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An approach to aggregating interval numbers into interval-valued intuitionistic fuzzy information for group decision making

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ABSTRACT

In this paper, we investigate the multiple attribute group decision making (MAGDM) problems, of which the attribute values in the group decision matrices provided by each decision maker (DM) is characterized by interval numbers. First, we define the concepts of attribute satisfactory interval and attribute dissatisfactory interval, respectively, according to the attribute values. Then we develop an approach for aggregating attribute satisfactory interval and attribute dissatisfactory interval into the collective attribute interval-valued intuitionistic fuzzy number (IVIFN), and then we obtain the collective interval-valued intuitionistic fuzzy decision matrix for group decision making. Next, we use the interval-valued intuitionistic fuzzy weighted averaging operator to aggregate all attribute values characterized by interval-valued intuitionistic fuzzy information to get the overall IVIFNs of alternatives. And then we use the score function and accuracy function to calculate the score and accuracy degree of each alternative value, and then rank the alternatives according to the score and accuracy degree of each alternative and select the most desirable one(s). And finally, we give an example for comprehensive pre-evaluation of air quality in Guangzhou, China during 16th Asian Olympic Games to illustrate in detail the decision process by the developed approach.

1. Introduction

Atanassov (1986, 1989, 1994a) introduced the concept of intuitionistic fuzzy set (IFS), which emerges from the simultaneous consideration of the degrees of membership and non-membership with a degree of hesitancy. Gau and Buehrer (1993) gave the notion of vague set, which is another generalization of fuzzy sets. But Bustince and Burillo (1996) showed that it is an equivalent of the IFS. In the process of decision making, the IFS has received more and more attention since its appearance, because the information about attribute values is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking. Many researchers have applied the IFS theory to the field of decision making. Chen and Tan (1994) presented some products for dealing with multiple attribute decision making problems based on vague sets. Szmidt and Kacprzky (2002) proposed some solution concepts in group decision making with intuitionistic (individual and social) fuzzy preference relations, such as intuitionistic fuzzy core and consensus winner, etc. Szmidt and Kacprzky (2003) investigated the consensus-reaching process in group decision making based on individual intuitionistic fuzzy preference relations. Herrera, Martínez, and Sánchez (1999) developed an aggregation process for combining numerical, interval valued and linguistic information, and then proposed different extensions of this process to deal with contexts in which can appear other type of information such as IFSs or multi-granular linguistic information. Park, Kwun, Park, and Park (2009) investigated the group decision making problems in which all the information provided by the decision makers (DMs) is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by interval-valued intuitionistic fuzzy number (IVIFN), and the information about attribute weights is partially known. Yue, Jia, and Ye (2009) introduced an approach for aggregating multiple attribute values characterized by precise numerical values into an intuitionistic fuzzy number and gave an application of this fusion to multiple attribute group decision making (MAGDM).

Atanassov and Gargov (1989) introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristics of IVIFS are that the values of its membership function and non-membership are intervals rather than exact numbers. Some researchers have applied the IVIFS theory to the field of decision making. Atanassov (1994b) defined some operations, relations and operators concerning IVIFSs. Xu and Chen (2007) developed some operators, such as the interval-valued intuitionistic fuzzy weighted averaging operator for aggregating interval-valued intuitionistic fuzzy information, and gave an application to MAGDM with interval-valued intuitionistic fuzzy information. Recently, Xu and Yager (2009) developed a new similarity measure between IVIFSs and utilized it to assess the

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consensus of experts' preferences in group decision making with interval-valued intuitionistic fuzzy information. Zhang, Zhang, and Mei (2009) proposed a new axiomatic definition of entropy of IVIFSs and discusses its relation with similarity measure. Ye (2009) proposed an accuracy function for IVIFSs by taking into account the hesitancy degree of IVIFSs to overcome the situation of difficult decision of existing accuracy functions to the alternatives in some cases.

However, many decision making processes, in the real world, take place in an environment in which the information is not precisely known. The DM cannot estimate attribute values of alternative with an exact numerical value, but with an interval number/value. Of which is called uncertain multiple attribute decision making problems. At present, a lot of methods have been proposed to deal with these problems. Bryson and Mobolurin (1996) proposed a systematic action learning evaluation procedure, which provides a structure for prioritizing and synthesizing interval preference scores in an ambiguous decision domain. Later, Lee, Park, Eum, and Park (2001) provided some methods for establishing dominance and potential optimality for alternatives. Yue (2011) developed an extended TOPSIS method for determining weights of decision makers with interval numbers. In this uncertain situation, the main difficulty for decision making is to rank the order of the interval numbers. In contrast, the task for ranking IVIFNs is simple than the interval numbers. The main problem, for MAGDM problem, is how to aggregate the interval numbers estimated an attribute of alternative into an IVIFN? As far as we know, currently, no approach has been developed to deal with the issue. In this paper, we shall develop an approach for solving this issue.

In order to do so, the rest of this paper is organized as follows. In Section 2, we review some basic notions, some operations and aggregation operators related to IVIFNs. It is followed by a description of the proposed approach is Section 3. For the given group decision matrixes in which the attribute values provided by each DM is characterized by interval numbers, the proposed approach first defines the concepts of attribute satisfactory interval and attribute dissatisfactory interval, respectively, according to attribute values. Then we obtain the overall attribute interval-valued intuitionistic fuzzy information by these concepts. Next, a numerical example and results are shown in Section 4. And the paper is concluded in Section 5.

