BT6270: Assignment - 2

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EE20B064

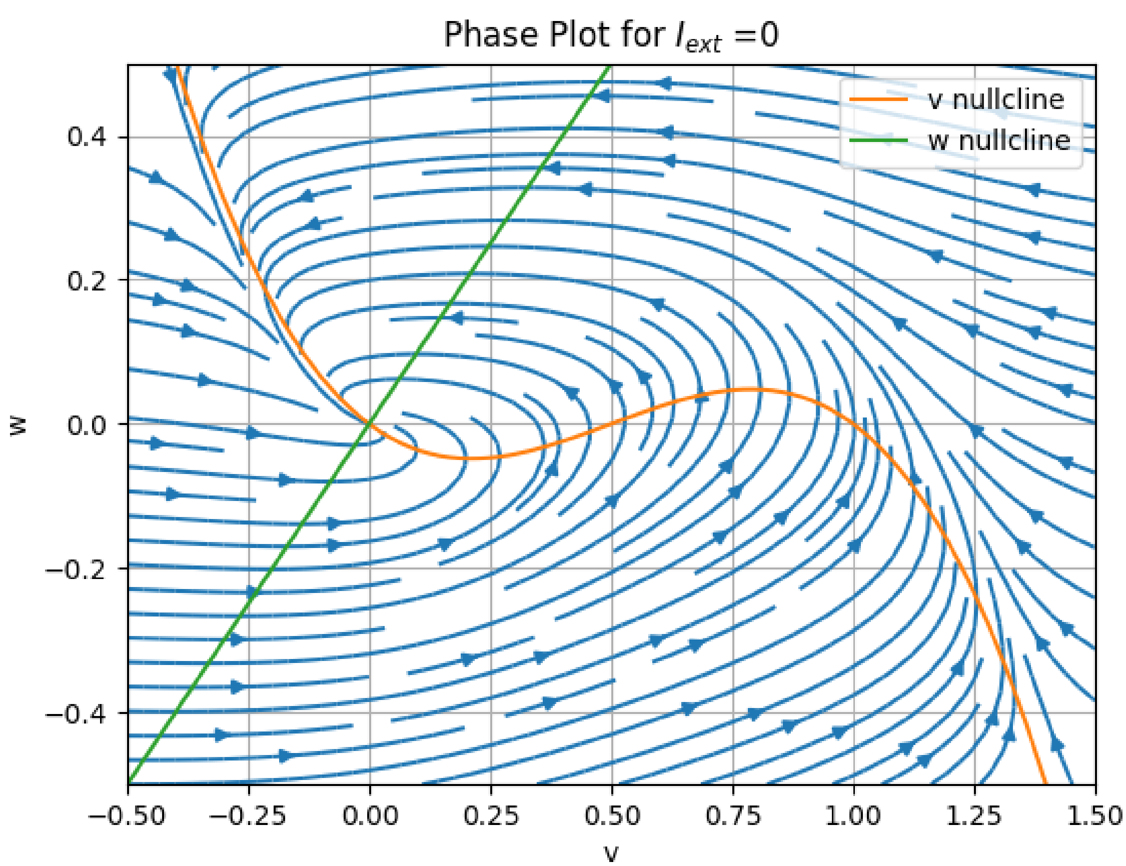
<u>FitzHugh Nagumo Neuron Model</u>

(Note : Parameter values: a = 0.5, b = 0.1, r = 0.1 for the first 3 cases)

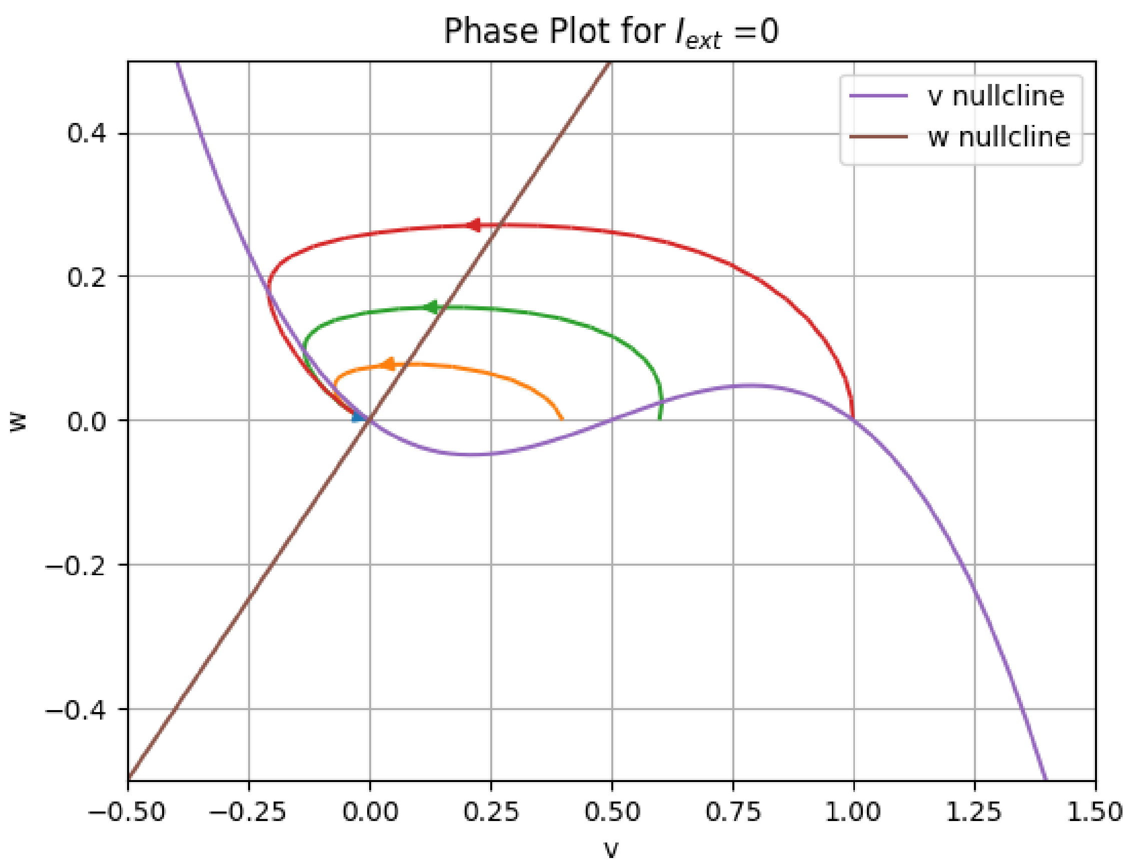
- Case 1 : $I_{ext} = 0$ (Excitability)

