_m^{k^+}}}) = ([(\tilde{\overline{r_1^{k^+}}})^L, (\tilde{\overline{r_1^{k^+}}})^U], [(\tilde{\overline{r_2^{k^+}}})^L, (\tilde{\overline{r_2^{k^+}}})^U], \ldots, [(\tilde{\overline{r_m^{k^+}}})^L, (\tilde{\overline{r_m^{k^+}}})^U]) \tag{7}$$

$$\tilde{\overline{R^{k^-}}} = (\tilde{\overline{r_1^{k^-}}}, \tilde{\overline{r_2^{k^-}}}, \ldots, \tilde{\overline{r_m^{k^-}}}) = ([(\tilde{\overline{r_1^{k^-}}})^L, (\tilde{\overline{r_1^{k^-}}})^U], [(\tilde{\overline{r_2^{k^-}}})^L, (\tilde{\overline{r_2^{k^-}}})^U], \ldots, [(\tilde{\overline{r_m^{k^-}}})^L, (\tilde{\overline{r_m^{k^-}}})^U]) \tag{8}$$

where $\tilde{\overline{r_j^{k^+}}} = [(\tilde{\overline{r_j^{k^+}}})^L, (\tilde{\overline{r_j^{k^+}}})^U] = [\max_i \overline{r_{ij}^k}^L, \max_i \overline{r_{ij}^k}^U]$, $\tilde{\overline{r_j^{k^-}}} = [\min_i \overline{r_{ij}^k}^L, \min_i \overline{r_{ij}^k}^U], i = 1, \ldots, n, k = 1, \ldots, K$.

Before developing a new TOPSIS method for group decision making based on deviation degree, the concept about deviation degree between two interval values is defined as follows:

Definition 1 (*Xu and Sun, 2002*). Let $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ be two interval values, then the distance between interval value $\tilde{a}$ and $\tilde{b}$ is defined as follows:

$$D(\tilde{a}, \tilde{b}) = \sqrt{(a^L - b^L)^2 + (a^U - b^U)^2} \tag{9}$$

Then $D(\tilde{a}, \tilde{b})$ is called the deviation degree between interval value $\tilde{a}$ and $\tilde{b}$.

Obviously, the larger $D(\tilde{a}, \tilde{b})$ is, the higher the deviation degree between interval values $\tilde{a}$ and $\tilde{b}$ will be. Specially, while $D(\tilde{a}, \tilde{b}) = 0$, then $\tilde{a} = \tilde{b}$.

According to Definition 1, we can define the positive deviation degree between candidate X_i and the positive ideal partner solution.

Definition 2. The deviation degree between $\tilde{\overline{r}}_{ij}^{k} = [\overline{r}_{ij}^{kL}, \overline{r}_{ij}^{kU}]$ $(i = 1,\dots,n; j = 1,\dots,m, k = 1,\dots,K)$ and $\tilde{r}_j^{k^+} = [(\tilde{r}_j^{k^+})^L, (\tilde{r}_j^{k^+})^U]$ is defined as follows:

$$d_{ij}^{k^+} = \sqrt{((\tilde{r}_j^{k^+})^L - \overline{r}_{ij}^{kL})^2 + ((\tilde{r}_j^{k^+})^U - \overline{r}_{ij}^{kU})^2} \tag{10}$$

Then the positive deviation between candidate X_i and the ideal partner solution is defined as

$$D_i^{k^+} = \sqrt{\sum_{j=1}^{m} (d_{ij}^{k^+})^2} \tag{11}$$

Similarly, we can define the negative deviation degree between candidate X_i and the negative ideal partner solution.

Definition 3. The deviation degree between $\tilde{\overline{r}}_{ij}^{k} = [\overline{r}_{ij}^{kL}, \overline{r}_{ij}^{kU}]$ $(i = 1,\dots,n; j = 1,\dots,m; k = 1,\dots,K)$ and $\tilde{r}_j^{k^-} = [(\tilde{r}_j^{k^-})^L, (\tilde{r}_j^{k^-})^U]$ is defined as follows:

$$d_{ij}^{k^-} = \sqrt{((\tilde{r}_j^{k^-})^L - \overline{r}_{ij}^{kL})^2 + ((\tilde{r}_j^{k^-})^U - \overline{r}_{ij}^{kU})^2} \tag{12}$$