2. Preliminaries

Let $X = \{x_1, x_2, \ldots, x_m\}$ be a discourse, an intuitionistic fuzzy set (IFS) A in X is given by

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}, \tag{1}$$

where μ_A: $X \to [0,1]$, ν_A: $X \to [0,1]$ with the condition $0 \leqslant \mu_A(x_i) + \nu_A(x_i) \leqslant 1$, for all $x_i \in X$.

The numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ denote the degree of membership and non-membership of x_i to A, respectively.

For each IFS A in X, if

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i), \tag{2}$$

then $\pi_A(x_i)$ is called the indeterminacy degree of x_i to the set A. Especially, if

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) = 0, \quad \text{for each } x_i \in X,$$

the IFS A is reduced to a fuzzy set.

Let $X = \{x_1, x_2, \ldots, x_m\}$ is a universe, an interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ over X is an object having the form:

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i)) | x_i \in X\}, \tag{3}$$

where $\mu_{\tilde{A}}(x_i) = [\mu_{ij}^L, \mu_{ij}^U] \subseteq [0, 1]$ and $\nu_{\tilde{A}}(x_i) = [\nu_{ij}^L, \nu_{ij}^U] \subseteq [0, 1]$ are intervals, $\mu_{\tilde{A}}^L(x_i) = \inf \mu_{\tilde{A}}(x_i)$, $\mu_{\tilde{A}}^U(x_i) = \sup \mu_{\tilde{A}}(x_i)$, $\nu_{\tilde{A}}^L(x_i) = \inf \nu_{\tilde{A}}(x_i)$, $\nu_{\tilde{A}}^U(x_i) = \sup \nu_{\tilde{A}}(x_i)$, and

$$\mu_{\tilde{A}}^U(x_i) + \nu_{\tilde{A}}^U(x_i) \leqslant 1, \quad \text{for all } x_i \in X \tag{4}$$

and $\pi_{\tilde{A}}(x_i) = [\pi_{\tilde{A}}^L(x_i), \pi_{\tilde{A}}^U(x_i)]$, where

$$\begin{aligned}\pi_{\tilde{A}}^L(x_i) &= 1 - \mu_{\tilde{A}}^U(x_i) - \nu_{\tilde{A}}^U(x_i),\ \pi_{\tilde{A}}^U(x_i) \\ &= 1 - \mu_{\tilde{A}}^L(x_i) - \nu_{\tilde{A}}^L(x_i), \quad \text{for all } x_i \in X.\end{aligned} \tag{5}$$

Especially, if $\mu_{\tilde{A}}(x_i) = \mu_{\tilde{A}}^L(x_i) = \mu_{\tilde{A}}^U(x_i)$ and $\nu_{\tilde{A}}(x_i) = \nu_{\tilde{A}}^L(x_i) = \nu_{\tilde{A}}^U(x_i)$, then, $\tilde{A}$ is reduced to an IFS.

Xu and Chen (2007) called the pair $\tilde{\alpha} = (\mu_{\tilde{\alpha}}(x_i), \nu_{\tilde{\alpha}}(x_i))$ an interval-valued intuitionistic fuzzy number (IVIFN), and denoted an IVIFN by $\tilde{\alpha} = ([a, b], [c, d])$, where

$$[a, b] \subseteq [0, 1],\ [c, d] \subseteq [0, 1],\ b + d \leqslant 1. \tag{6}$$

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ and $\tilde{\alpha} = ([a, b], [c, d])$ be three IVIFNs, then

(1) $\tilde{\alpha}_1 + \tilde{\alpha}_2 = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2],\ [c_1c_2, d_1d_2])$;
(2) $\lambda\tilde{\alpha} = ([1 - (1 - a)^\lambda, 1 - (1 - b)^\lambda],\ [c^\lambda, d^\lambda]),\ \lambda > 0$.

Both $\tilde{\alpha}_1 + \tilde{\alpha}_2$ and $\lambda\tilde{\alpha}$ are also IVIFNs.

Let $\tilde{\Theta}$ be the set of all IVIFNs. In order to aggregate the IVIFNs, we introduce the operator as following:

Definition 1. Let $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])(j = 1, 2, \ldots, n)$ be a collection of IVIFNs, and let IVIFWA: $\tilde{\Theta}^n \to \tilde{\Theta}$, if

$$IVIFWA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = w_1\tilde{\alpha}_1 + w_2\tilde{\alpha}_2 + \ldots + w_n\tilde{\alpha}_n, \tag{7}$$

then IVIFWA is called an interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator of dimension n, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $\tilde{\alpha}_j (j = 1, 2, \ldots, n)$, with $w_j \in [0,1]$, and $\sum_{j=1}^n w_j = 1$.

