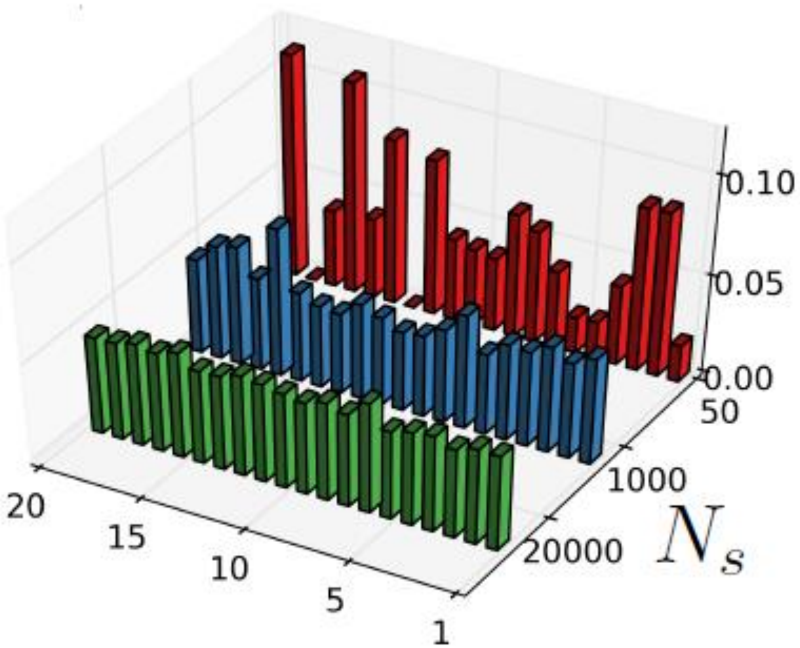
Many body quantum state tomography with neural networks

