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Extension of TOPSIS to determine weight of decision maker for group decision making problems with uncertain information

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ABSTRACT

In traditional TOPSIS method, the ideal solutions for alternatives are expressed in vectors. An important step in the process of group decision making is to determine the relative importance of each decision maker. In this paper, the weights of decision makers derived from individual decision are determined by using an extended TOPSIS method with interval numbers. The ideal decisions for all individual decisions are expressed in matrices. The positive ideal decision is the intersection of all individual decisions; the negative ideal decision is the union of all individual decisions. Comparisons with other methods are also made. A numerical example is examined to show the potential applications of the proposed method.

1. Introduction

Decision making is the process of finding the best option from all of the feasible alternatives. The increasing complexity of the socio-economic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem (Kim & Ahn, 1999). As a result, many decision making processes, in the real world, take place in group settings.

To determine the weights of every DMs is a very important step in multiple attribute group decision making (MAGDM) (Yue, Jia, & Ye, 2009; Yue, 2011b, c). There are many applications, which necessitate different weights (Ramanathan & Ganesh, 1994) because a DM cannot be expected to have sufficient expertise to comment on all aspects of the problem but on a part of the problem for which he/she is competent (Weiss & Rao, 1987). In this paper, we suppose that the weights of DMs are different and unknown. How to measure the weights of DMs? Up to now, many methods have been developed. French Jr (1956) proposed a method to determine the relative importance of the group's members by using the influence relations, which may exist between the members. Theil (1963) proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood (1975) and Keeney (1976) suggested the use of the interpersonal comparison to determine the scales constant values in an additive and weighted social choice function. Bodily (1979) and Mirkin and Fishburn (1979) proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Brock (1980) used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh (1994) proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Van den Honert (2001) used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by other group members. Jabeur and Martel (2002) proposed a procedure which exploits the idea of Zeleny (1982) to determine the relative importance coefficient of each member. Jabeur, Martel, and Khelifa (2004) proposed a distance-based collective preorder integrating the relative importance of the group's members. By using the deviation measures between additive linguistic preference relations, Xu (2008b) gave some straightforward formulas to determine the weights of DMs. Chen and Fan (2006, 2007) studied a method for the ranking of experts according to their levels in group decision. Recently, Yue (2011a) presented an approach for group decision making based on determining weights of DMs using TOPSIS (technique for order preference by similarity to an ideal solution) (Hwang & Yoon, 1981). And please refer to Yue (2011d, e, f) for some related research method.

The above methods have numerous advantages, however, most of the performance rating is quantified as crisp values. Under many circumstances, crisp data are inadequate to model real-life situations. Since human judgments including preferences are often uncertain, it is difficult to rate them as exact numerical values. In addition, in case of conflicting situations or attribute, a DM must also consider imprecise or uncertain data, which is very usual in this type of decision problems. A more realistic approach may be to use interval data instead of crisp values, that is, to suppose that the ratings of the attributes in the problem are assessed by means of interval data. In this paper, we will present a new TOPSIS method with interval data for MAGDM problems.

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The remaining paper is organized as follows. In Section 2, the concepts of interval number are presented and discussed, including the operations of interval numbers. Based on the concepts in Section 2, the proposed approach for determining the weights of DMs using an extended TOPSIS is shown in Section 3. Section 4 compares the proposed method with other methods. Then, an illustrative example is used to demonstrate the feasibility and practicability of the proposed method in Section 5. Finally, Section 6 concludes this paper.

2. Interval numbers and their operation

As aforementioned, in some cases, determining the exact decision information is difficult and, as a result, the obtained information from real world is always uncertain or incomplete. Hence, extending the applications from precise number to interval numbers is necessary for real-world applications.

We describe the basic definitions and operations of interval number as follows.

Definition 1 (*Xu, 2008a; Zhang, Wu, and Olson, 2005*). . Let $a = [a^l, a^u] = \{x | 0 < a^l \leqslant x \leqslant a^u\}$, then a is called a nonnegative interval number. Especially, a is a nonnegative real number, if $a^l = a^u$.

Note: For convenience of computation, throughout this paper, all the interval arguments are nonnegative interval numbers, and let Ω be the set of all interval arguments, $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots n\}$ and $T = \{1, 2, \ldots t\}$; $i \in M$, $j \in N$, and $k \in T$.

Definition 2 (*Xu, 2005, 2008a*). Let $a = [a^l, a^u]$, $b = [b^l, b^u]$ are interval numbers and $\lambda \geqslant 0$, then

(1) $a = b$ if and only if $a^l = b^l$ and $a^u = b^u$;
(2) $a + b = [a^l, a^u] + [b^l, b^u] = [a^l + b^l, a^u + b^u]$;
(3) $\lambda a = \lambda [a^l, a^u] = [\lambda a^l, \lambda a^u]$. Especially, $\lambda a = 0$ if $\lambda = 0$.

In order to aggregate interval numbers, we introduce the following weighted averaging operator (Harsanyi, 1955; Hwang & Yoon, 1981):

Definition 3. Let $a_j = \left[a_j^l, a_j^u\right] \in Q(j \in N)$, a weighted averaging operator of $\{a_j | j \in N\}$ is a mapping $WA : \Omega^n \to \Omega$ such that

$$WA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j a_j, \tag{1}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $\{a_j | j \in N\}$, $w_j \geqslant 0 (j \in N)$ and $\Sigma_{j=1}^{n} w_j = 1$.

