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The cycle inconsistency index in pairwise comparisons matrices

Jacek Szybowski

AGH University of Science and Technology, Address, al. Mickiewicza 30, 30-059 Krakow, Poland

State Higher Vocational School in Nowy Sacz, Staszica 1, 33-300, Nowy Sacz, Poland

Abstract

The method of pairwise comparisons is widely applied in the decision making process. The inconsistency of data may significantly affect the final result. Since the notion of consistency is based on triads or cycles, there is a great need for defining the measure of a triad or cycle inconsistency.

In the paper a set of properties of a good cycle inconsistency index is proposed. Two construction methods of a cycle-based inconsistency index for a pairwise comparisons matrix are introduced. All is supported by the examples.

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Keywords: pairwise comparisons, cycle, inconsistency;

1. Introduction

The first use of the method of Pairwise Comparisons (PC method) is attributed to Ramon Llull. In his recently discovered manuscript *Ars eleccionis*[13] this Catalan philosopher, logician and a pioneer of computation theory, described the Borda count (an election system) and the Condorcet cryterium (a cryterium for a voting system), which Jean-Charles de Borda[1] and Nicolas de Condorcet[5] independently discovered centuries later. In the 20th century the PC method was used by Thurstone[16], Kendall and Babington-Smith[6], as well as many others.

The method is used when we are supposed to order a set of alternatives. We compare them pairwise and write the numerical result of these comparisons to a square matrix called a pairwise comparisons matrix (a PC matrix). The problem occurs when it appears that an alternative A is considered to be better than B, B - better than C, and C - better than A. This situation was described as a Condorced's paradox[5] and it results in the so-called triad inconsistency.

Obviously, this paradox may involve more alternatives. For instance, A wins with B and C, B - with C and D, C - with D, but A loses with D. This results in the so-called cycle inconsistency.

* Jacek Szybowski. Tel.: +48-12-617-3587.
E-mail address: szybowsk@agh.edu.pl

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Many researchers describe different types of measures of inconsistency. Recently, some attempts to give a set of axioms for a good inconsistency index have been undertaken by Cavallo and d'Apuzzo[4] or Koczkodaj, Szybowski and Wajch[11,10] for the case of triads, and by Koczkodaj and Szwarc[8] or Brunelli and Fedrizzi[3] for the case of whole PC matrices. However, there has been no axiomatization for a cycle inconsistency index, so this paper tries to fill this gap.

2. Preliminaries

An $n \times n$ matrix $\mathbf{A} = [a_{ij}]$ with all positive elements is called a *pairwise comparisons matrix* (a PC matrix) if $n \geq 3$. A PC matrix $\mathbf{A}$ is called *reciprocal* if $a_{ij} = \frac{1}{a_{ji}}$ for every $i, j = 1, \ldots, n$ (then obviously $a_{ii} = 1$ for every $i = 1, \ldots, n$). When we compare two entities X and Y and we judge, that X is twice better (longer, more expensive etc.), it is quite natural to accept that Y is half as good (long, expensive) as X. That is why we will assume that each PC matrix is reciprocal.

Fix a PC matrix $\mathbf{A}$ and $s \in \mathbb{N}$, such that $3 \leq s \leq n$.

Definition 2.1. An ordered sequence (a_{ij}, a_{ik}, a_{jk}) of 3 elements of $\mathbf{A}$ is called a *triad* if i, j, k are pairwise different elements of the set $\{1, \ldots, n\}$.

We will denote the set of all triads in $\mathbf{A}$ by $\mathcal{T}_{\mathbf{A}}$.

Definition 2.2. A *triad* (a_{ij}, a_{ik}, a_{jk}) is said to be *consistent* if $a_{ij} \cdot a_{jk} = a_{ik}$.

Remark 2.3. *A triad* (a_{ij}, a_{ik}, a_{jk}) *is consistent* $\Leftrightarrow a_{ij} \cdot a_{jk} \cdot a_{ki} = 1$.