The aggregated value by using the IVIFWA operator is also an IVIFN, and satisfies:

$$IVIFWA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \left(\left[1 - \prod_{j=1}^n (1 - a_j)^{w_j}, 1 - \prod_{j=1}^n (1 - b_j)^{w_j}\right], \left[\prod_{j=1}^n c_j^{w_j}, \prod_{j=1}^n d_j^{w_j}\right]\right). \tag{8}$$

Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then

$$s(\tilde{\alpha}) = \frac{1}{2}(a - c + b - d) \tag{9}$$

is a score of $\tilde{\alpha}$, and s is a score function, where $s(\tilde{\alpha}) \in [-1, 1]$. The larger the score $s(\tilde{\alpha})$, the greater the IVIFN $\tilde{\alpha}$. Moreover,

$$h(\tilde{\alpha}) = \frac{1}{2}(a + b + c + d) \tag{10}$$

is an accuracy degree of $\tilde{\alpha}$, and h is an accuracy function, where $h(\tilde{\alpha}) \in [0, 1]$.

The larger the value of $h(\tilde{\alpha})$, the more the accuracy degree of $\tilde{\alpha}$.

In the following, we introduce a method for the comparison between two IVIFNs based on Eqs. (9) and (10):

Definition 2. Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs, then

- If $s(\tilde{\alpha}_1) < s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$;
- If $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then
  (1) If $h(\tilde{\alpha}_1) = h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ represent the same information, denoted by $\tilde{\alpha}_1 = \tilde{\alpha}_2$;
  (2) If $h(\tilde{\alpha}_1) < h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$.

The score and the accuracy degree of an IVIFN are similar to the mathematical expectation and variance of a random variable in probability, respectively. It is well known that the mathematical expectation is a criteria of evaluating the quality of a random variable, when mathematical expectations are equal between two random variables, variance will be a criteria of evaluating the quality between the two random variables.

3. The proposed approach for MAGDM

For convenience, let $M=\{1,2,\ldots,m\}$, $N=\{1,2,\ldots,n\}$ and $T=\{1,2,\ldots,t\}$.

Let $A=\{A_1,A_2,\ldots,A_m\}(m\geqslant 2)$ be a discrete set of m feasible alternatives, $U=\{U_1,U_2,\ldots,U_n\}$ be a finite set of attributes, $w=(w_1,w_2,\ldots,w_n)^T$ be the weight vector of attributes, with $0\leqslant w_j\leqslant 1$ and $\sum_{j=1}^{n}w_j=1$. And let $D=\{d_1,d_2,\ldots,d_t\}$ be a group of DMs, and $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_t)^T$ be the weight vector of DMs, where $\lambda_k\geqslant 0, \sum_{k=1}^{n}\lambda_k=1$.

A MAGDM problem can be described as follows:

Let

$$X_i=([(x_{kj}^i)^L,(x_{kj}^i)^U])_{t\times n}=\begin{array}{c}\\ d_1\\ d_2\\ \vdots\\ d_t\end{array}\begin{array}{c}\begin{array}{cccc}U_1 & U_2 & \cdots & U_n\end{array}\\ \begin{pmatrix}[(x_{11}^i)^L,(x_{11}^i)^U] & [(x_{12}^i)^L,(x_{12}^i)^U] & \cdots & [(x_{1n}^i)^L,(x_{1n}^i)^U]\\ [(x_{21}^i)^L,(x_{21}^i)^U] & [(x_{22}^i)^L,(x_{22}^i)^U] & \cdots & [(x_{2n}^i)^L,(x_{2n}^i)^U]\\ \vdots & \vdots & \vdots & \vdots\\ [(x_{t1}^i)^L,(x_{t1}^i)^U] & [(x_{t2}^i)^L,(x_{t2}^i)^U] & \cdots & [(x_{tn}^i)^L,(x_{tn}^i)^U]\end{pmatrix}\end{array},\quad \text{for all } i\in M, \tag{11}$$

be group decision matrix of the ith ($i\in M$) alternative with respect to the attributes, in which each of the elements is characterized by interval number. In general, there are benefit attributes and cost attributes in the multiple attribute decision making problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following formulas (Xu, 2005) to normalize each attribute value $[(x_{kj}^i)^L,(x_{kj}^i)^U]$ in group decision matrix $X_i=([(x_{kj}^i)^L,(x_{kj}^i)^U])_{t\times n}$ into a corresponding element $[(r_{kj}^i)^L,(r_{kj}^i)^U]$ in normalized group decision matrix $R_i=([(r_{kj}^i)^L,(r_{kj}^i)^U])_{t\times n}$ given by Eqs. (13) and (14).

