CHAPTER 8

HYPOTHESIS TESTING

Learning Objective

- Define what is meant by a hypothesis and hypothesis testing.
- Describe the five-step hypothesis-testing procedure.
- Describe the statistical errors that might result in testing a hypothesis.
- Distinguish between a one-tailed and a two-tailed test.

Hypothesis

In statistics, is a claim or statement about a property of a population

Examples

- The mean monthly income from all sources for senior citizens is RM900.
- Twenty percent of juvenile offenders ultimately are caught and sentenced to prison.
- The mean outside diameter of ball bearings produced during the day is 1.000 inch.
- Ninety percent of the federal income tax forms are filled out correctly.

Hypothesis Testing

A procedure used to test the claim or statement .

Five-Step Procedure for Testing A Hypothesis.

STEP I : Identify the **null hypothesis** and **alternate hypothesis**.

STEP II : Determine the **level of significance**.

STEP III : Find the **test statistics**.

STEP IV: Determine the **decision rule**.

STEP V : Make a **decision**.

STEP I

- **Null Hypothesis (denoted H_0):**

  is the statement being tested in a test of hypothesis.

  - Must contain condition of equality

- **Alternative Hypothesis (H_1):**

  is what is believe to be true if the null hypothesis is false.

  - 'opposite' of Null

Example:

$$H_0 : \mu = 30 \quad \text{versus } H_1 : \mu > 30$$

If you wish to **support** your claim, the claim must be stated so that it becomes **the alternative hypothesis.**

STEP II

- **The level of significance**

  The risk we assume of rejecting the null hypothesis when it is actually true.

  It is also referred to as the level of risk – it is the risk you take of rejecting the null hypothesis when it is really true.

  (*Traditionally, 0.05 level is selected for consumer research, 0.01 for quality assurance, and 0.10 for political polling*).

Ideally all claims should be stated that they are Null Hypothesis so that the most serious error would be a **Type I error**.

Type I Error

- The mistake of rejecting the null hypothesis when it is true.
- The probability of doing this is called the significance level, denoted by α (alpha).
- Common choices for α : 0.05 and 0.01

Example: *Rejecting a perfectly good parachute and refusing to jump*

Type I Error

- the mistake of failing to reject the null hypothesis when it is false.
- denoted by *ß* (beta)

Example: *Failing to reject a defective parachute and jumping out of a plane with it.*

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	Type I error (rejecting a true null hypothesis)	Correct decision
	We fail to reject the null hypothesis	Correct decision	Type II error (failing to reject a false null hypothesis)

STEP III

- **Test Statistic:**
  A sample statistic or value based on sample data, used to reject or not reject the null hypothesis.

  Example:

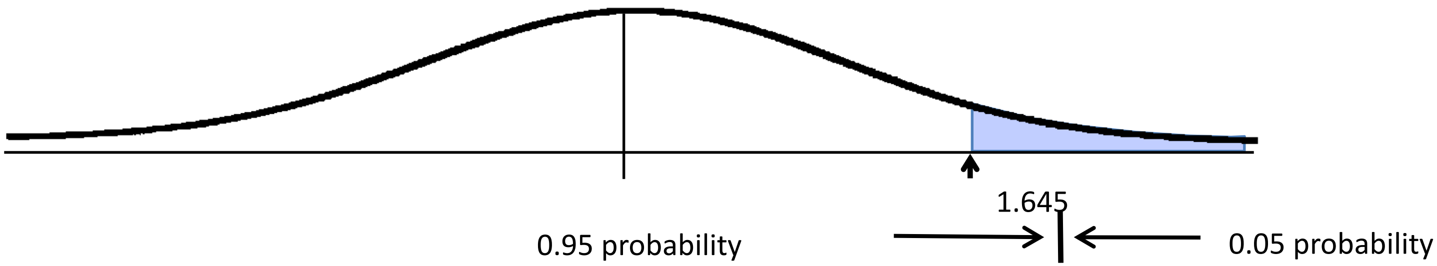
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}}$$

STEP IV

❖ Decision Rule:

States or condition of rejection or non-rejection.