We introduce the following formula in order to rank interval numbers.

Definition 4 Xu, 2008a. . Let $a = [a^l, a^u] \in \Omega$, $b = [b^l, b^u] \in \Omega$, $l_a = a^u - a^l$ and $l_b = b^u - b^l$, then the degree of possibility of $a \geqslant b$ is defined as

$$p(a \geqslant b) = \max\left\{1 - \max\left(\frac{b^u - a^l}{l_a + l_b}, 0\right), 0\right\}. \tag{2}$$

Moreover, we can get easily the following results (Xu, 2008a, 2005) from Eq. (2):

Let $a = [a^l, a^u] \in \Omega$, $b = [b^l, b^u] \in \Omega$, then.

(1) $0 \leqslant p(a \geqslant b) \leqslant 1$;
(2) $p(a \geqslant b) = 1$ if and only if $b^u \leqslant a^l$;
(3) $p(a \geqslant b) = 0$ if and only if $a^u \leqslant b^l$;
(4) $p(a \geqslant a) = \frac{1}{2}$;
(5) $p(a \geqslant b) + p(b \geqslant a) = 1$.

To rank the interval arguments $a_j = \left[a_j^l, a_j^u\right] \in Q(j \in N)$, we first compare each $a_i = [a_i^l, a_i^u]$ with all $a_j = \left[a_j^l, a_j^u\right] (j \in N)$ by using Eq. (2). For convenience, we let $p_{ij} = p(a_i \geqslant a_j)$, and then construct a complementary matrix (Xu, 2008a) as follows:

$$P = (p_{ij})_{n \times n} \tag{3}$$

where $p_{ij} \geqslant 0, p_{ij} + p_{ji} = 1, p_{ii} = \frac{1}{2}, \ i, j = 1, 2, \ldots, n$.

Summing all elements in each line of matrix P, we have

$$p_i = \sum_{j=1}^{n} p_{ij}, \quad i = 1, 2, \ldots, n. \tag{4}$$

Then we can reorder the interval arguments $a_j = \left[a_j^l, a_j^u\right] (j \in N)$ in descending order in accordance with the values of $p_i (i \in M)$ (Xu, 2008a).

Definition 5. Let $X = (\alpha_{ij})_{m \times n}$ be a matrix, where the elements α_{ij} are interval numbers, then X is called an interval matrix.

Definition 6. Let $a = [a^l, a^u] \in \Omega, b = [b^l, b^u] \in \Omega$, then

$$D(a, b) = \sqrt{(b^l - a^l)^2 + (b^u - a^u)^2} \tag{5}$$

is called the Euclidean distance between a and b.

Definition 7. Let $a = [a^l, a^u] \in \Omega, b = [b^l, b^u] \in \Omega$, if $a = \phi$ or $b = \phi$, then we difine $D(a, b) = 0$.

Definition 8. Let $a = [a^l, a^u] \in \Omega, b = [b^l, b^u], \in \Omega$, then

$$a \cup b = [\min\{a^l, b^l\}, \max\{a^u, b^u\}] \tag{6}$$

is called the union of a and b.

Definition 9. Let $a = [a^l, a^u] \in \Omega, b = [b^l, b^u] \in \Omega$, then

$$a \cap b = [\max\{a^l, b^l\}, \min\{a^u, b^u\}] \tag{7}$$

is called the intersection of a and b.

Theorem. *Let $a = [a^l, a^u] \in \Omega, b = [b^l, b^u] \in \Omega$, then $a \cap b = \phi$ if and only if $\max\{a^l, b^l\} > \min\{a^u, b^u\}$.*

Definition 10. Let $X_1 = (\alpha_{ij})_{m \times n}$ and $X_2 = (\beta_{ij})_{m \times n}$ be two interval matrices, where $\alpha_{ij} = [\alpha_{ij}^l, \alpha_{ij}^u], \beta_{ij} = [\beta_{ij}^l, \beta_{ij}^u]$, then

$$D(X_1, X_2) = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} (\beta_{ij}^l - \alpha_{ij}^l)^2 + (\beta_{ij}^u - \alpha_{ij}^u)^2} \tag{8}$$

is called the Euclidean distance between X_1 and X_2.

3. An extended TOPSIS to determine weights of decision makers with interval number

To aid in the elucidation of the proposed technique, in what follows, we first review the group decision making with interval number.

3.1. Multiple attribute group decision making with interval data

Let $A = \{A_1, A_2, \ldots A_m\} (m \geqslant 2)$ be a discrete set of m feasible alternatives, $U = \{u_1, u_2, \ldots u_n\}$ be a finite set of attributes, $w = (w_1, w_2, \ldots w_n)^T$ be the weight vector of attributes, with $0 \leqslant w_j \leqslant 1$ and $\sum_{j=1}^{n} w_j = 1$. And let $D = \{d_1, \ d_2, \ldots, d_t\}$ be a group of DMs, and

$\lambda = (\lambda_1, \lambda_2, \dots \lambda_t)^T$ be the weight vector of DMs, where $\lambda_k \geqslant 0$, $\sum_{k=1}^{n} \lambda_k = 1$.