Then the negative deviation between candidate X_i and the negative ideal partner solution is defined as

$$D_i^{k^-} = \sqrt{\sum_{j=1}^{m} (d_{ij}^{k^-})^2} \tag{13}$$

In addition, the group deviation measure of each candidate will be combined through an operation $\otimes$ for all decision makers, $k = 1, 2, \dots, K$. Thus, the two group deviation measures of the positive and negative ideal partner solution, R^{k+} and R^{k-}, respectively, can be calculated by the following formulas (Shiha & Shyurb, 2007):

$$D_i^+ = D_i^{1^+} \otimes D_i^{2^+} \cdots \otimes D_i^{K^+}, \quad \text{for candidate } i \tag{14}$$

$$D_i^- = D_i^{1^-} \otimes D_i^{2^-} \cdots \otimes D_i^{K^-}, \quad \text{for candidate } i \tag{15}$$

There are a few operations such as geometric mean, arithmetic mean, or their modification to be chosen. If the geometric mean of all individual deviation measures is taken, the group deviation measures of the positive and negative ideal partners will be Shiha and Shyurb (2007)

$$D_i^+ = \left(\prod_{k=1}^{K} D_i^{k^+}\right)^{\frac{1}{K}}, \quad \text{for candidate } i \tag{16}$$

$$D_i^- = \left(\prod_{k=1}^{K} D_i^{k^-}\right)^{\frac{1}{K}}, \quad \text{for candidate } i \tag{17}$$

where $i = 1, 2, \dots, n; k = 1, 2, \dots, K$.

A closeness coefficient is defined to determine the ranking order of all candidates once the D_i^+ and D_i^- of each candidate X_i has been calculated. The relative closeness of the candidate X_i is defined as

$$U_i = \frac{D_i^-}{D_i^- + D_i^+}, \quad i = 1, 2, \dots, n \tag{18}$$

Obviously, a candidate X_i is closer to the positive partner solution and farther from the negative partner solution as U_i approaches to 1. Therefore, according to the closeness coefficient, the ranking order of all candidates can be determined, and we can select the best one among a set of feasible candidates (Jahanshahloo et al., 2006).

We develop a procedure to find the most favored partner among all candidates with TOPSIS for group decision making based on deviation degree.

- Step1: Generate a set of possible candidates for sub-projects $X = \{X_i | i = 1,\dots,n\}$.
- Step2: Identify a set of attributes $A = \{A_j | j = 1,\dots,m\}$.
- Step3: Calculate normalized ratings. This step tries to transform various attribute dimensions into the non-dimensional attributes, which allows comparison across the attributes. Formulas (3) and (4) are used for computing the normalized interval value $[r_{ij}^{k^L}, r_{ij}^{k^U}]$.
- Step4: Calculate weighted normalized ratings. A set of attribute weights assessed from decision maker $k (k = 1, 2, \dots, K)$ is accommodated to the normalized decision matrix $\tilde{A}^k = [\tilde{a}_{ij}^k]_{n\times m}$ in this step. The weighted normalized interval decision matrix $\tilde{R}^k = [\tilde{\overline{r}}_{ij}^k]_{n\times m}$ can be calculated by using formulas (5) and (6).
- Step5: Identify positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) through by using formulas (7) and (8).
- Step6: Calculate positive deviation degree between the candidates and positive ideal partner solution (PIPS) for each decision maker by using formulas (10) and (11).
- Step7: Calculate negative deviation degree between the candidates and negative ideal partner solution (NIPS) for each decision maker by using formulas (12) and (13).
- Step8: Aggregate the deviation measures for the group by using formulas (16) and (17).
- Step9: Calculate the relative closeness of the candidate X_i by using formula (18).
- Step10: Rank the preference order of all candidates according to the relative closeness of the candidates, and select the best partner.

3.2. TOPSIS for group decision making based on risk factor

By introducing the risk attitude factor of decision makers, we can transfer an interval value into an exact value. The exact value $\hat{r}_{ij}^k$ is given as You and Fan (2000)

$$\hat{r}_{ij}^k = \overline{r_i^k(j)} + \varepsilon_k \cdot \underline{r_i^k(j)} \tag{19}$$

where $\overline{r_i^k(j)}$ represents the middle value of weighted standardized interval value $\tilde{\overline{r}}_{ij}^k = [\overline{r}_{ij}^{kL}, \overline{r}_{ij}^{kU}]$, and $\overline{r_i^k(j)}$ is computed as

$$\overline{r_i^k(j)} = \frac{\overline{r}_{ij}^{kL} + \overline{r}_{ij}^{kU}}{2} \tag{20}$$

$\underline{r_i^k(j)}$ represents the width of weighted standardized interval value $\tilde{\overline{r}}_{ij}^k = [\overline{r}_{ij}^{kL}, \overline{r}_{ij}^{kU}]$, and $\underline{r_i(j)}$ is calculated as

$$\underline{r_i^k(j)} = (\overline{r}_{ij}^{kU} - \overline{r}_{ij}^{kL}) \tag{21}$$

The risk factor ε_k represents the risk attitude of the decision maker $k (k = 1, 2, \dots, K)$, and $|\varepsilon_k| \leqslant 0.5$. If the decision maker k is risk averse, then the range of risk factor will be $-0.5 \leqslant \varepsilon_k < 0$. If the decision maker k is risk neutral, then the risk factor $\varepsilon_k = 0$. While the decision maker k is risk preference, then the range of risk factor will be $0 \leqslant \varepsilon_k < 0.5$.