Sim Jun Yan

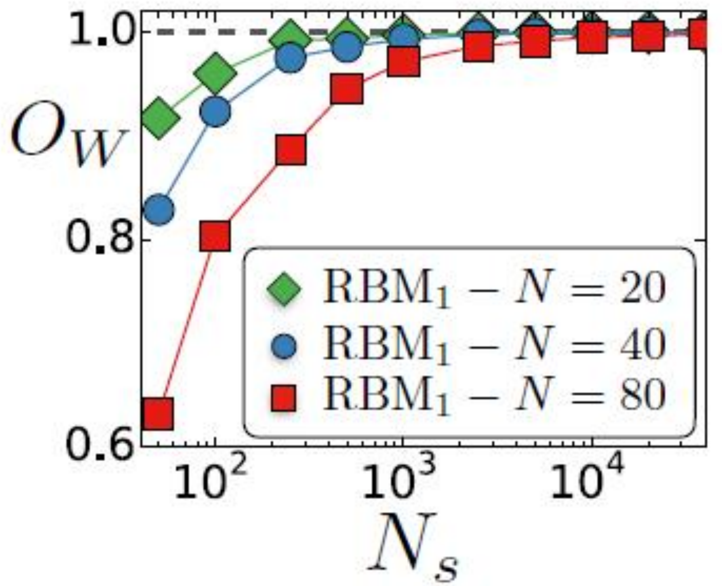
Numerical result

W state



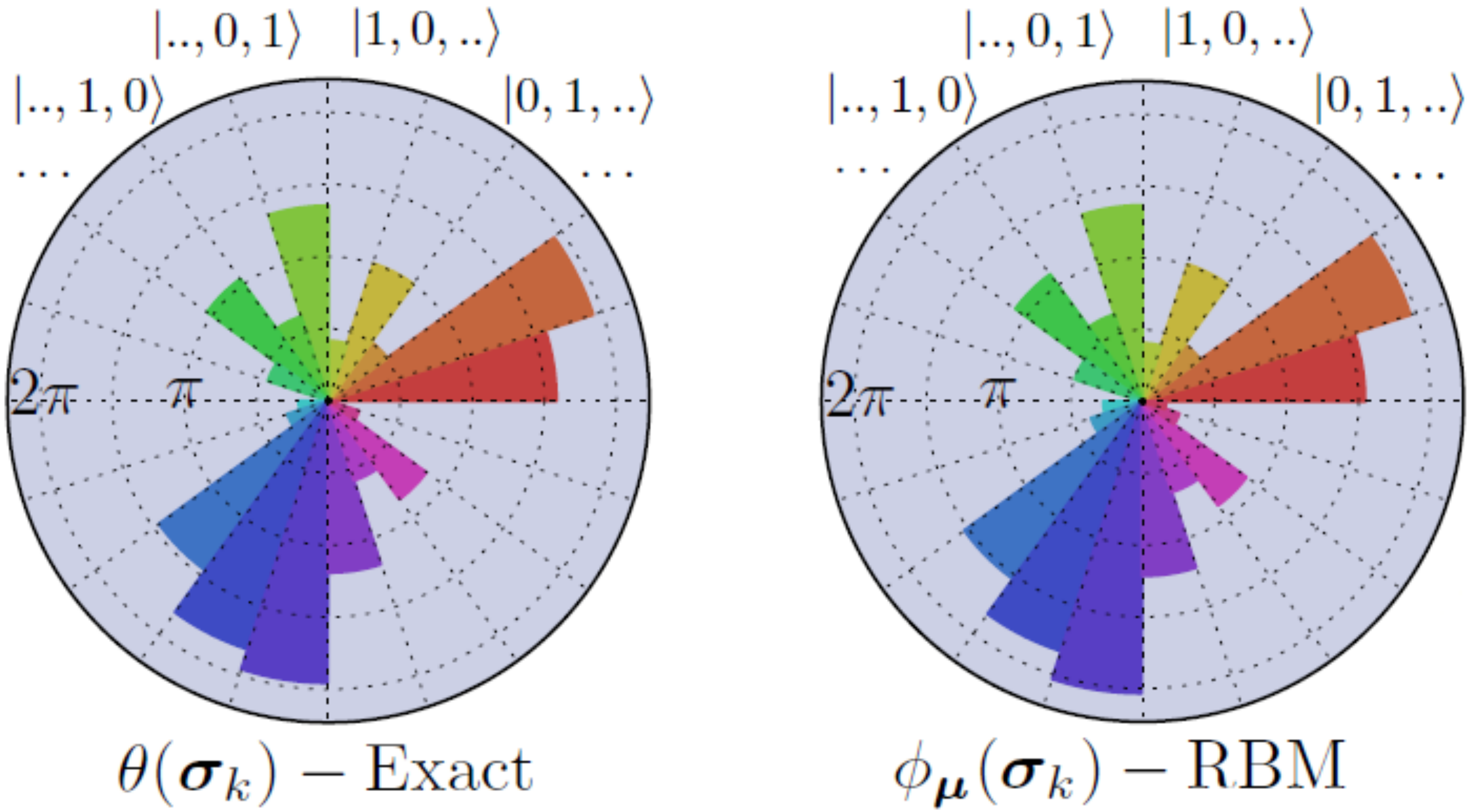
N_s is the number of training samples

Overlap (fidelity) between W state and the RBM wavefunction



For N=20, fidelity=0.99 with $N_s = 300$
Cf. MPS: fidelity=0.96 with $N_s = 400$

W state with random local phase shifts (N=20)

