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An extended TOPSIS for determining weights of decision makers with interval numbers

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ABSTRACT

In this paper, we develop a method for determining weights of decision makers under group decision environment, in which the each individual decision information is expressed by a matrix in interval numbers. We define the positive and negative ideal solutions of group decision, which are expressed by a matrix, respectively. The positive ideal solution is expressed by the average matrix of group decision and the negative ideal solution is maximum separation from positive ideal solution. The separation measures of each individual decision from the ideal solution and the relative closeness to the ideal solution are defined based on Euclidean distance. According to the relative closeness, we determine the weights of decision makers in accordance with the values of the relative closeness. Finally, we give an example for integrated assessment of air quality in Guangzhou during 16th Asian Olympic Games to illustrate in detail the calculation process of the developed approach.

1. Introduction

Multiple attribute decision making (MADM) occurs in a variety of actual situations, such as economic analysis, strategic planning, forecasting, medical diagnosis, venture capital and supply chain management. The increasing complexity of the socioeconomic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem [1]. As a result, many decision making processes, in the real world, take place in group settings. Moving from single DM's setting to group members' setting would lead to a great deal of complexity of the analysis. For example, consider that these DMs usually come from different specialty fields, and thus each DM has his/her unique characteristics with regard to knowledge, skills, experience and personality, which implies that each DM usually has different influence in overall decision result, i.e., the weights of DMs are different. Therefore, how to determine the weights of DMs will be an interesting and important research topic.

At present, many methods have been proposed to determine the weights of DMs. French [2] proposed a method to determine the relative importance of the group's members by using the influence relations, which may exist between the members. Theil [3] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [4] and Keeney [5] suggested the use of the interpersonal comparison to determine the scales constant values in an additive and weighted social choice function. Bodily [6] and Mirkin [7] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Brock [8] used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh [9] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Martel and Ben Khélifa [10] proposed a method to determine the relative importance of group's members by using individual outranking indexes. Van den Honert [11] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [12] proposed a procedure which exploits the idea of Zeleny [13] to determine the relative importance coefficient of each member. By using the deviation measures between additive linguistic preference relations, Xu [14] gave some straightforward formulas to determine the weights of DMs.

Many of literatures mentioned above described the individual decision information by a multiplicative preference matrix. Until now there has been little investigation of the weights of DMs based on individual decision information, in which the attribute values are given as observations in nonnegative real numbers, and the DMs have their subjective preferences on alternatives.

By considering the fact that, in some cases, determining precisely the exact values of the attributes is difficult and that, as a result of this, their values are considered as intervals. Therefore, in this article, we shall discuss the weights of DMs based on

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technique for order performance by similarity to ideal solution (TOPSIS) [15] with interval numbers.

The rest of the paper is organized as follows: Section 2 reviews multiple attribute group decision making (MAGDM) and the TOPSIS technique, the basic idea and main contributions of the developed method in this paper are presented. The preliminaries, including comparing and ranking interval numbers, are given in Section 3. The developed approach and its algorithm to determine the weights of DMs are presented in Section 4. Section 5 makes two comparisons between the proposed method in this paper and the literature of other methods. In Section 6, we illustrate our proposed algorithmic method with an example. Conclusions appear in Section 7.

2. Literature survey

In this part we review the MAGDM, which has become an important part of modern decision science [16–19]. The decision information provided by the DMs may take the various representation formats in group decision making problems, such as exact numerical values [20–22], interval numbers [23–26], fuzzy numbers [27–30], fuzzy linguistic [31,32], rough set theory [33] and evidence theory [34]. In this paper, we will focus on the proposed group TOPSIS model in order to determine the weights of DMs.

TOPSIS, one of known classical MADM method, was first developed by Hwang and Yoon [15] for solving a MADM problem. TOPSIS technique is a hot research topic, which has received a great deal of attention from researchers [35–39]. The basic idea of TOPSIS is rather straightforward. It originates from the concept of a displaced ideal point from which the compromise solution has the shortest distance. Hwang and Yoon [15] further propose that the ranking of alternatives will be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) or nadir. TOPSIS simultaneously considers the distances to both the PIS and the NIS, and a preference order is ranked according to their relative closeness, and a combination of these two distance measures. The PIS/NIS, as a benchmark of TOPSIS method, is expressed by a vector. The traditional TOPSIS is limited to compare vectors of alternatives (with respect to attributes) with the vector of PIS/NIS. However, this comparison can not reflect DM's overall decisional level, which is expressed by a decision matrix (see Eq. (5)). Suppose that $X_1, X_2, \ldots, X_t$ are the decision matrixes of k ($k = 1, 2, \ldots, t$) DMs. This article intend to extend the PIS/NIS to a matrix X, which is a benchmark of $X_1, X_2, \ldots, X_t$. The decisional level of kth DM is measured by the Euclid distance between X_k and X, and the weight of kth DM is determined by his/her decisional level.