A MAGDM problem can be described in detail as follows:

Let

$$X_k = \left(\left[x_{ij}^{kl}, x_{ij}^{ku}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \left(\begin{array}{cccc} [x_{11}^{kl}, x_{11}^{ku}] & [x_{12}^{kl}, x_{12}^{ku}] & \cdots & [x_{1n}^{kl}, x_{1n}^{ku}] \\ [x_{21}^{kl}, x_{21}^{ku}] & [x_{22}^{kl}, x_{22}^{ku}] & \cdots & [x_{2n}^{kl}, x_{2n}^{ku}] \\ \vdots & \vdots & \vdots & \vdots \\ [x_{m1}^{kl}, x_{m1}^{ku}] & [x_{m2}^{kl}, x_{m2}^{ku}] & \cdots & [x_{mn}^{kl}, x_{mn}^{ku}] \end{array}\right) \end{array}, \text{ for all } k \in T, \tag{9}$$

be decision matrix of the kth ($k \in T$) DM, in which each of the elements is characterized by interval number. In general, there are benefit attributes and cost attributes in the multiple attribute decision making problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following Eqs. (11) and (12) (Aghajani Bazzazi, Osanloo, & Karimi, 2011) to normalize each attribute value $\left[x_{ij}^{kl}, x_{ij}^{ku}\right]$ in decision matrix $X_k = \left(\left[x_{ij}^{kl}, x_{ij}^{ku}\right]\right)_{m\times n}$ into a corresponding element $\left[y_{ij}^{kl}, y_{ij}^{ku}\right]$ in normalized decision matrix $Y_k = \left(\left[y_{ij}^{kl}, y_{ij}^{ku}\right]\right)_{m\times n}$ given by Eq. (10).

$$Y_k = \left(\left[y_{ij}^{kl}, y_{ij}^{ku}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \left(\begin{array}{cccc} [y_{11}^{kl}, y_{11}^{ku}] & [y_{12}^{kl}, y_{12}^{ku}] & \cdots & [y_{1n}^{kl}, y_{1n}^{ku}] \\ [y_{21}^{kl}, y_{21}^{ku}] & [y_{22}^{kl}, y_{22}^{ku}] & \cdots & [y_{2n}^{kl}, y_{2n}^{ku}] \\ \vdots & \vdots & \vdots & \vdots \\ [y_{m1}^{kl}, y_{m1}^{ku}] & [y_{m2}^{kl}, y_{m2}^{ku}] & \cdots & [y_{mn}^{kl}, y_{mn}^{ku}] \end{array}\right) \end{array}, \text{ for all } k \in T, \tag{10}$$

where

$$\begin{cases} y_{ij}^{kl} = \dfrac{x_{ij}^{kl}}{\max\limits_{i\in M}\{x_{ij}^{ku}\}} \\ y_{ij}^{ku} = \dfrac{x_{ij}^{ku}}{\max\limits_{i\in M}\{x_{ij}^{ku}\}} \end{cases}, \quad \text{for benefit attribute } u_j, \quad i \in M,\ j \in N,\ k \in T, \tag{11}$$

and

$$\begin{cases} y_{ij}^{kl} = \dfrac{\min\limits_{i\in M}\{x_{ij}^{kl}\}}{x_{ij}^{ku}} \\ y_{ij}^{ku} = \dfrac{\min\limits_{i\in M}\{x_{ij}^{kl}\}}{x_{ij}^{kl}} \end{cases}, \quad \text{for cost attribute } u_j, \quad i \in M,\ j \in N,\ k \in T. \tag{12}$$

Obviously, the normalization method mentioned above is to preserve the characteristic that the ranges of normalized interval values belong to $[0, 1]$.

3.2. An extended TOPSIS method with interval number

Suppose that $w_k = (w_1^k, w_2^k, \dots, w_n^k)^T$ is the weight vector of attributes, with $0 \leqslant w_j^k \leqslant 1$ and $\sum_{j=1}^{n} w_j^k = 1$, which is provided by kth DM. For the normalized decision matrix Y_k of kth DM mentioned above, we can construct the weighted normalized decision matrix, multiplying each element of the decision matrix Y_k, by the weights w_j of the corresponding attribute, i.e.,

$$R_k = \left(\left[r_{ij}^{kl}, \upsilon_{ij}^{ku}\right]\right)_{m\times n} = \left(\left[w_j^k y_{ij}^{kl}, w_j y_{ij}^{ku}\right]\right)_{m\times n}$$

$$= \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \left(\begin{array}{cccc} [r_{11}^{kl}, r_{11}^{ku}] & [r_{12}^{kl}, r_{12}^{ku}] & \cdots & [r_{1n}^{kl}, r_{1n}^{ku}] \\ [r_{21}^{kl}, r_{21}^{ku}] & [r_{22}^{kl}, r_{22}^{ku}] & \cdots & [r_{2n}^{kl}, r_{2n}^{ku}] \\ \vdots & \vdots & \vdots & \vdots \\ [r_{m1}^{kl}, r_{m1}^{ku}] & [r_{m2}^{kl}, r_{m2}^{ku}] & \cdots & [r_{mn}^{kl}, r_{mn}^{ku}] \end{array}\right) \end{array}, \text{ for all } k \in T. \tag{13}$$

Based on the weighted normalized decision matrix $R_k (k \in T)$, we can consider further to determine the ideal decisions of group below.