$$R_i=([(r_{kj}^i)^L,(r_{kj}^i)^U])_{t\times n}=\begin{array}{c}\\ d_1\\ d_2\\ \vdots\\ d_t\end{array}\begin{array}{c}\begin{array}{cccc}U_1 & U_2 & \cdots & U_n\end{array}\\ \begin{pmatrix}[(r_{11}^i)^L,(r_{11}^i)^U] & [(r_{12}^i)^L,(r_{12}^i)^U] & \cdots & [(r_{1n}^i)^L,(r_{1n}^i)^U]\\ [(r_{21}^i)^L,(r_{21}^i)^U] & [(r_{22}^i)^L,(r_{22}^i)^U] & \cdots & [(r_{2n}^i)^L,(r_{2n}^i)^U]\\ \vdots & \vdots & \vdots & \vdots\\ [(r_{t1}^i)^L,(r_{t1}^i)^U] & [(r_{t2}^i)^L,(r_{t2}^i)^U] & \cdots & [(r_{tn}^i)^L,(r_{tn}^i)^U]\end{pmatrix}\end{array},\quad \text{for all }\ i\in M, \tag{12}$$

where

$$\begin{cases}(r_{kj}^i)^L=\dfrac{(x_{kj}^i)^L}{\sum_{k=1}^{t}(x_{kj}^i)^U},\\ (r_{kj}^i)^U=\dfrac{(x_{kj}^i)^U}{\sum_{k=1}^{t}(x_{kj}^i)^L}\end{cases},\quad \text{for benefit attribute } U_j,\ i\in M,\ j\in N,\ k\in T, \tag{13}$$

and

$$\begin{cases}(r_{kj}^i)^L=\dfrac{1/(x_{kj}^i)^U}{\sum_{k=1}^{t}(1/(x_{kj}^i)^L)}\\ (r_{kj}^i)^U=\dfrac{1/(x_{kj}^i)^L}{\sum_{k=1}^{t}(1/(x_{kj}^i)^U)}\end{cases},\ \text{for cost attribute } U_j,\ i\in M,\ j\in N,\ k\in T. \tag{14}$$

The attribute/colume vector $([(r_{1j}^i)^L,(r_{1j}^i)^U],[(r_{2j}^i)^L,(r_{2j}^i)^U],\ldots,[(r_{tj}^i)^L,(r_{tj}^i)^U])^T$ in normalized group decision matrix $R_i=([(r_{kj}^i)^L,(r_{kj}^i)^U])_{t\times n}$ represents group's evaluation. The larger the interval number $[(r_{kj}^i)^L,(r_{kj}^i)^U]$ $(k\in T)$, the higher the satisfactory degree of kth DM on ith alternative with respect jth attribute. The key problem is how to determine the satisfactory bounds by the each attribute/colume vector? That is to say, how to define the attribute satisfactory interval and the attribute dissatisfactory interval, respectively. we have

Definition 3. For each attribute/colume vector in normalized group decision matrix $R_i=([(r_{kj}^i)^L,(r_{kj}^i)^U])_{t\times n}$, if

$$\xi_{ij}^L=\min_{k\in T}\{(r_{kj}^i)^U\},\ \xi_{ij}^U=\max_{k\in T}\{(r_{kj}^i)^U\},\zeta_{ij}^L=\min_{k\in T}\{(r_{kj}^i)^L\},$$
$$\zeta_{ij}^L=\max_{k\in T}\{(r_{kj}^i)^L\},\quad i\in M,\ j\in N, \tag{15}$$

then, the $[\xi_{ij}^L,\xi_{ij}^U]$ and $[\zeta_{ij}^L,\zeta_{ij}^U]$ are called attribute satisfactory interval and attribute dissatisfactory interval, respectively. The framework $([\xi_{ij}^L,\xi_{ij}^U],\ [\zeta_{ij}^L,\zeta_{ij}^U])$ is called an ordered pair of the attribute satisfactory interval and attribute dissatisfactory interval.

Consider that, the smaller the values of endpoints ζ_{ij}^L and/or ζ_{ij}^U, the more the degree of group's dissatisfaction on the ith alternative with respect to the jth attribute. Therefore, we give the following linear transformation for both endpoints.

$$\zeta_{ij}^l=1-\zeta_{ij}^U,\ \zeta_{ij}^u=1-\zeta_{ij}^L,\quad i\in M,\ j\in N. \tag{16}$$

Actually, this is a key transformation of $[\zeta_{ij}^L,\zeta_{ij}^U]$ from an interval in $([\xi_{ij}^L,\xi_{ij}^U],\ [\zeta_{ij}^L,\zeta_{ij}^U])$ to a component of para-IVIFN $([\xi_{ij}^L,\xi_{ij}^U],\ [\zeta_{ij}^l,\zeta_{ij}^u])$. Further, in order to transform $([\xi_{ij}^L,\xi_{ij}^U],\ [\zeta_{ij}^l,\zeta_{ij}^u])$ into an IVIFN $([\mu_{ij}^L,\mu_{ij}^U],\ [v_{ij}^L,v_{ij}^U])$, we present the following definition:

Definition 4. We call

$$\tilde{y}_{ij}=([\mu_{ij}^L,\mu_{ij}^U],[v_{ij}^L,v_{ij}^U]) \tag{17}$$

interval-valued intutionstic fuzzy group evaluation value of the ith alternative with respect to the jth attribute, where

$$\mu_{ij}^L=\frac{\xi_{ij}^L}{\xi_{ij}^L+\xi_{ij}^U+\zeta_{ij}^l+\zeta_{ij}^u},\mu_{ij}^U=\frac{\xi_{ij}^U}{\xi_{ij}^L+\xi_{ij}^U+\zeta_{ij}^l+\zeta_{ij}^u},\qquad i\in M,\ j\in N, \tag{18}$$

and

$$v_{ij}^L=\frac{\zeta_{ij}^l}{\xi_{ij}^L+\xi_{ij}^U+\zeta_{ij}^l+\zeta_{ij}^u},v_{ij}^U=\frac{\zeta_{ij}^u}{\xi_{ij}^L+\xi_{ij}^U+\zeta_{ij}^l+\zeta_{ij}^u},\qquad i\in M,\ j\in N. \tag{19}$$

Obviously, the $\tilde{y}_{ij}(i\in M,j\in N)$ are IVIFNs.

