9. Optics

Can you recall?

1. What are laws of reflection and refraction?
2. What is dispersion of light?
3. What is refractive index?
4. What is total internal reflection?
5. How does light refract at a curved surface?
6. How does a rainbow form?

9.1 Introduction:

"See it to believe it" is a popular saying. In order to see, we need light. What exactly is light and how are we able to see anything? We will explore it in this and next standard. We know that acoustics is the term used for science of sound. Similarly, optics is the term used for science of light. There is a difference in the nature of sound waves and light waves which you have seen in chapter 8 and will learn in chapter 13.

9.2 Nature of light:

Earlier, light was considered to be that form of radiant energy which makes objects visible due to stimulation of retina of the eye. It is a form of energy that propagates in the presence or absence of a medium, which we now call waves. At the beginning of the 20th century, it was proved that these are electromagnetic (EM) waves. Later, using quantum theory, particle nature of light was established. According to this, photons are energy carrier particles. By an experiment using countable number of photons, it is now an established fact that light possesses dual nature. In simple words we can say that light consists of energy carrier photons guided by the rules of EM waves. In vacuum, these waves (or photons) travel with a speed of c = 299792458 m s^{-1} According to Einstein's special theory of relativity, this is the maximum possible speed for any object. For practical purposes we write it as $c = 3\times10^8$ m s^{-1}.

In a material medium, the speed of EM waves is given by $c = \sqrt{\frac{1}{\epsilon \mu}}$,
where permittivity ε and permeability μ are constants which depend on the electric and magnetic properties of the medium.

The ratio $n = \frac{c}{\text{v}}$ is called the absolute refractive index and is the property of the medium.

Commonly observed phenomena concerning light can be broadly split into three categories.

(I) **Ray optics or geometrical optics:** A particular direction of propagation of energy from a source of light is called a ray of light. We use ray optics for understanding phenomena like reflection, refraction, double refraction, total internal reflection, etc.

(II) **Wave optics or physical optics:** For explaining phenomena like interference, diffraction, polarization, Doppler effect, etc., we consider light energy to be in the form of EM waves. Wave theory will be further discussed in XIIth standard.

(III) **Particle nature of light:** Phenomena like photoelectric effect, emission of spectral lines, Compton effect, etc. cannot be explained by using classical wave theory. These involve the interaction of light with matter. For such phenomena we have to use quantum nature of light. Quantum nature of light will be discussed in XIIth standard.

9.3 Ray optics or geometrical optics:

In geometrical optics, we mainly study image formation by mirrors, lenses and prisms. It is based on four fundamental laws/ principles which you have learnt in earlier classes.

(i) Light travels in a straight line in a homogeneous and isotropic medium. Homogeneous means that the properties of the medium are same every where in the medium and isotropic means that the properties are the same in all directions.

(ii) Two or more rays can intersect at a point without affecting their paths beyond that point.

(iii) **Laws of reflection:**

(a) Reflected ray lies in the plane formed by incident ray and the normal drawn at the point of incidence; and the two rays are on either side of the normal.

(b) Angles of incidence and reflection are equal.

(iv) **Laws of refraction:** These apply at the boundary between two media

(a) Refracted ray lies in the plane formed by incident ray and the normal drawn at the point of incidence; and the two rays are on either side of the normal.

(b) Angle of incidence (θ_1 in a medium of refractive index n_1) and angle of refraction (θ_2 in medium of refractive index n_2) are related by Snell's law, given by

$(n_1)\sin\theta_1 = (n_2)\sin\theta_2$.

Do you know ?

Interestingly, all the four laws stated above can be derived from a single principle called Fermat's (pronounced "Ferma") principle. It says that "While travelling from one point to another by one or more reflections or refractions, a ray of light always chooses the path of least time".

Ideally it is the path of extreme time, i.e., path of minimum or maximum time. We strongly recommend you to go through a suitable reference book that will give you the proof of i = r during reflection and Snell's law during refraction using Fermat's principle.

Example 9.1: Thickness of the glass of a spectacle is 2 mm and refractive index of its glass is 1.5. Calculate time taken by light to cross this thickness. Express your answer with the most convenient prefix attached to the unit 'second'.

Solution:

Speed of light in vacuum, $c = 3\times10^8$ m/s

$n_{glass} = 10.5$

$\therefore$ Speed of light in glass =

$$\frac{c}{n_{glass}} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$

Distance to be travelled by light in glass,

s = 2 mm = 2×10^{-3} m

$\therefore$ Time t required by light to travel this distance,

$$t = \frac{s}{v_{glass}} = \frac{2\times10^{-3}}{2\times10^8} = 10^{-11}\text{ s}$$

Most convenient prefix to express this small time is pico (p) = 10^{-12}

$\therefore$ t = 10×10^{-12} = 10 ps

9.3.1 Cartesian sign convention:

While using geometrical optics it is necessary to use some sign convention. The relation between only the numerical values of u, v and f for a spherical mirror (or for a lens) will be different for different positions of the object and the type of mirror. Here u and v are the distances of object and image respectively from the optical center, and f is the focal length. Properly used suitable sign convention enables us to use the same formula for all different particular cases. Thus, while deriving a formula and also while using the formula it is necessary to use the same sign convention. Most convenient sign convention is Cartesian sign convention as it is analogous to coordinate geometry. According to this sign convention, (Fig. 9.1):

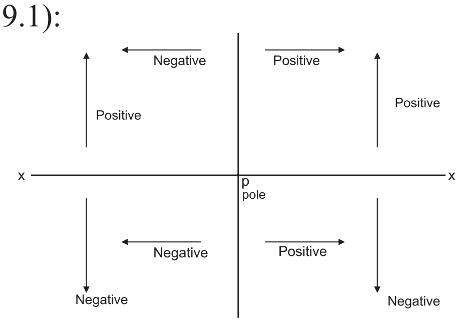


Fig. 9.1 Cartesian sign convention.

i) All distances are measured from the optical center or pole. For most of the optical objects such as spherical mirrors, thin lenses, etc., the optical centers coincides with their geometrical centers.

ii) Figures should be drawn in such a way that the incident rays travel from left to right. A diverging beam of incident rays corresponds to a real point object (Fig. 9.2 (a)), a converging beam of incident rays corresponds to a virtual object (Fig. 9.2 (b)) and a parallel beam corresponds to an object at infinity. Thus, a real object should be shown to the left of pole (Fig. 9.2 (a)) and virtual object or image to the right of pole. (Fig. 9.2 (b))

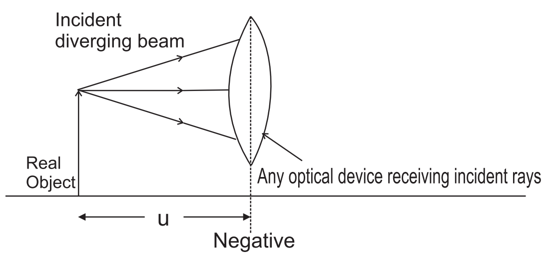


Fig. 9.2: (a) Diverging beam from a real object

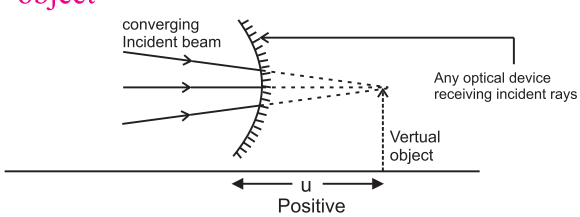


Fig. 9.2: (b) Converging beam towards a virtual object.

iii) x-axis can be conveniently chosen as the principal axis with origin at the pole.

iv) Distances to the left of the pole are negative and those to the right of the pole are positive.

v) Distances above the principal axis (x-axis) are positive while those below it are negative.

Unless specially mentioned, we shall always consider objects to be real for further discussion.

9.4 Reflection:

9.4.1 Reflection from a plane surface:

a) If the object is in front of a plane reflecting surface, the image is virtual and laterally inverted. It is of the same size as that of the object and at the same distance as that of object but on the other side of the reflecting surface.

b) If we are standing on the bank of a still water body and look for our image formed by water (or if we are standing on a plane mirror and look for our image formed by the mirror), the image is laterally reversed, of the same size and on the other side.

c) If an object is kept between two plane mirrors inclined at an angle θ (like in a kaleidoscope), a number of images are formed due to multiple reflections from both the mirrors. Exact number of images depends upon the angle between the mirrors and where exactly the object is kept. It can be obtained as follows (Table 9.1):

Calculate $n = \dfrac{360}{\theta}$

Let N be the number of images seen.

(I) If n is an even integer, $N = (n-1)$, irrespective of where the object is.

(II) If n is an odd integer and object is exactly on the angle bisector, $N = (n-1)$.

(III) If n is an odd integer and object is off the angle bisector, $N = n$

(IV) If n is not an integer, $N = m$, where m is integral part of n.

Table 9.1

Angle θ^0	$n = \dfrac{360}{\theta}$	**Position of the object**	***N***
120	3	On angle bisector	2
120	3	Off angle bisector	3
110	3.28	Anywhere	3
90	4	Anywhere	3
80	4.5	Anywhere	4
72	5	On angle bisector	4
72	5	Off angle bisector	5
60	6	Anywhere	5
50	7.2	Anywhere	7

Example 9.2: A small object is kept symmetrically between two plane mirrors inclined at 38°. This angle is now gradually increased to 41°, the object being symmetrical all the time. Determine the number of images visible during the process.

Solution: According to the convention used in the table above,

$$\theta = 38^0 \therefore n = \frac{360}{38} = 9.47$$

$\therefore N = 9$. This is valid till the angle is 40° as the object is kept symmetrically

Beyond 40°, $n < 9$ and it decreases upto

$$\frac{360}{41} = 8.78.$$

Hence now onwards there will be 8 images till 41°.

9.4.2 Reflection from curved mirrors:

In order to focus a parallel or divergent beam by reflection, we need curved mirrors. You might have noticed that reflecting mirrors for a torch or headlights, rear view mirrors of vehicles are not plane but concave or convex. Mirrors for a search light are parabolic. We shall restrict ourselves to spherical mirrors only which can be studied using simple mathematics. Such mirrors are parts of a sphere polished from outside (convex) or from inside (concave).

Radius of the sphere of which a mirror is a part is called as radius of curvature (R) of the mirror. Only for spherical mirrors, half of radius of curvature is focal length of the mirror $\left(f = \frac{R}{2}\right)$. For a concave mirror it is the distance at which parallel incident rays converge. For a convex mirror, it is the distance from where parallel rays appear to be diverging after reflection. According to sign convention, the incident rays are from left to right and they should face the polished surface of the mirror. Thus, focal length of a convex mirror is positive (Fig 9.3 (a)) while that of a concave mirror is negative (Fig. 9.3 (b)).

Relation between *f, u* and *v*:

For a point object or for a small finite object, the focal length of a *small* spherical (concave or convex) mirror is related to object distance and image distance as

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{--- (9.1)}$$

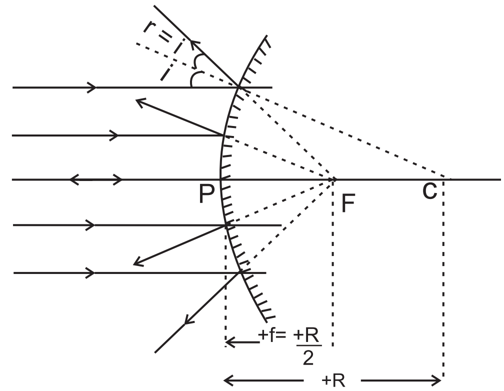


Fig. 9.3 (a): Parallel rays incident from left appear to be diverging from F, lying on the positive side of origin (pole).

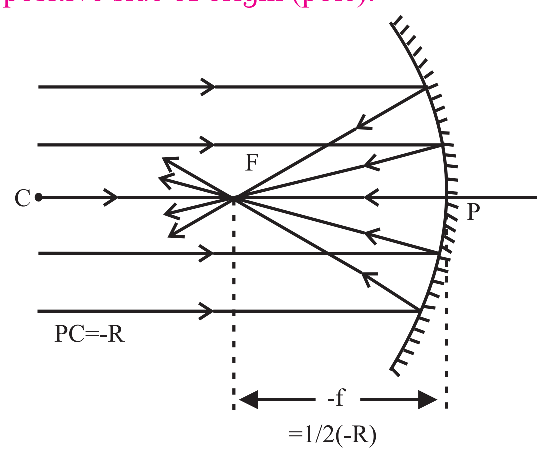


Fig. 9.3 (b): Parallel rays incident from left appear to converge at F, lying on the negative side of origin (pole).

By a *small* mirror we mean its aperture (diameter) is much smaller (at least one tenth) than the values of u, v and f.

Focal power: Converging or diverging ability of a lens or of a mirror is defined as its focal power. It is measured as $P = \frac{1}{f}$.

In SI units, it is measured as diopter. $\therefore 1\, dioptre(D) = 1m^{-1}$

Lateral magnification: Ratio of linear size of an image to that of the object, measured perpendicular to the principal axis, is defined as the lateral magnification $m = \frac{v}{u}$

For any position of the object, a convex mirror always forms virtual, erect and diminished image, $m<1$. In the case of a concave mirror it depends upon the position of the object. Following Table 9.2 will help you refresh your knowledge.