We further identify positive ideal partner solution and negative ideal partner solution for each decision maker. The positive ideal candidate R^{k^+} and negative ideal candidate R^{k^-} can be defined as

$$R^{k^+} = (r_1^{k^+}, r_2^{k^+}, \dots, r_m^{k^+}) \tag{22}$$

$$R^{k^-} = (r_1^{k^-}, r_2^{k^-}, \dots, r_m^{k^-}) \tag{23}$$

where $r_j^{k^+} = \max_i \hat{r}_{ij}^k$, $r_j^{k^-} = \min_i \hat{r}_{ij}^k$, $i = 1, 2, \ldots, n$.

By using the m-dimensional Euclidean distance, the separation of each candidate from the positive ideal partner R^{k^+} for decision maker k is given as

$$d_i^{k^+} = \left\{ \sum_{j=1}^{m} (\hat{r}_{ij}^k - r_j^{k^+})^2 \right\}^{\frac{1}{2}}, \quad i = 1, 2, \ldots, n \tag{24}$$

Similarly, the separation of each candidate from the negative ideal partner R^{k^-} for decision maker k is given as

$$d_i^{k^-} = \left\{ \sum_{j=1}^{m} (\hat{r}_{ij}^k - r_j^{k^-})^2 \right\}^{\frac{1}{2}}, \quad i = 1, 2, \ldots, n \tag{25}$$

The group separation measure of each candidate will be combined through the geometric mean for all decision makers, $k = 1, 2, \ldots, K$. The group separation measures from the positive and negative ideal partners will be

$$d_i^+ = \left(\prod_{k=1}^{K} d_i^{k^+} \right)^{\frac{1}{K}} \text{ for candidate } i \tag{26}$$

$$d_i^- = \left(\prod_{k=1}^{K} d_i^{k^-} \right)^{\frac{1}{K}} \text{ for candidate } i \tag{27}$$

where $i = 1, 2, \ldots, n; k = 1, 2, \ldots, K$.

The relative closeness of the candidate X_i is defined as

$$\mu_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, 2, \ldots, n \tag{28}$$

Hence, we also develop a procedure to find the most favored partner among all candidates with TOPSIS for group decision making based on risk factor.

Step1: Generate a set of possible candidates for sub-projects.
Step2: Identify a set of attributes.
Step3: Calculate normalized ratings.
Step4: Calculate weighted normalized ratings.
Step5: Transfer an interval value into an exact value for each decision maker by using formula (19).
Step6: Identify positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) for each decision maker by using formulas (22) and (23).
Step7: Calculate the separation of each candidate from the positive ideal partner solution (PIPS) for each decision maker by using formula (24).
Step8: Calculate the separation of each candidate from the negative ideal partner solution (NIPS) for each decision maker by using formula (25).

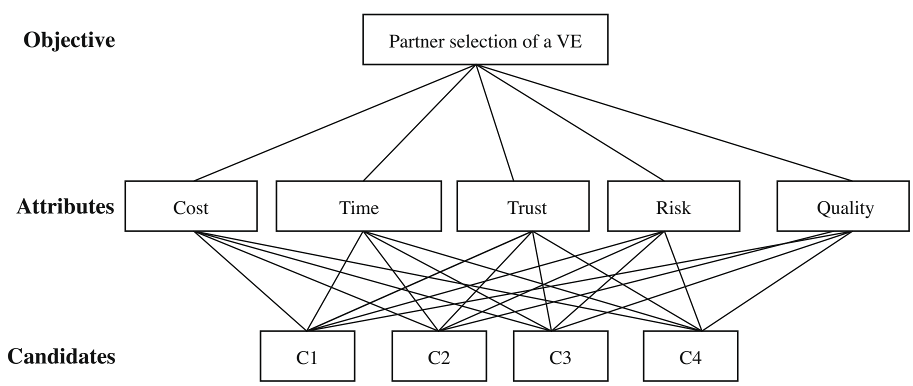

Fig. 1. Decision hierarchy of the partner selection problem of a VE.

Table 1
The decision matrix with interval values and weights of five attributes for each DM.