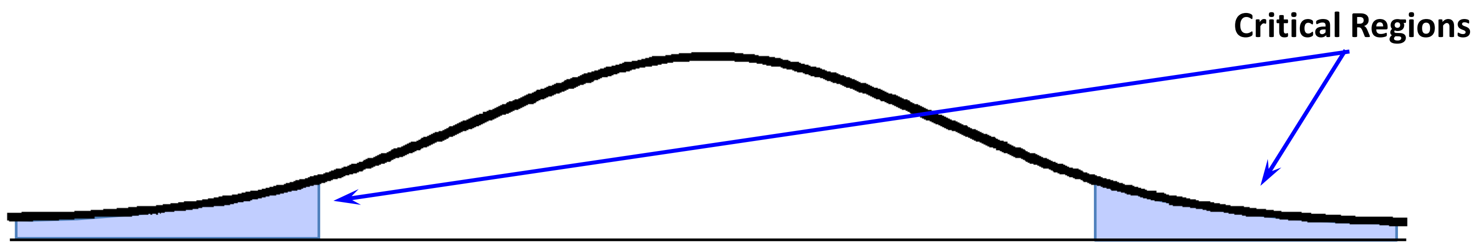
Sampling distribution for One-Tailed Test, 0.05 Level of Significance



1. The area where H_0 is not rejected includes the area to the left of 1.645.
2. The area of rejection is to the right of 1.645.
3. A one-tailed test is being applied.
4. The 0.05 level of significance was chosen.
5. The sampling distribution is for the test statistics z, the standard normal deviate.
6. The value 1.645 separates the regions where the null hypothesis is rejected and where it is not rejected.
7. The value 1.645 is called the critical value.

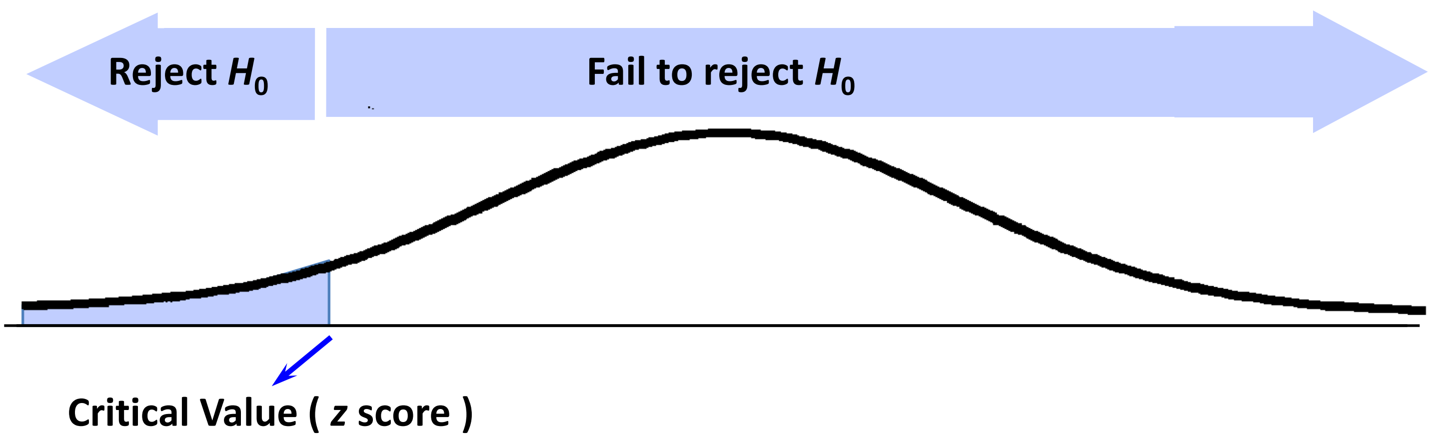
Critical Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis.



