

1. In a race competition the probability that Harry wins is 0.4, the probability that Krish wins is 0.2 and the probability that Jonny wins is 0.3.

Find the probability that

(a) Harry and Jonny wins

2 marks

(b) Harry or Krish or Jonny wins

2 marks

(c) Someone else wins.

2 marks

Solution: Since only one person wins, the events are mutually exclusive.

$$\begin{aligned}\text{a) } P(\text{Harry or Jonny wins}) &= P(\text{Harry wins}) + P(\text{Jonny wins}) \\ &= 0.4 + 0.3 \\ &= 0.7\end{aligned}$$

$$\begin{aligned}\text{b) } P(\text{Harry or Krish or Jonny wins}) &= P(\text{Harry wins}) + P(\text{Krish wins}) + P(\text{Jonny wins}) \\ &= 0.4 + 0.2 + 0.3 \\ &= 0.9\end{aligned}$$

$$\begin{aligned}\text{c) } P(\text{Someone else wins}) &= 1 - 0.9 \\ &= 0.1\end{aligned}$$

2. In a certain college, 25% of students failed in Economics, 15% of the students failed in Statistics and 10% failed in both Economics and Statistics. A student is selected at random.

(a) What is the probability that he has failed in Economics given that he has failed in Statistics?

3 marks

(b) What is the probability that he has failed in Statistics given that he has failed in Economics?

3marks

Solution: Let E and S be the events that a student failed in Economics and statistics respectively.

$$P(E) = 0.25, P(S) = 0.15 \text{ and } P(E \cap S) = 0.10$$

- a) The probability of a student failed in Economics given that he has failed in statistics is obtained as,

$$P\left(\frac{E}{S}\right) = \frac{P(E \cap S)}{P(S)} = \frac{0.10}{0.15} = 0.67$$

- b) The probability of a student failed in statistics given that he is failed in Economics is obtained as,

$$\begin{aligned} P\left(\frac{S}{E}\right) &= \frac{P(S \cap E)}{P(E)} \\ &= \frac{0.10}{0.25} \\ &= 0.40 \end{aligned}$$

3. The random variable X has probability distribution as shown in the table

X	1	2	3	4	5
P(X=x)	0.3	0.1	0.2	0.3	0.1

Find

- a) E(X) mean 2 marks
- (a) E(X²) 2 marks
- (b) Var(X) 2 marks
- (c) σ, the standard deviation of X. 2 marks

Solutions:

$$\begin{aligned} \text{a) } E(X) \text{ mean} &= 1 \times 0.3 + 2 \times 0.1 + 3 \times 0.2 + 4 \times 0.3 + 5 \times 0.1 \\ &= 2.8 \end{aligned}$$

$$\begin{aligned} \text{b) } E(X^2) &= \sum X^2 P(X = x) \\ &= 1^2 \times 0.3 + 2^2 \times 0.1 + 3^2 \times 0.2 + 4^2 \times 0.3 + 5^2 \times 0.1 \end{aligned}$$

$$=10.4$$

c) $\text{Var}(X) = E(X^2) - \mu^2$

$$=10.4 - 9$$

$$=1.4$$

d) Standard Deviation (σ) = $\sqrt{\text{Var}(X)}$

$$= \sqrt{1.4}$$

$$=1.18 \text{ (2 d.p.)}$$