Table 9.2

Concave mirror (f negative)			
Position of object	**Position of image**	**Real(R) or Virtual (V)**	**Lateral magnifi- -cation**
$u=\infty$	$v=f$	R	$m=0$
$u>2f$	$2f>v>f$	R	$m<1$
$u=2f$	$v=2f$	R	$m=1$
$2f>u>f$	$v>2f$	R	$m>1$
$u=f$	$v=\infty$	R	$m=\infty$
$u<f$	$\lvert v\rvert>\lvert u\rvert$	V	$m>1$

Example 9.3: A thin pencil of length 20 cm is kept along the principal axis of a concave mirror of curvature 30 cm. Nearest end of the pencil is 20 cm from the pole of the mirror. What will be the size of image of the pencil?

Solution: $R = 30$ cm

$f = R/2 = -15$ cm ... (Concave mirror)

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

For nearest end, $u = u_1 = -20$ cm . Let the image distance be v_1

$$\therefore \frac{1}{-15}=\frac{1}{v_1}+\frac{1}{-20} \quad \therefore v_1=-60\,\text{cm}$$

Nearest end is at 20 cm and pencil itself is 20 cm long. Hence farthest end is 20 + 20 = 40 cm $=-u_2$

Let the image distance be v_2

$$\therefore \frac{1}{-15}=\frac{1}{v_2}+\frac{1}{-40} \quad \therefore v_2=-24\,\text{cm}$$

$\therefore$ Length of the image = 60 -24 = 36 cm.

Defects or aberration of images: The theory of image formation by mirrors or lenses, and the formulae that we have used such as

$$f=\frac{R}{2} \text{ or } \frac{1}{f}=\frac{1}{v}+\frac{1}{u} \text{ etc.}$$

are based on the following assumptions: (i) Objects and images are situated close to the principal axis.

(ii) Rays diverging from the objects are confined to a cone of very small angle.

(iii) If there is a parallel beam of rays, it is paraxial, i.e., parallel and close to the principal axis.

However, in reality, these assumptions do not always hold good. This results into distorted or defective image. Commonly occurring defects are spherical aberration, coma, astigmatism, curvature, distortion. Except spherical aberration, all the other arise due to beams of rays inclined to principal axis. These are not discussed here.

Spherical aberration: As mentioned earlier, the relation $\left(f=\frac{R}{2}\right)$ giving a single point focus is applicable only for small aperture spherical mirrors and for paraxial rays. In reality, when the rays are farther from the principal axis, the focus gradually shifts towards pole (Fig. 9.4). This phenomenon (defect) arises due to spherical shape of the reflecting surface, hence called as spherical aberration. It results into a unsharp (fuzzy) image with unclear boundaries.

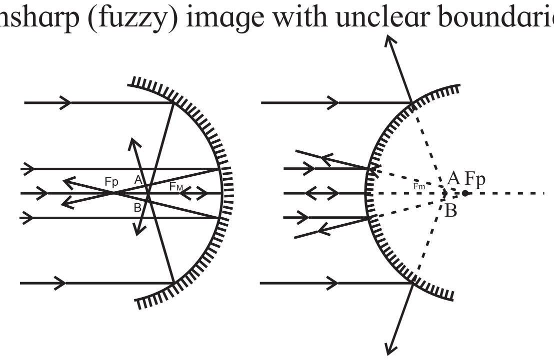


Fig. 9.4: Spherical aberration for curved mirrors.

The distance between F_M and F_P (Fig. 9.4) is measured as the longitudinal spherical aberration. If there is no spherical aberration, we get a single point image on a screen placed perpendicular to the principal axis at that location, for a beam of incident rays parallel to the axis. In the presence of spherical aberration, no such point is possible at any position of the screen and the image is always a circle. At a particular location of the screen, the diameter of this circle is minimum. This is called the circle of least confusion. In the figures it is across AB. Radius of this circle is transverse spherical aberration.

In the case of curved mirrors, this defect can be completely eliminated by using a parabolic mirror. Hence surfaces of mirrors used in a search light, torch, headlight of a car, telescopes, etc., are parabolic and not spherical.

Do you know ?

Why does a parabolic mirror not have spherical aberration?

Parabola is a geometrical shape drawn in such a way that every point on it is equidistant from a straight line and from a point. Figure 9.5 shows a parabola. Points A, B, C, ... on it are equidistant from line RS (called directrix) and point F (called focus). Hence A′A = AF, B′B = BF, C′C = CF, ...

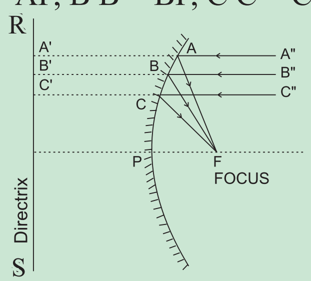


Fig. 9.5: Single focus for parabolic mirror.

If rays of equal optical path converge at a point, that point is the location of real image corresponding to that beam of rays.

Paths A″AA′, B″BB′. C″CC′, etc., are equal paths in the absence of mirror. If the parabola ABC... is a mirror then the respective optical paths will be A″AF, B″BF, C″CF, ... and from the definition of parabola, these are also equal. Thus, F is the single point focus for entire beam parallel to the axis with NO spherical aberration.

9.5 Refraction:

Being an EM wave, the properties of light (speed, wavelength, direction of propagation, etc.) depend upon the medium through which it is traveling. If a ray of light comes to an interface between two media and enters into another medium of different refractive index, it changes itself suitable to that medium. This phenomenon is defined as refraction of light. The extent to which these properties change is decided by the index of refraction, 'n'.

Absolute refractive index:

Absolute refractive index of a medium is defined as the ratio of speed of light in vacuum to that in the given medium.

$n = \frac{c}{v}$ where c and v are respective speeds of light in vacuum and in the medium. As n is the ratio of same physical quantities, it is a unitless and dimensionless physical quantity.

For any material medium (including air) $n > 1$, i.e., light travels fastest in vacuum than in any material medium. Medium having greater value of n is called optically denser. An optically denser medium need not be physically denser, e.g., many oils are optically denser than water but water is physically denser than them.

Relative refractive index:

Refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light v_1 in medium 1 to its speed v_2 in medium 2. Thus, ${}^1n_2 = \frac{n_2}{n_1} = \frac{v_1}{v_2}$

Do you know ?

(a) Logic behind the convention ${}^1n_2$: Letter n is the symbol for refractive index, n_2 corresponds to refractive index of medium 2 and ${}^1n_2$ indicates that it is with respect to medium 1. In this case, light travels from medium 1 to 2 so we need to discuss medium 2 in context to medium 1.

(b) Dictionary meaning of the word refract is to change the path`. However, in context of Physics, we should be more specific. We use the word *deviate* for changing the path. During refraction at normal incidence, there is no change in path. Thus, there is *refraction* but no *deviation*. Deviation is associated with refraction *only* during oblique incidence. Deviation or changing the path or bending is associated with many phenomena such as reflection, diffraction, scattering, gravitational bending due to a massive object, etc.

Illustrations of refraction: 1) When seen from outside, the bottom of a water body appears to be raised. This is due to refraction at the plane surface of water. In this case,

$$n_{water} \cong \frac{Real\ depth}{apparent\ depth}$$

This relation holds good for a plane parallel transparent slab also as shown below.

Figure 9.6 shows a plane parallel slab of a transparent medium of refractive index *n*. A point object O at real depth *R* appears to be at I at apparent depth *A*, when seen from outside (air). Incident rays OA (traveling undeviated) and OB (deviating along BC) are used to locate the image.

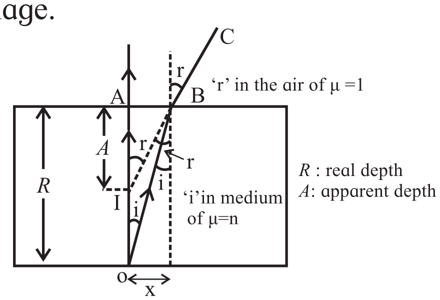


Fig. 9.6: Real and apparent depth.

By considering i and r to be small, we can write,

$$\tan(r) = \frac{x}{A} \cong \sin(r) \text{ and } \tan(i) = \frac{x}{R} \cong \sin(i)$$

$$\therefore n = \frac{\sin(r)}{\sin(i)} \cong \frac{\left(\frac{x}{A}\right)}{\left(\frac{x}{R}\right)} = \frac{R}{A} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

2) A stick or pencil kept obliquely in a glass containing water appears broken as its part in water appears to be raised.

> Small angle approximation: For small angles, expressed in radian, sin $\theta \cong \theta \cong \tan\theta$.
> For example, for
>
> $$\theta = 30^0 = \left(\frac{\pi}{6}\right)^c = 0.5236^c,$$
>
> $$\text{we have } \sin\theta = 0.5$$
>
> In this case the error is $0.5236 - 0.5 = 0.0236$ in 0.5, which is 4.72 %.
> For practical purposes we consider angles less than 10^0 where the error in using sin $\theta \cong \theta$ is less than 0.51 %. (Even for 60^0, it is still 15.7 %)
> It is left to you to verify that this is almost equally valid for $\tan\theta$ till 20^0 only.

Example 9. 4: A crane flying 6 m above a still, clear water lake sees a fish underwater. For the crane, the fish appears to be 6 cm below the water surface. How much deep should the crane immerse its beak to pick that fish?

For the fish, how much above the water surface does the crane appear? Refractive index of water = 4/3.

Solution: For crane, apparent depth of the fish is 6 cm and real depth is to be determined.

For fish, real depth (height, in this case) of the crane is 6 m and apparent depth (height) is to be determined.

$$n = \frac{R}{A} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

For crane, it is water with respect to air as real depth is in water and apparent depth is as seen from air

$$\therefore n = \frac{4}{3} = \frac{R}{A} = \frac{R}{6} \quad \therefore R = 8\,\text{cm}$$

For fish, it is air with respect to water as the real height is in air and seen from water.

$$\therefore n = \frac{3}{4} = \frac{R}{A} = \frac{6}{A} \quad \therefore A = 8\,\text{m}$$

9.6 Total internal reflection:

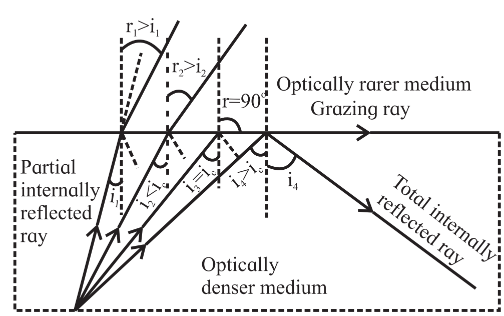


Fig. 9.7: Total internal reflection.

Figure 9.7 shows refraction of light emerging from a denser medium into a rarer medium for various angles of incidence. The angles of refraction in the rarer medium are larger than the corresponding angles of incidence. At a particular angle of incidence i_c in the denser medium, the corresponding angle of refraction in the rarer medium is 90^0. For angles of incidence greater than i_c, the angle of refraction become larger than 90^0 and the ray does not enter into rarer medium at all but is reflected totally into the denser medium. This is called total internal reflection. In general, there is always partial reflection and partial refraction at the interface. During total internal reflection TIR, it is total reflection and no refraction. The corresponding angle of incidence in the denser medium is greater than or equal to the critical angle.

Critical angle for a pair of refracting media can be defined as that angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

Do you know ?

In Physics the word *critical* is used when certain phenomena are not applicable or more than one phenomenon are applicable. Some examples are as follows.

(i) In case of total internal reflection, the phenomenon of reversibility of light is not applicable at critical angle and refraction is possible only for angles of incidence in the denser medium smaller than the critical angle.

(ii) At the *critical* temperature, a substance coexists into all the three states; solid, liquid and gas. At all the other temperatures, only two states are simultaneously possible.

(iii) For liquids, streamline flow is possible till *critical* velocity is achieved. At critical velocity it can be either streamline or turbulent.

Let μ be the relative refractive index of denser medium with respect to the rarer. Applying Snell's law at the critical angle of incidence, i_C, we can write $\sin(i_c) = \frac{1}{\mu}$ as,

$$(\mu)\sin(i_c) = (1)\sin 90°$$

For commonly used glasses of

$\mu = 1.5$, $i_c = 41° \; 49' \cong 42°$ and for water of

$\mu = \frac{4}{3}$, $i_c = 48° \; 35'$ (Both, with respect to air)

9.6.1 Applications of total internal reflection:

(i) Optical fibre: Though little costly for initial set up, optic fibre communication is undoubtedly the most effective way of telecommunication by way of EM waves.

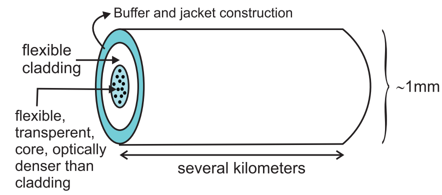


Fig. 9.8 (a): Optical fibre construction.

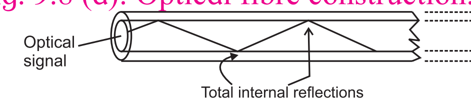


Fig. 9.8 (b): Optical fibre working.

An optical fibre essentially consists of an extremely thin (slightly thicker than a human hair), transparent, flexible core surrounded by optically rarer (smaller refractive index), flexible cover called cladding. This system is coated by a buffer and a jacket for protection. Entire thickness of the fibre is less than half a mm. (Fig. 9.8(a)). Number of such fibres may be packed together in an outer cover.

An optical signal (ray) entering the core suffers multiple total internal reflections (Fig. 9.8 (b)) and emerges after several kilometers with extremely low loss travelling with highest possible speed in that material ($\sim$2,00,000 km/s for glass). Some of the advantages of optic fibre communication are listed below.