Jahanshahloo et al. [40] have extended the concept of TOPSIS to develop a methodology for solving MADM problems with interval data. Ye and Li [41] extended the TOPSIS technique for solving MAGDM problems with interval data, in which the DMs' weights are same. Sayadi et al. [42] developed an extension of VIKOR method [43,44] for MAGDM problem with interval numbers, in which the DMs' weights are also same. To overcome this limitation of same weights of DMs, we report a further extension of TOPSIS method in MAGDM environment with interval numbers, in which the DMs' weights are different. This paper focuses on determining the weights of DMs in MAGDM environment with interval data. The paper has the following main contributions:

1. The extended TOPSIS technique is also called group TOPSIS with interval data in this article. For the given individual decision matrixes, the PIS of group opinion is depicted by a matrix, in which every element is expressed in average of each individual decision interval; similarly, the NIS of group opinion is also depicted by a matrix, in which the decision information is expressed in maximum separation from the corresponding interval of positive case. The ranking of DMs (based on their decision matrixes) will be based on the shorter distance from the PIS and the farther from the NIS. That is, a DM's decision matrix is closer to the PIS and farther from the NIS, and then the DM is more weight.
2. In this paper, each DM has a decision matrix. The weight of DM is determined by both distances, which one is between the DM's decision matrix and the PIS, another is between the DM's decision matrix and the NIS. The second contribution of this paper is that this paper extends alternatives'/vectors' ranking based on ideal solutions to DMs'/matrixes' ranking based on ideal solutions. Here, the former ideal solutions are vectors while the latter ideal solutions are matrixes. TOPSIS technique focuses on the set of alternatives including PIS and NIS, while the extended TOPSIS technique focuses on the set of decision matrixes including PIS and NIS. Furthermore, the extended TOPSIS is clear in algorithm and without loss of information in aggregation, which no investigation has been devoted to.
3. The proposed method is suitable for determining the weights of attributes of group decision making when the exchange takes place between corresponding positions of DMs and attributes in each individual decision matrix. In this sense, the third contribution of this paper is that the proposed method not only can determine the weights of DMs, but also can determine the weights of attributes.

The TOPSIS method introduces two "reference" points: PIS and NIS in order to ranking of alternatives. The extended TOPSIS method in this paper, a key issue is determination of two "reference" points (or a benchmark) of all individual decision matrixes for comparison of the decisional levels among DMs. The reasons why the PIS is defined as the average matrix of group decision are that: (1) the PIS is the maximum compromise (in mean sense) among all individual decisions of group; (2) the "average" is adopted as the final decision (outcome) of group in most of the situations where a group decision must be taken. For example, for a teaching competition participated by young teachers in a university, if there is t DMs, the final score of each competitor is the average of t scores given by the DMs; and (3) the NIS is the maximum individual regret (the farthest distance from PIS).

TOPSIS method is suitable for cautious (risk avoider) DM(s), because the DM(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible [42]. The developed approach in this paper assigns high weights to those DMs if the DMs want to have maximum group utility (majority/group), and minimum individual risk (minority/individual) in mean sense.

In order to realize the idea above, in the following, we will establish an extended TOPSIS model with interval data in a group decision environment.

3. Preliminaries

In the following, we first review the notion of the nonnegative interval number and some operational laws.

Definition 1 [45]. Let $a = [a^l, a^u] = \{x | 0 < a^l \leqslant x \leqslant a^u\}$, then a is called a nonnegative interval number. Especially, a is a nonnegative real number, if $a^l = a^u$.

Note: For convenience of computation, throughout this paper, all the interval arguments are nonnegative interval numbers.

Definition 2 [45,46]. Let $a = [a^l, a^u]$, $b = [b^l, b^u]$ are interval numbers and $\lambda \geqslant 0$, then

(1) $a = b$ if and only if $a^l = b^l$ and $a^u = b^u$;
(2) $a + b = [a^l, a^u] + [b^l, b^u] = [a^l + b^l, a^u + b^u]$;
(3) $\lambda a = \lambda[a^l, a^u] = [\lambda a^l, \lambda a^u]$. Especially, $\lambda a = 0$ if $\lambda = 0$.

For convenience, we let Ω be the set of all interval arguments.

In order to aggregate interval numbers, we introduce the following weighted averaging operator [15,47]:

Definition 3. Let $a_j = [a_j^l, a_j^u]$ $(j = 1, 2, \dots, n)$ are interval numbers, a weighted averaging operator of $\{a_j | j = 1, 2, \dots, n\}$ is a mapping WA: $\Omega^n \to \Omega$ such that

$$WA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j a_j, \tag{1}$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $\{a_j | j = 1, 2, \dots, n\}$, $w_j \geqslant 0$ $(j = 1, 2, \dots, n)$ and $\sum_{j=1}^{n} w_j = 1$.

We introduce the following formula in order to rank interval numbers.

Definition 4 [45]. Let $a = [a^l, a^u] \in \Omega$, $b = [b^l, b^u] \in \Omega$, $l_a = a^u - a^l$ and $l_b = b^u - b^l$, then the degree of possibility of $a \geqslant b$ is defined as

$$p(a \geqslant b) = \max\left\{1 - \max\left(\frac{b^u - a^l}{l_a + l_b}, 0\right), 0\right\}. \tag{2}$$

Moreover, we can get easily the following results [45,46] from Eq. (2).

Let $a = [a^l, a^u] \in \Omega$, $b = [b^l, b^u] \in \Omega$, then

(1) $0 \leqslant p(a \geqslant b) \leqslant 1$;
(2) $p(a \geqslant b) = 1$ if and only if $b^u \leqslant a^l$;
(3) $p(a \geqslant b) = 0$ if and only if $a^u \leqslant b^l$;
(4) $p(a \geqslant a) = \frac{1}{2}$;
(5) $p(a \geqslant b) + p(b \geqslant a) = 1$.

To rank the interval arguments $a_j = \left[a_j^l, a_j^u\right]$ $(j = 1, 2, \dots, n)$, we first compare each $a_i = \left[a_i^l, a_i^u\right]$ with all $a_j = \left[a_j^l, a_j^u\right]$ $(j = 1, 2, \dots, n)$ by using Eq. (2). For convenience, we let $p_{ij} = p\ (a_i \geqslant a_j)$, and then construct a complementary matrix as follows:

$$P = (p_{ij})_{n \times n}, \tag{3}$$

where $p_{ij} \geqslant 0,\ p_{ij} + p_{ji} = 1,\ p_{ii} = \frac{1}{2},\ i, j = 1, 2, \dots, n$.

