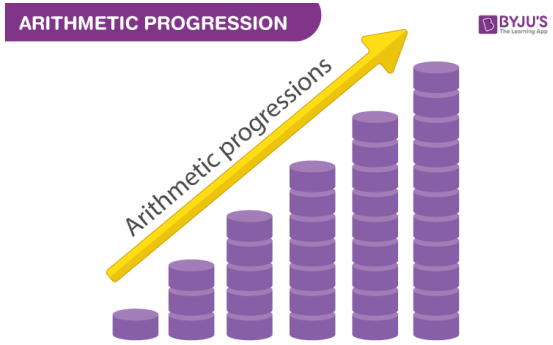
Arithmetic Progression

Arithmetic Progression (AP) is a sequence of numbers in order in which the difference of any two consecutive numbers is a constant value. For example, the series of natural numbers (https://byjus.com/maths/natural-numbers/): 1, 2, 3, 4, 5, 6,... is an AP, which has a common difference between two successive terms (say 1 and 2) equal to 1 (2 -1). Even in the case of odd numbers and even numbers, we can see the common difference between two successive terms will be equal to 2.

Check: Mathematics for Grade 11 (https://byjus.com/maths/what-is-mathematics/)

If we observe in our regular lives, we come across Arithmetic progression quite often. For example, Roll numbers of students in a class, days in a week or months in a year. This pattern of series and sequences (https://byjus.com/maths/sequence-and-series/) has been generalized in Maths as progressions.



Definition

In mathematics, there are three different types of progressions. They are:

- Arithmetic Progression (AP)
- Geometric Progression (GP)
- Harmonic Progression (HP)

A progression is a special type of sequence for which it is possible to obtain a formula for the nth term. The Arithmetic Progression is the most commonly used sequence in maths with easy to understand formulas. Let's have a look at its three different types of definitions.

Definition 1: A mathematical sequence in which the difference between two consecutive terms is always a constant and it is abbreviated as AP.

Definition 2: An arithmetic sequence or progression is defined as a sequence of numbers in which for every pair of consecutive terms, the second number is obtained by adding a fixed number to the first one.

Definition 3: The fixed number that must be added to any term of an AP to get the next term is known as the common difference of the AP. Now, let us consider the sequence, 1, 4, 7, 10, 13, 16,... is considered as an arithmetic sequence with common difference 3.

Notation in AP

In AP, we will come across three main terms, which are denoted as:

- Common difference (d)
- nth Term (a_n)
- Sum of the first n terms (S_n)

All three terms represent the property of Arithmetic Progression. We will learn more about these three properties in the next section.

Common Difference in Arithmetic Progression

In this progression, for a given series, the terms used are the first term, the common difference between the two terms and nth term. Suppose, $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n$ is an AP, then; the **common difference " d "** can be obtained as;

$$d = a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_n - a_{n-1}$$

Where "d" is a common difference. It can be positive, negative or zero.

First Term of AP

The AP can also be written in terms of common difference, as follows;

$$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots\ldots, a + (n - 1)\, d$$

where "a" is the **first term** of the progression.

Also, check:

- Geometric Progression Sum Of Gp (https://byjus.com/maths/geometric-progression-sum-of-gp/)
- Arithmetic Progression For Class 10 (https://byjus.com/maths/arithmetic-progression-for-class-10/)
- Important Questions Class 10 Maths Chapter 5 Arithmetic Progressions (https://byjus.com/maths/important-questions-class-10-maths-chapter-5-arithmetic-progressions/)

General Form of an A. P

Consider an AP to be: $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n$

Position of Terms	Representation of Terms	Values of Term
1	a_1	a = a + (1-1) d
2	a_2	a + d = a + (2-1) d
3	a_3	a + 2d = a + (3-1) d
4	a_4	a + 3d = a + (4-1) d
.	.	.
.	.	.
.	.	.
.	.	.
n	a_n	a + (n-1)d

Formulas

There are two major formulas we come across when we learn about Arithmetic Progression, which is related to:

- The nth term of AP
- Sum of the first n terms

Let us learn here both the formulas with examples.

nth Term of an AP

The formula for finding the n-th term of an AP is:

$$\mathbf{a_n = a + (n - 1) \times d}$$

Where

a = First term

d = Common difference

n = number of terms

a_n = nth term

Example: Find the nth term of AP: 1, 2, 3, 4, 5...., a_n if the number of terms are 15.

Solution: Given, AP: 1, 2, 3, 4, 5...., a_n

n=15

By the formula we know, $a_n = a+(n-1)d$

First-term, a =1

Common difference, d=2-1 =1

Therefore, $a_n = 1+(15-1)1 = 1+14 = 15$

Note: The finite portion of an AP is known as finite AP and therefore the sum of finite AP is known as arithmetic series. The behaviour of the sequence depends on the value of a common difference.