The ideal decision should reflect the common decision aspirations and consistent judgments. So we define

$$R^+ = \left(\left[r_{ij}^{+l}, r_{ij}^{+u}\right]\right)_{m\times n}, \quad i \in M,\ j \in N, \tag{14}$$

as the positive ideal decision (PID) of group, where $[r_{ij}^{+l}, r_{ij}^{+u}] = \bigcap_{k=1}^{t} [r_{ij}^{kl}, r_{ij}^{ku}] (i \in M, j \in N)$

And the negative ideal decision (NID) of group should has the maximum separation from the PID. So we define

$$R^- = \left(\left[r_{ij}^{-l}, r_{ij}^{-u}\right]\right)_{m\times n}, \quad i \in M,\ j \in N, \tag{15}$$

as the NID of all individual decisions, where $\left[r_{ij}^{-l}, r_{ij}^{-u}\right] = \bigcup_{k=1}^{t} \left[r_{ij}^{kl}, r_{ij}^{ku}\right] (i \in M, j \in N)$. In fact, $\bigcup_{k=1}^{t} \left[r_{ij}^{kl}, r_{ij}^{ku}\right] = \left[\min_{k\in T}\left\{r_{ij}^{kl}\right\}, \max_{k\in T}\left\{r_{ij}^{ku}\right\}\right] (i \in M, j \in N)$.

The positive separation of each individual decision from the PID, using the n-dimensional Euclidean distance, can be currently calculated as

$$S_k^+ = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left(r_{ij}^{kl} - r_{ij}^{+l}\right)^2 + \left(r_{ij}^{ku} - r_{ij}^{+u}\right)^2\right)}, k \in T. \tag{16}$$

Similarly, the negative separation from the NID is given as

$$S_k^- = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \left(\left(r_{ij}^{kl} - r_{ij}^{-l}\right)^2 + \left(r_{ij}^{ku} - r_{ij}^{-u}\right)^2\right)}, \quad k \in T. \tag{17}$$

The next step combines the two separation measures S_k^+ and S_k^- in order to obtain the relative closeness. The relative closeness of each individual decision with respect to ideal decisions is defined as

$$C_k = \frac{S_k^-}{S_k^+ + S_k^-}, \quad k \in T. \tag{18}$$

Since $S_k^- \geqslant 0$ and $S_k^+ \geqslant 0$, clearly, the range of C_k belongs to the closed interval $[0, 1]$ for all $k \in T$.

Obviously, a decision matrix R_k is closer to the A^+ and farther from A^- as R_k approaches to 1. Therefore, according to the relative closeness, we can determine the ranking order of all DMs and select the best one from among a set of DMs.

So, we can define

$$\lambda_k = \frac{C_k}{\sum_{k=1}^{t} C_k}, \quad k \in T, \tag{19}$$

as weight of kth ($k \in T$) DM, such that $\lambda_k \geqslant 0, \sum_{k=1}^{t} \lambda_k = 1$.

Further, we can aggregate all individual decisions $R_k (k \in T)$ into a collective decision R once the DMs' weight vector $\lambda = (\lambda_1, \lambda_2, \dots \lambda_t)^T$ has been determined by using Eq. (19).

$$R = \sum_{k=1}^{t} \lambda_k R_k = \left(\left[r_{ij}^{l}, r_{ij}^{u}\right]\right)_{m\times n}. \tag{20}$$

Then, we can sum all intervals in each line of the collective decision R, the overall interval assessment of each alternative $A_i (i \in M)$ is obtained:

$$r_i = \left[r_i^l, r_i^u\right] = \sum_{j=1}^{n} \left[r_{ij}^{l}, r_{ij}^{u}\right], \quad i \in M. \tag{21}$$

Now, we can construct the complementary matrix $P = (p(r_i \geqslant r_j))_{m\times m} = (p_{ij})_{m\times m}$ by Eq. (3). Then, summing all elements in each line of matrix P by Eq. (4), we can reorder all $r_i (i \in M)$ in descending order in accordance with the values of $p_i (i \in M)$. Finally, we can rank alternatives $A_i (i \in M)$ according to $p_i (i \in M)$ in descending order.

In sum, an algorithm for MAGDM based on determining the weights of DMs, when data is interval number, using the extended TOPSIS approach, is given in the following steps:

Step 1. Establish individual decision matrix.
Each DM d_k provides his/her decision matrix $X_k = \left(x_{ij}^k\right)_{m\times n}$ on alternatives with respect to attributes (see Eq. (9)).

Step 2. Normalize the individual decision matrix.
Normalize the individual decision $X_k = \left(x_{ij}^k\right)_{m\times n}$ into $Y_k = \left(y_{ij}^k\right)_{m\times n}$ in Eq. (10) by using Eqs. (11) and/or (12).

Step 3. Calculate weighted normalized decision matrix.
First, each DM d_k provides his/her weight vector $W^k = (w_1^k, w_2^k, \ldots, w_n^k)^T$ of the attributes. Then, the weighted decision matrix R_k is constructed by Eq. (13).

Step 4. Determine ideal decisions.
The PID and NID of group are determined by Eqs. (14) and (15), respectively.

Step 5. Calculate the separation measures between the individual decisions and the ideal decisions.
The positive and negative separation measures of each individual decision from the PID R^+ and NID R^-, S_k^+ and S_k^-, are calculated by Eqs. (16) and (17), respectively.

Step 6. Calculate the relative closeness of each individual decision.
A relative closeness combining the separation measures of each individual decision and the ideal decisions is calculated by using Eq. (18).

Step 7. Determine weight vector of DMs.
The weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ of DMs is determined by using Eq. (19).