Summing all elements in each line of matrix P, we have

$$p_i = \sum_{j=1}^{n} p_{ij}, \quad i = 1, 2, \dots, n. \tag{4}$$

Then we can reorder the interval arguments $a_j = \left[a_j^l, a_j^u\right]$ $(j = 1, 2, \dots, n)$ in descending order in accordance with the values of p_j $(j = 1, 2, \dots, n)$ [45].

4. The developed approach

To aid in the elucidation of the proposed technique, in what follows, we first review the group decision making with interval number.

4.1. Multiple attribute group decision making with interval number

For convenience, let $M = \{1, 2, \dots, m\}$, $N = \{1, 2, \dots, n\}$ and $T = \{1, 2, \dots, t\}$. A MAGDM problem can be described in detail as follows.

Let $A = \{A_1, A_2, \dots, A_m\}$ $(m \geqslant 2)$ be a discrete set of m feasible alternatives, $U = \{u_1, u_2, \dots, u_n\}$ be a finite set of attributes, $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector of attributes, satisfies $0 \leqslant w_j \leqslant 1$ and $\sum_{j=1}^{n} w_j = 1$. And let $D = \{d_1, d_2, \dots, d_t\}$ be a group of DMs, and $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)^T$ be the weight vector of DMs, where $\lambda_k \geqslant 0,\ \sum_{k=1}^{n} \lambda_k = 1$.

In the process of MADM, the DMs are usually asked to provide their preference information on attributes, and the attribute values are not precisely known but value ranges can be obtained. Therefore, it is significative that we consider MAGDM problem with interval number.

First we let

$$X_k = \left(\left[x_{ij}^{k(l)}, x_{ij}^{k(u)}\right]\right)_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[x_{11}^{k(l)}, x_{11}^{k(u)}\right] & \left[x_{12}^{k(l)}, x_{12}^{k(u)}\right] & \cdots & \left[x_{1n}^{k(l)}, x_{1n}^{k(u)}\right] \\ \left[x_{21}^{k(l)}, x_{21}^{k(u)}\right] & \left[x_{22}^{k(l)}, x_{22}^{k(u)}\right] & \cdots & \left[x_{2n}^{k(l)}, x_{2n}^{k(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[x_{m1}^{k(l)}, x_{m1}^{k(u)}\right] & \left[x_{m2}^{k(l)}, x_{m2}^{k(u)}\right] & \cdots & \left[x_{mn}^{k(l)}, x_{mn}^{k(u)}\right] \end{pmatrix} \end{array}, \tag{5}$$

for all $k \in T$,

be decision matrix of the kth $(k \in T)$ DM, in which each of the elements is characterized by interval number. In general, there are benefit attributes and cost attributes in the MADM problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following Eqs. (7) and (8) [46] to normalize each attribute value $\left[x_{ij}^{k(l)}, x_{ij}^{k(u)}\right]$ in decision matrix $X_k = \left(\left[x_{ij}^{k(l)}, x_{ij}^{k(u)}\right]\right)_{m \times n}$ into a corresponding element $\left[\left(y_{ij}^{k(l)}, y_{ij}^{k(u)}\right]\right.$ in normalized decision matrix $Y_k = (\left[\left(y_{ij}^{k(l)}, y_{ij}^{k(u)}\right]\right)_{m \times n}$ given by Eq. (6).

$$Y_k = \left(\left[y_{ij}^{k(l)}, y_{ij}^{k(u)}\right]\right)_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[y_{11}^{k(l)}, y_{11}^{k(u)}\right] & \left[y_{12}^{k(l)}, y_{12}^{k(u)}\right] & \cdots & \left[y_{1n}^{k(l)}, y_{1n}^{k(u)}\right] \\ \left[y_{21}^{k(l)}, y_{21}^{k(u)}\right] & \left[y_{22}^{k(l)}, y_{22}^{k(u)}\right] & \cdots & \left[y_{2n}^{k(l)}, y_{2n}^{k(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[y_{m1}^{k(l)}, y_{m1}^{k(u)}\right] & \left[y_{m2}^{k(l)}, y_{m2}^{k(u)}\right] & \cdots & \left[y_{mn}^{k(l)}, y_{mn}^{k(u)}\right] \end{pmatrix} \end{array}, \tag{6}$$

for all $k \in T$,

where

$$\begin{cases} y_{ij}^{k(l)} = \dfrac{x_{ij}^{k(l)}}{\sum_{i=1}^{m} x_{ij}^{k(u)}} \\ y_{ij}^{k(u)} = \dfrac{x_{ij}^{k(u)}}{\sum_{i=1}^{m} x_{ij}^{k(l)}} \end{cases}, \quad \text{for benefit attribute } u_j,\ i \in M,\ j \in N,\ k \in T, \tag{7}$$

and

$$\begin{cases} y_{ij}^{k(l)} = \dfrac{1/x_{ij}^{k(u)}}{\sum_{i=1}^{m} 1/x_{ij}^{k(l)}} \\ y_{ij}^{k(u)} = \dfrac{1/x_{ij}^{k(l)}}{\sum_{i=1}^{m} 1/x_{ij}^{k(u)}} \end{cases}, \quad \text{for cost attribute } u_j,\ i \in M,\ j \in N,\ k \in T. \tag{8}$$

We take notice of the fact that the property that the ranges belong to $[0, 1]$ is not preserved for the normalized interval numbers.

For example, given three interval numbers on a cost attribute:

$$x_1 = [0.044, 0.291], \quad x_2 = [0.001, 0.038], \quad x_3 = [0.008, 0.039].$$

The normalized interval numbers are calculated by Eq. (8) as follows:

$y_1 = [0.0030, 0.4103], \quad y_2 = [0.0229, 18.0527],$

$y_3 = [0.0223, 2.2566],$

where y_2 and y_3 do not belong to $[0,1]$.

