

Category	Email text
Not Spam	"Hi there, how are you?"
Not Spam	"Meeting at 3 PM tomorrow"
Not Spam	"Please send the report"
Spam	"Win a free prize now!"
Spam	"Claim your discount today"
Spam	"Limited time offer: click here"
?	"Free meeting tomorrow"
9.	"Claim your free prize"

- ①  $\text{Total unique words in spam} = 14$   
 $\text{Total unique words in not spam} = 14$   
 $\text{Vocabulary size} = 28$

Using Laplace Smoothing ("free meeting tomorrow")

$$P(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{meeting} | \text{spam}) = \frac{0+1}{14+28} = \frac{0.1}{42}$$

$$P(\text{tomorrow} | \text{spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{free} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{meeting} | \text{not spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{tomorrow} | \text{not spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

Now

$$P(\text{Spam} | \text{free, meeting, tomorrow}) \propto P(\text{Spam})$$

# Prior spam probabilities

$$P(\text{spam}) = \frac{3}{6} = 0.5$$

$$P(\text{not spam}) = \frac{3}{6} = 0.5$$

{ 3 spam and  
3 not spam  
emails }

Now,

$$P(\text{spam} | \text{free, meeting, tomorrow}) \propto P(\text{spam}) \cdot P(\text{free} | \text{spam}) \cdot P(\text{meeting} | \text{spam}) \cdot P(\text{tomorrow} | \text{spam})$$

$$= 0.5 \times \frac{2}{42} \times \frac{1}{42} \times \frac{1}{42}$$

$$\approx 0.0000235$$

$$P(\text{Not spam} | \text{free, meeting, tomorrow}) \propto P(\text{not spam}) \cdot P(\text{free} | \text{not spam}) \cdot P(\text{meeting} | \text{not spam}) \cdot P(\text{tomorrow} | \text{not spam})$$

$$= 0.5 \times \frac{1}{42} \times \frac{2}{42} \times \frac{2}{42}$$

$$\approx 0.000027$$

Since  $P(\text{not spam}) > P(\text{spam})$  The email is not spam.

$$\text{Normalization} = \frac{0.000027}{0.000027 + 0.000013}$$

$$\approx 67.5\% \text{ not spam}$$

② for email "Claim your free prize"

Using Laplace smoothing

$$p(\text{claim} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{your} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{prize} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$p(\text{claim} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{your} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{free} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$p(\text{prize} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

Now

$$P(\text{spam} | \text{Claim, your, free, prize}) \propto P(\text{spam}) \cdot P(\text{Claim} | \text{spam}) \cdot P(\text{your} | \text{spam}) \cdot P(\text{free} | \text{spam}) \cdot P(\text{prize} | \text{spam})$$

$$= 0.5 \times \frac{2}{42} \times \frac{2}{42} \times \frac{2}{42} \times \frac{2}{42}$$

$$\approx 0.0000026$$

$$P(\text{not spam} | \text{Claim, your, free, prize}) \propto P(\text{not spam}) \cdot P(\text{Claim} | \text{not spam}) \cdot P(\text{your} | \text{not spam}) \cdot P(\text{free} | \text{not spam}) \cdot P(\text{prize} | \text{not spam})$$

$$= 0.5 \times \frac{1}{42} \times \frac{1}{42} \times \frac{1}{42} \times \frac{1}{42}$$

$$\approx 0.00000016$$

Since  $P(\text{spam}) > P(\text{not spam})$ , the email "Claim your free prize" is spam

$$\text{Normalize} = \frac{0.0000026}{0.0000026 + 0.00000016}$$

$$\approx 90.5\% \text{ spam}$$