(a) Broad bandwidth (frequency range): For TV signals, a single optical fibre can carry over 90000 channels (independent signals).

(b) Immune to EM interference: Being electrically non-conductive, it is not able to pick up nearby EM signals.

(c) Low attenuation loss: The loss is lower than 0.2 dB/km so that a single long cable can be used for several kilometers.

(d) Electrical insulator: No issue with ground loops of metal wires or lightning.

(e) Theft prevention: It is does not use copper or other expensive material.

(f) Security of information: Internal damage is most unlikely.

(ii) Prism binoculars: Binoculars using only two cylinders have a limitation of field of view as the distance between the two cylinders can't be greater than that between the two eyes. This limitation can be overcome by using two right angled glass prisms ($i_C \sim 42^0$) used for total internal reflection as shown in the Fig. 9.9. Total internal reflections occur inside isosceles, right angled prisms.

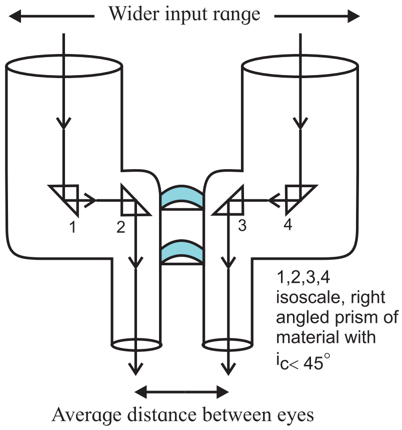


Fig. 9.9: Prism binoculars

(iii) Periscope: It is used to see the objects on the surface of a water body from inside water. The rays of light should be reflected twice through right angle. Reflections are similar to those in the binoculars (Fig 9.10) and total internal reflections occur inside isosceles, right angled prisms.

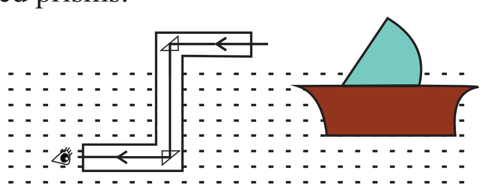
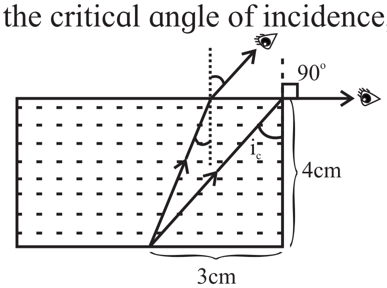


Fig. 9.10: Periscope.

Example 9.5: There is a tiny LED bulb at the center of the bottom of a cylindrical vessel of diameter 6 cm. Height of the vessel is 4 cm. The beaker is filled completely with an optically dense liquid. The bulb is visible from any inclined position but just visible if seen along the edge of the beaker. Determine refractive index of the liquid.

Solution: As seen from the accompanying figure, if the bulb is just visible from the edge, angle of incidence in the liquid (at the edge) must be the critical angle of incidence, i_C



From the dimensions given,

$$\tan(i_c) = \frac{3}{4} \therefore \sin(i_c) = \frac{3}{5} \therefore n_{\text{liquid}} = \frac{\sin 90°}{\sin(i_c)} = \frac{5}{3}$$

9.7 Refraction at a spherical surface and lenses:

In the section 9.5 we saw that due to refraction, the bottom of a water body appears to be raised and $n_{water} = \dfrac{Real\ depth}{apparent\ depth}$.

However, this is valid only if we are dealing with refraction at a plane surface. In many cases such as liquid drops, lenses, ellipsoid paper weights, etc, curved surfaces are present and the formula mentioned above may not be true. In such cases we need to consider refraction at one or more spherical surfaces. This will involve parameters including the curvature such as radius of curvature, in addition to refractive indices.

Lenses: Commonly used lenses can be visualized to be consisting of intersection of two spheres of radii of curvature R_1 and R_2 or of one sphere and a plane surface ($R = \infty$) . A lens is said to convex if it is thicker in the middle and narrowing towards the periphery. A lens is concave if it is thicker at periphery and narrows down towards center. Convex lens is visualized to be internal cross section of two spheres (or one sphere and a plane surface) while concave lens is their external cross section (Figs. 9.11-a to 9.11-f). Concavo-convex and convexo-concave lenses are commonly used for spectacles of positive and negative numbers, respectively.

For lenses of material optically denser than the medium in which those are kept, convex lenses have positive focal length [according to Cartesian sign convention] and converge the incident beam while concave lenses have negative focal length and diverge the incident beam.

For most of the applications of lenses, maximum thickness of lens is negligible (at least 50 times smaller) compared with all the other distances such as R_1 and R_2, u, v, f, etc. Such a lens is called as a thin lens and physical center of such a lens can be assumed to be the common pole (or optical center) for both its refracting surfaces.

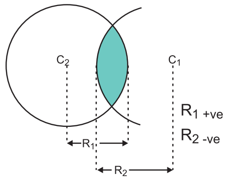


Fig. 9.11 (a): Convex lens as internal cross section of two spheres.

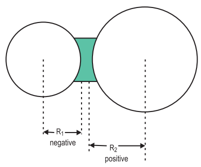


Fig. 9.11 (b): Concave lens as external cross section of two spheres.

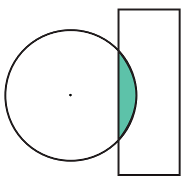


Fig. 9.11 (c): Plano convex lens

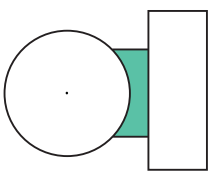


Fig. 9.11 (d): Plano concave lens

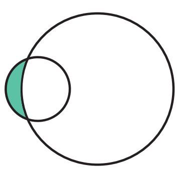


Fig. 9.11 (e): concave-convex lens

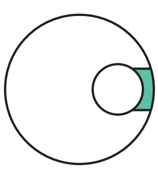


Fig. 9.11 (f) convex-concave lens

For any thin lens, $\frac{1}{f}=\frac{1}{\text{v}}-\frac{1}{u}$ --- (9.2)

If necessary, we can have a number of thin lenses in contact with each other having common principal axis. Focal power of such combination is given by the algebraic addition (by considering ± signs) of individual focal powers.

$$\therefore \frac{1}{f}=\sum\left(\frac{1}{f_i}\right)=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots$$

$$=P_1+P_2+P_3\ldots..=\sum P_i=P$$

For only two thin lenses, separated in air by distance d,

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}=P_1+P_2-dP_1P_2=P$$

For lenses, the relations between u, v, R and f depend also upon the refractive index n of the material of the lens. The relation $\left(f=\frac{R}{2}\right)$ does NOT hold good for lenses. Below we shall derive the necessary relation by considering refraction at the two surfaces of a lens independently.

> Unless mentioned specifically, we assume lenses to be made up of optically denser material compared to the medium in which those are kept, e.g., glass lenses in air or in water, etc. As special cases we may consider lenses of rarer medium such as an air lens in water or inside a glass. A spherical hole inside a glass slab is also a lens of rarer medium. In such case, physically (or geometrically or shape-wise) convex lens diverges the incident beam while concave lens converges the incident beam.

Refraction at a single spherical surface: Consider a spherical surface YPY' of radius of curvature R, separating two transparent media of refractive indices n_1 and n_2 respectively with $n_1 < n_2$. P is the pole and X'PX is the principal axis. A point object O is at an object distance $-u$ from the pole, in the medium of refractive index n_1. Convexity or concavity of a surface is always with respect to the incident rays, i.e., with respect to a real object. Hence in this case the surface is convex (Fig. 9.12).

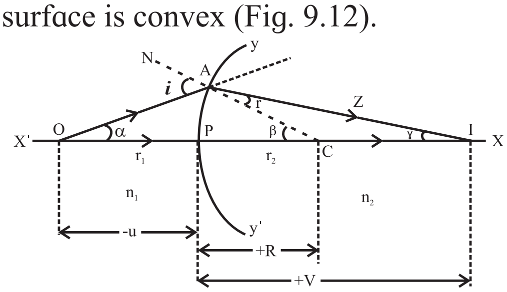


Fig. 9.12: Refraction at a single refracting surface.

To locate its image and in order to minimize spherical aberration, we consider two paraxial rays. The ray OP along the principal axis travels undeviated along PX. Another ray OA strikes the surface at A. CAN is the normal from center of curvature C of the surface at A. Angle of incidence in the medium n_1 at A is i.

As $n_1 < n_2$, the ray deviates towards the normal, travels along AZ and cuts the principal axis at I. Thus, real image of point object O is formed at I. Angle of refraction in medium n_2 is r. According to Snell's law,

$$n_1 \sin(i) = n_2 \sin(r) \quad \text{--- (9.3)}$$

Let α, β and γ be the angles subtended by incident ray, normal and refracted ray with the principal axis.

$\therefore i = \alpha + \beta$ and $r = \beta - \gamma$

For paraxial rays, all these angles are small and PA can be considered as an arc for α, β and γ.

$\therefore \sin(i) \cong i$ and $\sin(r) \cong r$

Also, $\alpha \cong \dfrac{arc\ AP}{PO} = \dfrac{arc\ AP}{-u}$,

$\beta = \dfrac{arc\ AP}{PC} = \dfrac{arc\ AP}{R}$ and

$\gamma \cong \dfrac{arc\ AP}{PI} = \dfrac{arc\ AP}{\text{v}}$

$\therefore n_1 i = n_2 r$

$\therefore n_1(\alpha + \beta) = n_2(\beta - \gamma)$

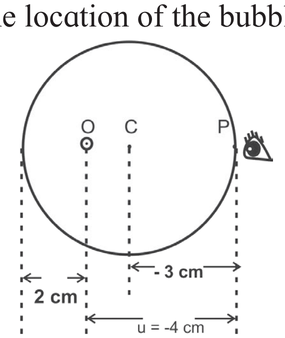
$\therefore (n_2 - n_1)\beta = n_2\gamma + n_1\alpha$

Substituting α, β and γ and canceling 'arc AP', we get

$$\frac{n_2 - n_1}{R} = \frac{n_2}{\text{v}} - \frac{n_1}{u} \quad \text{--- (9.4)}$$

Example 6: A glass paper-weight (n =1.5) of radius 3 cm has a tiny air bubble trapped inside it. Closest distance of the bubble from the surface is 2 cm. Where will it appear when seen from the other end (from where it is farthest)?

Solution: Accompanying Figure below illustrates the location of the bubble.



According to the symbols used in the Eq. (9.4), we get,

n_1 = refractive index of the medium of real object(medium of incident rays) = 1.5

n_2 = refractive index of the other medium = 1

$u = -4\,cm$

v = ?

$R = -3\,cm$

$\dfrac{n_2 - n_1}{R} = \dfrac{n_2}{\text{v}} - \dfrac{n_1}{u}$

$\therefore \dfrac{1-1.5}{-3} = \dfrac{1}{\text{v}} - \dfrac{1.5}{-4} \quad \therefore \dfrac{1}{6} = \dfrac{1}{\text{v}} + \dfrac{3}{8} \quad \therefore \text{v} = -4.8\,\text{cm}$

In this case apparent depth is NOT less than real depth. This is due to curvature of the refracting surface.

In this case (Fig. 9.12) we had considered the object placed in rarer medium, real image in denser medium and the surface facing the object to be convex. However, while deriving the relation, all the symbolic values (which could be numeric also) were substituted as per the Cartesian sign convention (e.g. 'u' as negative, etc.). Hence the final expression (Eq. 9.4) is applicable to any surface separating any two media, and real or virtual image provided you substitute your values (symbolic or numerical) as per Cartesian sign convention. The *only restriction* is that n_1 is for medium of real object and n_2 is the *other* medium (*not necessarily* the medium of image). Only in the case of real image, it will be in medium n_2. If virtual, it will be in the medium n_1 (with image distance negative how do you justify this?).

We strongly suggest you to do the derivations yourself for any other special case such as object placed in the denser medium, virtual image, concave surface, etc. It must be remembered that in any case you will land up with the same expression as in Eq. (9.4).

Lens makers' equation: Relation between refractive index (n), focal length (f) and radii of curvature R_1 and R_2 for a thin lens.

Consider a lens of radii of curvature R_1 and R_2 kept in a medium such that n is refractive index of material of the lens with respect to the outside medium. Assuming the lens to be thin, P is the common pole for both the surfaces. O is a point object on the principal axis at a distance u from P. First refracting surface of the lens of radius of curvature R_1 faces the object (Fig 9.13).

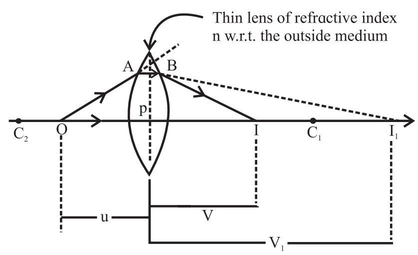


Fig. 9.13: Lens maker's equation.

Axial ray OP travels undeviated. Paraxial ray OA deviates towards normal and would intersect axis at I_1, in the absence of second refracting surface. $PI_1 = v_1$ is the image distance for intermediate image I_1.

Thus, the symbols to be used in Eq. (9.4) are

$$n_2 = n,\ \ n_1 = 1,\ \ R = R_1,\ \ u = u,\ \ v = v_1$$

$$\therefore \frac{n-1}{R_1} = \frac{n}{v_1} - \frac{1}{(u)} \qquad \text{--- (9.5)}$$

(Not that, in this case, we are not substituting the algebraic values but just using different symbols.)