In order to preserve the property that the ranges of normalized interval numbers are contained in $[0,1]$, we can further transform the decision matrix $\left(\left[y_{ij}^{k(l)}, y_{ij}^{k(u)}\right]\right)_{m\times n}$ into the normalized decision matrix $\left(\left[r_{ij}^{k(l)}, r_{ij}^{k(u)}\right]\right)_{m\times n}$ using the following formulas [40]:

$$\begin{cases} r_{ij}^{k(l)} = \dfrac{y_{ij}^{k(l)}}{\sqrt{\sum_{i=1}^{m}\left(\left(y_{ij}^{k(l)}\right)^2 + \left(y_{ij}^{k(u)}\right)^2\right)}} \\ r_{ij}^{k(u)} = \dfrac{y_{ij}^{k(u)}}{\sqrt{\sum_{i=1}^{m}\left(\left(y_{ij}^{k(l)}\right)^2 + \left(y_{ij}^{k(u)}\right)^2\right)}} \end{cases}, \quad \text{for all } i \in M,\ j \in N,\ k \in T. \tag{9}$$

Therefore, we get the normalized decision matrix of kth DM as follows:

$$R_k = \left(\left[r_{ij}^{k(l)}, r_{ij}^{k(u)}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[r_{11}^{k(l)}, r_{11}^{k(u)}\right] & \left[r_{12}^{k(l)}, y_{12}^{k(u)}\right] & \cdots & \left[r_{1n}^{k(l)}, r_{1n}^{k(u)}\right] \\ \left[r_{21}^{k(l)}, r_{21}^{k(u)}\right] & \left[r_{22}^{k(l)}, r_{22}^{k(u)}\right] & \cdots & \left[r_{2n}^{k(l)}, r_{2n}^{k(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[r_{m1}^{k(l)}, r_{m1}^{k(u)}\right] & \left[r_{m2}^{k(l)}, r_{m2}^{k(u)}\right] & \cdots & \left[r_{mn}^{k(l)}, r_{mn}^{k(u)}\right] \end{pmatrix} \end{array}, \quad \text{for all } k \in T, \tag{10}$$

Based on described above, in what follows, we will discuss the TOPSIS with interval number, which is our main work.

4.2. TOPSIS method with interval number

For the normalized decision matrix R_k of kth DM, considering the different importance of each attribute, we can construct the weighted normalized interval decision matrix as

$$V_k = \left(\left[v_{ij}^{k(l)}, v_{ij}^{k(u)}\right]\right)_{m\times n} = \left(\left[w_j r_{ij}^{k(l)}, w_j r_{ij}^{k(u)}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[v_{11}^{k(l)}, v_{11}^{k(u)}\right] & \left[v_{12}^{k(l)}, v_{12}^{k(u)}\right] & \cdots & \left[v_{1n}^{k(l)}, v_{1n}^{k(u)}\right] \\ \left[v_{21}^{k(l)}, v_{21}^{k(u)}\right] & \left[v_{22}^{k(l)}, v_{22}^{k(u)}\right] & \cdots & \left[v_{2n}^{k(l)}, v_{2n}^{k(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[v_{m1}^{k(l)}, v_{m1}^{k(u)}\right] & \left[v_{m2}^{k(l)}, v_{m2}^{k(u)}\right] & \cdots & \left[v_{mn}^{k(l)}, v_{mn}^{k(u)}\right] \end{pmatrix} \end{array}, \quad \text{for all } k \in T, \tag{11}$$

where w_j is the weight of the jth attribute, such that $0 \leqslant w_j \leqslant 1$ and $\sum_{j=1}^{n} w_j = 1$.

As described in the Literature reviews section, in mean sense, the best decision result of group should be the average of group decision matrix:

$$A^+ = \left(\left[v_{ij}^{+(l)}, v_{ij}^{+(u)}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[v_{11}^{+(l)}, v_{11}^{+(u)}\right] & \left[v_{12}^{+(l)}, v_{12}^{k(u)}\right] & \cdots & \left[v_{1n}^{+(l)}, v_{1n}^{+(u)}\right] \\ \left[v_{21}^{+(l)}, v_{21}^{+(u)}\right] & \left[v_{22}^{+(l)}, v_{22}^{k(u)}\right] & \cdots & \left[v_{2n}^{+(l)}, v_{2n}^{+(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[v_{m1}^{+(l)}, v_{m1}^{+(u)}\right] & \left[v_{m2}^{+(l)}, v_{m2}^{+(u)}\right] & \cdots & \left[v_{mn}^{+(l)}, v_{mn}^{+(u)}\right] \end{pmatrix} \end{array}, \tag{12}$$

where $v_{ij}^{+(l)} = \frac{1}{t}\sum_{k=1}^{t} v_{ij}^{k(l)}$, $v_{ij}^{+(u)} = \frac{1}{t}\sum_{k=1}^{t} v_{ij}^{k(u)}$. So, we define $A^+ = \left(\left[v_{ij}^{+(l)}, v_{ij}^{+(u)}\right]\right)_{m\times n}$ as the PIS of all individual decisions.