Step 8. Aggregate the all individual decisions into a collective decision.
A collective decision can be aggregated by using the weight vector $\lambda = (\lambda_1, \lambda_2, \ldots \lambda_t)^T$ of DMs according to Eq. (20).

Step 9. Calculate the overall evaluations of alternatives.
Summing all interval numbers in each line of the collective decision, a overall evaluation of each alternative A_i, expressed in interval numbers, is obtained by using Eq. (21).

Step 10. Construct the complementary matrix of overall evaluations of alternatives.
A complementary matrix of overall evaluations of alternatives is constructed by Eq. (3).

Step 11. Rank the preference of alternatives.
Summing all elements in each line of the complementary matrix, denoted as p_i, shown in Eq. (4), then we can rank the alternatives in descending order in accordance with the values of p_i.

The extended TOPSIS method to determine the weights of DMs for MAGDM is presented graphically in Fig. 1.

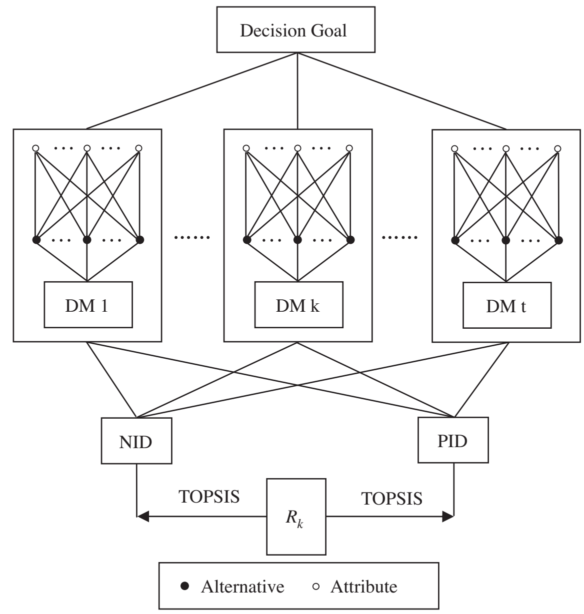

Fig. 1. Hierarchical structure of the extended TOPSIS.

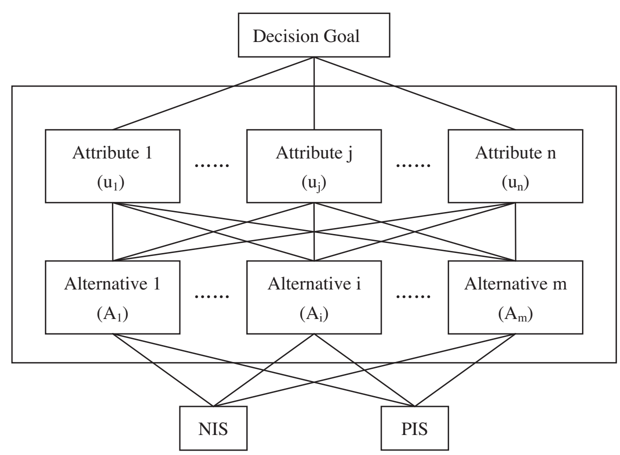

Fig. 2. Hierarchical structure of the traditional TOPSIS.

4. Comparing the proposed approach with other methods

In this section we compare the extended TOPSIS method proposed in this paper with other methods. The methods to be compared here are the traditional TOPSIS method (Hwang & Yoon, 1981) and another extended TOPSIS method proposed by Lin, Lee, Chang, and Ting (2008).

The traditional TOPSIS method is presented graphically in Fig. 2.

These are two "reference" points: positive ideal solution (PIS) and negative ideal solution (NIS) introduced in the traditional TOPSIS method in order to rank alternatives. It is suitable for cautious (risk avoider) DM (s), because the DM (s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible (Sayadi, Heydari, & Shahanaghi, 2009).

The traditional TOPSIS has solved some multiple attribute decision making problems with just one DM; whereas the extended TOPSIS technique in this paper has solved some MAGDM problems with multiple DMs. The weights of attributes are priori in the traditional TOPSIS method; whereas the weights of attributes in each individual decision are given by DM. Two "reference" points: PIS and NIS are vectors in the traditional TOPSIS method; whereas two "reference" points: PID and NID are matrices in the extended TOPSIS technique. PIS and NIS are "reference" points for all alternatives in the traditional TOPSIS method; PID and NID are 'reference" points for all individual decisions in the extended TOPSIS tech-

Table 1
Comparison with the traditional TOPSIS.

Characteristics	Traditional TOPSIS	Extended TOPSIS
No. of DMs	One	More than one
Weights on attributes	Priori	Given by DM
Cardinal information	Alternatives with respect to attributes	Alternatives with respect to attributes of multiple DMs
PIS/PID	The best alternative expressed by a vector	The consistent judgments expressed by a matrix
NIS/NID	The worst alternative expressed by a vector	The maximum separation from the PID expressed by a matrix
Core process	The separations from each alternative to PIS and NIS (between vectors)	The separations from each individual decision to PID and NID (between matrices)
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives

nique. The separation measures of each alternative from ideal solutions is compared between vectors in the traditional TOPSIS method; The separation measures of each individual decision from ideal decisions are compared between matrices in the extended TOPSIS technique. These differences are presented in Table 1.

Lin et al. (2008) presented an approach for group decision making using an extended TOPSIS with interval numbers. This is a similar methodology to this paper. These similarities and differences between two methodologies are shown in Table 2.