Critical Value

Value (s) that separates the critical region from the values that would not lead to a rejection of H_0



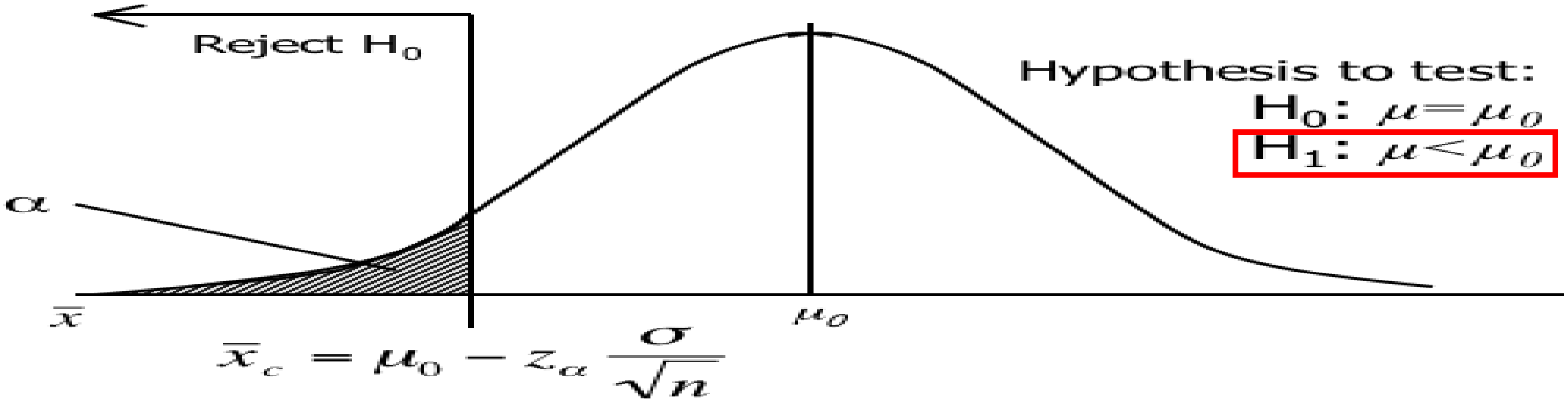
STEP V

- **Decision :**

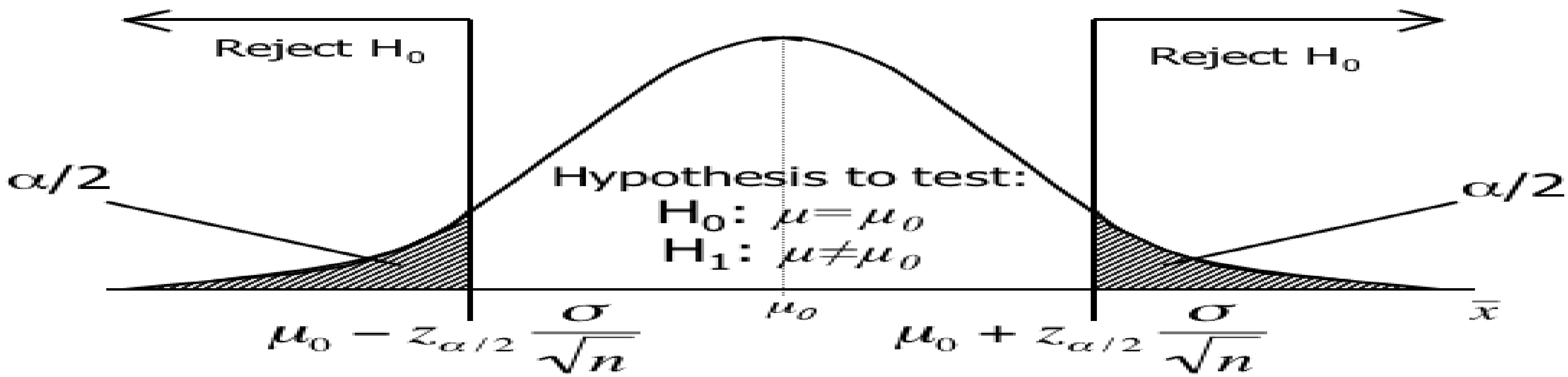
  A process whether to reject or not to reject the null hypothesis.
  In this process, we can use tests to assist us in decision making;