Decision maker	Candidate and weight	Cost	Time	Trust	Risk	Quality
DM#1	C1	[10, 12]	[21, 25]	[80, 84]	[0.95, 0.98]	[0.95, 0.96]
	C2	[11, 15]	[24, 25]	[84, 85]	[0.92, 0.93]	[0.96, 0.97]
	C3	[12, 13]	[22, 24]	[87, 89]	[0.88, 0.91]	[0.96, 0.97]
	C4	[14, 16]	[18, 20]	[91, 93]	[0.89, 0.90]	[0.99, 1.00]
	Weight	0.22	0.17	0.25	0.15	0.21
DM#2	C1	[9, 13]	[24, 25]	[79, 82]	[0.93, 0.94]	[0.96, 0.98]
	C2	[11, 12]	[21, 23]	[83, 84]	[0.92, 0.94]	[0.97, 0.98]
	C3	[10, 12]	[22, 23]	[88, 89]	[0.89, 0.91]	[0.98, 0.99]
	C4	[15, 16]	[19, 20]	[89, 92]	[0.90, 0.92]	[0.99, 1.00]
	Weight	0.19	0.18	0.22	0.16	0.25
DM#3	C1	[11, 13]	[19, 22]	[74, 78]	[0.96, 0.97]	[0.93, 0.96]
	C2	[12, 14]	[18, 25]	[76, 80]	[0.93, 0.96]	[0.94, 0.96]
	C3	[12, 15]	[21, 22]	[82, 85]	[0.90, 0.92]	[0.95, 0.96]
	C4	[13, 17]	[18, 23]	[86, 88]	[0.91, 0.94]	[0.97, 0.98]
	Weight	0.21	0.19	0.23	0.17	0.20
DM#4	C1	[13, 14]	[22, 23]	[76, 78]	[0.95, 0.96]	[0.94, 0.95]
	C2	[13, 15]	[19, 23]	[81, 82]	[0.94, 0.95]	[0.93, 0.94]
	C3	[16, 18]	[20, 22]	[84, 86]	[0.89, 0.92]	[0.94, 0.95]
	C4	[15, 17]	[19, 21]	[87, 88]	[0.88, 0.91]	[0.95, 0.96]
	Weight	0.24	0.18	0.21	0.18	0.19

Step9: Aggregate the separation measures for the group by using formulas (26) and (27).

Step10: Calculate the relative closeness of the candidate X_i by using formula (28).

Step11: Rank the preference order of all candidates according to the relative closeness of the candidates, and select the best partner.

4. Numerical example

In order to demonstrate the application of two extended TOPSIS methods for group decision making to partner selection in the formation of a new virtual enterprise (VE), we will consider an example where the core enterprise of the virtual enterprise has to select a partner for a sub-project. The partner selection decision is made on the basis of five main attributes including Cost, Time, Trust, Risk and Quality. Cost, Time and Risk are cost type, while Trust and Quality are benefit type.

There are four partners have been identified as candidates, and four decision makers are responsible for the partner selection problem. The objective here is to select a partner, which can satisfy all attributes in the best way.

Since it is useful to construct a hierarchical structure showing the overall objective, the attributes and candidates, this hierarchy for the partner selection problem is shown in Fig. 1. And these data and the vector of corresponding weight of each attribute are given in Table 1.

The normalized decision matrix with interval values and the weighted normalized decision matrix with interval values are given in Tables 2 and 3, respectively. Next we will utilize two extended TOPSIS methods to solve the partner selection problem.

4.1. TOPSIS for group decision making based on deviation degree applied into partner selection

First, the positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) for each decision maker are given in Table 4 by using formulas (7) and (8).

Next, the positive deviation degree between each candidate and the positive ideal partner for each decision maker is calculated by using formulas (10) and (11) and is given in Table 5. Similarly, the negative deviation degree between each candidate and the negative ideal partner for each decision maker is calculated by using formulas (12) and (13), and is also shown in Table 5.

In the end, the group deviation degree measures are aggregated by using formulas (16) and (17) and are given in Table 6. Moreover, the relative closeness of each candidate is calculated by formula (18) (that is also shown in Table 6). According to the relative closeness, we can rank the preference order of all candidates, which is

Table 2
The normalized decision matrix with interval values for each DM.