And the worst result of group decision making should be the result of maximum separation from the PIS

$$A^- = \left(\left[v_{ij}^{-(l)}, v_{ij}^{-(u)}\right]\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \left[v_{11}^{-(l)}, v_{11}^{-(u)}\right] & \left[v_{12}^{-(l)}, v_{12}^{-(u)}\right] & \cdots & \left[v_{1n}^{-(l)}, v_{1n}^{-(u)}\right] \\ \left[v_{21}^{-(l)}, v_{21}^{-(u)}\right] & \left[v_{22}^{-(l)}, v_{22}^{-(u)}\right] & \cdots & \left[v_{2n}^{-(l)}, v_{2n}^{-(u)}\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[v_{m1}^{-(l)}, v_{m1}^{-(u)}\right] & \left[v_{m2}^{-(l)}, v_{m2}^{-(u)}\right] & \cdots & \left[v_{mn}^{-(l)}, v_{mn}^{-(u)}\right] \end{pmatrix} \end{array}, \tag{13}$$

where $v_{ij}^{-(l)} = \min_{1\leqslant k\leqslant t}\left\{v_{ij}^{k(l)}\right\}$, $v_{ij}^{-(u)} = \max_{1\leqslant k\leqslant t}\left\{v_{ij}^{k(u)}\right\}$.

So, we define $A^- = \left(\left[v_{ij}^{-(l)}, v_{ij}^{-(u)}\right]\right)_{m\times n}$ as the NIS of all individual decisions.

The separation of each individual decision from the PIS, using the n-dimensional Euclidean distance, can be currently calculated as

$$S_k^+ = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left(v_{ij}^{k(l)} - v_{ij}^{+(l)}\right)^2 + \left(v_{ij}^{k(u)} - v_{ij}^{+(u)}\right)^2\right)}, \quad k \in T. \tag{14}$$

Similarly, the separation from the NIS is given as

$$S_k^- = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left((v_{ij}^{k(l)} - v_{ij}^{-(l)})^2 + \left(v_{ij}^{k(u)} - v_{ij}^{-(u)}\right)^2\right)}, \quad k \in T. \tag{15}$$

A relative closeness is defined to determine the ranking order of all DMs once the S_k^+ and S_k^- of each individual decision has been calculated. The relative closeness of each individual decision with respect to A^+ is defined as

$$RC_k = \frac{S_k^-}{S_k^+ + S_k^-}, \quad k \in T. \tag{16}$$

Since $S_k^- \geqslant 0$ and $S_k^+ \geqslant 0$, then, clearly, $RC_k \in [0,1]$ for all $k \in T$.

Obviously, a decision matrix V_k is closer to the A^+ and farther from A^- as RC_k approaches to 1. Therefore, according to the relative closeness, we can determine the ranking order of all DMs and select the best one from among a set of DMs. For example, for a teaching competition participated by young teachers in a university, if there is t DMs, then the score given by kth DM is closer to the average of t scores given by the DMs, the better decision of kth DM.

So, we can define

$$\lambda_k = \frac{RC_k}{\sum_{k=1}^{t} RC_k}, \quad k \in T, \tag{17}$$

as weight of kth ($k \in T$) DM, such that $\lambda_k \geqslant 0$, $\sum_{k=1}^{t}\lambda_k = 1$.

Further, for the DMs' weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ given in Eq. (17), we can aggregate all the group decision matrixes V_k ($k \in T$) into a collective matrix V by

$$V = \sum_{k=1}^{t} \lambda_k V_k = ([v_{ij}^{(l)}, v_{ij}^{(u)}])_{m\times n}. \tag{18}$$

Table 1
Comparison with the traditional TOPSIS.

Characteristics	TOPSIS	Extended TOPSIS
Evaluation objective	Selection and ranking of a number of alternatives	Selection and ranking of a number of DMs
No. of DMs	One	More than one
Weights on attributes	Given	Given
Cardinal information	Alternatives with respect to attributes	Alternatives with respect to attributes of multiple DMs
PIS/NIS	The best/worst alternative expressed by a vector	The best/worst decision expressed by a matrix
Core process	The distances from each alternative to PIS and NIS (between two vectors)	The distances from each decision to PIS and NIS (between two matrixes)

Table 2
Comparison with the method proposed by Ye and Li.

Characteristics	Proposed by Ye and Li	Proposed by this paper
Decision information	Decision matrixes $X_1,X_2,\ldots,X_t$ of alternatives with respect to attributes	Decision matrixes $X_1,X_2,\ldots,X_t$ of alternatives with respect to attributes
No. of DMs	$t \geqslant 1$	$t \geqslant 1$
Weights on attributes	Given	Given
PIS/NIS	The best/worst alternative expressed by a vector	The best/worst decision expressed by a matrix
Core process	The distances from each alternative to PIS and NIS (between two vectors)	The distances from each decision matrix to PIS and NIS (between two matrixes)
Weights on DMs	Same weight	Different weight
Key decision	Ranking of a number of alternatives	Ranking of a number of DMs
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives

Then, we can sun all interval numbers in each line of the collective matrix V, the overall interval assessment of each alternative A_i ($i \in M$) is obtained:

$$v_i = [v_i^{(l)}, v_i^{(u)}] = \sum_{j=1}^{n}[v_{ij}^{(l)}, v_{ij}^{(u)}], \quad i \in M. \tag{19}$$

Now, we can construct the complementary matrix $P = (p(v_i \geqslant v_j))_{m\times m} = (p_{ij})_{m\times m}$ by Eq. (3). Then, summing all elements in each line of matrix P by Eq. (4), we can rank all v_i ($i \in M$) in descending order in accordance with the values of p_i ($i \in M$). Finally, we can rank alternatives A_i ($i \in M$) according to p_i ($i \in M$) in descending order.