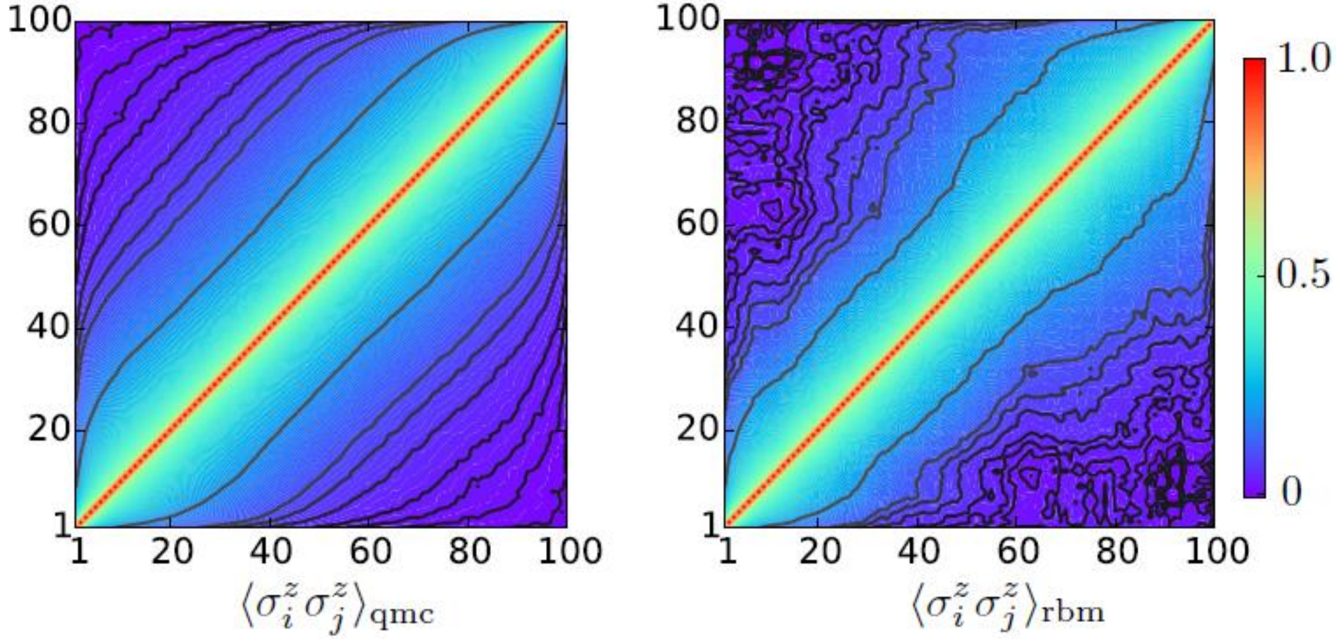


6400 samples per basis, for 2N-1 total basis

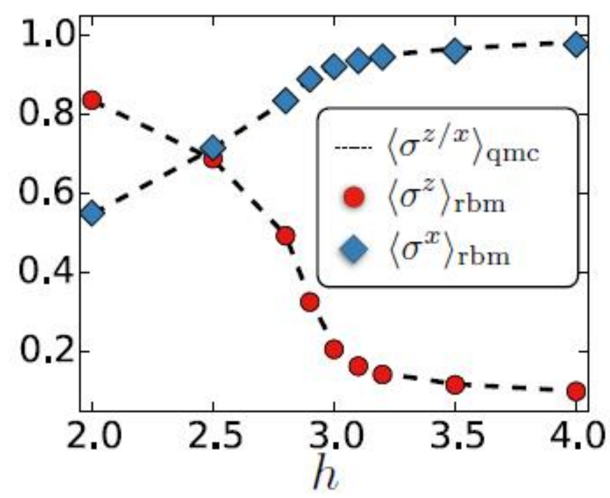
Comparison of the phase

Ground state of Transverse Field Ising Model (TFIM)