  - Two-tailed
  - Left-tailed
  - Right-tailed
  - P-value

One tail test with rejection region on left



Two tail test with rejection region in both tails

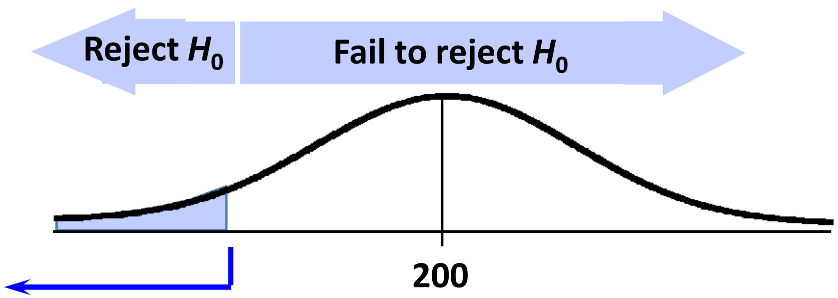


Left-tailed & Right-tailed Test

H_0: μ = 200

H_1: μ < 200

Points Left

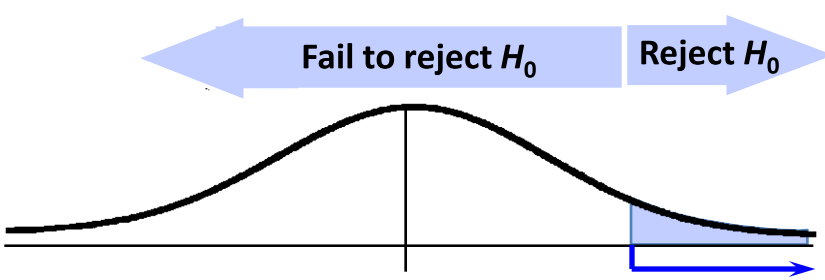


Values that differ significantly from 200

H_0: μ = 200

H_1: μ > 200

Points Right



Values that differ significantly from 200

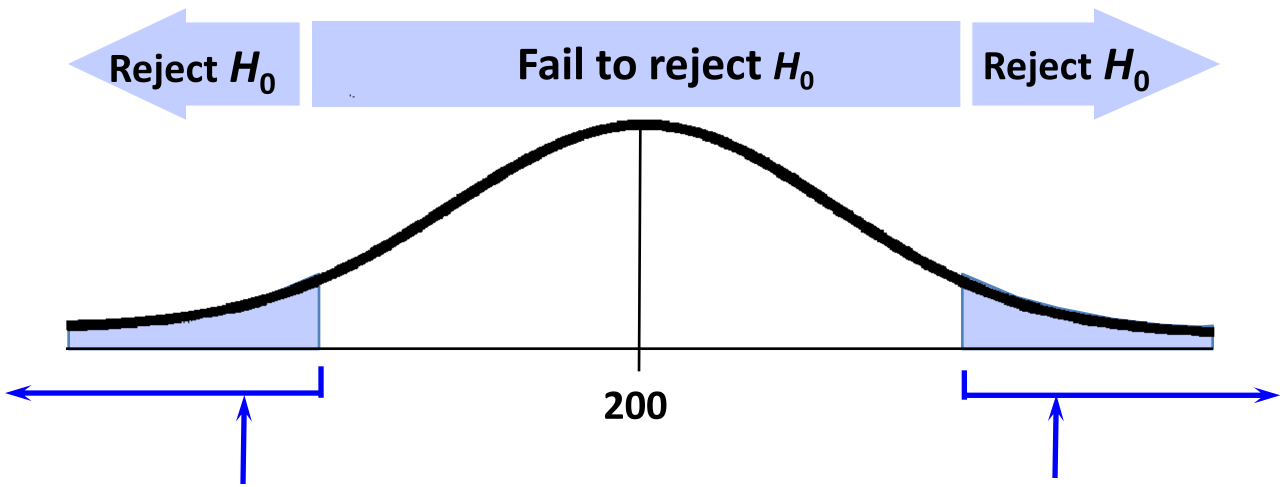
Two-tailed Test

H_0: $\mu = 200$

H_1: $\mu \neq 200$

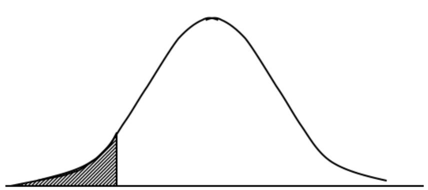
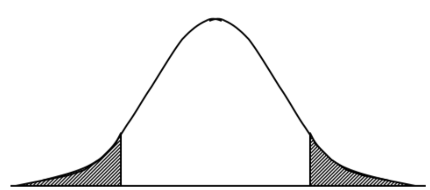
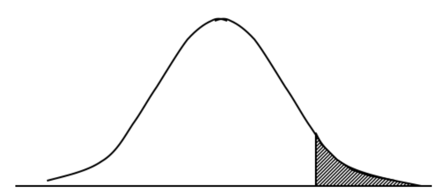
Means less than or greater than

α is divided equally between the two tails of the critical region



Values that differ significantly from 200

Summary of One and Two-Tail Tests…

One-Tail Test (left tail)	Two-Tail Test	One-Tail Test (right tail)
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$
		