Definition 2.4. An ordered sequence $(a_{i_1 i_2}, a_{i_2 i_3}, \ldots, a_{i_{s-1} i_s}, a_{i_1 i_s})$ of s elements of $\mathbf{A}$ is called an $s-$*cycle* if $i_1, \ldots, i_s$ are pairwise different elements of the set $\{1, \ldots, n\}$.

We will denote the set of all $s-$cycles in $\mathbf{A}$ by $C^s_{\mathbf{A}}$.

Definition 2.5. An $s-$*cycle* $(a_{i_1 i_2}, a_{i_2 i_3}, \ldots, a_{i_{s-1} i_s}, a_{i_1 i_s})$ is said to be *consistent* if $a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{s-1} i_s} = a_{i_1 i_s}$.

Remark 2.6. *An* $s-$*cycle* $(a_{i_1 i_2}, a_{i_2 i_3}, \ldots, a_{i_{s-1} i_s}, a_{i_1 i_s})$ *is consistent* $\Leftrightarrow a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{s-1} i_s} \cdot a_{i_s i_1} = 1$.

Definition 2.7. A PC matrix is *consistent* if all its triads are consistent.

Proposition 2.8. *A PC matrix* $\mathbf{A}$ *is consistent* $\Leftrightarrow$ *all its* $s-$*cycles are consistent.*

Proof. The proof is inductive. It is enough to show that $\forall p \in \{3, \ldots, s-1\}$

$$\text{All } p\text{-cycles of } \mathbf{A} \text{ are consistent} \Leftrightarrow \text{all } (p+1)\text{-cycles of } \mathbf{A} \text{ are consistent.}$$

To prove the $\Rightarrow$ notice that

$$a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_{p+1}} \cdot a_{i_{p+1} i_1} = a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_1} = 1$$

For the reverse implcation take any $j \notin \{i_1, , \ldots, i_p\}$ and multiply the below equations by themselves:

$$\begin{aligned}
a_{i_1 j} \cdot a_{j i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_1} &= 1 \\
a_{i_1 i_2} \cdot a_{i_2 j} \cdot a_{j i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_1} &= 1 \\
&\ldots \\
a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot a_{i_3 i_4} \cdot \ldots \cdot a_{i_{p-1} j} \cdot a_{j i_1} &= 1.
\end{aligned}$$

It follows that

$$a_{i_1 j} \cdot a_{j i_1} \cdot \ldots \cdot a_{i_{p-1} j} \cdot a_{j i_{p-1}} \cdot a^{p-2}_{i_1 i_2} \cdot a^{p-2}_{i_2 i_3} \cdot \ldots \cdot a^{p-2}_{i_{p-1} i_p} \cdot a^{p-2}_{i_p i_1} = 1,$$

so

$$(a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_1})^{p-2} = 1,$$

and finally,

$$a_{i_1 i_2} \cdot a_{i_2 i_3} \cdot \ldots \cdot a_{i_{p-1} i_p} \cdot a_{i_p i_1} = 1.$$

□

3. The triad inconsistency index

Recall some definitions and results from[10].

Definition 3.1. A function $ii : \mathbb{R}_+^3 \to [0, +\infty)$ is called *a triad inconsistency index*, if there exists a metric

$$d : \mathbb{R}_+^2 \to [0, +\infty)$$

such that $\forall x, y, z$ it holds

$$ii(x, y, z) = d(xz, y). \tag{1}$$

Definition 3.2. We say that a triad inconsistency index ii satysfying (1) is *induced by a metric d*.

Straight from the definition of a triad inconsistency index we get

Proposition 3.3. *For all numbers a, b, c, d and e a triad inconsistency index ii satisfies the conditions*

$$ii(a, b, c) = 0 \Leftrightarrow ac = b \tag{2}$$
$$ii(a, b, c) = ii(b, ac, 1) \tag{3}$$
$$ii(a, de, c) \leq ii(a, b, c) + ii(d, b, e) \tag{4}$$

Example 3.4. Since function $d : \mathbb{R}_+^2 \to [0, 1)$ given by formula

$$d(x, y) = 1 - \min\left(\frac{x}{y}, \frac{y}{x}\right) \tag{5}$$

is a metric, the Koczkodaj triad inconsistency index

$$KI(a, b, c) = 1 - \min\left(\frac{b}{ac}, \frac{ac}{b}\right),$$

is a triad inconsistency index induced by d.