1d-TFIM N=100 h=1

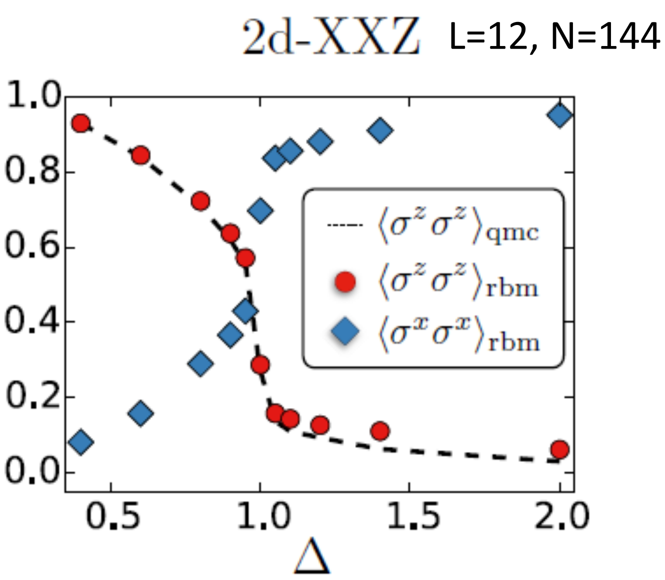


2d-TFIM L=12, N=144



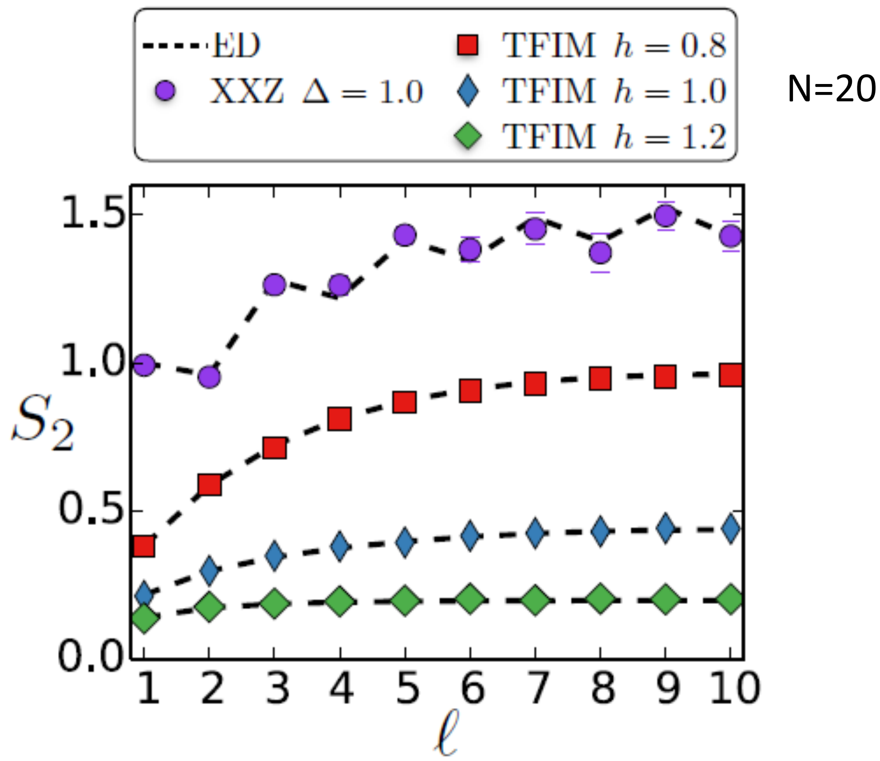
$$\langle\sigma^x\rangle = \sum_{i=1}^{N}\langle\sigma_i^x\rangle$$