Conclusions in Hypothesis Testing

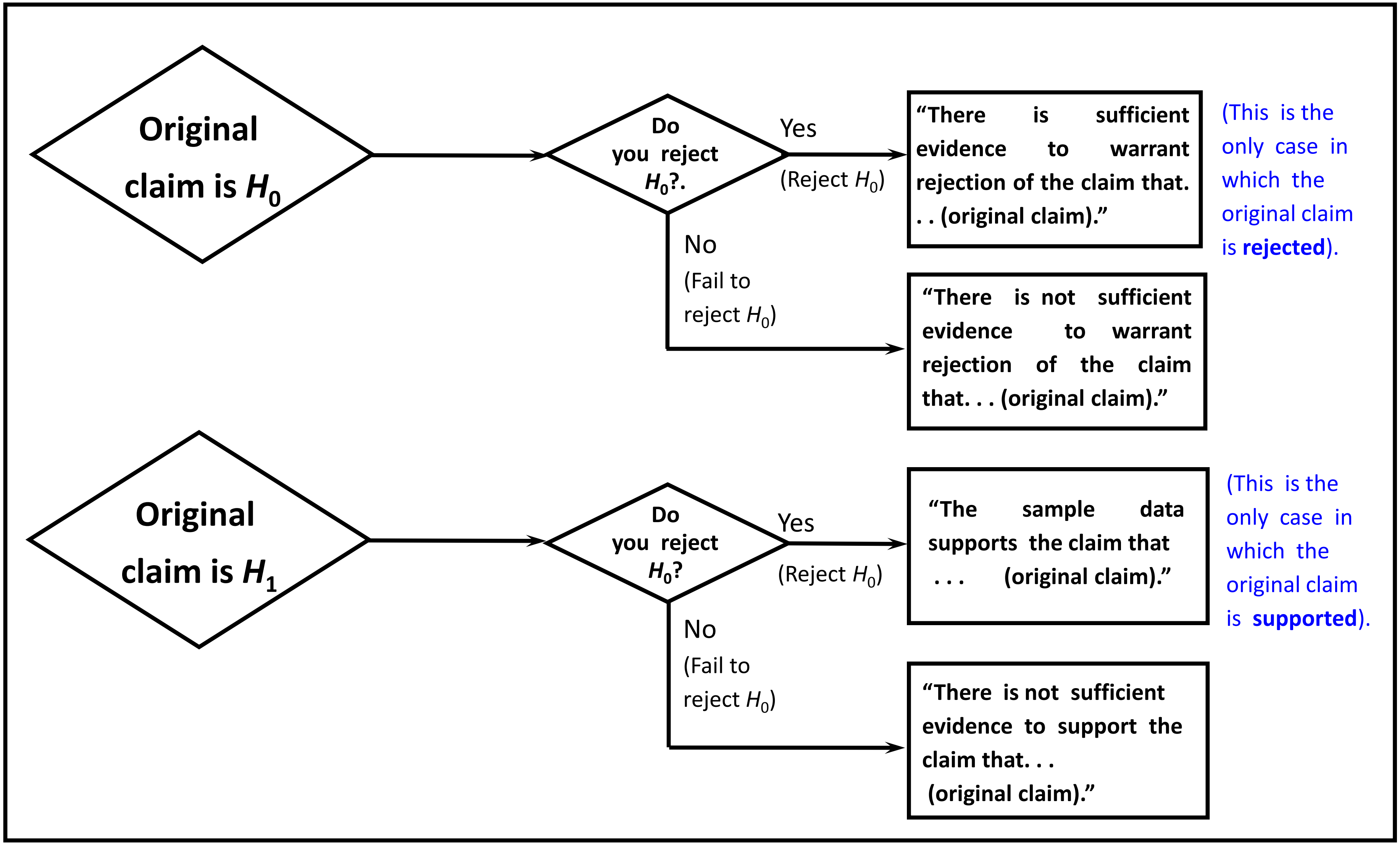
- always test the null hypothesis

  1. Fail to reject the H_0

  2. Reject the H_0

- need to formulate **correct wording of final conclusion**