Before reaching I_1, the ray PI_1 is intercepted at B by the second refracting surface. In this case, the incident rays AB and OP are in the medium of refractive index n and converging towards I_1. Thus, I_1 acts as *virtual* object for second surface of radius of curvature (R_2) and object distance is $(u = v_1)$. As the incident rays are in the medium of refractive index n, this is the medium of (virtual) object $\therefore n_1 = n$ and refractive index of the other medium is $n_2 = 1$.

After refraction, the ray bends away from the normal and intersects the principal axis at I which is the real image of object O formed due to the lens. $\therefore$ PI = v.

Substituting all these *symbols* in Eq. (9.4), we get

$$\frac{1-n}{R_2} = \frac{n-1}{-R_2} = \frac{1}{v} - \frac{n}{(v_1)} \qquad \text{--- (9.6)}$$

Adding Eq. (9.5) and (9.6), we get,

$$(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{v} - \frac{1}{u}$$

For $u = \infty, v = f$

$$\therefore \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{--- (9.7)}$$

For preparing spectacles, it is necessary to grind the glass (or acrylic, etc.) for having the desired radii of curvature. Equation (9.7) can be used to calculate the radii of curvature for the lens, hence it is called the lens makers' equation. (It should be remembered that while solving problems when you are using equations 9.1, 9.2, 9.4, 9.7, etc., we will be substituting the values of the corresponding quantities. Hence this time it is algebraic substitution, i.e., with proper $\pm$ sign)

Special cases:

Most popular and most common special case is the one in which we have a thin, symmetric, double lens. In this case, R_1 and R_2 are numerically equal.

(A) Thin, symmetric, double convex lens: R_1 is positive, R_2 is negative and numerically equal. Let $|R_1| = |R_2| = R$.

$$\therefore \frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(n-1)}{R}$$

Further, for popular variety of glasses, $n \cong 1.5$. In such a case, $f = R$.

(B) Thin, symmetric, double concave lens: R_1 is negative, R_2 is positive and numerically equal. Let $|R_1| = |R_2| = R$.

$$\therefore \frac{1}{f} = (n-1)\left(\frac{1}{-R} - \frac{1}{R}\right) = \frac{2(n-1)}{-R}$$

Further if $n \cong 1.5, f = -R$

(C) Thin, planoconvex lenses: One radius is R and the other is ∞. $\therefore \frac{1}{f} = \frac{n-1}{R}$

Further if $n \cong 1.5, f = 2|R|$

Example 7: A dense glass double convex lens $(n = 2)$ designed to reduce spherical aberration has $|R_1|:|R_2|=1:5$. If a point object is kept 15 cm in front of this lens, it produces its real image at

7.5 cm. Determine R_1 and R_2.

Solution: u = - 15 cm, v = + 7.5 cm (real image is on opposite side).

$$\frac{1}{f}=\frac{1}{\text{v}}-\frac{1}{u} \quad \therefore \frac{1}{f}=\frac{1}{7.5}-\frac{1}{-15} \quad \therefore f=+5\,\text{cm}$$

The lens is double convex. Hence, R_1 is positive and R_2 is negative. Also, $|R_2|=5|R_1|$ and $n=2$.

$$(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$

$$\therefore (2-1)\left(\frac{1}{R_1}-\frac{1}{-(5R_1)}\right)=\frac{1}{5}$$

$$\therefore (1)\left(\frac{6}{5R_1}\right)=\frac{1}{5} \quad \therefore R_1=6\,\text{cm} \quad \therefore R_2=30\,\text{cm}$$

9.8 Dispersion of light and prisms:

The colour of light that we see depends upon the frequency of that ray (wave). The refractive index of a material also depends upon the frequency of the wave and increases with frequency. Obviously refractive index of light is different for different colours. As a result, for an obliquely incident ray, the angles of refraction are different for each colour and they separate (disperse) as they travel along different directions. This phenomenon is called angular dispersion Fig 9.14.

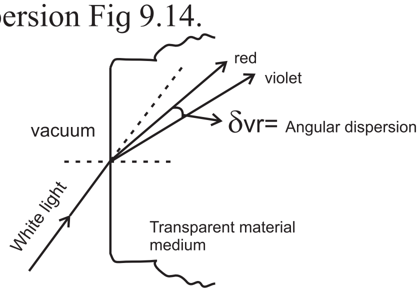


Fig. 9.14: Angular dispersion at a single surface.

If a polychromatic beam of light (bundle of rays of different colours) is obliquely incident upon a plane parallel transparent slab, emergent beam consists of all component colours separated out. However, in this case all those are parallel to each other and also parallel to initial direction. This is lateral dispersion which is measured as the perpendicular distance between the direction of incident ray and respective directions of dispersed emergent rays (L_R and L_V) Fig 9.15. For it to be easily detectable, the parallel surfaces must be separated over very large distance and i should be large.

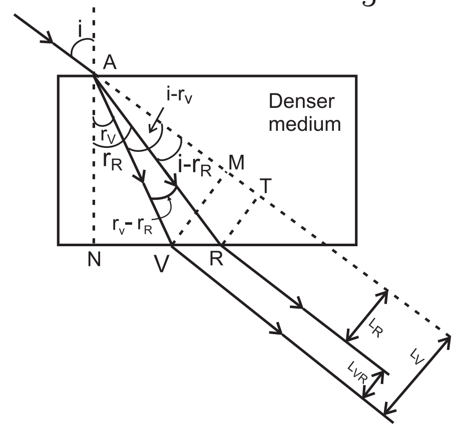


Fig. 9.15: Lateral dispersion due to plane parellal slab.

Example 8: A fine beam of white light is incident upon the longer side of a plane parallel glass slab of breadth 5 cm at angle of incidence 60^0. Calculate angular deviation of red and violet rays within the slab and lateral dispersion between them as they emerge from the opposite side. Refractive indices of the glass for red and violet are 1.51 and 1.53 respectively.

Solution: As shown in the Fig. 9.15 above, VM = L_V and RT = L_R give respective lateral deviations for violet and red colours and L_{VR} = $L_V - L_R$ is the lateral dispersion between these colours. n_R = 1.51, n_V = 1.53 and i = 60°

$$\therefore \sin r_R=\frac{\sin i}{n_R}=\frac{\sin 60^0}{1.51}=0.5735$$

$$\sin r_V=\frac{\sin i}{n_V}=\frac{\sin 60^0}{1.53}=0.566$$

$$\therefore r_R=35^0 \text{ and } R_V=34^0 28' \;\therefore \delta_{RV}=r_R-r_V\, 32'$$

$$\therefore i-r_R=25^0,\; i-r_V=25^0 32'$$

$$\therefore \text{AR}=\frac{\text{AN}}{\cos r_R}=6.104\,\text{cm}$$

$$\text{AV}=\frac{\text{AN}}{\cos r_V}=6.063\,\text{cm}$$

$$\therefore L_R=\text{RT}=\text{AR}\left(\sin\left[i-r_R\right]\right)=2.58\,\text{cm}$$

$$L_V=\text{VM}=\text{AV}\left(\sin\left[i-r_V\right]\right)=2.58\,\text{cm}$$

$$\therefore L_{VR}=L_V-L_R=0.033\,\text{cm}=0.33\text{ mm}$$

It shows that the lateral dispersion is too small to detect.

In order to have appreciable and observable dispersion, two parallel surfaces are not useful. In such case we use prisms, in which two refracting surfaces inclined at an angle are used. Popular variety of prisms are having three rectangular surfaces forming a triangle. At a time two of these are taking part in the refraction. The one, not involved in refraction is called base of the prism. Fig 9.16.

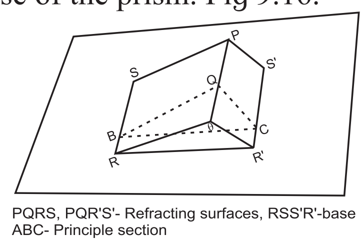


Fig. 9.16: Prism consisting of three plane surfaces.

Any section of prism perpendicular to the base is called principal section of the prism. Usually we consider all the rays in this plane. Fig 9.17 a and 9.17 b show refraction through a prism for monochromatic and white beams respectively. Angular dispersion is shown for white beam.

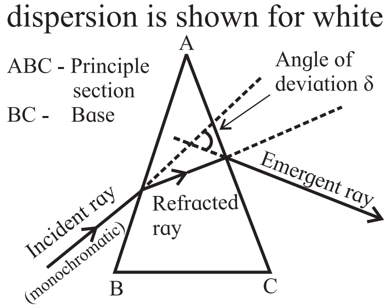


Fig. 9.17 (a): Refraction through a prism (monocromatic light).

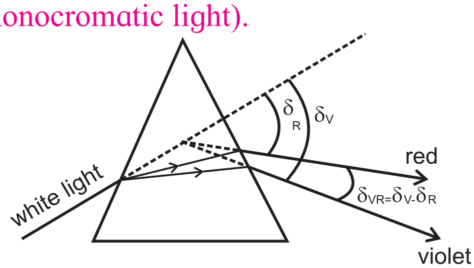


Fig. 9.17 (b): Angular dispersion through a prism. (white light).

Relations between the angles involved: Figure 9.18 shows principal section ABC of a prism of absolute refractive index n kept in air. Refracting surfaces AB and AC are inclined at angle A, which is refracting angle of prism or simply 'angle of prism'. Surface BC is the base. A monochromatic ray PQ obliquely strikes first reflecting surface AB. Normal passing through the point of incidence Q is MQN. Angle of incidence at Q is i. After refraction at Q, the ray deviates towards the normal and strikes second refracting surface AC at R which is the point of emergence. MRN is the normal through R. Angles of refraction at Q and R are r_1 and r_2 respectively.

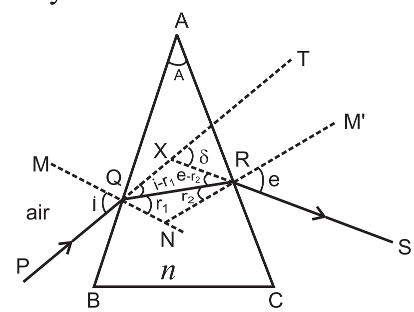


Fig. 9.18: Deviation through a prism.

After R, the ray deviates away from normal and finally emerges along RS making e as the angle of emergence. Incident ray PQ is extended as QT. Emergent ray RS meets QT at X if traced backward. Angle TXS is angle of deviation δ .

$\angle$ AQN = $\angle$ ARN $= 90^0$ …… (Angles at normal)

$\therefore$ From quadrilateral AQNR,

$$A + \angle QNR = 180^0 \quad \text{--- (9.8)}$$

From Δ QNR, $r_1 + r_2 + \angle QNR = 180^0$ --- (9.9)

$\therefore$ From Eqs. (9.8) and (9.9),

$$A = r_1 + r_2 \quad \text{--- (9.10)}$$

Angle δ is exterior angle for triangle XQR.

$$\therefore \angle XQR + \angle XRQ = \delta$$

$$\therefore (i - r_1) + (e - r_2) = \delta$$

$$\therefore (i + e) - (r_1 + r_2) = \delta$$

Hence, using Eq. (9.10), $(i + e) - (A) = \delta$

$$\therefore i + e = A + \delta \quad \text{--- (9.11)}$$

Deviation curve, minimum deviation and prism formula: From the relations (9.10) and (9.11), it is clear that δ, e, r_1 and r_2 depend upon i, A and n. After a certain minimum value of angle of incidence i_{min}, the emergent ray is possible. This is because of the fact that for $i < i_{min}$, $r_2 > i_c$ and there is total internal reflection at the second surface and there is no emergent ray. This will be shown later. Then onwards, as i increases, r_1 increases as $\frac{\sin i}{\sin r_1}=n$ but r_2 and e decrease. However, variation in δ with increasing i is different. It is as plotted in the Fig. 9.19.

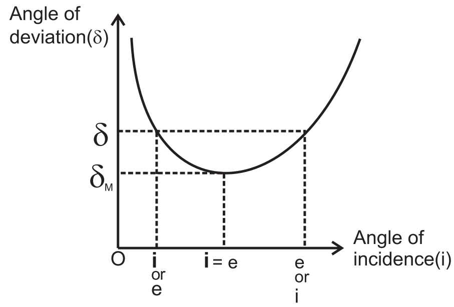


Fig. 9.19: Deviation curve for a prism.

It shows that, with increasing values of i, the angle of deviation δ decreases initially to a certain minimum (δ_m) and then increases. It should also be noted that the curve is not a symmetric parabola, but the slope in the part after is less. It is clear that except at $\delta=\delta_m$, (Angle of minimum deviation) there are two values of i for any given δ. By applying the principle of reversibility of light to path PQRS it is obvious that if one of these values is i, the other must be e and vice versa. Thus at $\delta=\delta_m$, we have $i=e$. Also, in this case, $r_1=r_2$ and $A=r_1+r_2=2r \therefore r=\frac{A}{2}$

Only in this case QR is parallel to base BC and the figure is symmetric.

Using these in Eq. (9.11), we get,

$$i+i=A+\delta_m \therefore i=\frac{(A+\delta_m)}{2}$$

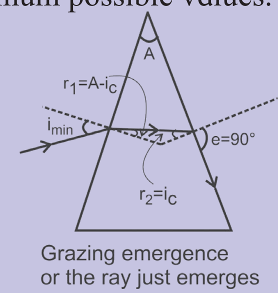
According to Snell's law,

$$\therefore n=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \text{--- (9.12)}$$

Equation (9.12) is called prism formula.