4.3. The presented algorithm

In sum, an algorithm to determine the weights of DMs, when data is interval, with extended TOPSIS approach is given in the following:

Step 1. Utilize Eqs. (7)–(9) to normalize $X_k = \left(\left[x_{ij}^{k(l)}, x_{ij}^{k(u)}\right]\right)_{m\times n}$ into $R_k = \left(\left[r_{ij}^{k(l)}, r_{ij}^{k(u)}\right]\right)_{m\times n}$, where $X_k = \left(\left[x_{ij}^{k(l)}, x_{ij}^{k(u)}\right]\right)_{m\times n}$ is decision matrix of kth DM ($k \in T$).
Step 2. Utilize Eq. (11) to calculate weighted normalized interval decision matrix $V_k = \left(\left[v_{ij}^{k(l)}, v_{ij}^{k(u)}\right]\right)_{m\times n} (k \in T)$.
Step 3. Utilize Eqs. (12) and (13) to determine PIS and NIS, respectively.
Step 4. Utilize Eqs. (14) and (15) to calculate the separation measures of each individual decision from PIS and NIS, respectively.
Step 5. Utilize Eq. (16) to calculate the relative closeness of each individual decision to PIS, then rank the order of all DMs in descending order in accordance with the values of the relative closeness.
Step 6. Utilize Eq. (17) to determine the weights of DMs.

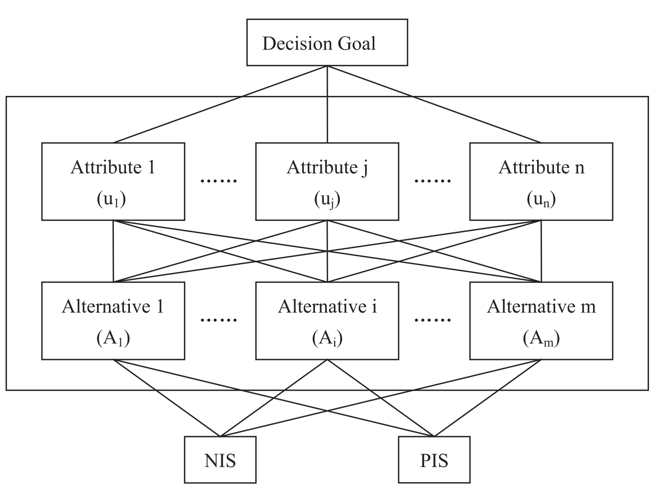

Fig. 1. Hierarchical structure for TOPSIS.

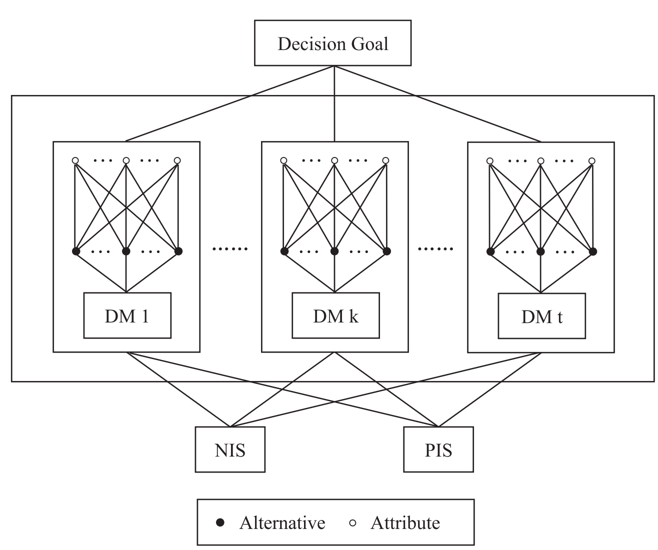

Fig. 2. Hierarchical structure for the extended TOPSIS.

5. Comparing the proposed approach with other methods

As a future step to this paper could be the comparisons of the proposed approach to other methods. Here the proposed method in this study is compared with two different MADM methods, the traditional TOPSIS and the extended TOPSIS proposed by Ye and Li [41], which is a similar to approach of this research in the literature. These methods are selected as the background of proposed method. The comparisons are shown in Tables 1 and 2.

Table 1 illustrates the differences and similarities between the proposed method and the traditional TOPSIS. The figures of hierarchical structure for TOPSIS and extended TOPSIS are shown in Figs. 1 and 2, respectively. Table 2 shows the differences and similarities between the method proposed in this paper and the approach proposed by Ye and Li. Compared with the method of traditional TOPSIS and the approach proposed by Ye and Li, this method has several differences. First, the PIS and NIS in traditional TOPSIS are vectors, which are derived from alternatives, while in the proposed method the PIS and NIS are matrixes, which are derived from decision matrixes of all DMs. This description of procedure of the proposed method is clear and simple for high dimensional TOPSIS in framework. Second, the relative importance of DMs are different and the weight of each DM is determined by his/her own decision matrix. The decision matrix is closer to the PIS and farther away from the NIS, the decision is better, furthermore, the more the weight. The best decision is done by a (some) pseudo-DM(s), whose decision is PIS (the average matrix of all group decision matrixes). From this point of view, a DM's decision matrix is closer to the PIS, that is to say, a decision matrix is closer to the average matrix of group decision matrixes, then it is better to represent majority in mean sense; a DM's decision matrix is closer to the NIS, the decision is larger bias in mean sense, meanwhile, the DM is with maximum regret, and the proposed method assigns low weights to those "false" or "biased" ones. Therefore, it is suitable for those situations in which the DM wants to have maximum group utility and minimum individual risk in mean sense.

6. Illustrative example

The Pearl River Delta Regional Air Quality Monitoring Network (the Network) was jointly established by the Guangdong Provincial Environmental Monitoring Center (GDEMC) and the Environmental Protection Department of the Hong Kong Special Administrative Region (HKEPD) from 2003 to 2005. It came into operation on November 30, 2005 and has been providing data for reporting of Regional Air Quality Index to the public since then.

The Network comprises 16 automatic air-quality monitoring stations across the Pearl River Delta region. All stations are installed with equipment to measure the ambient concentrations of respirable suspended particulate (PM_{10} or RSP), sulphur dioxide (SO_2) and nitrogen dioxide (NO_2).