- If the value of "d" is positive, then the member terms will grow towards positive infinity
- If the value of "d" is negative, then the member terms grow towards negative infinity

Sum of N Terms of AP

For any progression, the sum of n terms can be easily calculated. For an AP, the sum of the first n terms (https://byjus.com/maths/sum-of-n-terms/) can be calculated if the first term and the total terms are known. The formula for the arithmetic progression sum is explained below:

Consider an AP consisting "n" terms.

S = n/2[2a + (n − 1) × d]

This is the AP sum formula to find the sum of n terms in series.

Proof: Consider an AP consisting "n" terms having the sequence a, a + d, a + 2d, ……,a + (n − 1) × d

Sum of first n terms = a + (a + d) + (a + 2d) + …… + [a + (n − 1) × d] ————-(i)

Writing the terms in reverse order,we have:

S = [a + (n − 1) × d] + [a + (n − 2) × d] + [a + (n − 3) × d] + …… (a) ———-(ii)

Adding both the equations term wise, we have:

2S = [2a + (n − 1) × d] + [2a + (n − 1) × d] + [2a + (n − 1) × d] + …… + [2a + (n − 1) ×d] (n-terms)

2S = n × [2a + (n − 1) × d]

S = n/2[2a + (n − 1) × d]

Example: Let us take the example of adding natural numbers up to 15 numbers.

AP = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15

Given, a = 1, d = 2-1 = 1 and an = 15

Now, by the formula we know;

S = n/2[2a + (n − 1) × d] = 15/2[2.1+(15-1).1]

S = 15/2[2+14] = 15/2 [16] = 15 x 8

S = 120

Hence, the sum of the first 15 natural numbers is 120.

Sum of AP when the Last Term is Given

Formula to find the sum of AP when first and last terms are given as follows:

S = n/2 (first term + last term)

Formula Lists

The list of formulas is given in a tabular form used in AP. These formulas are useful to solve problems based on the series and sequence concept.

General Form of AP	a, a + d, a + 2d, a + 3d, . . .
The nth term of AP	a_n = a + (n − 1) × d
Sum of n terms in AP	S = n/2[2a + (n − 1) × d]
Sum of all terms in a finite AP with the last term as 'l'	n/2(a + l)

Arithmetic Progressions Questions and Solutions

Below are the problems to find the nth terms and sum of the sequence are solved using AP sum formulas in detail. Go through them once and solve the practice problems to excel your skills.

Example 1: Find the value of n. If a = 10, d = 5, a_n = 95.

Solution: Given, a = 10, d = 5, a_n = 95

From the formula of general term, we have:

a_n = a + (n − 1) × d

95 = 10 + (n − 1) × 5

(n − 1) × 5 = 95 − 10 = 85

(n − 1) = 85/ 5

(n − 1) = 17

n = 17 + 1

n = 18

Example 2: Find the 20th term for the given AP:3, 5, 7, 9, ……

Solution: Given,

3, 5, 7, 9, ……

a = 3, d = 5 − 3 = 2, n = 20

a_n = a + (n − 1) × d

a_{20} = 3 + (20 − 1) × 2

a_{20} = 3 + 38

$\Rightarrow a_{20}$ = 41

Example 3: Find the sum of first 30 multiples of 4.

Solution: Given, a = 4, n = 30, d = 4

We know,

S = n/2 [2a + (n − 1) × d]

S = 30/2[2 (4) + (30 − 1) × 4]

S = 15[8 + 116]

S = 1860

Problems on AP

Find the below questions based on Arithmetic sequence formulas and solve it for good practice.

Question 1: Find the a_n and 10th term of the progression: 3, 1, 17, 24, ……

Question 2: If a = 2, d = 3 and n = 90. Find a_n and S_n.

Question 3: The 7th term and 10th terms of an AP are 12 and 25. Find the 12th term.

To learn more about different types of formulas with the help of personalised videos, download BYJU'S-The Learning App and make learning fun.

Frequently Asked Questions – FAQs

What is the Arithmetic Progression Formula?

The arithmetic progression general form is given by a, a + d, a + 2d, a + 3d, ... Hence, the formula to find the nth term is:
an = a + (n − 1) × d

What is arithmetic progression? Give an example.

A sequence of numbers which has a common difference between any two consecutive numbers is called an arithmetic progression (A.P.). The example of A.P. is 3,6,9,12,15,18,21, ...

How to find the sum of arithmetic progression?

To find the sum of arithmetic progression, we have to know the first term, the number of terms and the common difference between each term. Then use the formula given below:
S = n/2[2a + (n − 1) × d]

What are the types of progressions in Maths?

There are three types of progressions in Maths. They are:
Arithmetic Progression (AP)
Geometric Progression (GP)
Harmonic Progression (HP)

What is the use of Arithmetic Progression?

An arithmetic progression is a series which has consecutive terms having a common difference between the terms as a constant value. It is used to generalise a set of patterns, that we observe in our day to day life.

Quiz on Arithmetic Progression

Put your understanding of this concept to test by answering a few MCQs. Click 'Start Quiz' to begin!

Select the correct answer and click on the "Finish" button
Check your score and answers at the end of the quiz

Start Quiz

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