Example 9.9: For a glass (n =1.5) prism having refracting angle 60^0, determine the range of angle of incidence for which emergent ray is possible from the opposite surface and the corresponding angles of emergence. Also calculate the angle of incidence for which $i = e$. How much is the corresponding angle of minimum deviation?

(I) Grazing emergence and minimum angle of incidence: At the point of emergence, the ray travels form a denser medium into rarer (popular prisms are of denser material, kept in rarer). Thus if $r_2=\sin^{-1}\left(\frac{1}{n}\right)$ is the critical angle, the angle of emergence $e=90^0$. This is called grazing emergence or we say that the ray just emerges. Angle of prism A is constant for a given prism and $A=r_1+r_2$. Hence the corresponding r_1 and i will have their minimum possible values.



Grazing emergence
or the ray just emerges

(II) For commonly used glass prisms,

$$n=1.5,\ \sin^{-1}\left(\frac{1}{n}\right)=\sin^{-1}\left(\frac{1}{1.5}\right)$$
$$=41^0 49' = (r_2)_{max}$$

If prism is symmetric (equilateral),

$$A=60^0 \therefore r_1=60^0-41^0 49'=18^0 11'$$

$$\therefore n=1.5=\frac{\sin(i_{min})}{\sin 18^0 11'} \quad \therefore i_{min}=27^0 55' \cong 28^0.$$

(III) For a symmetric (equilateral) prism, the prism formula can be written as

$$n=\frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin\left(\frac{60}{2}\right)}=\frac{\sin\left(30+\frac{\delta_m}{2}\right)}{\sin(30)}$$
$$=2\sin\left(30+\frac{\delta_m}{2}\right)$$

(IV) For a prism of denser material, kept in a rarer medium, the incident ray deviates towards the normal during the first refraction and away from the normal during second refraction. However, during both the refractions it deviates towards the base only.

Solution: As shown in the box above, $i_{min} = 27^0 55'$. Angle of emergence for this is $e_{max} = 90^0$.

From the principle of reversibility of light, $i_{max} = 90^0$ *and* $e_{min} = 27^0 55'$

Also, from the box above,

$$n = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$= \frac{\sin\left(30+\frac{\delta_m}{2}\right)}{\sin(30)} = 2\sin\left(30+\frac{\delta_m}{2}\right)$$

$$\therefore 1.5 = 2\sin\left(30+\frac{\delta_m}{2}\right) \quad \therefore 0.75 = \sin\left(30+\frac{\delta_m}{2}\right)$$

$$\therefore \left(30+\frac{\delta_m}{2}\right) = 48^0 35'$$

$$\therefore \frac{\delta_m}{2} = 18^0 35' \quad \therefore \delta_m = 37^0 10'$$

$i + e = A + \delta$ and $i = e$ for $\delta = \delta_m$

$\therefore\ i + i = 60 + 37° \ 10' = 97°10' \ \therefore\ i = 48°35'$

Thin prisms: Prisms having refracting angle less than 10^0 $\left(A < 10^0\right)$ are called thin prisms. For such prisms we can comfortably use $\sin\theta \cong \theta$. For such prisms to deviate the incident ray towards the base during both refractions, it is essential that i should also be less than 10^0 so that all the other angles will also be small.

Thus

$$n = \frac{\sin i}{\sin r_1} \cong \frac{i}{r_1} \text{ and } n = \frac{\sin e}{\sin r_2} \cong \frac{e}{r_2}$$

$\therefore i \cong nr_1$ and $e \cong nr_2$

Using these in Eq. (9.11), we get,

$$i + e = nr_1 + nr_2 = n\left(r_1 + r_2\right) = nA = A + \delta$$

$$\therefore \delta = A\left(n-1\right) \qquad \text{--- (9.13)}$$

A and n are constant for a given prism. Thus, for a thin prism, for small angles of incidences, angle of deviation is constant (independent of angle of incidence).

Angular dispersion and mean deviation: As discussed earlier, if a polychromatic beam is incident upon a prism, the emergent beam consists of all the individual colours angularly separated. This is angular dispersion (Fig. 9.20).

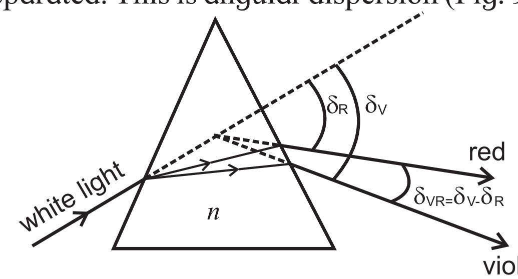


Fig. 9.20: Angular dispersion through a prism.

It is measured for any two component colours.

$$\therefore \delta_{21} = \delta_2 - \delta_1$$

Normally we do it for extreme colours.

For white light, violet and red are the extreme colours.

$$\therefore \delta_{VR} = \delta_V - \delta_R$$

Using deviation for thin prism (Eq. 9.13), we can write

$$\therefore \delta_{21} = \delta_2 - \delta_1 = A\left(n_2 - 1\right) - A\left(n_1 - 1\right)$$

$$= A\left(n_2 - n_1\right)$$

where n_1 and n_2 are refractive indices for the two colours.

Also,

$$\delta_{VR} = \delta_V - \delta_R = A\left(n_V - 1\right) - A\left(n_R - 1\right)$$

$$= A\left(n_V - n_R\right) \qquad \text{--- (9.14)}$$

Yellow is practically chosen to be the mean colour for violet and red.

This gives mean deviation

$$\delta_{VR} = \frac{\delta_V + \delta_R}{2} \cong \delta_Y = A\left(n_Y - 1\right) \qquad \text{--- (9.15)}$$

Do you know ?

(i) If you see a rainbow widthwise, yellow appears to be centrally located. Hence angular deviation of yellow is average for the entire colour span. This may be the reason for choosing yellow as the mean colour. Remember, red band is widest and violet is much thinner than blue.

(ii) While obtaining the expression for ω, we have used thin prism formula for δ. However, the expression for ω (equation 9.16) is valid as well for equilateral prisms or right-angled prisms.

Dispersive power: Ability of an optical material to disperse constituent colours is its dispersive power. It is measured for any two colours as the ratio of angular dispersion to the mean deviation for those two colours. Thus, for the extreme colours of white light, dispersive power is given by

$$\omega = \frac{[\delta_V - \delta_R]}{\left[\frac{\delta_V + \delta_R}{2}\right]} \cong \frac{\delta_V - \delta_R}{\delta_Y}$$

$$= \frac{A(n_V - n_R)}{A(n_Y - 1)} = \frac{n_V - n_R}{n_Y - 1} \quad \text{--- (9.16)}$$

As ω is the ratio of same physical quantities, it is unitless and dimensionless quantity. From the expression in terms of refractive indices it should be understood that dispersive power depends only upon refractive index (hence material only) and not upon the dimensions of prism. For commonly used glasses it is around 0.03.

Example 10: For a dense flint glass prism of refracting angle 10^0, obtain angular deviation for extreme colours and dispersive power of dense flint glass. ($n_{red} = 1.712$, $n_{violet} = 1.792$)

$\delta_V = A(n_V - 1) = 10(1.792 - 1) = (7.92)°$

$\delta_R = A(n_R - 1) = 10(1.712 - 1) = (7.12)°$

$\therefore$ Angular dispersion, $\delta_{VR} = \delta_V - \delta_R = (0.8)^0$

dispersive power, ω =

$$= \frac{\delta_V - \delta_R}{\left(\frac{\delta_V + \delta_R}{2}\right)}$$

$$= 2\left(\frac{7.92 - 7.12}{7.92 + 7.12}\right)$$

$$= \frac{2 \times 0.8}{15.04} = 0.1064$$

(This is much higher than popular crown glass)

9.9 Some natural phenomena due to Sunlight:

Mirage: On a hot clear Sunny day, along a level road, a pond of water appears to be there ahead. However, if we physically reach the spot, there is nothing but the dry road and water pond again appears ahead. This illusion is called a mirage (Fig. 9.21).

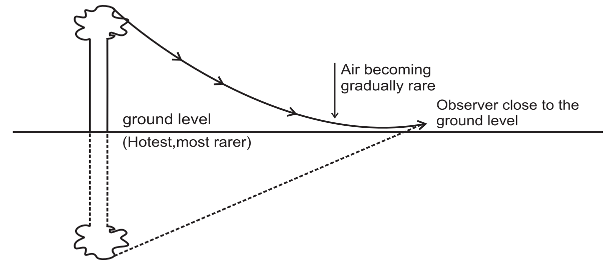


Fig. 9.21: The Mirage.

On a hot day the air in contact with the road is hottest and as we go up, it gets gradually cooler. The refractive index of air thus increases with height. As shown in the figure, due to this gradual change in the refractive index, the ray of light coming from the top of an object becomes more and more horizontal as it almost touches the road. For some reason (mentioned later) it bends above. Then onwards, upward bending continues due to denser air. As a result, for an observer, it appears to be coming from below thereby giving an illusion of reflection from an (imaginary) water surface.

Rainbow: Undoubtedly, rainbow is an eye-catching phenomenon occurring due to rains and Sunlight. It is most popular because it is observable from anywhere on the Earth. A few lucky persons might have observed two rainbows simultaneously one above the other. Some might have seen a complete circular rainbow from an aeroplane (Of course, this time it's not a bow!). Optical phenomena discussed till now are sufficient to explain the formation of a rainbow.

The facts to be explained are:

(i) It is seen during rains and on the opposite side of the Sun.

(ii) It is seen only during mornings and evenings and not throughout the day.

(iii) In the commonly seen rainbow red arch is outside and violet is inside.

(iv) In the rarely occurring concentric secondary rainbow, violet arch is outside and red is inside.

(v) It is in the form of arc of a circle.

(vi) Complete circle can be seen from a higher altitude, i.e., from an aeroplane.

(vii) Total internal reflection is not possible in this case.

Conditions necessary for formation of a rainbow: Light shower with relatively large raindrops, morning or evening time and enough Sunlight.

Optical phenomena involved: During the formation of a rainbow, the rays of Sunlight incident on water drops, deviate and disperse during refraction, internally (NOT total internally) reflect once (for primary rainbow) or twice (for secondary rainbow) and finally refract again into air. At all stages there is angular dispersion which leads to clear separation of the colours.

Primary rainbow: Figure 9.22 (a) shows the optical phenomena involved in the formation of a primary rainbow due to a spherical water drop.

Do you know ?

Possible reasons for the upward bending at the road during mirage could be:

(i) Angle of incidence at the road is glancing. At glancing incidence, the reflection coefficient is very large which causes reflection.

(ii) Air almost in contact with the road is not steady. The non-uniform motion of the air bends the ray upwards and once it has bent upwards, it continues to do so.

(iii) Using Maxwell's equations for EM waves, correct explanation is possible for the reflection.

It may be pointed out that total internal reflection is NEVER possible here because the relative refractive index is just less than 1 and hence the critical angle (discussed in the article 9.6) is also approaching 90^0.

White ray AB from the Sun strikes from upper portion of a water drop at an incident angle *i*. On entering into water, it deviates and disperses into constituent colours. Extreme colours violet(V) and red(R) are shown. Refracted rays BV and BR strike the opposite inner surface of water drop and suffer internal (NOT total internal) reflection. These reflected rays finally emerge from V′ and R′ and can be seen by an observer on the ground. For the observer they appear to be coming from opposite side of the Sun. Minimum deviation rays of red and violet colour are inclined to the ground level at $\theta_R = 42.8° \cong 43°$ and $\theta_V = 40.8 \cong 41°$ respectively. As a result, in the 'bow' or arch, the red is above or outer and violet is lower or inner.

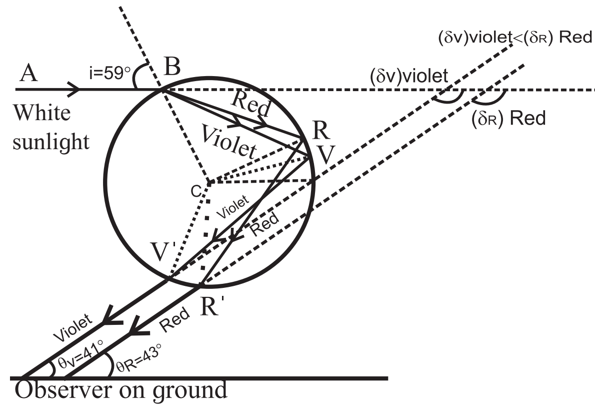


Fig. 9.22 (a): Formation of primary rainbow.

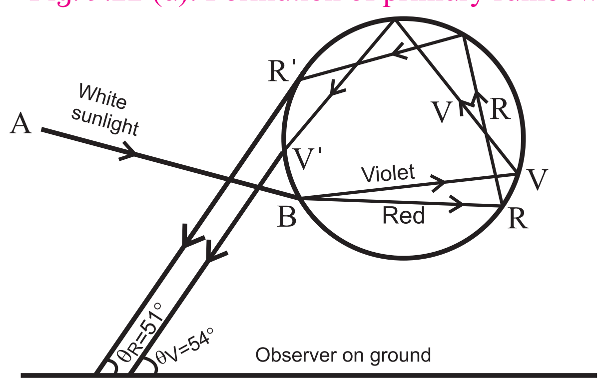


Fig. 9.22 (b): Formation of secondary rainbow.

