

Category	Email text
Not Spam	"Hi there, how are you?"
Not Spam	"Meeting at 3PM tomorrow"
Not Spam	"Please send the report"
Spam	"Join a free prize now!"
Spam	"Claim your discount today"
Spam	"Limited time offer : click here"
?	"Free meeting tomorrow"
9.	"Claim your free prize"

① $\text{UniTotal unique words in spam} = 14$

$\text{Total unique words in not spam} = 14$

Vocabulary size = 28

Using Laplace smoothing ("free meeting tomorrow")

$$P(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{meeting} | \text{spam}) = \frac{0+1}{14+28} = \frac{0.1}{42}$$

$$P(\text{tomorrow} | \text{spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{free} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{meeting} | \text{not spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{tomorrow} | \text{not spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

Now

$$P(\text{Spam} \mid \text{free, meeting, tomorrow}) \propto P(\text{spam})$$

Prior spam probabilities

$$P(\text{spam}) = \frac{3}{6} = 0.5$$

$$P(\text{not spam}) = \frac{3}{6} = 0.5$$

$\left. \begin{array}{l} 3 \text{ spam and } 3 \\ 3 \text{ not spam} \end{array} \right\} \text{ emails}$

Now,

$$P(\text{spam} \mid \text{free, meeting, tomorrow}) \propto p(\text{spam}) \cdot p(\text{free} \mid \text{spam}) \cdot p(\text{meeting} \mid \text{spam}) \cdot p(\text{tomorrow} \mid \text{spam})$$

$$= 0.5 \times \frac{2}{4^2} \times \frac{1}{4^2} \times \frac{1}{4^2}$$

$$\approx 0.0000135$$

$$P(\text{not spam} \mid \text{free, meeting, tomorrow}) \propto p(\text{not spam}) \cdot p(\text{free} \mid \text{not spam}) \cdot p(\text{meeting} \mid \text{not spam}) \cdot p(\text{tomorrow} \mid \text{not spam})$$

$$= 0.5 \times \frac{1}{4^2} \times \frac{2}{4^2} \times \frac{2}{4^2}$$

$$\approx 0.000027$$

Since $P(\text{not spam}) > P(\text{spam})$ the email is not spam.

$$\text{Normalization} = \frac{0.000027}{0.000027 + 0.000013}$$

$\approx 67.5\%$ not spam

② for email "Claim your free prize"

Using Laplace smoothing

$$p(\text{claim} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{your} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{prize} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{claim} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{your} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{free} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{prize} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

Now

$P(\text{spam} \mid (\text{claim}, \text{your free prize}) \wedge P(\text{spam})) \cdot P(\text{claim} \mid \text{spam})$

$$= 0.5 \times \frac{2}{42} \times \frac{2}{42} \times \frac{2}{42} \times \frac{2}{42}$$

$$\approx 0.0000026$$

$P(\text{not spam} \mid (\text{claim}, \text{your free prize}) \wedge P(\text{not spam})) \cdot P(\text{claim} \mid \text{not spam}) \cdot P(\text{your} \mid \text{not spam}) \cdot P(\text{free} \mid \text{not spam}) \cdot P(\text{prize} \mid \text{not spam})$

$$= 0.5 \times \frac{1}{42} \times \frac{1}{42} \times \frac{1}{42} \times \frac{1}{42}$$

$$\approx 0.00000016$$

$P(\text{spam}) > P(\text{not spam})$, the email "Claim your free gift/prize" is spam

$$\text{Normalize} \approx \frac{0.0000026}{0.0000026 + 0.00000016}$$

$$\approx 90.5\% \text{ spam}$$