Hence, the collective group decision matrix based on the aggregated IVIFNs is obtained in Eq. (20).

$$Y=(\tilde{y}_{ij})_{m\times n}=\begin{array}{c}\\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}U_1 & U_2 & \cdots & U_n\end{array}\\ \begin{pmatrix}([\mu_{11}^L,\mu_{11}^U],[v_{11}^L,v_{11}^U]) & ([\mu_{12}^L,\mu_{12}^U],[v_{12}^L,v_{12}^U]) & \cdots & ([\mu_{1n}^L,\mu_{1n}^U],[v_{1n}^L,v_{1n}^U])\\ ([\mu_{21}^L,\mu_{21}^U],[v_{21}^L,v_{21}^U]) & ([\mu_{22}^L,\mu_{22}^U],[v_{22}^L,v_{22}^U]) & \cdots & ([\mu_{2n}^L,\mu_{2n}^U],[v_{2n}^L,v_{2n}^U])\\ \vdots & \vdots & \vdots & \vdots\\ ([\mu_{m1}^L,\mu_{m1}^U],\ [v_{m1}^L,v_{m1}^U]) & ([\mu_{m2}^L,\mu_{m2}^U],[v_{m2}^L,v_{m2}^U]) & \cdots & ([\mu_{mn}^L,\mu_{mn}^U],[v_{mn}^L,v_{mn}^U])\end{pmatrix}\end{array} \tag{20}$$

Weighted averaging all elements in each line of matrix Y based on Eqs. (7) and (8), we have:

$$\tilde{y}_i = \left(\left[1 - \prod_{j=1}^{n} (1-\mu_{ij}^{L})^{w_j},\ 1 - \prod_{j=1}^{n} (1-\mu_{ij}^{U})^{w_j} \right], \left[\prod_{j=1}^{n} (\nu_{ij}^{L})^{w_j}, \prod_{j=1}^{n} (\nu_{ij}^{U})^{w_j} \right] \right),\quad i \in M. \tag{21}$$

Based on the analysis above, in the following we develop an approach to MAGDM:

Step 1. Establish group decision matrix $X_i (i \in M)$ by Eq. (11), for each alternative.

Step 2. Normalize the group decision matrix $X_i (i \in M)$ into $R_i (i \in M)$ by Eq. (12), for each alternative.

Table 1
Air quality in Pearl River Delta in November, 2006 (X_1).

Stations	SO_2	NO_2	PM_{10}
d_1	[0.013,0.129]	[0.028,0.144]	[0.021,0.136]
d_2	[0.040,0.161]	[0.034,0.093]	[0.047,0.199]
d_3	[0.006,0.118]	[0.004,0.053]	[0.003,0.174]
d_4	[0.014,0.076]	[0.060,0.153]	[0.029,0.157]
d_5	[0.023,0.204]	[0.040,0.104]	[0.035,0.068]
d_6	[0.052,0.191]	[0.038,0.142]	[0.028,0.207]
d_7	[0.066,0.391]	[0.048,0.195]	[0.046,0.458]
d_8	[0.023,0.116]	[0.016,0.073]	[0.025,0.153]
d_9	[0.044,0.291]	[0.030,0.103]	[0.024,0.182]
d_{10}	[0.001,0.038]	[0.001,0.118]	[0.039,0.218]
d_{11}	[0.008,0.039]	[0.005,0.015]	[0.017,0.137]
d_{12}	[0.027,0.216]	[0.027,0.135]	[0.035,0.263]
d_{13}	[0.004,0.152]	[0.041,0.122]	[0.008,0.115]
d_{14}	[0.008,0.076]	[0.055,0.140]	[0.040,0.155]
d_{15}	[0.010,0.036]	[0.008,0.034]	[0.025,0.142]
d_{16}	[0.015,0.082]	[0.030,0.135]	[0.031,0.160]

Table 2
Air quality in Pearl River Delta in November, 2007 (X_2).

Stations	SO_2	NO_2	PM_{10}
d_1	[0.013,0.107]	[0.038,0.139]	[0.047,0.155]
d_2	[0.047,0.127]	[0.040,0.081]	[0.102,0.206]
d_3	[0.015,0.046]	[0.001,0.026]	[0.021,0.157]
d_4	[0.012,0.045]	[0.043,0.092]	[0.039,0.148]
d_5	[0.018,0.131]	[0.032,0.083]	[0.037,0.100]
d_6	[0.030,0.182]	[0.035,0.143]	[0.048,0.203]
d_7	[0.060,0.308]	[0.052,0.225]	[0.100,0.444]
d_8	[0.036,0.214]	[0.020,0.091]	[0.064,0.260]
d_9	[0.052,0.239]	[0.031,0.143]	[0.048,0.235]
d_{10}	[0.017,0.051]	[0.008,0.059]	[0.028,0.231]
d_{11}	[0.014,0.043]	[0.007,0.020]	[0.061,0.187]
d_{12}	[0.011,0.065]	[0.031,0.104]	[0.044,0.167]
d_{13}	[0.045,0.135]	[0.035,0.107]	[0.066,0.192]
d_{14}	[0.010,0.046]	[0.048,0.117]	[0.043,0.173]
d_{15}	[0.014,0.046]	[0.008,0.029]	[0.050,0.125]
d_{16}	[0.011,0.049]	[0.037,0.092]	[0.044,0.173]

Table 3
Air quality in Pearl River Delta in November, 2008 (X_3).