Decision maker	Candidate	Cost	Time	Trust	Risk	Quality
DM#1	C1	[0.479, 0.687]	[0.418, 0.552]	[0.455, 0.491]	[0.464, 0.489]	[0.487, 0.497]
	C2	[0.383, 0.624]	[0.435, 0.483]	[0.478, 0.497]	[0.489, 0.505]	[0.492, 0.503]
	C3	[0.442, 0.572]	[0.475, 0.527]	[0.495, 0.520]	[0.499, 0.528]	[0.492, 0.503]
	C4	[0.359, 0.490]	[0.581, 0.644]	[0.518, 0.543]	[0.505, 0.522]	[0.508, 0.513]
DM#2	C1	[0.411, 0.722]	[0.425, 0.469]	[0.455, 0.483]	[0.484, 0.494]	[0.486, 0.503]
	C2	[0.445, 0.590]	[0.462, 0.536]	[0.478, 0.495]	[0.484, 0.504]	[0.491, 0.503]
	C3	[0.445, 0.649]	[0.462, 0.512]	[0.507, 0.524]	[0.450, 0.521]	[0.496, 0.508]
	C4	[0.334, 0.433]	[0.532, 0.593]	[0.512, 0.542]	[0.495, 0.515]	[0.501, 0.513]
DM#3	C1	[0.459, 0.661]	[0.429, 0.603]	[0.447, 0.490]	[0.476, 0.493]	[0.482, 0.507]
	C2	[0.426, 0.606]	[0.378, 0.636]	[0.459, 0.502]	[0.481, 0.509]	[0.487, 0.507]
	C3	[0.398, 0.606]	[0.429, 0.545]	[0.495, 0.534]	[0.502, 0.526]	[0.492, 0.507]
	C4	[0.351, 0.559]	[0.411, 0.521]	[0.519, 0.552]	[0.492, 0.520]	[0.503, 0.517]
DM#4	C1	[0.503, 0.606]	[0.432, 0.505]	[0.445, 0.475]	[0.476, 0.492]	[0.495, 0.500]
	C2	[0.469, 0.606]	[0.432, 0.584]	[0.485, 0.499]	[0.481, 0.497]	[0.489, 0.495]
	C3	[0.391, 0.493]	[0.452, 0.555]	[0.502, 0.524]	[0.496, 0.525]	[0.495, 0.500]
	C4	[0.414, 0.526]	[0.474, 0.584]	[0.520, 0.536]	[0.502, 0.531]	[0.500, 0.505]

Table 3
The weighted normalized decision matrix with interval values for each DM.

Decision maker	Candidate	Cost	Time	Trust	Risk	Quality
DM#1	C1	[0.105, 0.151]	[0.071, 0.094]	[0.114, 0.123]	[0.07, 0.073]	[0.102, 0.104]
	C2	[0.084, 0.137]	[0.074, 0.082]	[0.120, 0.124]	[0.073, 0.076]	[0.103, 0.106]
	C3	[0.097, 0.126]	[0.081, 0.090]	[0.124, 0.130]	[0.075, 0.079]	[0.103, 0.106]
	C4	[0.079, 0.108]	[0.099, 0.111]	[0.130, 0.136]	[0.076, 0.078]	[0.107, 0.108]
DM#2	C1	[0.078, 0.137]	[0.077, 0.084]	[0.100, 0.106]	[0.077, 0.080]	[0.122, 0.126]
	C2	[0.085, 0.112]	[0.083, 0.097]	[0.105, 0.109]	[0.077, 0.081]	[0.123, 0.126]
	C3	[0.085, 0.123]	[0.083, 0.092]	[0.111, 0.115]	[0.080, 0.083]	[0.124, 0.127]
	C4	[0.063, 0.082]	[0.096, 0.107]	[0.113, 0.119]	[0.079, 0.082]	[0.125, 0.128]
DM#3	C1	[0.096, 0.139]	[0.082, 0.115]	[0.103, 0.113]	[0.081, 0.084]	[0.096, 0.101]
	C2	[0.090, 0.127]	[0.072, 0.121]	[0.105, 0.116]	[0.082, 0.87]	[0.097, 0.101]
	C3	[0.084, 0.127]	[0.082, 0.104]	[0.114, 0.123]	[0.085, 0.089]	[0.098, 0.101]
	C4	[0.074, 0.117]	[0.078, 0.099]	[0.119, 0.127]	[0.084, 0.088]	[0.101, 0.103]
DM#4	C1	[0.121, 0.146]	[0.078, 0.091]	[0.095, 0.100]	[0.086, 0.089]	[0.094, 0.095]
	C2	[0.113, 0.146]	[0.078, 0.105]	[0.102, 0.105]	[0.087, 0.089]	[0.093, 0.094]
	C3	[0.094, 0.118]	[0.081, 0.100]	[0.106, 0.110]	[0.089, 0.094]	[0.094, 0.095]
	C4	[0.099, 0.126]	[0.085, 0.105]	[0.109, 0.113]	[0.090, 0.096]	[0.095, 0.096]

Table 4
The positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) for each DM.