4. The cycle inconsistency index

Now we may define the inconsistency index for cycles by analogy to the case of triads.

Definition 4.1. A function $ii : \mathbb{R}_+^s \to [0, +\infty)$ is called *a cycle inconsistency index*, if there exists a metric

$$d : \mathbb{R}_+^2 \to [0, +\infty)$$

such that $\forall x_1, \ldots x_s$ it holds

$$ii(x_1, \ldots, x_s) = d(x_1 \cdot \ldots \cdot x_{s-1}, x_s). \tag{6}$$

Definition 4.2. We say that a cycle inconsistency index ii satysfying (6) is *induced by a metric d*.

Straight from the definition of a cycle inconsistency index we get

Proposition 4.3. *For all numbers $x_1, \ldots, x_{2s-1}$ a cycle inconsistency index ii satisfies the conditions*

$$ii(x_1, \ldots, x_s) = 0 \Leftrightarrow x_1 \cdot \ldots \cdot x_{s-1} = x_s \tag{7}$$
$$ii(x_1, \ldots, x_s) = ii(x_s, 1, \ldots, 1, x_1 \cdot \ldots \cdot x_{s-1}) \tag{8}$$
$$ii(x_1, \ldots, x_{s-1}, x_{s+1} \cdot \ldots \cdot x_{2s-1}) \leq ii(x_1, \ldots, x_{s-1}, x_s) + ii(x_{s+1}, \ldots, x_{2s-1}, x_s) \tag{9}$$

Proof. The first statement is obvious. For the proof of the second one notice that

$$ii(x_1,\dots,x_s) = d(x_1\cdot\ldots\cdot x_{s-1}, x_s) = d(x_s, x_1\cdot\ldots\cdot x_{s-1}) = d(x_s\cdot 1\cdot\ldots\cdot 1, x_1\cdot\ldots\cdot x_{s-1}) = ii(x_s, 1,\dots,1, x_1\cdot\ldots\cdot x_{s-1}).$$

Finally,

$$\begin{aligned} ii(x_1,\dots,x_{s-1}, x_{s+1}\cdot\ldots\cdot x_{2s-1}) &= d(x_1\cdot\ldots\cdot x_{s-1}, x_{s+1}\cdot\ldots\cdot x_{2s-1}) \le \\ &\le d(x_1\cdot\ldots\cdot x_{s-1}, x_s) + d(x_{s+1}\cdot\ldots\cdot x_{2s-1}, x_s) = \\ &= ii(x_1,\dots,x_{s-1},x_s) + ii(x_{s+1},\dots,x_{2s-1},x_s). \end{aligned}$$

□

Remark 4.4. *We may treat conditions (7) – (9) as axioms for a cycle inconsistency index.*

As a consequence of (8) we obtain

Corollary 4.5. *For all numbers* $x_1,\dots,x_s$ *and any permutation* j *of a set* $\{1,\dots,s-1\}$

$$ii(x_1,\dots,x_{s-1},x_s) = ii(x_{j(1)},\dots,x_{j(s-1)},x_s). \tag{10}$$

Proposition 4.6. *For each function* $ii:\ \mathbb{R}_+^s \to [0,+\infty)$ *satisfying* (7) – (9) *function* $d_{ii}:\ \mathbb{R}_+^2 \to [0,+\infty)$ *given by*

$$d_{ii}(x,y) = ii(x,1,\dots,1,y) \tag{11}$$

is a metric. Moreover, index ii is induced by the metric d_{ii}*.*

Definition 4.7. We say that a metric d_{ii} satysfying (11) is *induced by a cycle inconsistency index ii*.

Definition 4.8. We say that a cycle inconsistency index *ii* is *bounded* if $\exists M>0\ \forall x_1,\dots,x_s\in\mathbb{R}_+\ ii(x_1,\dots,x_s)\le M$.

Fix a natural number $s\ge 3$.