Stations	SO_2	NO_2	PM_{10}
d_1	[0.003,0.042]	[0.018,0.054]	[0.014,0.150]
d_2	[0.014,0.113]	[0.016,0.086]	[0.030,0.187]
d_3	[0.009,0.034]	[0.005,0.019]	[0.011,0.103]
d_4	[0.003,0.034]	[0.031,0.103]	[0.032,0.129]
d_5	[0.003,0.079]	[0.019,0.070]	[0.023,0.138]
d_6	[0.019,0.124]	[0.024,0.098]	[0.025,0.189]
d_7	[0.019,0.125]	[0.034,0.097]	[0.041,0.156]
d_8	[0.028,0.160]	[0.023,0.100]	[0.019,0.179]
d_9	[0.030,0.201]	[0.028,0.094]	[0.012,0.143]
d_{10}	[0.005,0.027]	[0.009,0.056]	[0.030,0.121]
d_{11}	[0.003,0.019]	[0.005,0.019]	[0.022,0.114]
d_{12}	[0.012,0.091]	[0.016,0.071]	[0.029,0.159]
d_{13}	[0.014,0.126]	[0.031,0.094]	[0.026,0.213]
d_{14}	[0.014,0.056]	[0.047,0.120]	[0.032,0.129]
d_{15}	[0.007,0.026]	[0.004,0.019]	[0.032,0.123]
d_{16}	[0.005,0.044]	[0.027,0.095]	[0.029,0.132]

Table 4
Normalized air quality in Pearl River Delta in November, 2006 (R_1).

Stations	SO_2	NO_2	PM_{10}
d_1	[0.0035,0.4327]	[0.0036,0.1602]	[0.0078,0.4652]
d_2	[0.0028,0.1406]	[0.0055,0.1319]	$[0.0053,\underline{0.2078}]$
d_3	[0.0039,0.9375]	[0.0097,1.1211]	$[0.0061,\underline{3.2561}]$
d_4	[0.0060,0.4018]	$[0.0034,\underline{0.0747}]$	[0.0068,0.3368]
d_5	[0.0022,0.2446]	[0.0050,0.1121]	$[\underline{0.0156},0.2791]$
d_6	[0.0024,0.1082]	[0.0036,0.1180]	[0.0051,0.3489]
d_7	$[\underline{0.0012},\underline{0.0852}]$	$[\underline{0.0026},0.0934]$	$[\underline{0.0023},0.2124]$
d_8	[0.0039,0.2446]	[0.0071,0.2803]	[0.0070,0.3907]
d_9	[0.0016,0.1278]	[0.0050,0.1495]	[0.0058,0.4070]
d_{10}	$[0.0120,\underline{5.6252}]$	$[0.0044,\underline{4.4843}]$	[0.0049,0.2505]
d_{11}	[0.0117,0.7031]	$[\underline{0.0344},0.8969]$	[0.0078,0.5746]
d_{12}	[0.0021,0.2083]	[0.0038,0.1661]	[0.0040,0.2791]
d_{13}	[0.0030,1.4063]	[0.0042,0.1094]	[0.0093,1.2210]
d_{14}	[0.0065,0.7031]	[0.0037,0.0815]	[0.0069,0.2442]
d_{15}	$[\underline{0.0127},0.5625]$	[0.0152,0.5605]	[0.0075,0.3907]
d_{16}	[0.0056,0.3750]	[0.0038,0.1495]	[0.0067,0.3151]

Table 5
Normalized air quality in Pearl River Delta in November, 2007 (R_2).

Stations	SO_2	NO_2	PM_{10}
d_1	[0.0103,0.3562]	[0.0041,0.1052]	[0.0182,0.2360]
d_2	[0.0087,0.0685]	[0.0071,0.0999]	$[0.0137,\underline{0.1088}]$
d_3	[0.0240,0.3087]	$[0.0221,\underline{3.9976}]$	$[0.0180,\underline{0.5282}]$
d_4	[0.0245,0.3859]	[0.0063,0.0930]	[0.0191,0.2844]
d_5	[0.0084,0.2573]	[0.0069,0.1249]	$[\underline{0.0282},0.2998]$
d_6	[0.0061,0.1544]	[0.0040,0.1142]	[0.0139,0.2311]
d_7	$[\underline{0.0036},\underline{0.0772}]$	$[\underline{0.0026},\underline{0.0769}]$	$[\underline{0.0064},0.1109]$
d_8	[0.0052,0.1286]	[0.0063,0.1999]	[0.0108,0.1733]
d_9	[0.0046,0.0890]	[0.0040,0.1290]	[0.0120,0.2311]
d_{10}	[0.0216,0.2724]	[0.0098,0.4997]	[0.0122,0.3962]
d_{11}	$[\underline{0.0257},0.3308]$	$[\underline{0.0288},0.5711]$	[0.0151,0.1819]
d_{12}	[0.0170,0.4210]	[0.0055,0.1290]	[0.0169,0.2521]
d_{13}	[0.0082,0.1029]	[0.0054,0.1142]	[0.0147,0.1682]
d_{14}	$[0.0240,\underline{0.4631}]$	[0.0049,0.0833]	[0.0163,0.2580]
d_{15}	[0.0240,0.3308]	[0.0198,0.4997]	[0.0226,0.2219]
d_{16}	[0.0225,0.4210]	[0.0063,0.1087]	[0.0163,0.2521]