Decision maker	Ideal partner solution	Cost	Time	Trust	Risk	Quality
DM#1	PIPS	[0.105, 0.151]	[0.099, 0.111]	[0.130, 0.136]	[0.076, 0.079]	[0.107, 0.108]
	NIPS	[0.079, 0.108]	[0.071, 0.082]	[0.114, 0.123]	[0.070, 0.073]	[0.102, 0.104]
DM#2	PIPS	[0.085, 0.137]	[0.096, 0.107]	[0.113, 0.119]	[0.080, 0.083]	[0.125, 0.128]
	NIPS	[0.063, 0.082]	[0.077, 0.084]	[0.100, 0.106]	[0.077, 0.080]	[0.122, 0.126]
DM#3	PIPS	[0.096, 0.139]	[0.082, 0.121]	[0.119, 0.127]	[0.085, 0.089]	[0.101, 0.103]
	NIPS	[0.074, 0.117]	[0.072, 0. 099]	[0.103, 0.113]	[0.081, 0.084]	[0.096, 0.101]
DM#4	PIPS	[0.121, 0.146]	[0.085, 0.105]	[0.109, 0.113]	[0.090, 0.096]	[0.095, 0.096]
	NIPS	[0.094, 0.118]	[0.078, 0. 091]	[0.095, 0.100]	[0.086, 0.089]	[0.093, 0.094]

Table 5
Deviation degree for each DM.

Candidate	DM#1		DM#2		DM#3		DM#4	
C1	$D_1^{1^+} = .0403$	$D_1^{1^-} = .0518$	$D_1^{2^+} = .0360$	$D_1^{2^-} = .0571$	$D_1^{3^+} = .0241$	$D_1^{3^-} = .0362$	$D_1^{4^+} = .0262$	$D_1^{4^-} = .0384$
C2	$D_2^{1^+} = .0486$	$D_2^{1^-} = .0308$	$D_2^{2^+} = .0328$	$D_2^{2^-} = .0401$	$D_2^{3^+} = .0251$	$D_2^{3^-} = .0291$	$D_2^{4^+} = .0174$	$D_2^{4^-} = .0371$
C3	$D_3^{1^+} = .0397$	$D_3^{1^-} = .0319$	$D_3^{2^+} = .0242$	$D_3^{2^-} = .0503$	$D_3^{3^+} = .0255$	$D_3^{3^-} = .0240$	$D_3^{4^+} = .0397$	$D_3^{4^-} = .0186$
C4	$D_4^{1^+} = .0504$	$D_4^{1^-} = .0450$	$D_4^{2^+} = .0589$	$D_4^{2^-} = .0351$	$D_4^{3^+} = .0383$	$D_4^{3^-} = .0236$	$D_4^{4^+} = .0294$	$D_4^{4^-} = .0279$

Table 6
Deviation degree, closeness coefficients and ranking.

Candidate	Positive deviation degree	Negative deviation degree	U_i	Ranking
C1	$D_1^+ = 0.0309$	$D_1^- = 0.0450$	0.5929	1
C2	$D_2^+ = 0.0289$	$D_2^- = 0.0340$	0.5405	2
C3	$D_3^+ = 0.0314$	$D_3^- = 0.0291$	0.4810	3
C4	$D_4^+ = 0.0427$	$D_4^- = 0.0319$	0.4276	4

Table 7
Exact values by transferring for each DM.

Case	Decision maker and risk factor	Candidate	Cost	Time	Trust	Risk	Quality
Case#1	DM#1 $\varepsilon_1 = 0.3$	C1	0.1419	0.0893	0.1209	0.0726	0.1040
		C2	0.1267	0.0805	0.1232	0.0752	0.1051
		C3	0.1201	0.0878	0.1287	0.0783	0.1051
		C4	0.1021	0.1074	0.1346	0.0777	0.1075
	DM#2 $\varepsilon_2 = 0.2$	C1	0.1194	0.0821	0.1044	0.0791	0.1244
		C2	0.1066	0.0939	0.1082	0.0800	0.1251
		C3	0.1156	0.0904	0.1146	0.0827	0.1263
		C4	0.0785	0.1045	0.1180	0.0818	0.1276
	DM#3 $\varepsilon_3 = 0.35$	C1	0.1324	0.1096	0.1111	0.0834	0.1006
		C2	0.1216	0.1135	0.1140	0.0858	0.1007
		C3	0.1207	0.1003	0.1214	0.0888	0.1009
		C4	0.1109	0.0958	0.1259	0.0877	0.1030
	DM#4 $\varepsilon_4 = 0.25$	C1	0.1393	0.0876	0.0987	0.0878	0.0947
		C2	0.1373	0.0983	0.1041	0.0887	0.0937
		C3	0.1121	0.0953	0.1089	0.0932	0.0947
		C4	0.1194	0.1002	0.1117	0.0943	0.0958
Case#2	DM#1 $\varepsilon_1 = -0.35$	C1	0.1122	0.0745	0.1152	0.0701	0.1026
		C2	0.0922	0.0752	0.1202	0.0736	0.1037
		C3	0.1015	0.0821	0.1248	0.0755	0.1037
		C4	0.0833	0.1003	0.1305	0.0761	0.1068
	DM#2 $\varepsilon_2 = 0.00$	C1	0.1076	0.0805	0.1032	0.0786	0.1236
		C2	0.0984	0.0899	0.1070	0.0791	0.1242
		C3	0.1040	0.0877	0.1134	0.0817	0.1255
		C4	0.0728	0.1012	0.1160	0.0808	0.1268
	DM#3 $\varepsilon_3 = 0.25$	C1	0.1282	0.1063	0.1102	0.0831	0.1001
		C2	0.1178	0.1086	0.1130	0.0854	0.1003
		C3	0.1163	0.0981	0.1205	0.0884	0.1006
		C4	0.1065	0.0937	0.1251	0.0872	0.1027
	DM#4 $\varepsilon_3 = 0.3$	C1	0.1406	0.0882	0.0989	0.0879	0.0948
		C2	0.1389	0.0997	0.1042	0.0889	0.0938
		C3	0.1134	0.0962	0.1091	0.0935	0.0948
		C4	0.1208	0.1012	0.1119	0.0945	0.0958