Example 4.9. The function $D:\ \mathbb{R}_+^s\to\{0,1\}$ given by

$$D(x_1,\dots,x_{s-1},y) = \begin{cases} 0, & x_1\cdot\ldots\cdot x_{s-1} = y \\ 1, & \text{otherwise} \end{cases}.$$

is a bounded s-cycle inconsistency index induced by a discrete metric.

Example 4.10. The function $E:\ \mathbb{R}_+^s\to[0,+\infty)$ given by

$$E(x_1,\dots,x_{s-1},y) = |x_1\cdot\ldots\cdot x_{s-1} - y|$$

is an unbounded s-cycle inconsistency index induced by a Euclidean metric.

Example 4.11. The function $I_1:\ \mathbb{R}_+^s\to[0,1)$ given by

$$I_1(x_1,\dots,x_{s-1},y) = \frac{|x_1\cdot\ldots\cdot x_{s-1}-y|}{1+|x_1\cdot\ldots\cdot x_{s-1}-y|}$$

is a bounded s-cycle inconsistency index induced by a metric d_1 given by formula $d_1(x,y)=\frac{|x-y|}{1+|x-y|}$.

Example 4.12. The function $K:\ \mathbb{R}_+^s\to[0,1)$ given by

$$K(x_1,\dots,x_{s-1},y) = 1-\min\left(\frac{y}{x_1\cdot\ldots\cdot x_{s-1}}, \frac{x_1\cdot\ldots\cdot x_{s-1}}{y}\right)$$

is a bounded s-cycle inconsistency index induced by the metric (5).

Notice, that a given triad (cycle) inconsistency index *ii* may be applied to the triads (cycles) of a PC matrix **A**. Basing on the inconsistency indices for triads or s-cycles, we can introduce the inconsistency s-indices for the whole PC-matrices. Obviouslly, we can do it in various ways. However, two approaches seem the most natural.

The first one, introduced in [14] or [4] (for the case of triads) involves the arithmetic or geometric means of triad indices.

Example 4.13. The Peláez-Lamata index was defined as

$$PLI(\mathbf{A}) = \frac{6 \sum\limits_{(a_{ij},a_{ik},a_{jk})\in\mathcal{T}_{\mathbf{A}}} PL(a_{ij}, a_{ik}, a_{jk})}{n(n-1)(n-2)},$$

where

$$PL(a_{ij}, a_{ik}, a_{jk}) = \frac{a_{ik}}{a_{ij}a_{jk}} + \frac{a_{ij}a_{jk}}{a_{ik}} - 2.$$

It is natural to generalize the above definition to a cycle-based Peláez-Lamata inconsistency index:

$$PLI_s(\mathbf{A}) = \frac{\sum\limits_{(a_{i_1i_2},\ldots,a_{i_{s-1}i_s},a_{i_1i_s})\in C^s_{\mathbf{A}}} PL_s(a_{i_1i_2}, \ldots, a_{i_{s-1}i_s}, a_{i_1i_s})}{\binom{n}{s}},$$

where

$$PL_s(a_{i_1i_2}, \ldots, a_{i_{s-1}i_s}, a_{i_1i_s}) = \frac{a_{i_1i_s}}{a_{i_1i_2} \cdot \ldots \cdot a_{i_{s-1}i_s}} + \frac{a_{i_1i_2} \cdot \ldots \cdot a_{i_{s-1}i_s}}{a_{i_1i_s}} - 2.$$

Proposition 4.14. *PL_s does not satisfy (9).*

Proof. Consider $s = 4$.

$$PL_4(1,4,1,1\cdot 2\cdot 1) = \frac{1\cdot 2\cdot 1}{1\cdot 4\cdot 1} + \frac{1\cdot 4\cdot 1}{1\cdot 2\cdot 1} - 2 = \frac{1}{2} > \frac{1}{4} = \frac{3}{4} + \frac{4}{3} - 2 + \frac{3}{2} + \frac{2}{3} - 2 = PL_4(1,4,1,3) + PL_4(1,2,1,3).$$

□

The second type of definitions uses the maximum function. This approach was used (for triads and cycles) for the Koczkodaj inconsistency indices proposed in [7] and simplified in [8].