Tables 1 and 2 show that the suggested methodology in this paper extends the two ideal solutions expressed as a vector, respectively, in the traditional TOPSIS method to two ideal decisions expressed as a matrix, respectively; the weights of DMs are priori in the traditional TOPSIS method to be extended to derive from individual decision (measured data); ideal decisions are mac-

Table 2
Comparison with the method proposed by Lin et al.

Characteristics	Proposed by Lin et al.	Proposed by this paper
Decision information	Decision matrices	Decision matrices
	$X_1, X_2, \ldots X_t$ of alternatives with respect to attributes	$X_1, X_2, \ldots X_t$ of alternatives with respect to attributes
No. of DMs	$t > 1$	$t > 1$
Weights on attributes	Given by DM	Given by DM
PISs/PID	The best alternative of each individual decision expressed by a vector	The consistent judgment of all individual decisions expressed by a matrix
NISs/NID	The worst alternative of each individual decision expressed by a vector	The union of all individual decision expressed by a matrix
No. of PIS/PID	$t > 1$	One
No. of NIS/NID	$t > 1$	One
Decision function	The separations from each alternative to PIS and NIS (between vectors)	The separations from each individual decision to PID and NID (between matrices)
Weights of DMs	Priori	Derived from individual decision
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives

Table 3
Four decision matrices and five attributes' weights given by each DM.

DMs	Candidates and weights	Cost	Time	Trust	Risk	Quality
d_1	A_1	[10,12]	[21,25]	[80,84]	[0.95,0.98]	[0.95,0.96]
	A_2	[11,15]	[24,25]	[84,85]	[0.92,0.93]	[0.96,0.97]
	A_3	[12,13]	[22,24]	[87,89]	[0.88,0.91]	[0.96,0.97]
	A_4	[14,16]	[18,20]	[91,93]	[0.89,0.90]	[0.99,1.00]
	Weights	0.22	0.17	0.25	0.15	0.21
d_2	A_1	[9,13]	[24,25]	[79,82]	[0.93,0.94]	[0.96,0.98]
	A_2	[11,12]	[21,23]	[83,84]	[0.92,0.94]	[0.97,0.98]
	A_3	[10,12]	[22,23]	[88,89]	[0.89,0.91]	[0.98,0.99]
	A_4	[15,16]	[19,20]	[89,92]	[0.90,0.92]	[0.99,1.00]
	Weights	0.19	0.18	0.22	0.16	0.25
d_3	A_1	[11,13]	[19,22]	[74,78]	[0.96,0.97]	[0.93,0.96]
	A_2	[12,14]	[18,25]	[76,80]	[0.93,0.96]	[0.94,0.96]
	A_3	[12,15]	[21,22]	[82,85]	[0.90,0.92]	[0.95,0.96]
	A_4	[13,17]	[18,23]	[86,88]	[0.91,0.94]	[0.97,0.98]
	Weights	0.21	0.19	0.23	0.17	0.20
d_4	A_1	[13,14]	[22,23]	[76,78]	[0.95,0.96]	[0.94,0.95]
	A_2	[13,15]	[19,23]	[81,82]	[0.94,0.95]	[0.93,0.94]
	A_3	[16,18]	[20,22]	[84,86]	[0.89,0.92]	[0.94,0.95]
	A_4	[15,17]	[19,21]	[87,88]	[0.88,0.91]	[0.95,0.96]
	Weights	0.24	0.18	0.21	0.18	0.19

Table 4
Normalized decision matrices.

DMs	Candidates	Cost	Time	Trust	Risk	Quality
d_1	A_1	[0.63, 0.75]	[0.84, 1.00]	[0.95, 1.00]	[0.97, 1.00]	[0.99, 1.00]]
	A_2	[0.69, 0.94]	[0.96, 1.00]	[0.94, 0.95]	[0.94, 0.95]	[0.98, 0.99]
	A_3	[0.75, 0.81]	[0.88, 0.96]	[0.90, 0.92]	[0.90, 0.93]	[0.98, 0.99]
	A_4	[0.88, 1.00]	[0.72, 0.80]	[0.86, 0.88]	[0.91, 0.92]	[0.95, 0.96]
d_2	A_1	[0.56, 0.81]	[0.96, 1.00]	[0.96, 1.00]	[0.99, 1.00]	[0.98, 1.00]
	A_2	[0.69, 0.75]	[0.84, 0.92]	[0.94, 0.95]	[0.98, 1.00]	[0.98, 0.99]
	A_3	[0.63, 0.75]	[0.88, 0.92]	[0.89, 0.90]	[0.95, 0.97]	[0.97, 0.98]
	A_4	[0.94, 1.00]	[0.76, 0.80]	[0.86, 0.89]	[0.96, 0.98]	[0.96, 0.97]
d_3	A_1	[0.65, 0.76]	[0.76, 0.88]	[0.95, 1.00]	[0.99, 1.00]	[0.97, 1.00]
	A_2	[0.71, 0.82]	[0.72, 1.00]	[0.93, 0.97]	[0.96, 0.99]	[0.97, 0.99]
	A_3	[0.71, 0.88]	[0.84, 0.88]	[0.87, 0.90]	[0.93, 0.95]	[0.97, 0.98]
	A_4	[0.76, 1.00]	[0.72, 0.92]	[0.84, 0.86]	[0.94, 0.97]	[0.95, 0.96]
d_4	A_1	[0.72, 0.78]	[0.96, 1.00]	[0.97, 1.00]	[0.99, 1.00]	[0.98, 0.99]
	A_2	[0.72, 0.83]	[0.83, 1.00]	[0.93, 0.94]	[0.98, 0.99]	[0.99, 1.00]
	A_3	[0.89, 1.00]	[0.87, 0.96]	[0.88, 0.90]	[0.93, 0.96]	[0.98, 0.99]
	A_4	[0.83, 0.94]	[0.83, 0.91]	[0.86, 0.87]	[0.92, 0.95]	[0.97, 0.98]

Table 5
Weighted normalized decision matrices.