shown in Table 6. Based on the results, the priority of candidates is $c_1 \succ c_2 \succ c_3 \succ c_4$, where the symbol "$\succ$" means superior to. Therefore, C1 will be the best candidate.

4.2. TOPSIS based on risk factor applied into partner selection

Let us take Case#1 and Case#2 with different risk attitudes of different decision makers for example. First, we transfer the interval values into exact values for each decision maker by using formula (19) in both cases, which is shown in Table 7.

Next, we identify the positive ideal partner and negative ideal partner by using formulas (22) and (23), which is shown in Table 8.

Then, the separation of each candidate from the positive ideal partner is calculated by using formula (24), and the separation of each candidate from the negative ideal partner is also calculated by using formula (25). These results are shown in Table 9.

In the end, the separation measures for group are aggregated by using formulas (26) and (27) and the relative closeness of each candidate is calculated and shown in Table 10. According to the relative closeness, the preference ranking orders of all candidates are also shown in Table 10. According to the results, C1 will be the best candidate both in the Case#1 and Case#2. And the results are consistent with that of TOPSIS for group decision making based on deviation degree.

5. Conclusion

A major issue in the formation of the VE is to select suitable partners in the process of the virtual enterprise establishing, and it has attracted much research attention recently. In reality, in the partner selection process, the decision makers are generally

Table 8
The positive ideal partner solution (PIPS) and negative ideal partner solution (NIPS) for each DM.

Case	Decision maker	Ideal partner solution	Cost	Time	Trust	Risk	Quality
Case#1	DM#1	PIPS	0.1419	0.1074	0.1346	0.0783	0.1075
		NIPS	0.1021	0.0805	0.1209	0.0726	0.1040
	DM#2	PIPS	0.1194	0.1045	0.1180	0.0827	0.1276
		NIPS	0.0785	0.0821	0.1044	0.0791	0.1244
	DM#3	PIPS	0.1324	0.1135	0.1259	0.0888	0.1030
		NIPS	0.1109	0.0958	0.1111	0.0834	0.1006
	DM#4	PIPS	0.1393	0.1002	0.1117	0.0943	0.0958
		NIPS	0.1194	0.0876	0.0987	0.0878	0.0937
Case#2	DM#1	PIPS	0.1122	0.1003	0.1305	0.0761	0.1068
		NIPS	0.0833	0.0745	0.1152	0.0701	0.1026
	DM#2	PIPS	0.1076	0.1012	0.1160	0.0817	0.1268
		NIPS	0.0728	0.0805	0.1032	0.0786	0.1236
	DM#3	PIPS	0.1282	0.1086	0.1251	0.0884	0.1027
		NIPS	0.1065	0.0937	0.1102	0.0831	0.1001
	DM#4	PIPS	0.1406	0.1012	0.1119	0.0945	0.0958
		NIPS	0.1134	0.0882	0.0989	0.0879	0.0938

Table 9
Separation of each candidate from the positive ideal partner (PIPS) and negative ideal partner solution (NIPS) for each DM.