Secondary rainbow: Figure 9.22 (b) shows some optical phenomena involved in the formation of a secondary rainbow due to a spherical water drop. White ray AB from the Sun strikes from lower portion of a water drop at an incident angle *i*. On entering into water, it deviates and disperses into constituent colours. Extreme colours violet(V) and red(R) are shown. Refracted rays BV and BR finally emerge the drop from V' and R' after suffering two internal reflections and can be seen by an observer on the ground. Minimum deviation rays of red and violet colour are inclined to the ground level at $\theta_R \cong 51°$ and $\theta_V \cong 53°$ respectively. As a result, in the 'bow' or arch, the violet is above or outer and red is lower or inner.

Do you know ?

(I) Why total internal reflection is not possible during formation of a rainbow?

Angle of incidence i in air, at the water drop, can't be greater than 90°. As a result, angle of refraction r in water will *always less than the critical angle*. From Fig a and b and by simple geometry, it is clear that this r itself is the angle of incidence at any point for one or more internal reflections. Obviously, total internal reflection is not possible.

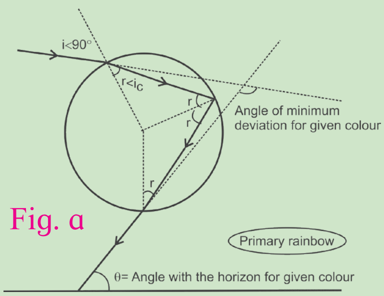


Fig. a

(II) Why is rainbow seen only for a definite angle range with respect to the ground?

For clear visibility we must have a beam of enough intensity. From the deviation curve (Fig 9.19) it is clear that near minimum deviation the curve is almost parallel to x-axis, i.e., for majority of angles of incidence in this range, the angle of deviation is nearly the same and those are almost parallel forming a beam of enough intensity. Thus, the rays in the near vicinity of minimum deviation are almost parallel to each other. Rays beyond this range suffer wide angular dispersion and thus will not have enough intensity for visibility.

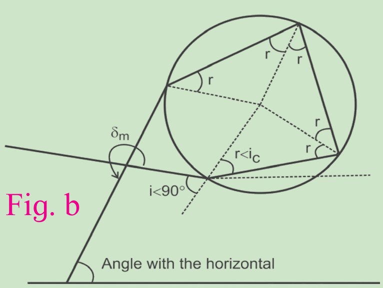
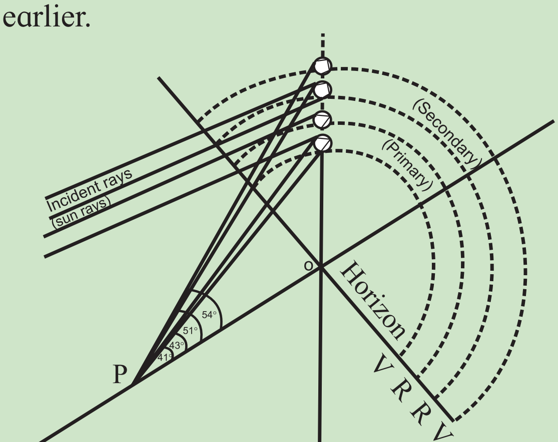


Fig. b

By using simple geometry for Figs. a and b it can be shown that the angle of deviation between final emergent ray and the incident ray is $\delta = \pi + 2i - 4r$ during primary rainbow, and $\delta = 2\pi + 2i - 6r$ during secondary rainbow. Using these relations and Snell's law $\sin i = n \sin r$, we can obtain derivatives of δ. Second derivative of δ comes out to be negative, which shows that it is the minima condition. Equating first derivative to zero we can obtain corresponding values of i and r. Again, by using Figs. a and b, we can obtain the corresponding angles θ_R and θ_V at the horizontal, which is the visible angular position for the rainbow.

(III) Why is the rainbow a bow or an arch? Can we see a complete circular rainbow?

Figure c illustrates formation of primary and secondary rainbows with their common centre O is the point where the line joining the sun and the observer meets the Earth when extended. P is location of the observer. Different colours of rainbows are seen on arches of cones of respective angles described earlier.



Symmetrical angles of respective curves of colours

Fig. c

Smallest half angle refers to the cone of violet colour of primary rainbow, which is 41^0. As the Sun rises, the common centre of the rainbows moves down. Hence as the Sun comes up, smaller and smaller part of the rainbows will be seen. If the Sun is above 41^0, violet arch of primary rainbow cannot be seen. Obviously beyond 53^0, nothing will be seen. That is why rainbows are visible only during mornings and evenings.

However, if observer moves up (may be in an aeroplane), the line PO itself moves up making lower part of the arches visible. After a certain minimum elevation, entire circle for all the cones can be visible.

(IV) Size of water drops convenient for rainbow: Water drops responsible for the formation of a rainbow should not be too small. For too small drops the phenomenon of diffraction (redistribution of energy due to obstacles, discussed in XIIth standard) dominates and clear rainbow can't be seen.

9.10 Defects of lenses (aberrations of optical images):

As mentioned in the section 9.4 for aberration for curved mirrors, while deriving various relations, we assume most of the rays to be paraxial by using lenses of small aperture. In reality, we have objects of finite sizes. Also, we need optical devices of large apertures (lenses and/or mirrors of size few meters for telescopes, etc.). In such cases the beam of rays is no more paraxial, quite often not parallel also. As a result, the spherical oberration discussed for spherical mirrors can occur for lenses also. Only one defect is mentioned corresponding to monochromatic beam of light.

Chromatic aberration: In case of mirrors there is no dispersion of light due to refractive index. However, lenses are prepared by using a transparent material medium having different refractive index for different colours. Hence angular dispersion is present. A convex lens can be approximated to two thin prisms connected base to base and for a concave lens those are vertex to vertex. (Fig. 9.23 (a) and 9.23 (b))

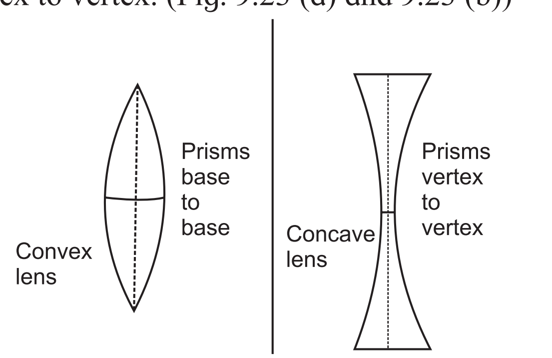


Fig. 9.23: (a) Convex lens (b) Concave lens

If the lens is thick, this will result into notably different foci corresponding to each colour for a polychromatic beam, like a white light. This defect is called chromatic aberration, violet being focused closest to pole as it has maximum deviation. (Fig 9.24 (a) and 9.24 (b)) Longitudinal chromatic aberration, transverse chromatic aberration and circle of least confusion are defined in the same manner as that of spherical aberration for spherical mirrors.

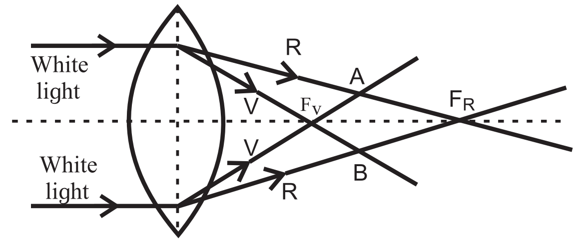


Fig. 9.24: Chromatic aberration: (a) Convex lens.

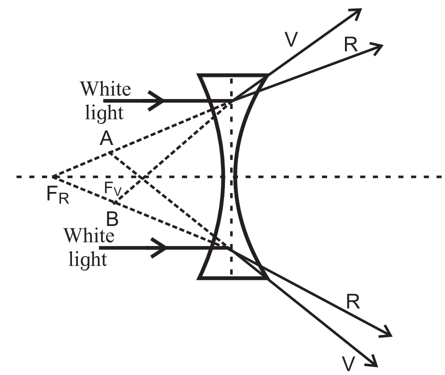


Fig. 9.24: Chromatic aberration: (b) Concave lens

Reducing/eliminating chromatic aberration:

Eliminating chromatic aberration simultaneously for all the colours is impossible. We try to eliminate it for extreme colours which reduces it for other colours. Convenient methods to do it use either a convex and a concave lens in contact or two thin convex lenses with proper separation. Such a combination is called achromatic combination.

Achromatic combination of two lenses in contact: Let ω_1 and ω_2 be the dispersive powers of materials of the two component lenses used in contact for an achromatic combination. Their focal lengths f for violet, red and yellow (assumed to be the mean colour) are suffixed by respective letters V, R and Y.

Also, let $K_1 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1$ for lens 1 and

$K_2 = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2$ for lens 2.

For two thin lenses in contact,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots\ldots$$

To be used separately for respective colours.

For the combination to be achromatic, the resultant focal length of the combination must be the same for both the colours, i.e.,

$$f_V = f_R \text{ or } \frac{1}{f_V} = \frac{1}{f_R}$$

$$\therefore \frac{1}{f_{1V}} + \frac{1}{f_{2V}} = \frac{1}{f_{1R}} + \frac{1}{f_{2R}}$$

$(n_{1V}-1)\,K_1 + (n_{2V}-1)\,K_2 = (n_{1R}-1)\,K_1 + (n_{2R}-1)\,K_2$

...... using lens makers' Eq. (9.7)

$$\therefore \frac{K_1}{K_2} = \frac{n_{2V} - n_{2R}}{n_{1V} - n_{1R}} \quad \text{--- (9.17)}$$

For mean colour yellow,

$$\frac{1}{f_Y} = \frac{1}{f_{1Y}} + \frac{1}{f_{2Y}}$$

$$\text{with } \frac{1}{f_{1Y}} = (n_{1Y} - 1) K_1$$

$$\text{and } \frac{1}{f_{2Y}} = (n_{2Y} - 1) K_2$$

$$\therefore \frac{K_1}{K_2} = \left(\frac{n_{2Y} - 1}{n_{1Y} - 1}\right)\left(\frac{f_{2Y}}{f_{1Y}}\right) \quad \text{--- (9.18)}$$

Equating R.H.S. of (9.17) and (9.18) and rearranging, we can write

$$\frac{f_{2Y}}{f_{1Y}} = -\left(\frac{n_{2V} - n_{2R}}{n_{2Y} - 1}\right) \div \left(\frac{n_{1R} - n_{1R}}{n_{1Y} - 1}\right)$$

$$= -\frac{\omega_2}{\omega_1} \quad \text{--- (9.19)}$$

Equation (9.19) is the condition for achromatic combination of two lenses, in contact.

Dispersive power ω is always positive. Thus, one of the lenses must be convex and the other concave.

If second lens is concave, f_{2Y} is negative.

$$\therefore \frac{1}{f_Y} = \frac{1}{f_{1Y}} - \frac{1}{f_{2Y}}$$

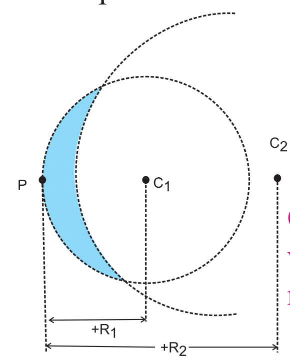
For this combination to be converging, f_Y should be positive.

Hence, $f_{1Y} < f_{2Y}$ **and** $\omega_1 < \omega_2$

Thus, for an achromatic combination if there is a choice between flint glass ($n =$ 1.655) and crown glass ($n = 1.517$), the convergent (convex) lens must be of crown glass and the divergent (concave) lens of flint glass.

Example 9.11: After Cataract operation, a person is recommended with concavo-convex spectacles of curvatures 10 cm and 50 cm. Crown glass of refractive indices 1.51 for red and 1.53 for violet colours is used for this. Calculate the lateral chromatic aberration occurring due to these glasses.

Solution: For a concavo-concave lens, both the radii of curvature are either positive or both negative. If convex shape faces object, both will be positive. See the accompanying figure.



Concavo-convex lens with convex face receiving incident rays

$\therefore R_1 = 10\,\text{cm}$ and $R_2 = 50\,\text{cm}$

$$\therefore \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{+10} - \frac{1}{+50}\right) = 0.08\ \text{cm}^{-1}$$

$$\therefore \frac{1}{f_R} = (n_R - 1)\left(\frac{1}{R_1} \ \frac{1}{R_2}\right)$$

$$= (1.51 - 1) \times 0.08 = 0.0408$$

$$\therefore f_R = 25.51\,\text{cm}$$

$$\text{and } \frac{1}{f_V} = (n_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.53 - 1) \times 0.08 = 0.0424$$

$$\therefore f_V = 23.58\,\text{cm}$$

$\therefore$ Longitudinal chromatic aberration

$= f_V - f_R = 25.51 - 23.58$

= 1.93 cm,... (quite appreciable!)

Verify that you get the same answer even if you consider the concave surface facing the incident rays.

Spherical aberration: Longitudinal spherical aberration, transverse spherical aberration and circle of least confusion are defined in the same manner as that for spherical mirrors. (Fig 9.25 (a) and 9.25 (b))

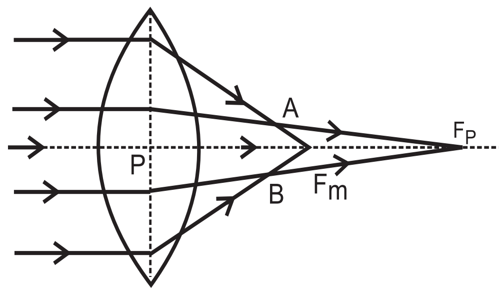


Fig. 9.25 (a): Spherical aberration, Convex lens.