The 16th Asian Olympic Games will be held in Guangzhou, during November 12–27, 2010. In what follows, we will present a comprehensive evaluation of the air quality in Guangzhou for the Novembers of 2006, 2007, and 2008 for the 16th Asian Olympic Games. The air-quality monitoring stations can be considered as DMs. For convenience, we select three air-quality monitoring stations located in Guangzhou from the 16 air-quality monitoring stations across the Pearl River Delta region, i.e., $D = \{d_1, d_2, d_3\} = \{\text{Luhu Park}, \text{Wanqingsha}, \text{Tianhu}\}$. The measured values [48–50] are shown in Tables 3–5. The monthly air quality for the Novembers of 2006, 2007 and 2008, respectively, can be considered as alternative. For convenience, let $A = \{A_1, A_2, A_3\} = \{\text{November of 2006}, \text{November of 2007}, \text{November of 2008}\}$ be the set of alternatives, $U = \{u_1, u_2, u_3\} = \{SO_2, NO_2, PM_{10}\}$ be the set of attributes.

Since all attributes are cost, we first normalize Tables 3–5 into the corresponding Tables 6–8 by Eqs. (8) and (9) according to Step 1.

The weights of SO_2, NO_2 and PM_{10} are determined according to Implementing Details for Urban Environmental Comprehensive Treatment and Quantitative Examination during the 11th Five-Year Plan (General Office of State Environmental Protection

Table 3
Air quality data derived from Luhu Park monitoring station (X_1).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.013, 0.129]	[0.028, 0.144]	[0.021, 0.136]
A_2	[0.013, 0.107]	[0.038, 0.139]	[0.047, 0.155]
A_3	[0.003, 0.042]	[0.018, 0.054]	[0.014, 0.150]

Table 4
Air quality data derived from Wanqingsha monitoring station (X_2).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.040, 0.161]	[0.034, 0.093]	[0.047, 0.199]
A_2	[0.047, 0.127]	[0.040, 0.081]	[0.102, 0.206]
A_3	[0.014, 0.113]	[0.016, 0.086]	[0.030, 0.187]

Table 5
Air quality data derived from Tianhu monitoring station (X_3).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.006, 0.118]	[0.004, 0.053]	[0.003, 0.174]
A_2	[0.015, 0.046]	[0.001, 0.026]	[0.021, 0.157]
A_3	[0.009, 0.034]	[0.005, 0.019]	[0.011, 0.103]

Table 6
Normalized air quality data derived from Luhu Park monitoring station (R_1).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.0019, 0.2194]	[0.0270, 0.5007]	[0.0121, 0.5383]
A_2	[0.0022, 0.2194]	[0.0280, 0.3689]	[0.0106, 0.2405]
A_3	[0.0057, 0.9506]	[0.0721, 0.7788]	[0.0110, 0.8075]

Table 7
Normalized air quality data derived from Wanqingsha monitoring station (R_2).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.0154, 0.3178]	[0.0433, 0.3991]	[0.0291, 0.5215]
A_2	[0.0195, 0.2705]	[0.0498, 0.3392]	[0.0281, 0.2403]
A_3	[0.0219, 0.9081]	[0.0469, 0.8480]	[0.0310, 0.8171]

Table 8
Normalized air quality data derived from Tianhu monitoring station (R_3).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.0069, 0.7891]	[0.0014, 0.2381]	[0.0008, 0.9557]
A_2	[0.0178, 0.3156]	[0.0028, 0.9524]	[0.0008, 0.1365]
A_3	[0.0241, 0.5261]	[0.0038, 0.1905]	[0.0013, 0.2607]

Table 9
Weighted normalized air quality data derived from Luhu Park monitoring station (V_1).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00074, 0.08775]	[0.00541, 0.10013]	[0.00485, 0.21532]
A_2	[0.00090, 0.08775]	[0.00560, 0.07378]	[0.00426, 0.09621]
A_3	[0.00228, 0.38025]	[0.01442, 0.15576]	[0.00440, 0.32298]

Table 10
Weighted normalized air quality data derived from Wanqingsha monitoring station (V_2).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00615, 0.12714]	[0.00867, 0.07981]	[0.01165, 0.20862]
A_2	[0.00780, 0.10820]	[0.00995, 0.06784]	[0.01125, 0.09613]
A_3	[0.00877, 0.36325]	[0.00937, 0.16960]	[0.01240, 0.32684]

Table 11
Weighted normalized air quality data derived from Tianhu monitoring station (V_3).

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00278, 0.31564]	[0.00027, 0.04762]	[0.00030, 0.38229]
A_2	[0.00713, 0.12626]	[0.00056, 0.19047]	[0.00034, 0.05461]
A_3	[0.00964, 0.21043]	[0.00076, 0.03809]	[0.00052, 0.10426]

Table 12
Positive ideal solution A^+.

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00323, 0.17684]	[0.00478, 0.07585]	[0.00560, 0.26874]
A_2	[0.00527, 0.10740]	[0.00537, 0.11070]	[0.00528, 0.08232]
A_3	[0.00690, 0.31798]	[0.00818, 0.12115]	[0.00577, 0.25136]

Table 13
Negative ideal solution A^-.

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00074, 0.31564]	[0.00027, 0.10013]	[0.00030, 0.38229]
A_2	[0.00090, 0.12626]	[0.00056, 0.19047]	[0.00034, 0.09621]
A_3	[0.00228, 0.38025]	[0.00076, 0.16960]	[0.00052, 0.32684]

Administration (GOSEPA)[2006], No. 36) [51], which is provided to the public by State Environmental Protection Administration. The weights of SO_2, NO_2 and PM_{10} are 0.4, 0.2 and 0.4, respectively.