Table 6
Normalized air quality in Pearl River Delta in November, 2008 (R_3).

Stations	SO_2	NO_2	PM_{10}
d_1	$[0.0097,\underline{1.0447}]$	[0.0146,0.1766]	[0.0093,0.6350]
d_2	[0.0036,0.2239]	[0.0092,0.1986]	[0.0074,0.2963]
d_3	[0.0120,0.3482]	$[\underline{0.0415},0.6357]$	$[\underline{0.0135},\underline{0.8082}]$
d_4	[0.0120,1.0447]	[0.0077,0.1025]	[0.0108,0.2778]
d_5	[0.0051,1.0447]	[0.0113,0.1673]	[0.0101,0.3865]
d_6	[0.0033,0.1650]	[0.0081,0.1324]	[0.0074,0.3556]
d_7	[0.0033,0.1650]	[0.0081,0.0935]	$[0.0089,\underline{0.2168}]$
d_8	[0.0025,0.1119]	[0.0079,0.1382]	[0.0078,0.4679]
d_9	$[\underline{0.0020},\underline{0.1045}]$	[0.0084,0.1135]	[0.0097,0.7409]
d_{10}	[0.0151,0.6268]	[0.0141,0.3532]	[0.0115,0.2963]
d_{11}	$[\underline{0.0214},1.0447]$	[0.0415,0.6357]	[0.0122,0.4041]
d_{12}	[0.0045,0.2612]	[0.0111,0.1986]	[0.0087,0.3066]
d_{13}	[0.0032,0.2239]	[0.0084,0.1025]	$[\underline{0.0065},0.3419]$
d_{14}	[0.0073,0.2239]	$[\underline{0.0066},\underline{0.0676}]$	[0.0108,0.2778]
d_{15}	[0.0156,0.4477]	$[0.0415,\underline{0.7946}]$	[0.0113,0.2778]
d_{16}	[0.0092,0.6268]	[0.0083,0.1177]	[0.0105,0.3066]

Table 7
Ordered pairs of satisfactory and dissatisfactory intervals of air quality in Guangzhou for Novembers of 2006, 2007 and 2008.

Alternatives	SO_2	NO_2	PM_{10}
A_1	([0.0852,5.6252],[0.0012,0.0127])	([0.0747,4.4843],[0.0026,,0.0344])	([0.2078,3.2561],[0.0023, 0.0156])
A_2	([0.0772,0.4631],[0.0036,0.0257])	([0.0769,3.9976],[0.0026,0.0288])	([0.1088,0.5282],[0.0064,0.0282])
A_3	([0.1045,1.0447],[0.0020,0.0214])	([0.0676,0.7946],[0.0066,0.0415])	([0.2168,0.8082],[0.0065,0.0135])

Table 8
Air quality in Guangzhou for Novembers of 2006, 2007 and 2008 based on IVIFNs.

Alternatives	SO_2	NO_2	PM_{10}
A_1	([0.0111,0.7309],[0.1283,0.1298])	([0.0115,0.6876],[0.1481,0.1529])	([0.0382,0.5979],[0.1808,0.1832])
A_2	([0.0307,0.1844],[0.3880,0.3968])	([0.0127,0.6615],[0.1607,0.1651])	([0.0418,0.2030],[0.3734,0.3818])
A_3	([0.0334,0.3342],[0.3131,0.3193])	([0.0240,0.2824],[0.3406,0.3530])	([0.0722,0.2690],[0.3283,0.3306])

Table 9
Overall evaluations, scores and rankings of alternatives.

Alternatives	Overall evaluations	Scores	Ranking
A_1	([0.0221,0.6744],[0.1514,0.1539])	0.2602	1
A_2	([0.0316,0.3222],[0.3204,0.3279])	−0.0028	2
A_3	([0.0473,0.2984],[0.3245,0.3303])	−0.0160	3

Step 3. Construct a collective interval-valued intuitionistic fuzzy decision matrix Y in Eq. (20) by Eqs. (15)–(19).

Step 4. Calculate the overall weighted averaging evaluation value $\tilde{y}_i (i \in M)$ of each alternative by Eq. (21), for a given weight vector $w = (w_1, w_2, \ldots, w_n)^T$ of the attributes.

Step 5. Utilize the score function Eq. (9) to calculate the scores of the overall values $\tilde{y}_i (i \in M)$ of the alternatives.