Example 4.15.

$$KI(\mathbf{A}) = \max_{(a_{ij},a_{ik},a_{jk})\in\mathcal{T}_{\mathbf{A}}} K(a_{ij}, a_{ik}, a_{jk}),$$

where

$$K(a_{ij}, a_{ik}, a_{jk}) = 1 - \min\left(\frac{a_{ik}}{a_{ij}a_{jk}}, \frac{a_{ij}a_{jk}}{a_{ik}}\right),$$

and

$$KI_s(\mathbf{A}) = \max_{(a_{i_1i_2},\ldots,a_{i_{s-1}i_s},a_{i_1i_s})\in C^s_{\mathbf{A}}} K(a_{i_1i_2}, \ldots, a_{i_{s-1}i_s}, a_{i_1i_s}),$$

where

$$K(a_{i_1i_2}, \ldots, a_{i_{s-1}i_s}, a_{i_1i_s}) = 1 - \min\left(\frac{a_{i_1i_s}}{a_{i_1i_2} \cdot \ldots \cdot a_{i_{s-1}i_s}}, \frac{a_{i_1i_2} \cdot \ldots \cdot a_{i_{s-1}i_s}}{a_{i_1i_s}}\right).$$

Remark 4.16. *As it was shown in [12] indices PLI and KI are not equivalent, which means that there are no positive constants α and β such that*

$$\alpha KI \le PLI \le \beta KI.$$

Example 4.17. Similarly, using the inconsistency index from Ex. 4.9 we can define

$$DI(\mathbf{A}) = \max_{(a_{ij},a_{ik},a_{jk})\in\mathcal{T}_\mathbf{A}} D(a_{ij}, a_{jk}, a_{ik})$$

and

$$DI_s(\mathbf{A}) = \max_{(a_{i_1i_2},\ldots,a_{i_{s-1}i_s},a_{i_1i_s})\in C^s_\mathbf{A}} D(a_{i_1i_2}, \ldots, a_{i_{s-1}i_s}, a_{i_1i_s})$$

However, from Proposition 2.8, we get

Corollary 4.18. $\forall s \in \{4, \ldots, n\}$

$$DI(\mathbf{A}) = DI_s(\mathbf{A}).$$

This means that defining dicrete cycle inconsistency index DI_s is useless, since it carries the same information as the simpler one based on triads.

On the other hand, this is not the case for other indices, for example KI, as the following example shows.

Example 4.19. Consider the PC matrices

$$A = \begin{bmatrix} 1 & \boxed{2} & 1 & \boxed{1} \\ \frac{1}{2} & 1 & \boxed{\mathbf{3}} & \mathbf{1} \\ 1 & \frac{1}{3} & 1 & \boxed{\mathbf{5}} \\ 1 & 1 & \frac{1}{5} & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & \boxed{\mathbf{9}} & \mathbf{1} & 2 \\ \frac{1}{9} & 1 & \mathbf{9} & 2 \\ 1 & \boxed{\frac{1}{9}} & 1 & \boxed{1} \\ \boxed{\frac{1}{2}} & \frac{1}{2} & 1 & 1. \end{bmatrix}$$

A simple calculation shows that

$$K(a_{23}, a_{24}, a_{34}) = 1 - \min\left(\frac{1}{3\cdot 5}, \frac{3\cdot 5}{1}\right) = \frac{14}{15}$$

(a triad in bold), and

$$K(a_{12}, a_{23}, a_{34}, a_{14}) = 1 - \min\left(\frac{1}{2\cdot 3\cdot 5}, \frac{2\cdot 3\cdot 5}{1}\right) = \frac{29}{30}$$

(a cycle marked in frames).