Ground state of XXZ



Entanglement Entropy

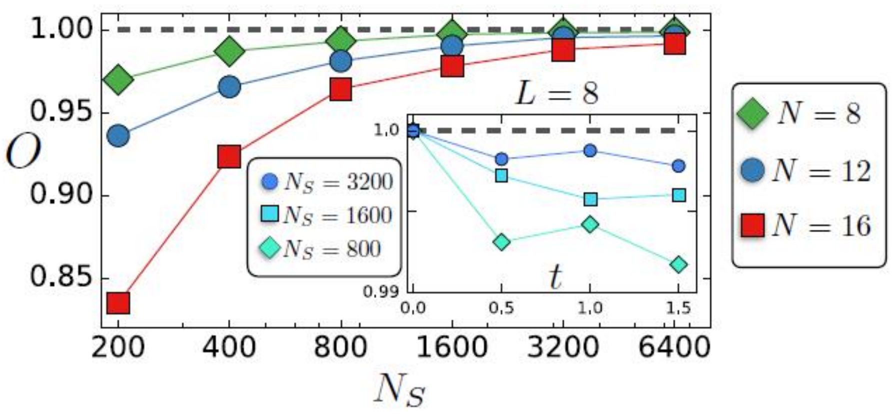
$S_2(\rho_A) = -\log(\mathrm{Tr}(\rho_A^2))$



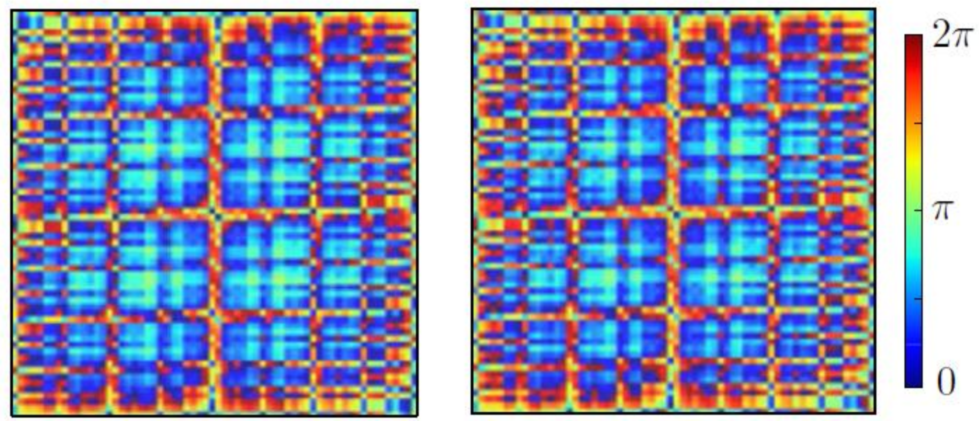
ℓ is the size of the subsystem ρ_A

Quantum dynamics of Long Range Ising Model (LRIM)