Step 6. Rank the alternatives in accordance with the scores of the overall values $\tilde{y}_i (i \in M)$, and then select the most desirable one(s) (if there is no difference between two scores of the overall values, then we rank the alternatives according as Definition 2).

4. Illustrative example

The Pearl River Delta (PRD) Regional Air Quality Monitoring Network (the Network) was jointly established by the Guangdong Provincial Environmental Monitoring Centre (GDEMC) (formerly named as Guangdong Provincial Environmental Protection Monitoring Centre) and the Environmental Protection Department of the Hong Kong Special Administrative Region (HKEPD) from 2003 to 2005. It came into operation on November 30, 2005 and has been providing data for reporting of Regional Air Quality Index (RAQI) to the public since then.

The Network comprises 16 automatic air quality monitoring stations across the PRD region. Ten of these stations are operated by the Environmental Monitoring Centers of the individual cities in Guangdong while the 3 stations located in Hong Kong are managed by the HKEPD. The remaining 3 regional stations in the Network are operated by the GDEMC. All stations are installed with equipment to measure the ambient concentrations of respirable suspended particulate (PM_{10} or RSP), sulphur dioxide (SO_2), nitrogen dioxide (NO_2) and ozone (O_3).

The 16th Asian Olympic Games will be held in Guangzhou, during November 12–27, 2010. In what follows, we will present a comprehensive evaluation of the air quality in Guangzhou for the Novembers of 2006, 2007, and 2008 in order to forecast the air quality during the 16th Asian Olympic Games in the November of 2010. The each air-quality monitoring station can be considered as DM, i.e., $D = \{d_1, d_2, \ldots, d_{16}\}$ = {Luhu Park (Guangzhou), Wanqingsha (Guangzhou), Tianhu (Guangzhou), Liyuan (Shenzhen), Tangjia (Zhuhai), Shunde Dangxiao (Foshan), Huijingcheng (Foshan), Donghu (Jiangmen), Chengzhong (Zhaoqing), Xiapu (huizhou), Jinguowan (Huizhou), Haogang (Dongguan), Zimaling Park (Zhongshan), Tsuen Wan (HKEPD), Tap Mun (HKEPD), Tung Chung (HKEPD)}. The measured values (GDEMC & HKEPD, 2006; GDEMC & HKEPD, 2007, & GDEMC & HKEPD, 2008) are shown in Tables 1–3. The monthly air quality for the Novembers of 2006, 2007 and 2008, respectively, can be considered as alternatives. For convenience, let $A = \{A_1, A_2, A_3\}$ = {November of 2006, November of 2007, November of 2008} be the set of alternatives. $U = \{U_1, U_2, U_3\} = \{SO_2, NO_2, PM_{10}\}$ is the set of attributes.

Since all attributes are cost, we first normalize Tables 1–3 into the corresponding Tables 4–6 by Eq. (14) according to Step 2. The underlines in Tables 4–6 show either satisfactory lower bound or satisfactory upper bound, or dissatisfactory lower bound or dissatisfactory upper bound. These bounds are summarized in Table 7 in which elements are expressed by the ordered pairs of satisfactory and dissatisfactory intervals.

The each evaluation result in Table 7 can be transformed to corresponding the IVIFN by Step 3 as shown in Table 8.

The $(0.4, 0.2, 0.4)^T$ is weight vector of $\{SO_2, NO_2, PM_{10}\}$. Of which is determined according to the Implementing Details for Urban Environmental Comprehensive Treatment and Quantitative Examination during the *11*th Five-Year Plan (General Office of State Environmental Protection Administration (GOSEPA)[2006], No. 36) (GOSEPA, 2006), which is provided to the public by State Environmental Protection Administration. The overall weighted averaging values of monthly air quality are calculated by Step 4. The overall evaluation results characterized by IVIFNs, scores and their rankings are summarized in Table 9, which also shows comprehensive evaluation of air quality in Guangzhou for the Novembers of 2006, 2007 and 2008.

5. Conclusions

Many practical problems are often characterized by MAGDM. In this paper, we have proposed an approach for handling the MAGDM problems. For each alternative, assume that it has given the group decision matrix in which the attribute values provided by each DM are characterized by interval numbers. The proposed approach has first defined the concepts of attribute satisfactory interval and attribute dissatisfactory interval, respectively, according to the given attribute values. Then we have used the obtained attribute satisfactory interval and attribute dissatisfactory interval to get the overall attribute interval-valued intuitionistic fuzzy values. Thereby the collective interval-valued intuitionistic fuzzy decision matrix has been obtained. Next, for given attribute weights, we have used the weighted averaging operator to get the overall interval-valued intuitionistic fuzzy value of each alternative, and have used the score function and accuracy function to rank the alternatives and then to select the most desirable one.

It is worth pointing out that: (1) the ranking of the alternatives by the proposed approach is simple as compare with ranking interval numbers; (2) the proposed approach has not limits for the data distribution, the number of indexes and the sample size. It suits for the data not only of fewer alternatives, fewer indexes and small sample, but also of multi-alternative, multi-index and large sample. So, it is convenient to operate and aggregate.

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