It is easy to notice that

$$KI(\mathbf{A}) = K(a_{23}, a_{24}, a_{34}) = \frac{14}{15} < \frac{29}{30} = K(a_{12}, a_{23}, a_{34}, a_{14}) = KI_4(\mathbf{A})$$

On the other hand,

$$AV(\mathbf{A}) := \frac{1}{|\mathcal{T}_\mathbf{A}|} \sum_{(a_{ij},a_{ik},a_{jk})\in\mathcal{T}_\mathbf{A}} K(a_{ij}, a_{ik}, a_{jk}) = \frac{1}{24}\cdot 6\cdot\left(\frac{5}{6} + \frac{14}{15} + \frac{1}{2} + \frac{4}{5}\right) = \frac{23}{30} \approx 0.7667$$

and

$$AV_4(\mathbf{A}) := \frac{1}{|C^4_\mathbf{A}|} \sum_{(a_{ij},a_{jk},a_{kl},a_{il})\in C^4_\mathbf{A}} K(a_{ij}, a_{jk}, a_{kl}, a_{il}) = \frac{1}{24}\cdot 8\cdot\left(\frac{29}{30} + \frac{3}{5} + \frac{2}{3}\right) = \frac{67}{90} \approx 0.7444$$

so

$$AV(\mathbf{A}) > AV_4(\mathbf{A}).$$

Similarly,

$$K(b_{12}, b_{13}, b_{23}) = 1 - \min\left(\frac{1}{9\cdot 9}, \frac{9\cdot 9}{1}\right) = \frac{80}{81}$$

(a triad in bold), and

$$K(b_{34}, b_{41}, b_{12}, b_{32}) = 1 - \min\left(\frac{\frac{1}{9}}{1\cdot\frac{1}{2}\cdot 9}, \frac{1\cdot\frac{1}{2}\cdot 9}{\frac{1}{9}}\right) = \frac{79}{81}$$

(a cycle marked in frames).

It is also easy to notice that

$$KI(\mathbf{B}) = K(b_{12}, b_{13}, b_{23}) = \frac{80}{81} > \frac{79}{81} = K(b_{34}, b_{41}, b_{12}, b_{32}) = KI_4(\mathbf{B}).$$

On the other hand,

$$AV(\mathbf{B}) = \frac{1}{24}\cdot 6\cdot\left(\frac{80}{81} + \frac{7}{9} + \frac{8}{9} + \frac{1}{2}\right) = \frac{511}{648} \approx 0.7886$$

and

$$AV_4(\mathbf{B}) = \frac{1}{24}\cdot 8\cdot\left(\frac{79}{81} + \frac{17}{18} + \frac{8}{9}\right) = \frac{455}{486} \approx 0.9362$$

so

$$AV(\mathbf{B}) < AV_4(\mathbf{B}).$$

Note that definitions of some inconsistency indices are based neither on triads nor on cycles, like the Saaty's consistency index defined in [15] as

$$CI(\mathbf{A}) = \frac{\lambda_{max} - n}{n - 1},$$

where λ_{max} is the principle eigenvalue of $\mathbf{A}$.

However, as indicated in [10], one may easily identify a triad with a 3×3 PC matrix, so we can introduce a triad inconsistency index on the base of a matrix inconsistency index. A detailed comparison of indices CI and KI for 3×3 matrices was done in [2].

5. Conclusion

It seems very natural to measure the level of a cycle $(a_{i_1 i_2}, \ldots, a_{i_{s-1} i_s}, a_{i_1 i_s})$ inconconsistency by means of a distance between $a_{i_1 i_2} \cdot \ldots \cdot a_{i_{s-1} i_s}$ and $a_{i_1 i_s}$. The formulas (7) – (9) allow to easily check if a given index is distance-based. Applying different metrics gives an opportunity to define various triad, cycle or PC matrix inconsistency indices. Althouth the definitition of a matrix consistency bases on triads, Example 4.19 shows that the level of inconsistency may increase or decrease if we consider longer cycles instead.

The high level of inconsistency indicates errors in comparison of alternatives. This is why the inconsistency reduction is desirable before prioritization. An algorithm of the reconstruction of a consistent PC matrix from an $(n-1)$- subset of its elements (named a base) was proposed in [9]. A cycle inconsistency index might be applied to choose the least inconsistent n-cycle as a base for the construction of the most reliable consistent PC matrix.

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