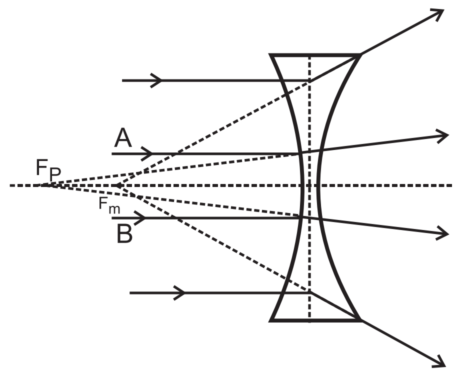


Fig. 9.25 (b): Spherical aberration, Concave lens

Methods to reduce/eliminate spherical aberration of lenses:

(i) Cheapest method to reduce the spherical aberration is to use a planoconvex or planoconcave lens with curved side facing the incident rays (real object). Reversing it increases the aberration appreciably.

(ii) Certain ratio of radii of curvature for a given refractive index almost eliminates the spherical aberration. For n = 1.5, the ratio is $\frac{R_1}{R_2}=\frac{1}{6}$ and for n = 2, it is $\frac{1}{5}$

(iii) Use of two thin converging lenses separated by distance equal to difference between their focal lengths with lens of larger focal length facing the incident rays considerably reduces spherical aberration.

(iv) Spherical aberration of a convex lens is positive (for real image), while that of a concave lens is negative. Thus, a suitable combination of them (preferably a double convex lens of smaller focal length and a planoconcave lens of greater focal length) can completely eliminate spherical aberration.

9.11 Optical instruments:

Introduction: Whether an object appears bigger or not does not necessarily depend upon its own size. Huge mountains far off may appear smaller than a small tree close to us. This is because the angle subtended by the mountain at the eye from that distance (called the visual angle) is smaller than that subtended by the tree from its position. Hence, apparent size of an object depends upon the visual angle subtended by the object from its position. Obviously, for an object to appear bigger, we must bring it closer to us or we should go closer to it.

However, due to the limitation for focusing the eye lens it is not possible to take an object closer than a certain distance. This distance is called least distance of distance vision D. For a normal, unaided human eye D = 25cm. If an object is brought closer than this, we cannot see it clearly. If an object is too small (like the legs of an ant), the corresponding visual angle from 25 cm is not enough to see it and if we bring it closer than that, its image on the retina is blurred. Also, the visual angle made by cosmic objects far away from us (such as stars) is too small to make out minor details and we cannot bring those closer. In such cases we need optical instruments such as a microscope in the former case and a telescope in the latter. It means that microscopes and telescopes help us in increasing the visual angle. This is called angular magnification or magnifying power.

Magnifying power: Angular magnification or magnifying power of an optical instrument is defined as the ratio of the visual angle made by the image formed by that optical instrument (β) to the visual angle subtended by the object when kept at the least distance of distinct vision (α). (Figure 9.26 (a) and 9.26 (b)) In the case of telescopes, α is the angle subtended by the object from its own position as it is not possible to get it closer.

Simple microscope or a reading glass: In order to read very small letters in a newspaper, sometimes we use a convex lens. You might have seen watch-makers using a special type of small convex lens while looking at very tiny parts of a wrist watch. Convex lens used for this purpose is a simple microscope.

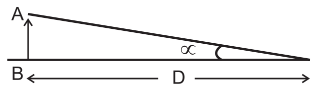


Fig. 9.26: (a) Visual Angle α.

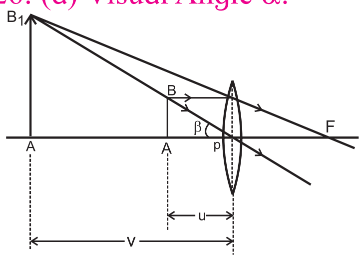


Fig. 9.26: (b) Visual Angle β.

Figure 9.26 (a) shows visual angle α made by an object, when kept at the least distance of distinct vision D. Without an optical instrument this is the greatest possible visual angle as we cannot get the object closer than this. Figure 9.26 (b) shows a convex lens forming erect, virtual and magnified image of the same object, when placed within the focus. The visual angle β of the object and the image in this case are the same. However, this time the viewer is looking at the image which is not closer than D. Hence the same object is now at a distance smaller than D. It makes β greater then α and the same object appears bigger.

Angular magnification or magnifying power, in this case, is given by

$$M = \frac{\text{Visual angle of the image}}{\text{Visual angle of the object, when at } D} = \frac{\beta}{\alpha}$$

For small angles α and β, we can write,

$$M = \frac{\beta}{\alpha} \cong \frac{\tan(\beta)}{\tan(\alpha)} = \frac{\left(\frac{BA}{PA}\right)}{\left(\frac{BA}{D}\right)} = \frac{D}{u}$$ - (numerically)

Limiting cases:

(i) For maximum magnifying power, the image should be nearest possible, i.e., at D.

For a thin lens, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

In this case, $f = +f, v = v_{min} = -D, u = -u$

and $M = M_{max}$

$$\therefore \frac{1}{f} = \frac{1}{-D} - \frac{1}{-u} \quad \therefore \frac{D}{f} = \frac{D}{-D} + \frac{D}{u}$$

$$\therefore M_{max} = \frac{D}{u} = 1 + \frac{D}{f}$$

(ii) For minimum magnifying power, $v = \infty$, i.e., $u = f$ (numerically)

$$\therefore M_{min} = \frac{D}{u} = \frac{D}{f}$$

Thus the angular magnification by a lens of focal length f is between $\left(\frac{D}{f}\right)$ and $\left(1 + \frac{D}{f}\right)$ only.

For common human eyesight, D = 25 cm. Thus, if f = 5 cm,

$$M_{min} = \left(\frac{D}{f}\right) = 5 \text{ and } M_{max} = \left(1 + \frac{D}{f}\right) = 6.$$

Hence image appears to be only 5 to 6 times bigger for a lens of focal length 5 cm.

For $M_{min} = \left(\frac{D}{f}\right) = 5$, $v = \infty$. $\therefore$ $m = \frac{v}{u} = \infty$.

Thus, the image size is infinite times that of the object, but appears only 5 times bigger.

For

$$M_{max} = 1 + \left(\frac{D}{f}\right) = 6,$$

$v = -25\,\text{cm}$. Corresponding $u = \frac{-25}{6}$ cm

$\therefore$ $m = \frac{v}{u} = 6$. Thus, image size is 6 times that of the object, and appears also 6 times larger.

Example 9.12: A magnifying glass of focal length 10 cm is used to read letters of thickness 0.5 mm held 8 cm away from the lens. Calculate the image size. How big will the letters appear? Can you read the letters if held 5 cm away from the lens? If yes, of what size would the letters appear? If no, why not?

$f = +10\,\text{cm}, u = -8\,\text{cm}, v = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \therefore \frac{1}{10} = \frac{1}{v} - \frac{1}{-8} \quad \therefore v = -40\,\text{cm}$$

$$m = \frac{v}{u} = \frac{\text{Image size } h_i}{\text{Object size } h_o} \therefore \frac{40}{8} = \frac{h_i}{0.5}$$

$\therefore h_i = 2.5$ cm (5 times that of the object)

$$M = \frac{D}{u} = \frac{25}{8} = 3.125$$

$\therefore$ Image will appear to be 3.125 times bigger. i.e., $3.125 \times 0.5 = 1.5625$ cm

For μ = - 5 cm, v will be - 10 cm.

For an average human being to see clearly, the image must be at or beyond 25 cm. Thus, it will not possible to read the letters if held 5 cm away from the lens.

Compound microscope: As seen above, the magnifying power of a simple microscope is inversely proportional to its focal length. However, if we need focal length to be smaller and smaller, the corresponding lens becomes thicker and thicker. For such a lens both spherical as well as chromatic aberrations are dominant. Thus, if higher magnifying power is needed, we go for using more than one lenses. The instrument is then called a compound microscope. It is used to view very small objects (sizes $\sim 10^{-1}$ mm to 10^{-3} mm). Also, whether the image is erect or inverted is immaterial.

A compound microscope essentially uses two convex lenses of suitable focal lengths fit into a cylindrical tube with some adjustment possible for its length. The smaller lens ($\sim$ 4 mm to 6 mm aperture) facing the object is called the objective. Other lens with which the observer jams her/his eye is litter larger and called as the eye lens. (Fig 9.27) During this discussion we consider the eye lens to be a single lens, but in practice it is an eyepiece, itself consisting of two planoconvex lenses.

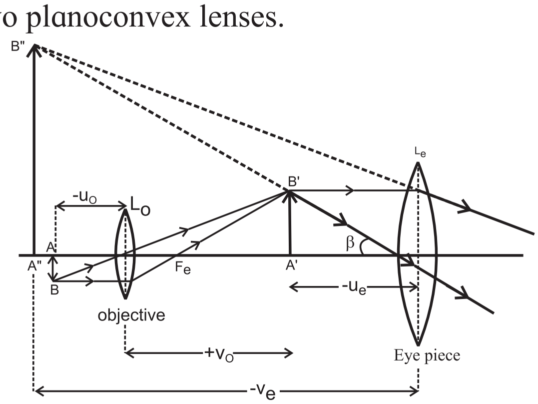


Fig. 9.27: Compound Microscope.

As shown in the Fig. 9.27, a tiny object AB is placed between f and $2f$ of the objective which produces its real, inverted and magnified image A′ B′ in front of the eye lens. Position of the eye lens is so adjusted that the (intermediate) image A′ B′ is within its focus. Hence, for this object A′ B′, the eye lens behaves as a simple microscope and produces its virtual and magnified image A″ B″, which is inverted with respect to original object AB.

Magnifying power of a compound microscope with two lenses: From its position, the final image A″ B″ makes a visual angle β at the eye (jammed at the eye lens). Visual angle made by the object from distance D is α.

$$\therefore \tan\beta = \frac{A''B''}{v_e} = \frac{A'B'}{u_e}$$

$$\tan\alpha = \frac{AB}{D} \quad \text{(Fig. 9.29 (a))}$$

$\therefore$ Angular magnification or magnifying power,

$$M = \frac{\beta}{\alpha} \cong \frac{\tan\beta}{\tan\alpha} = \left(\frac{A'B'}{u_e}\right) \times \left(\frac{D}{AB}\right)$$

$$= \left(\frac{A'B'}{AB}\right) \times \left(\frac{D}{u_e}\right)$$

$$\therefore M = m_o \times M_e$$

Where, $\left(\frac{A'B'}{AB}\right) = m_o = \frac{v_o}{u_o}$ is the linear (lateral) magnification of the objective and

$\left(\frac{D}{u_e}\right) = M_e$ is the angular magnification or magnifying power of the eye lens. Length of the compound microscope then becomes L = distance between the two lenses $v_0 + u_e$.

Remarks:

(i) In order to increase m_o, we need to decrease u_o. Thereby, the object comes closer and closer to the focus of the objective. This increases v_0 and hence length of the microscope. Thus m_o can be increased only within the limitation of length of the microscope.

(ii) Minimum value of M_e is $\left(\frac{D}{f_e}\right)$ for final image at infinity and maximum value of M_e is $\left(1+\frac{D}{f_e}\right)$ for final image at D respectively. M_e and m_o together decide the minimum and maximum magnifying power of the microscope.

Example 9.13: The pocket microscope used by a student consists of eye lens of focal length 6.25 cm and objective of focal length 2 cm. At microscope length 15 cm, the final image appears biggest. Estimate distance of the object from the objective and magnifying power of the microscope.

Solution:

$$f_e = 6.25\,\text{cm},\ f_o = 2\,\text{cm},\ L = |\text{v}_o| + |u_e|$$

$$= 15\,\text{cm},\ \text{v}_e = 25\,\text{cm}\ (\text{Image appears largest})$$

$$\frac{1}{f_e} = \frac{1}{\text{v}_e} - \frac{1}{u_e} \quad \therefore \frac{1}{6.25} = \frac{4}{25}$$

$$= \frac{1}{-25} - \frac{1}{u_e} \quad \therefore |u_e| = 5\,\text{cm}$$

$$\therefore |\text{v}_o| = L - |u_e| = 15 - 5 = 10\,\text{cm}$$

$$\frac{1}{f_o} = \frac{1}{\text{v}_o} - \frac{1}{u_o} \quad \therefore \frac{1}{2} = \frac{1}{+10} - \frac{1}{u_o} \quad \therefore |u_o| = 2.5\,\text{cm}$$

$$M = m_o \times M_e = \left(\frac{\text{v}_0}{u_o}\right)\left(\frac{D}{u_e}\right)$$

$$= \left(\frac{10}{2.5}\right)\left(\frac{25}{5}\right) = 4 \times 5 = 20$$

Telescope: Telescopes are used to see terrestrial or astronomical bodies. A telescope essentially uses two lenses (or one large parabolic mirror and a lens). The lens facing the object (called objective) is of aperture as large as possible. For Newtonian telescopes, a large parabolic mirror faces the object.

For terrestrial telescopes the objects to be seen are on the Earth , like mountains, trees, players playing a match in a stadium, etc. In such case, the final image must be erect. Eye lens used for this purpose must be concave and such a telescope is popularly called a binocular. A variety of binoculars use three convex lenses with proper separation. The third lens again inverts the second intermediate image and makes final image erect with respect to the object. In this text we shall be discussing astronomical telescope.

