Probability

We Going To Learn..

- Probability definition
- Conceptual approach to probability
- Mutual exclusive events
- Independent and dependent events
- Conditional probability

Definitions and Terms

- A probability is a measure of the likelihood that an event in the future will happen.
- It can only assume a value between 0 and 1.
- A value near zero means the event is not likely to happen.
- A value near one means it is likely.

Experiment. Outcomes. Sample space. Events. Venn Diagram.

Sample Space

The set which contains all possible outcomes is known as **sample space**

Example :

The outcome of an experiment consists in the determination of the gender of a newborn child can be written as

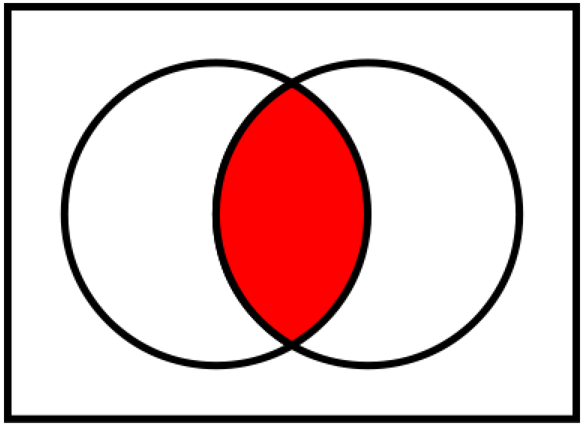
$$\boxed{S = \{g, b\}}$$

Event

A subset of sample space is called an **event**

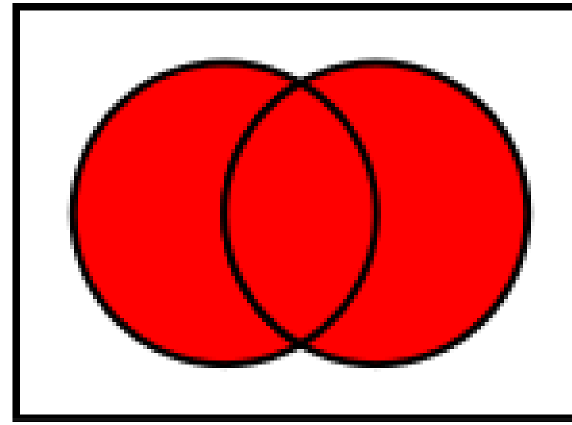
Event of getting baby girl

Venn diagram



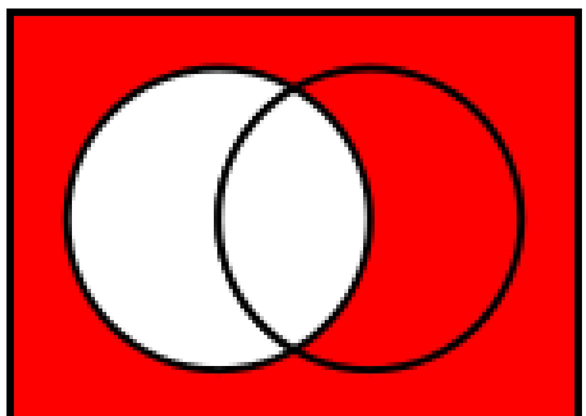
Intersection of two set:

$A \cap B$



Union of two set :

$A \cup B$



Absolute complement:

A^c

Two properties of probability

1. The probability of an event always lies in the range 0 to1

$$0 \leq P(E_i) \leq 1$$

2. The sum of the probabilities of all simple events (or final outcomes) for an experiment is always 1

$$\Sigma P(E_i) = P(E_1) + P(E_2) + P(E_3) + \ldots = 1$$

Three Conceptual Approaches to Probability

- Classical Probability
  – all outcomes are equally likely

- Empirical / Relative Frequency Concept of probability
  – outcomes are not equally likely

- Subjective Probability
  -based on subjective judgment, experience, information, and belief.

Classical Probability

- According to the **classical probability rule**, the probability of a simple event is equal to 1 divided by the total number of outcomes for the experiment.

- The probability of event *A* is the number of ways event *A* can occur divided by the total number of possible outcomes

- **Probability of an Event *A***

$$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of possible outcomes}}$$

Empirical / Relative Frequency Probability

- Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = \frac{\text{Frequency for the class}}{\text{Total frequencies in the distribution}} = \frac{f}{n}$$

Subjective Concept of Probability

- ***Subjective probability*** is the probability assigned to an event based on subjective judgment, experience, information, and belief.

- It is usually influenced by the biases, preferences, and experience of the person assigning the probability.

Summary

- If $P(A) > P(B)$ then event A is more likely to occur than event B.
- If $P(A) = P(B)$ then events A and B are equally likely to occur.