1d-LRIM $\gamma = \frac{3}{4}$ t=0.5



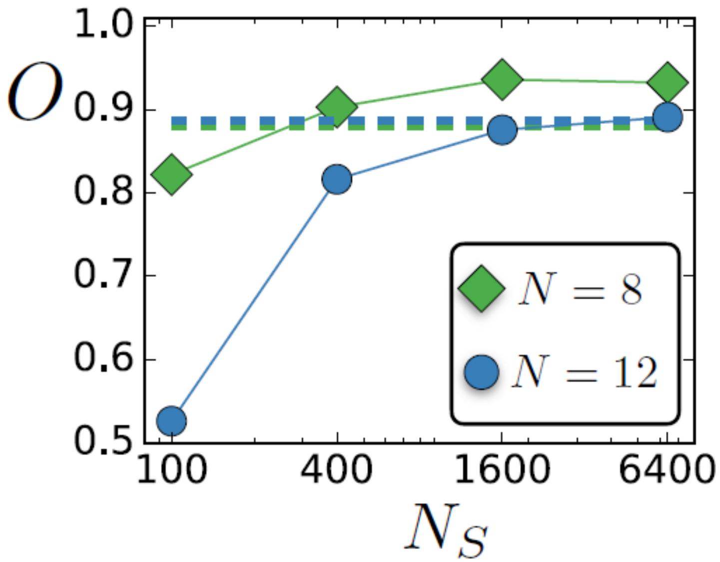
Comparison of the phase

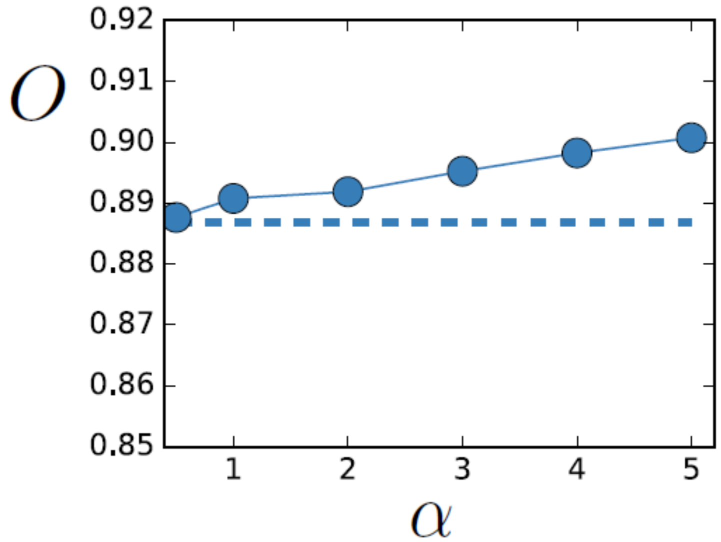


$\theta(\boldsymbol{\sigma}_k)$ – Exact $\phi_{\boldsymbol{\mu}}(\boldsymbol{\sigma}_k)$ – RBM

N=12, t=0.5

Random State





The wavefunctions in the previous examples all have some kind of structure which explains the good performance. In this example, consider an unstructured wavefunction, i.e. a randomly chosen state.

In the first figure, the overlap saturates rather quickly.

In the second figure, the overlap shows little improvement even when α which is the ratio of hidden units to the visible units increase to 5. Whereas previous examples shows pretty good result with $\frac{1}{4} \leq \alpha \leq 1$.

Applications of machine learning technique generally fails when there is little structure in the wavefunction.

Representational power and limitations of RBM

List from Z.-A. Jia, B. Yi, R. Zhai, Y.-C. Wu, G.-C. Guo, and G.-P. Guo, "Quantum Neural Network States," arxiv: 1808.10601

States that can be efficiently represented by RBM:

- $\mathbb{Z}_2$-toric code states[23];
- Graph states[25];
- Stabilizer states with generators of pure type, $\mathbf{S}_X, \mathbf{S}_Y, \mathbf{S}_Z$ and their arbitrary union[24];
- Perfect surface code states, surface code states with boundary, defect and twist[24];
- Kitaev's $D(\mathbb{Z}_d)$ quantum double ground states[24];

States that cannot be efficiently represented by RBM:

universal quantum computational states, PEPS, and ground states of k-local Hamiltonians

States that can be efficiently represented by Deep Boltzmann Machine (DBM):

- Any n-qubit quantum states generated by a quantum circuit of depth T, the number of hidden neurons is $O(nT)$[25];
- Tensor network states consist of n-local tensors with bound dimension D and maximum coordination number d, the number of hidden neurons is $O(nD^{2d})$[25];
- The ground states of Hamiltonian with gap Δ, the number of hidden neurons is $O(\frac{m^2}{\Delta}(n - \log \epsilon))$ where ϵ is the representational error[25];

D.-L. Deng, X. Li, and S. Das Sarma, "Machine learning topological states," Phys. Rev. B 96, 195145 (2017).
Z.-A. Jia, Y.-H. Zhang, Y.-C. Wu, G.-C. Guo, and G.-P. Guo, "Efficient machine learning representations of surface code with boundaries, defects, domain walls and twists," arXiv preprint arXiv:1802.03738 (2018).
X. Gao and L.-M. Duan, "Efficient representation of quantum many-body states with deep neural networks," Nature Communications 8, 662 (2017).