Case	Candidate	DM#1		DM#2		DM#3		DM#4	
Case#1	C1	$d_1^{1^+} = .0237$	$d_1^{1^-} = .0408$	$d_1^{2^+} = .0196$	$d_1^{2^-} = .0409$	$d_1^{3^+} = .0164$	$d_1^{3^-} = .0257$	$d_1^{4^+} = .0193$	$d_1^{4^-} = .0199$
	C2	$d_2^{1^+} = .0332$	$d_2^{1^-} = .0249$	$d_2^{2^+} = .0196$	$d_2^{2^-} = .0307$	$d_2^{3^+} = .0165$	$d_2^{3^-} = .0211$	$d_2^{4^+} = .0101$	$d_2^{4^-} = .0216$
	C3	$d_3^{1^+} = .0300$	$d_3^{1^-} = .0217$	$d_3^{2^+} = .0150$	$d_3^{2^-} = .0396$	$d_3^{3^+} = .0183$	$d_3^{3^-} = .0160$	$d_3^{4^+} = .0278$	$d_3^{4^-} = .0157$
	C4	$d_4^{1^+} = .0398$	$d_4^{1^-} = .0308$	$d_4^{2^+} = .0409$	$d_4^{2^-} = .0265$	$d_4^{3^+} = .0279$	$d_4^{3^-} = .0154$	$d_4^{4^+} = .0199$	$d_4^{4^-} = .0193$
Case#2	C1	$d_1^{1^+} = .0309$	$d_1^{1^-} = .0289$	$d_1^{2^+} = .0247$	$d_1^{2^-} = .0348$	$d_1^{3^+} = .0162$	$d_1^{3^-} = .0251$	$d_1^{4^+} = .0196$	$d_1^{4^-} = .0272$
	C2	$d_2^{1^+} = .0339$	$d_2^{1^-} = .0109$	$d_2^{2^+} = .0175$	$d_2^{2^-} = .0275$	$d_2^{3^+} = .0164$	$d_2^{3^-} = .0190$	$d_2^{4^+} = .0100$	$d_2^{4^-} = .0285$
	C3	$d_3^{1^+} = .0221$	$d_3^{1^-} = .0226$	$d_3^{2^+} = .0143$	$d_3^{2^-} = .0338$	$d_3^{3^+} = .0167$	$d_3^{3^-} = .0158$	$d_3^{4^+} = .0278$	$d_3^{4^-} = .0142$
	C4	$d_4^{1^+} = .0289$	$d_4^{1^-} = .0309$	$d_4^{2^+} = .0348$	$d_4^{2^-} = .0246$	$d_4^{3^+} = .0158$	$d_4^{3^-} = .0157$	$d_4^{4^+} = .0198$	$d_4^{4^-} = .0210$

Table 10
Aggregated separation, closeness coefficient of each candidate and ranking.

Case	Candidate	Positive separation	Negative separation	Closeness coefficient	Ranking
Case#1	C1	$d_1^+ = 0.0309211$	$d_1^- = 0.0304$	$u_1 = 0.5898$	1
	C2	$d_2^+ = 0.0181$	$d_2^- = 0.0243$	$u_2 = 0.5726$	2
	C3	$d_3^+ = 0.0219$	$d_3^- = 0.0216$	$u_3 = 0.4960$	3
	C4	$d_4^+ = 0.0308$	$d_4^- = 0.0222$	$u_4 = 0.4189$	4
Case#2	C1	$d_1^+ = 0.0222$	$d_1^- = 0.0288$	$u_1 = 0.5649$	1
	C2	$d_2^+ = 0.0177$	$d_2^- = 0.0201$	$u_2 = 0.5319$	2
	C3	$d_3^+ = 0.0196$	$d_3^- = 0.0203$	$u_3 = 0.5098$	3
	C4	$d_4^+ = 0.0269$	$d_4^- = 0.0244$	$u_4 = 0.4538$	4

unsure of there preferences because the information about the candidates and their performances are incomplete and uncertain. Therefore, in this paper we study the partner selection problem with interval values in the formation of a new virtual enterprise (VE) under incomplete information. And two extended TOPSIS methods for group decision making including "TOPSIS for group decision making based on deviation degree" and "TOPSIS or group decision making based on risk factor" are proposed to solve the partner selection problem with interval values. In two extended TOPSIS methods for group decision making, as well as considering the distance of a candidate from the positive ideal partner solution (PIPS), its distance from the negative ideal partner solution (NINS) is also considered. That is to say, the less the distance of the candidate from the positive ideal partner solution (PIPS) and the more its distance from the negative ideal partner solution (NINS), the better its ranking will be.

This paper also has taken an illustrative example to demonstrate the feasibility and practicability of the two extended TOPSIS methods for group decision making. Results show that these two extended TOPSIS methods for group decision making can effectively deal with the partner selection problem under incomplete information.

These two extended TOPSIS methods for group decision making can not only be applied into solving the partner selection problem of the virtual enterprise, but also be utilized in other areas, such as investment, subcontractor selection, construction technique alternative evaluation and human resource arrangement so on.

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