Probability laws

Properties of probability

- The probability of an event ranges from 0 to 1
- The sum of probabilities of all possible events equals 1

$$P(A) = \frac{n(A)}{n(S)}$$

Rule of subtraction

The probability that event A will occur is equal to **1 minus the probability that event A will not occur.**

$$P(A) = 1 - P(A^{'})$$

Example :

The probability that Ted will graduate from college is 0.80. What is the probability that Ted will not graduate from the college?

Rule of multiplication

The probability that Events A and B both occur is equal to the probability that Event A occurs times the probability that Event B occurs, given that A has occurred.

$$\boxed{P(A \cap B) = P(A)P(B \mid A)}$$

- Applies to the situation when we want to know the probability of the intersection of two events; that is, we want to know the **probability that two events (Even A and Event B) both occur.**

Rule of addition

The probability that Event A or Event B occurs is equal to the **probability that Event A occurs plus the probability that Event B occurs minus the probability that both Event A and B occur**

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Rule Of Addition : EXAMPLE

A student goes to the library. The probability that she checks out (a) a work of fiction is 0.40, (b) a work of non-fiction is 0.30, and (c) both fiction is 0.20. What is the probability that the student checks out a work of fiction, non fiction, or both?

Solution:

Let F= the event that the student checks out fiction; and let N= the event that the student checks out non-fiction. Thus,

$$P(F \cup N) = P(F) + P(N) - P(F \cap N)$$

$$P(F \cup N) = 0..40 + 0.30 - 0.20 = 0.50$$

Independent and mutually exclusive events.

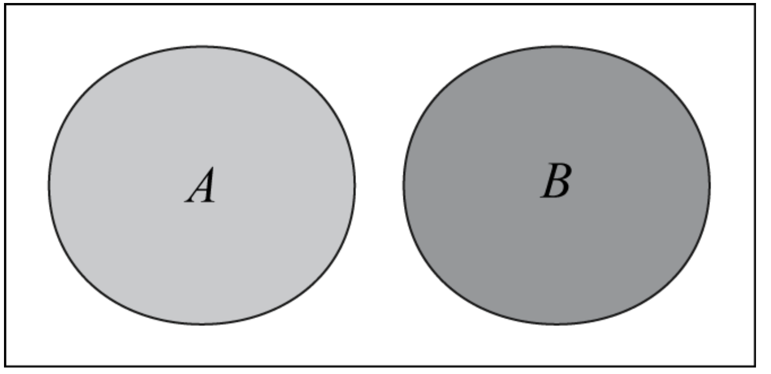
Mutually exclusive event

Definition of a mutually exclusive event

If event A happens, then event B cannot, or vice-versa. The two events **"it rained on Tuesday"** and **"it did not rain on Tuesday"** are mutually exclusive events. When calculating the probabilities for exclusive events you add the probabilities

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Mutually exclusive event



Two events A and B are **mutually exclusive** if they have no elements in common or,

$$\boxed{A \cap B = \varnothing}$$

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Independent event

.

Independent events

The outcome of event A, has no effect on the outcome of event B. Such as **"It rained on Tuesday"** and **"My chair broke at work"**. When calculating the probabilities for independent events you multiply the probabilities. You are effectively saying what is the chance of both events happening bearing in mind that the two were unrelated.

$$\boxed{P\left(A \cap B\right) = P(A) \times P(B)}$$

Thus,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A) \times P(B)}$$

Example:

A and B are two events such that $P(A) = \dfrac{2}{5}$ and $P(B) = \dfrac{1}{4}$.

Find $P(A \cup B)$ if A and B are

a) mutually exclusive

b) independent events.

Example:

A weather forecast from radio announced that the probabilities of raining today and tomorrow are 0.5 and 0.3 respectively.
What is the probability that both days will rain ?

Let event A = it will rain today
event B = it will rain tomorrow

** the outcome of weather for today would not affect the weather for tomorrow. So, **events A and B are independent.**

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

$$\boxed{= 0.5 \times 0.3 = 0.15}$$

Hence, the probability that both days will rain is 0.15.

Conditional probability

Conditional probability

Defined as :

The probability of event *A* happening given that **event *B* has happened**.

Denoted by :

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

If A and B are independent,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

Conditional probability : Example

A and B are two events such that :