For the weight vector $w = (w_1, w_2, w_3)^T = (0.4, 0.2, 0.4)^T$ of attributes, the next step is to calculate the weighted normalized interval decision matrix $V_k = \left(\left[v_{ij}^{k(l)}, v_{ij}^{k(u)}\right]\right)_{m\times n} (k = 1, 2, 3)$ by Step 2, which are show in Tables 9–11, respectively.

By Step 3, the positive ideal and negative ideal solutions, A^+ and A^-, are shown as Tables 12 and 13, respectively.

Then, the separation measures, S_k^+ and S_k^-, are calculated by Step 4, which are shown in Table 14.

The relative closeness and weights of air-quality monitoring stations are calculated by Steps 5 and 6, respectively. The relative closeness, weights and their ranking are summarized in Table 15.

For the weight vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)^T = (0.3563, 0.3625, 0.2812)^T$ of air-quality monitoring stations, we can aggregate the individual decision matrixes V_1, V_2 and V_3 into a collective decision matrix V by Eq. (18), whose information is shown in Table 16.

Summing all interval arguments in each line of Table 16, the overall interval assessment of alternatives A_i ($i = 1, 2, 3$) are obtained:

$$\alpha_1 = [0.01446, 0.01660], \quad \alpha_2 = [0.02166, 0.50397],$$
$$\alpha_3 = [0.29492, 0.71689].$$

Table 14
Separation measures of each air-quality monitoring station.

Separation measures	d_1	d_2	d_3
S_k^+	0.1537	0.1356	0.2841
S_k^-	0.3089	0.2872	0.3166

Table 15
Relative closeness, weights and ranking of air-quality monitoring stations.

Monitoring stations	RC_k	λ_k	Ranking
d_1	0.6678	0.3563	2
d_2	0.6793	0.3625	1
d_3	0.5270	0.2812	3

Table 16
Collective decision matrix V.

Alternatives	SO_2	NO_2	PM_{10}
A_1	[0.00328, 0.16612]	[0.00514, 0.07800]	[0.00604, 0.25985]
A_2	[0.00515, 0.10599]	[0.00576, 0.10445]	[0.00569, 0.08448]
A_3	[0.00670, 0.32633]	[0.00875, 0.12769]	[0.00620, 0.26287]

Table 17
DMs' rankings derived from three different measurements.

Measurements	d_1	d_2	d_3	Rankings
S_k^+	0.1537	0.1356	0.2841	$d_2 \succ d_1 \succ d_3$
S_k^-	0.3089	0.2872	0.3166	$d_3 \succ d_1 \succ d_2$
λ_k	0.3563	0.3625	0.2812	$d_2 \succ d_1 \succ d_3$

To get the order of these overall interval arguments α_i ($i = 1, 2, 3$), we first compare each argument α_i with all arguments α_j ($j = 1, 2, 3$) by using Eq. (2). Then we can construct a complementary matrix by Eq. (3) as follows:

$$P = \begin{pmatrix} 0.5000 & 0.6347 & 0.4071 \\ 0.3653 & 0.5000 & 0.2807 \\ 0.5929 & 0.7193 & 0.5000 \end{pmatrix}.$$

Summing all elements in each line of matrix P by Eq. (4), we have:

$$p_1 = 1.5418, \quad p_2 = 1.1460, \quad p_3 = 1.8122.$$

Then we can rank the arguments α_i ($i = 1, 2, 3$) in descending order in accordance with the value of p_i ($i = 1, 2, 3$):

$$\alpha_3 > \alpha_1 > \alpha_2.$$

And then rank all the alternatives A_i ($i = 1, 2, 3$) in accordance with α_i ($i = 1, 2, 3$):

$$A_3 \succ A_1 \succ A_2,$$

where the symbol "$\succ$ " means superior to. And thus, the best alternative is A_3, i.e., the best air quality in Guangzhou is November, 2008 among the Novembers of 2006, 2007 and 2008.

In addition, there are two individual separation measures, S_k^+ and S_k^-, to be considered in extended TOPSIS in this example, which are the individual separation measure of each DM from the PIS and NIS. The smaller the value S_k^+, the better the decision of kth DM. The larger the value S_k^-, the better the decision of kth DM. So, we can also rank DMs according to the S_k^+ or S_k^-. These rankings are shown in Table 17.

Table 17 shows that the final ranking, as shown in the last line λ_k, are different from the ranking of "farther away from the anti-ideal is better", as shown in the middle line S_k^-. The largest difference occurs at d_2, where the λ_k calculated it 0.3625 while the S_k^- calculated it 0.2872. Different measurements usually lead to different results. It is inappropriate to say which method is better because every method has a different underlying theory or assertion. However, extended TOPSIS, the DMs' ranking by weights λ_k, is more suitable for compromise of "closer to the ideal " and "farther away from the anti-ideal".

Since the DMs' weights of the proposed method are generated from DMs' observations, a "false" or "biased" observations lead to a low weight.

7. Conclusions

We have presented a new algorithm to determining the weights of DMs for MAGDM problem. Via the proposed method, some representation formats such as the positive ideal and negative ideal solutions of group opinion, the separation measures and the relative closeness from the PIS, have been given. The proposed method is straightforward and can be performed on computer easily. A numerical example has demonstrated the feasibility of the method. Compared to the existing MADM approaches, the method proposed in this paper has certain distinguishing characteristics. The method can avoid comparing and ranking interval numbers, and can reflect both the group's objective information and the DM's subjective considerations.

However, it should be made clear that the use of the proposed method is limited by the requirement that the attribute data is in the form of interval numbers. The proposed method should be extended to support situations in which the information is in other forms, e.g., linguistic variables or fuzzy numbers.

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