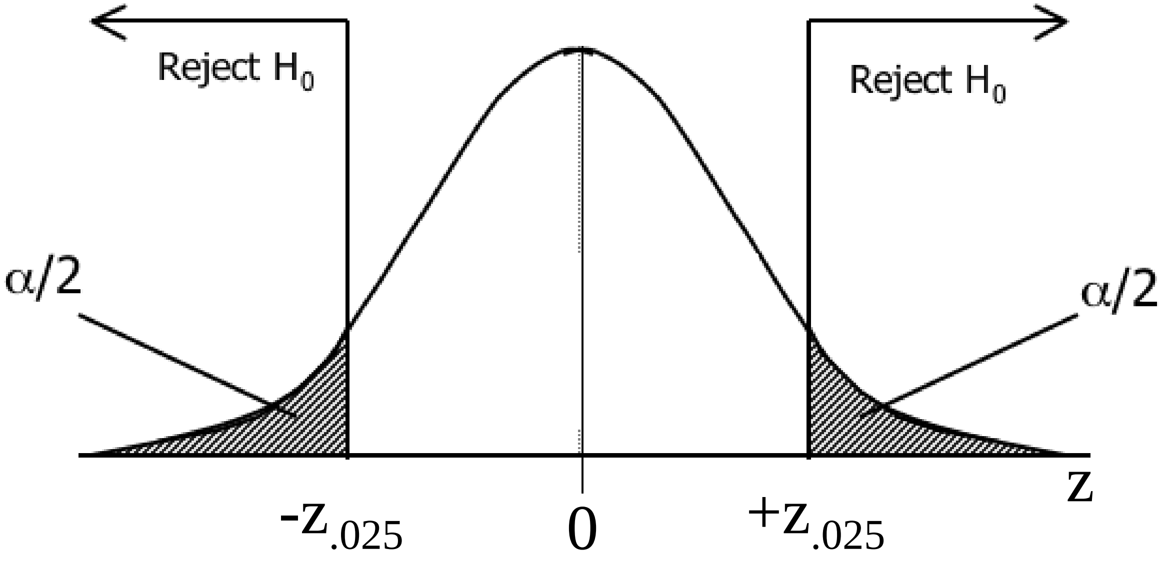
Wording of Conclusions in Hypothesis Tests



Example

At a 5% significance level (i.e. α = .05), we have α /2 = .025. Thus, $z_{.025}$ = 1.96 and our rejection region is:

$z < -1.96$ **-or-** $z > 1.96$



Example

From the data, we calculate $\bar{x}$ = 17.55

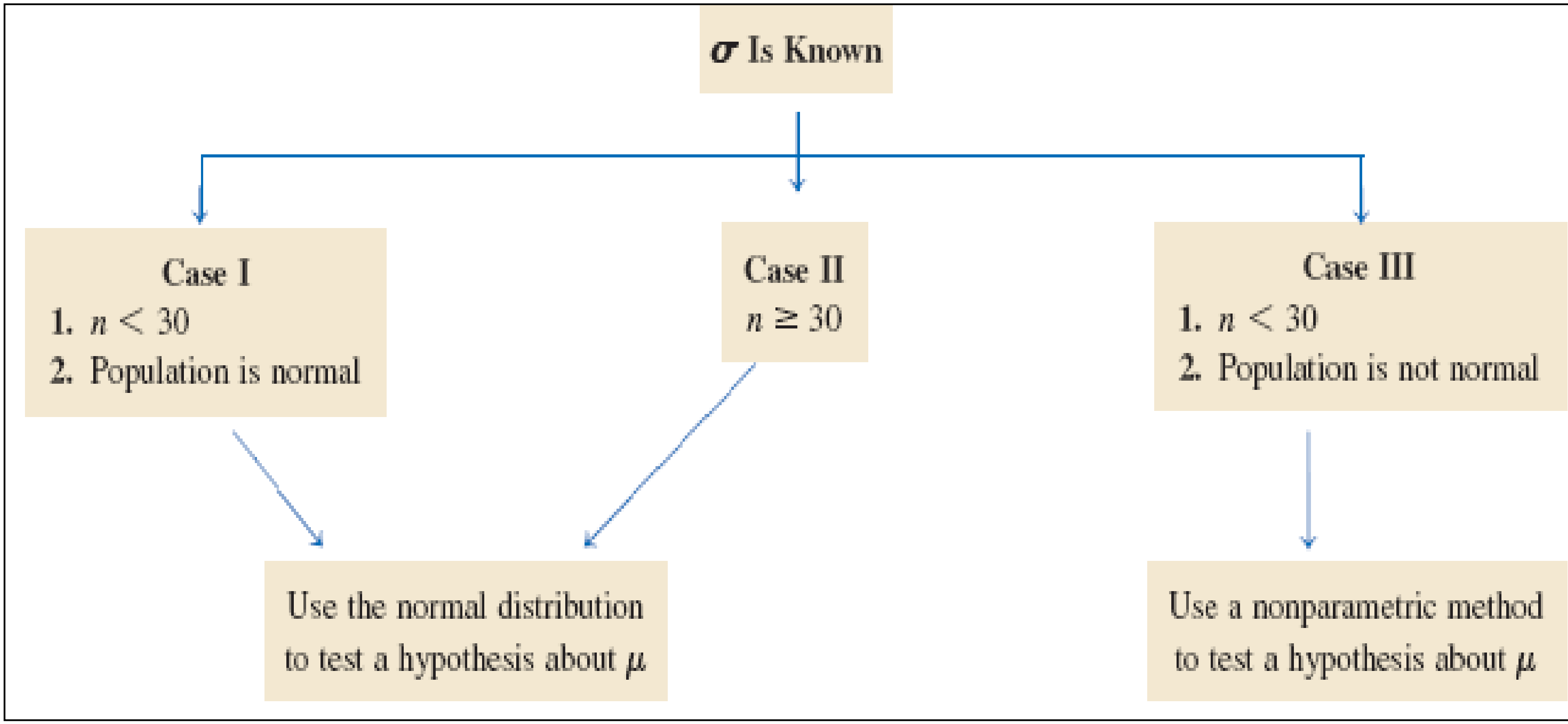
Using our standardized test statistic: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

We find that: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{17.55 - 17.09}{3.87 / \sqrt{100}} = 1.19$

Since z = 1.19 is not greater than 1.96, nor less than –1.96 we cannot reject the null hypothesis in favor of H_1. That is ***"there is insufficient evidence to infer that there is a difference between the bills of AT&T and the competitor."***

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Three Possible Cases



EXAMPLE

The management of Priority Health Club claims that its members lose an average of 10 pounds or more within the first month after joining the club. A consumer agency that wanted to check this claim took a random sample of 36 members of this health club and found that they lost an average of 9.2 pounds within the first month of membership with a standard deviation of 2.4 pounds. Find the p–value for this test. What will you decision be if $\alpha = .01$? What if $\alpha = .05$?

Solution:

Step 1: *State the null and alternative hypotheses*

Step 2: *Select the distribution to use*

Step 3: *Calculate the p - value*

Step 4: *Make a decision*