DMs	Candidates	Cost	Time	Trust	Risk	Quality
d_1	A_1	[0.1375, 0.1650]	[0.1428, 0.1700]	[0.2381, 0.2500]	[0.1454, 0.1500]	[0.2078, 0.2100]
	A_2	[0.1513, 0.2062]	[0.1632, 0.1700]	[0.2353, 0.2381]	[0.1408, 0.1423]	[0.2057, 0.2078]
	A_3	[0.1650, 0.1787]	[0.1496, 0.1632]	[0.2247, 0.2299]	[0.1347, 0.1393]	[0.2057, 0.2078]
	A_4	[0.1925, 0.2200]	[0.1224, 0.1360]	[0.2151, 0.2198]	[0.1362, 0.1378]	[0.1995, 0.2015]
d_2	A_1	[0.1069, 0.1544]	[0.1728, 0.1800]	[0.2120, 0.2200]	[0.1583, 0.1600]	[0.2449, 0.2500]
	A_2	[0.1306, 0.1425]	[0.1512, 0.1656]	[0.2069, 0.2094]	[0.1566, 0.1600]	[0.2449, 0.2474]
	A_3	[0.1187, 0.1425]	[0.1584, 0.1656]	[0.1953, 0.1975]	[0.1515, 0.1549]	[0.2424, 0.2449]
	A_4	[0.1781, 0.1900]	[0.1368, 0.1440]	[0.1889, 0.1953]	[0.1532, 0.1566]	[0.2400, 0.2424]
d_3	A_1	[0.1359, 0.1606]	[0.1444, 0.1672]	[0.2182, 0.2300]	[0.1682, 0.1700]	[0.1938, 0.2000]
	A_2	[0.1482, 0.1729]	[0.1368, 0.1900]	[0.2128, 0.2239]	[0.1630, 0.1682]	[0.1938, 0.1979]
	A_3	[0.1482, 0.1853]	[0.1596, 0.1672]	[0.2002, 0.2076]	[0.1577, 0.1612]	[0.1938, 0.1958]
	A_4	[0.1606, 0.2100]	[0.1368, 0.1748]	[0.1934, 0.1979]	[0.1595, 0.1647]	[0.1898, 0.1918]
d_4	A_1	[0.1733, 0.1867]	[0.1722, 0.1800]	[0.2046, 0.2100]	[0.1781, 0.1800]	[0.1860, 0.1880]
	A_2	[0.1733, 0.2000]	[0.1487, 0.1800]	[0.1946, 0.1970]	[0.1762, 0.1781]	[0.1880, 0.1900]
	A_3	[0.2133, 0.2400]	[0.1565, 0.1722]	[0.1856, 0.1900]	[0.1669, 0.1725]	[0.1860, 0.1880]
	A_4	[0.2000, 0.2267]	[0.1487, 0.1643]	[0.1814, 0.1834]	[0.1650, 0.1706]	[0.1841, 0.1860]

Table 6
Ideal decision (ID) matrices.

IDs	Candidates	Cost	Time	Trust	Risk	Quality
PID	A_1	[0.1544, 0.1733]	[0.1672, 0.1728]	[0.2100, 0.2381]	[0.1500, 0.1781]	[0.1880, 0.2449]
	A_2	[0.1425, 0.1733]	Φ	[0.1970, 0.2353]	[0.1423, 0.1762]	[0.1900, 0.2449]
	A_3	[0.1425, 0.2133]	Φ	[0.1900, 0.2247]	[0.1393, 0.1669]	[0.1880, 0.2424]
	A_4	[0.1900, 0.2000]	[0.1360, 0.1487]	[0.1834, 0.2151]	[0.1378, 0.1650]	[0.1860, 0.2400]
NID	A_1	[0.1069, 0.1867]	[0.1428, 0.1800]	[0.2046, 0.2500]	[0.1454, 0.1800]	[0.1860, 0.2500]
	A_2	[0.1306, 0.2062]	[0.1368, 0.1900]	[0.1946, 0.2381]	[0.1408, 0.1781]	[0.1880, 0.2474]
	A_3	[0.1187, 0.2400]	[0.1496, 0.1722]	[0.1856, 0.2299]	[0.1347, 0.1725]	[0.1860, 0.2449]
	A_4	[0.1606, 0.2267]	[0.1224, 0.1748]	[0.1814, 0.2198]	[0.1362, 0.1706]	[0.1841, 0.2424]

roscopic instead of local and microscopic, global instead of individual in the traditional TOPSIS method; ideal decisions are high dimension expressed as matrixes in suggested methodology in this paper instead of ideal solutions in low dimension expressed as vectors in the traditional TOPSIS method.