For an astronomical telescope, the objects to be seen are planets, stars, galaxies, etc. In this case there is no necessity of erect image. Such telescopes use convex lens as eye lens. (Fig. 9.27).

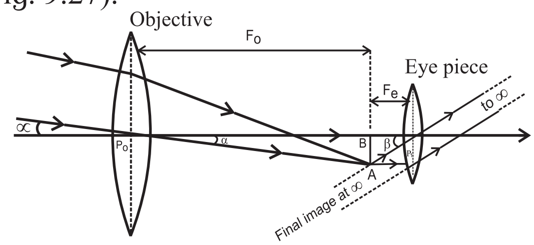


Fig. 9.28: Telescope.

Magnifying power of a telescope: Objects to be seen through a telescope cannot be brought to distance D from the objective, like in microscopes. Hence, for telescopes, α is the visual angle of the object from its own position, which is practically at infinity. Visual angle of the final image is β and its position can be adjusted to be at D. However, under normal adjustments, the final image is also at infinity but making a greater visual angle than that of the object. (If the image is really at infinity, there will not be any parallax at the cross wires). Beam of incident rays is now inclined at an angle α with the principal axis while emergent beam is inclined at a greater angle β with the principal axis causing angular magnification. (Fig. 9.28)

Objective of focal length f_o focusses the parallel incident beam at a distance f_o from the objective giving an inverted image AB. For normal adjustment, the eye lens is so adjusted that the intermediate image AB happens to be at the focus of the eye lens. Rays refracted beyond the eye lens form a parallel beam inclined at an angle β with the principal axis resulting into the image also at infinity.

$\therefore$ Angular magnification or magnifying power,

$$M = \frac{\beta}{\alpha} \cong \frac{\tan\beta}{\tan\alpha} = \frac{\left(\frac{\text{BA}}{\text{P}_e\text{B}}\right)}{\left(\frac{\text{BA}}{\text{P}_o\text{B}}\right)} = \frac{\left(\frac{\text{BA}}{f_e}\right)}{\left(\frac{\text{BA}}{f_o}\right)}$$

$$\therefore M = \frac{f_o}{f_e}$$

Length of the telescope for normal adjustment is $L = f_o + f_e$

Under the allowed limit of length objective of maximum possible focal length f_o and eye lens of minimum possible focal length f_e can be chosen for maximum magnifying power.

Example 14: Focal length of the objective of an astronomical telescope is 1 m. Under normal adjustment, length of the telescope is 1.05 m. Calculate focal length of the eyepiece and magnifying power under normal adjustment.

Solution: For astronomical telescope,

$L = f_0 + f_e \therefore 1.05 = 1 + f_0 \therefore f_e = 0.05$ m = 5 cm

Under normal adjustments,

$$M = \frac{f_o}{f_e} = \frac{1}{0.05} = 20$$

Exercises

1. Choose the correct option

i. As per recent understanding *light* consists of

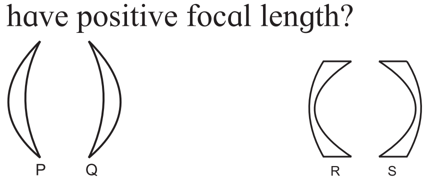
(A) rays

(B) waves

(C) corpuscles

(D) photons obeying the rules of waves

ii. Consider optically denser lenses P, Q, R and S drawn below. According to Cartesian sign convention which of these have positive focal length?



(A) Only P

(B) Only P and Q

(C) Only P and R

(D) Only Q and S

iii. Two plane mirrors are inclined at angle 40^0 between them. Number of images seen of a tiny object kept between them is

(A) Only 8 (B) Only 9

(C) 8 or 9 (D) 9 or 10

iv. A concave mirror of curvature 40 cm, used for *shaving purpose* produces image of double size as that of the object. Object distance must be

(A) 10 cm only

(B) 20 cm only

(C) 30 cm only

(D) 10 cm or 30 cm

v. Which of the following aberrations will NOT occur for spherical mirrors?

(A) Chromatic aberration

(B) Coma

(C) Distortion

(D) Spherical aberration

vi. There are different fish, monkeys and water on the habitable planet of the star Proxima b. A fish swimming underwater feels that there is a monkey at 2.5 m on the top of a tree. The same monkey feels that the fish is 1.6 m below the water surface. Interestingly, height of the tree and the depth at which the fish is swimming are exactly same. Refractive index of that water must be

(A) 6/5 (B) 5/4

(C) 4/3 (D) 7/5

vii. Consider following phenomena/ applications: P) Mirage, Q) rainbow, R) Optical fibre and S) glittering of a diamond. Total internal reflection is involved in

(A) Only R and S (B) Only R

(C) Only P, R and S (D) all the four

viii. A student uses spectacles of number -2 for seeing distant objects. Commonly used lenses for her/his spectacles are

(A) bi-concave

(B) double concave

(C) concavo-convex

(D) convexo-concave

ix. A spherical marble of refractive index 1.5 and curvature 1.5 cm, contains a tiny air bubble at its centre. Where will it appear when seen from outside?

(A) 1 cm inside (B) at the centre

(C) 5/3 cm inside (D) 2 cm inside

x. Select the WRONG statement.

(A) Smaller angle of prism is recommended for greater angular dispersion.

(B) Right angled isosceles glass prism is commonly used for total internal reflection.

(C) Angle of deviation is practically constant for thin prisms.

(D) For emergent ray to be possible from the second refracting surface, certain minimum angle of incidence is necessary from the first surface.

xi. Angles of deviation for extreme colours are given for different prisms. Select the one having maximum dispersive power of its material.

(A) 7°, 10° (B) 8°, 11°

(C) 12°, 16° (D) 10°, 14°

xii. Which of the following is not involved in formation of a rainbow?

(A) refraction

(B) angular dispersion

(C) angular deviation

(D) total internal reflection

xiii. Consider following statements regarding a simple microscope:

(P) It allows us to keep the object within the least distance of distant vision.

(Q) Image appears to be biggest if the object is at the focus.

(R) It is simply a convex lens.

(A) Only (P) is correct

(B) Only (P) and (Q) are correct

(C) Only (Q) and (R) are correct

(D) Only (P) and (R) are correct

2. Answer the following questions.

i) As per recent development, what is the nature of light? Wave optics and particle nature of light are used to explain which phenomena of light, respectively?

ii) Which phenomena can be satisfactorily explained using ray optics? State the assumptions on which ray optics is based.

iii) What is focal power of a spherical mirror or of a lens? What may be the reason for using $P = \frac{1}{f}$ as its expression?

iv) At which positions of the objects do spherical mirrors produce (i) diminished image, (ii) magnified image?

v) State the restrictions for having images produced by spherical mirrors to be appreciably clear.

vi) Explain spherical aberration for spherical mirrors. How can it be minimized? Can it be eliminated by some curved mirrors?

vii) Define absolute refractive index and relative refractive index. Explain in brief, with an illustration for each.

viii) Explain 'mirage' as an illustration of refraction.

ix) Under what conditions is total internal reflection possible? Explain it with a suitable example. Define critical angle of incidence and obtain an expression for it.

x) Describe construction and working of an optical fibre. What are the advantages of optical fibre communication over electronic communication?

xi) Why is a prism binoculars preferred over traditional binoculars? Describe its working in brief.

xii) A spherical surface separates two transparent media. Derive an expression that relates object and image distances with the radius of curvature for a point object. Clearly state the assumptions, if any.

xiii) Derive lens makers' equation. Why is it called so? Under which conditions focal length f and radii of curvature R are numerically equal for a lens?

2. Answer the following questions in detail.

i) What are different types of dispersions of light? Why do they occur?

ii) Define angular dispersion for a prism. Obtain its expression for a thin prism. Relate it with the refractive indices of the material of the prism for corresponding colours.

iii) Explain and define dispersive power of a transparent material. Obtain its expressions in terms of angles of deviation and refractive indices.

iv) (i) State the conditions under which a rainbow can be seen.

(ii) Explain the formation of a primary rainbow. For which angular range with the horizontal is it visible?

(iii) Explain the formation of a secondary rainbow. For which angular range with the horizontal is it visible?

(iv) Is it possible to see primary and secondary rainbow simultaneously? Under what conditions?

v) (i) Explain chromatic aberration for spherical lenses. State a method to minimize or eliminate it.

(ii) What is achromatism? Derive a condition to achieve achromatism for a lens combination. State the conditions for it to be converging.

vi) Describe spherical aberration for spherical lenses. What are different ways to minimize or eliminate it?

vii) Define and describe magnifying power of an optical instrument. How does it differ from linear or lateral magnification?

viii) Derive an expression for magnifying power of a simple microscope. Obtain its minimum and maximum values in terms of its focal length.

ix) Derive the expressions for the magnifying power and the length of a compound microscope using two convex lenses.

x) What is a terrestrial telescope and an astronomical telescope?

xi) Obtain the expressions for magnifying power and the length of an astronomical telescope under normal adjustments.

xii) What are the limitations in increasing the magnifying powers of (i) simple microscope (ii) compound microscope (iii) astronomical telescope?

3. Solve the following numerical examples

i) A monochromatic ray of light strike the water (n = 4/3) surface in a cylindrical vessel at angle of incidence 53^0. Depth of water is 36 cm. After striking the water surface, how long will the light take to reach the bottom of the vessel? [Angles of the most popular Pythagorean triangle of sides in the ratio 3:4:5 are nearly 37^0, 53^0 and 90^0]

[Ans: 2 ns]

ii) Estimate the number of images produced if a tiny object is kept in between two plane mirrors inclined at 35^0, 36^0, 40^0 and 45^0.

[Ans: 10, 9, 9 or 8, 7 respectively]

iii) A rectangular sheet of length 30 cm and breadth 3 cm is kept on the principal axis of a concave mirror of focal length 30 cm. Draw the image formed by the mirror on the same ray diagram, as far as possible on scale.

[Ans: Inverted image starts from 50 cm and ends at 90 cm. Its height in the beginning is 2 cm and at the end it is 6 cm. At 60 cm, image height is 3 cm. Thus, outer boundary if the image is a curve]

iv) A car uses a convex mirror of curvature 1.2 m as its rear-view mirror. A minibus of cross section 2.4 m × 2.4 m is 6.6 m away from the mirror. Estimate the image size.

[Ans: A square of edge 0.2 m]

v) A glass slab of thickness 2.5 cm having refractive index 5/3 is kept on an ink spot. A transparent beaker of very thin bottom, containing water of refractive index 4/3 up to 8 cm, is kept on the glass block. Calculate apparent depth of the ink spot when seen from the outside air.

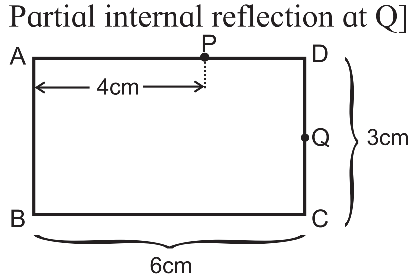
[Ans: 7.5 cm]

vi) A convex lens held some distance above a 6 cm long pencil produces its image of SOME size. On shifting the lens by a distance equal to its focal length, it again produces the image of the SAME size as earlier. Determine the image size.

[Ans: 12 cm]

vii) Figure below shows the section ABCD of a transparent slab. There is a tiny green LED light source at the bottom left corner B. A certain ray of light from B suffers total internal reflection at nearest point P on the surface AD and strikes the surface CD at point Q. Determine refractive index of the material of the slab and distance DQ. At Q, the ray PQ will suffer partial or total internal reflection? [You may use the approximation given in Q 1 above].

[Ans: n = 5/4, DQ = 1.5 cm, Partial internal reflection at Q]



viii) A point object is kept 10 cm away from one of the surfaces of a thick double convex lens of refractive index 1.5 and radii of curvature 10 cm and 8 cm. Central thickness of the lens is 2 cm. Determine location of the final image considering paraxial rays only.

Hint : Single spherical surface formula to be used twice.

[Ans: 64 cm away from the other surface]

ix) A monochromatic ray of light is incident at 37^0 on an equilateral prism of refractive index 3/2. Determine angle of emergence and angle of deviation. If angle of prism is adjustable, what should its value be for emergent ray to be just possible for the same angle of incidence.

[Ans: $e = 63°$, $\delta = 40°$, $A = 65°\ 24'$ for $e = 90°$ (just emerges)]

x) From the given data set, determine angular dispersion by the prism and dispersive power of its material for extreme colours. $n_R = 1.62$ $n_V = 1.66$, $\delta_R = 3.1°$

[Ans: $\delta_{VR} = 0.2°$, $\omega_{VR} = \frac{1}{16} = 0.0625$]

xi) Refractive index of a flint glass varies from 1.60 to 1.66 for visible range. Radii of curvature of a thin convex lens are 10 cm and 15 cm. Calculate the chromatic aberration between extreme colours.

[Ans: 10/11 cm]

xii) A person uses spectacles of 'number' 2.00 for reading. Determine the range of magnifying power (angular magnification) possible. It is a concavo-convex lens ($n = 1.5$) having curvature of one of its surfaces to be 10 cm. Estimate that of the other.

[Ans: $M_{min} = 0.5$, $M_{max} = 1.5$ $R_2 = 50/3$ cm]

xiii) Focal power of the eye lens of a compound microscope is 6 dioptre. The microscope is to be used for maximum magnifying power (angular magnification) of at least 12.5. The packing instructions demand that length of the microscope should be 25 cm. Determine minimum focal power of the objective. How much will its radius of curvature be if it is a biconvex lens of n = 1.5.

[Ans: 40 dioptre, 2.5 cm]

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