$P(A) = 0.5, P(B) = 0.7$ and $P(A \cap B) = 0.4$. Calculate

a) $P(A \mid B)$ b) $P(B \mid A)$ c) $P(A \cup B)$

Independent Vs Dependent Event

If A and B are independent, then

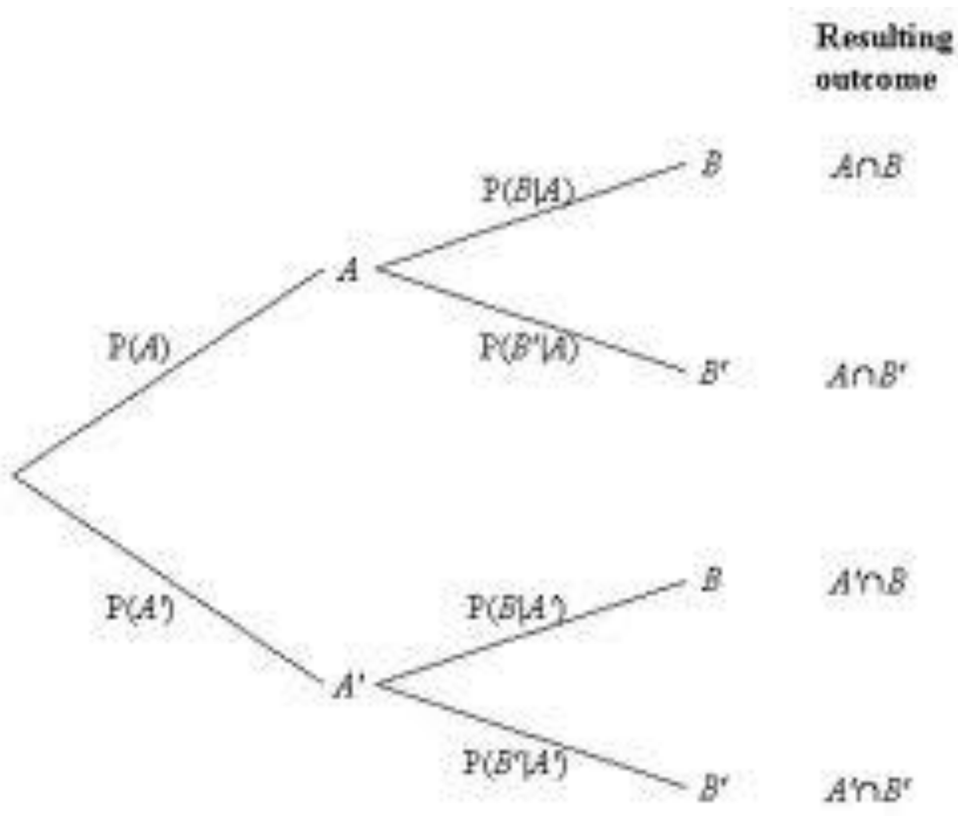
$$P(A \cap B) = P(A)P(B)$$

If A and B are dependent, then

$$P(A \cap B) = P(A)P(B \mid A)$$

Tree Diagram

- Useful way to compute the conditional probabilities.



Conditional probability: example

Joe plans to buy a new car. The probability that he will get a buyer for his old car is $\frac{4}{5}$. The probability that Joe will buy a new car if he gets a buyer for his old car is $\frac{7}{8}$. On the other hand, the probability that Joe will still buy a new car even though he fails to get a buyer for his old car is $\frac{5}{9}$.

a) Find the probability that Joe will buy a new car.

b) If it is known that Joe did not buy a new car, what is the probability that he will fail to get a buyer for his old car?

$a)\frac{73}{90}$ b) $\frac{8}{17}$

Conditional probability: example

Step 1 : List down all the information given.

Let C = Joe buy new car

B = Joe find buyer for his old car

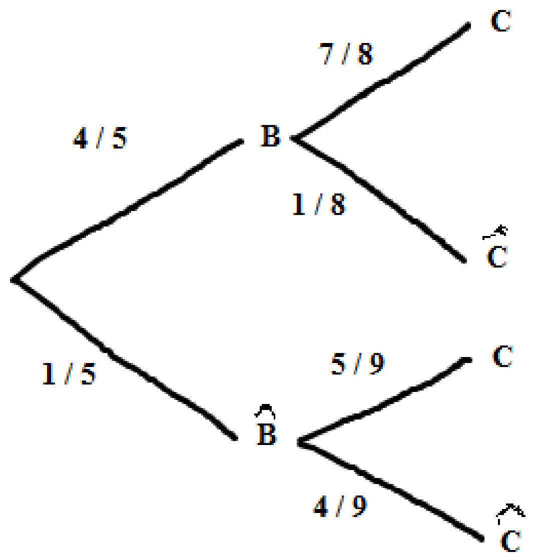
Conditional probability: example

Step 2 : Draw a tree diagram

Conditional probability: example

Step 3 : Find the probability of Joe buying new car

$$P(C) = P(B)P(C \mid B) + P(\widehat{B})P(C \mid \widehat{B})$$



Conditional probability: example

b) If it is known that Joe did not buy a new car, what is the probability that he will fail to get a buyer for his old car?

TRY THIS!

In an examination, the probability that a student will pass Mathematics I is 0.8. If the student passes Mathematics I, the probability that the student will pass Mathematics II is 0.9. If the student fails Mathematics I, the probability that the student will fail Mathematics II is 0.4.

Calculate the probability that the student will pass

a) Mathematics II.

b) Mathematics I given that the student passes Mathematics II.

$$\boxed{a)\frac{21}{25} \quad \text{b)}\frac{6}{7}}$$