5. Numerical example

While fierce competition impels many of the commercial markets into a low-profit environment, virtual enterprise appears as a business strategy for small and medium-sized enterprises to alley together (Hsu & Hsu, 2008). A virtual enterprise is a team composed of several enterprises with different core competitions driven by some transient market opportunity.

Table 7
Separation measures between individual decisions and ideal decisions.

Ideal decisions	R_1	R_2	R_3	R_4
PID	0.1383	0.1599	0.1242	0.1715
NID	0.1703	0.1955	0.1503	0.2066

Table 8
Relative closeness, weights and ranking of DMs.

DMs	Relative closeness	Weights	Ranking
d_1	0.5519	0.2513	1
d_2	0.5501	0.2505	2
d_3	0.5477	0.2494	3
d_4	0.5465	0.2488	4

Table 9
Collective decision matrix.

Candidates	Cost	Time	Trust	Risk	Quality
A_1	[0.1383,0.1666]	[0.1580,0.1743]	[0.2183,0.2275]	[0.1625,0.1650]	[0.2082,0.2120]
A_2	[0.1508,0.1804]	[0.1500,0.1764]	[0.2124,0.2172]	[0.1591,0.1621]	[0.2081,0.2108]
A_3	[0.1613,0.1865]	[0.1560,0.1670]	[0.2015,0.2063]	[0.1527,0.1569]	[0.2070,0.2092]
A_4	[0.1828,0.2117]	[0.1361,0.1547]	[0.1947,0.1991]	[0.1534,0.1574]	[0.2034,0.2055]

Recently, Ministry of Transport of the People's Republic of China has taken a huge project in road construction. A core enterprise, which is one of the Chinese construction companies, catches this market opportunity. However, it does not own all the competencies and resources needed to realize the market opportunity. That is to say, to support the operations of a virtual enterprise, the partners selection is required. In order to illustrate the proposed method introduced above, in the following, we utilize the proposed method to deal with the partners selection problem faced by this core enterprise.

There are five main attributes including cost, time, trust, risk and quality in this process of the partners selection. Cost, time and risk are cost type, while trust and quality are benefit type. There are four partners who have been identified as candidates/alternatives, and four DMs are responsible for the partner selection problem. The objective here is to select a partner, which can satisfy all attributes in the best way. Each DM provides his/her decision matrix and weights of the attributes as shown in Table 3.

By Step 2, we can normalize four individual decisions in Table 3, respectively, as shown in Table 4.

Next, for the attributes' weights given by DMs, we can construct further weighted normalized decision matrix according to Step 3, which are shown in Table 5.

In the following we shall start to determine the weights of DMs according to individual decision. Firstly, the positive and negative ideal decisions of group are determined by Step 4, which are shown in Table 6. Secondly, we utilize Eqs. (16) and (17) to derive the positive and negative separations, respectively, according to Step 5, which are shown in Table 7. Then, the relative closeness of each individual decision and weight vector of DMs are determined by Steps 6 and 7, respectively, these result are summarized in Table 8.

After determining weights of DMs, the next is ranking the alternatives (or candidates). Firstly, we can aggregate the all individual decisions into a collective decision by using the weights of DMs according to Step 8, the collective decision are shown in Table 9. Secondly, by Step 9, summing all interval arguments in each line of Table 9, the overall interval assessments of alternatives $A_i(i=1,2,3,4)$ are obtained:

$$r_1=[0.8853,0.9455],\quad r_2=[0.8805,0.9469],$$
$$r_3=[0.8785,0.9260],\quad r_4=[0.8705,0.9284].$$

To get the order of these overall interval arguments $r_i(i=1,2,3,4)$, by Step 10, we first compare each argument r_i with all arguments $r_j(j=1,2,3,4)$ by using Eq. (2). Then we can construct a complementary matrix by Eq. (3) as follows:

$$P=\begin{pmatrix}0.5000 & 0.5131 & 0.6222 & 0.6349\\ 0.4869 & 0.5000 & 0.6010 & 0.6148\\ 0.3778 & 0.3990 & 0.5000 & 0.5262\\ 0.3651 & 0.3852 & 0.4738 & 0.5000\end{pmatrix}.$$

Summing all elements in each line of matrix P by Eq. (4), we have:

$$p_1=2.2702,\quad p_2=2.2027,\quad p_3=1.8030,\quad p_4=1.7241.$$

Then we can rank the arguments $r_i(i=1,2,3,4)$ in descending order in accordance with the value of $p_i(i=1,2,3,4)$:

$$r_1>r_2>r_3>r_4.$$

In the end, by Step 11, all the candidates/alternatives $A_i(i=1,2,3,4)$ can be ranked in accordance with $r_i(i=1,2,3)$:

$$A_1\succ A_2\succ A_3\succ A_4,$$

where the symbol "$\succ$" means superior to. And thus, the best candidate is A_1.

6. Conclusion

In this paper we have presented a new TOPSIS method with interval data for MAGDM problems. We have extended the positive and negative ideal solutions expressed as a vector, respectively, in original TOPSIS method to positive and negative ideal decisions expressed as a matrix, respectively, in extended TOPSIS method in this paper. So, this article has established an approach for determining the weights of DMs using the extended TOPSIS in group settings. The proposed method is clear in concept, simple in computation and able to be performed on computer easily. The developed methodology should be extended to support situations where the information is in other forms, e.g., fuzzy numbers, intuitionistic fuzzy numbers, interval-valued intuitionistic fuzzy numbers and linguistic variables.

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