(a) Phase Portrait plots

The fixed point is the intersection of the two nullclines ie. (0, 0) which is clearly a stable fixed point.



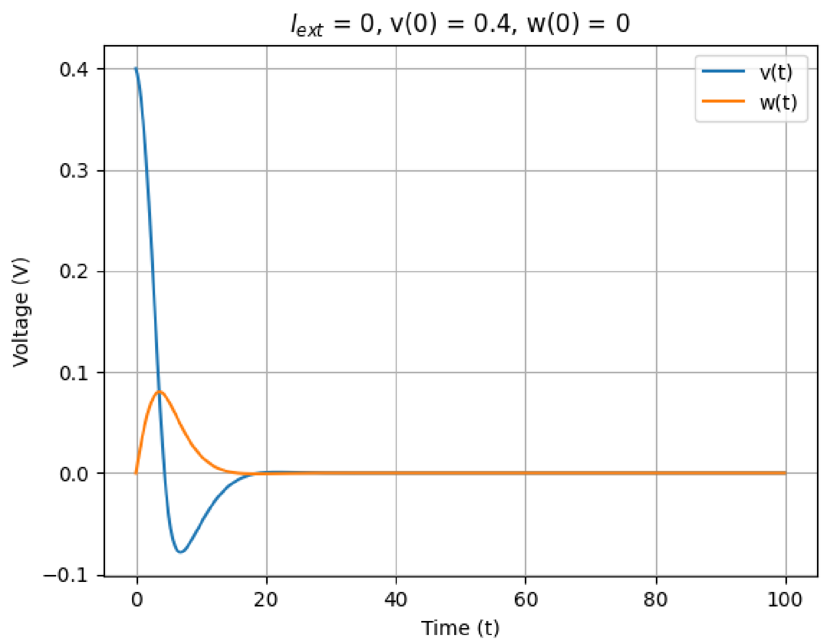
Plotting the trajectories for various starting points, again confirms the fact that (0, 0) is a stable fixed point, as irrespective of the initial conditions, all the trajectories eventually converge at (0, 0) as shown in the plot below.



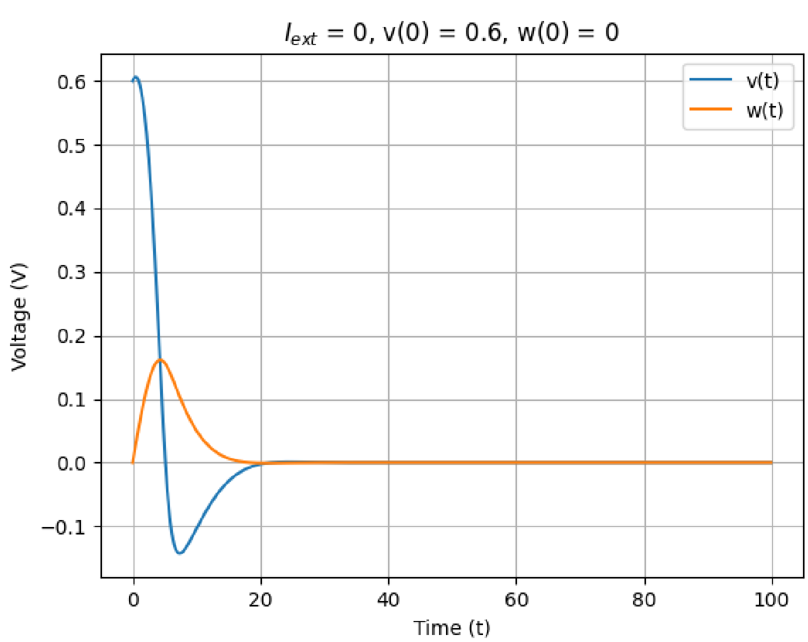
The same is also evident from the below obtained plots of v(t) and w(t) vs time for the same conditions, since the values of v and w become 0 eventually, in both the cases shown below.

(b) v(t) and w(t) vs time plots

(i) v(0) = 0.4 < a, w(0) = 0



(ii) v(0) = 0.6 > a, w(0) = 0



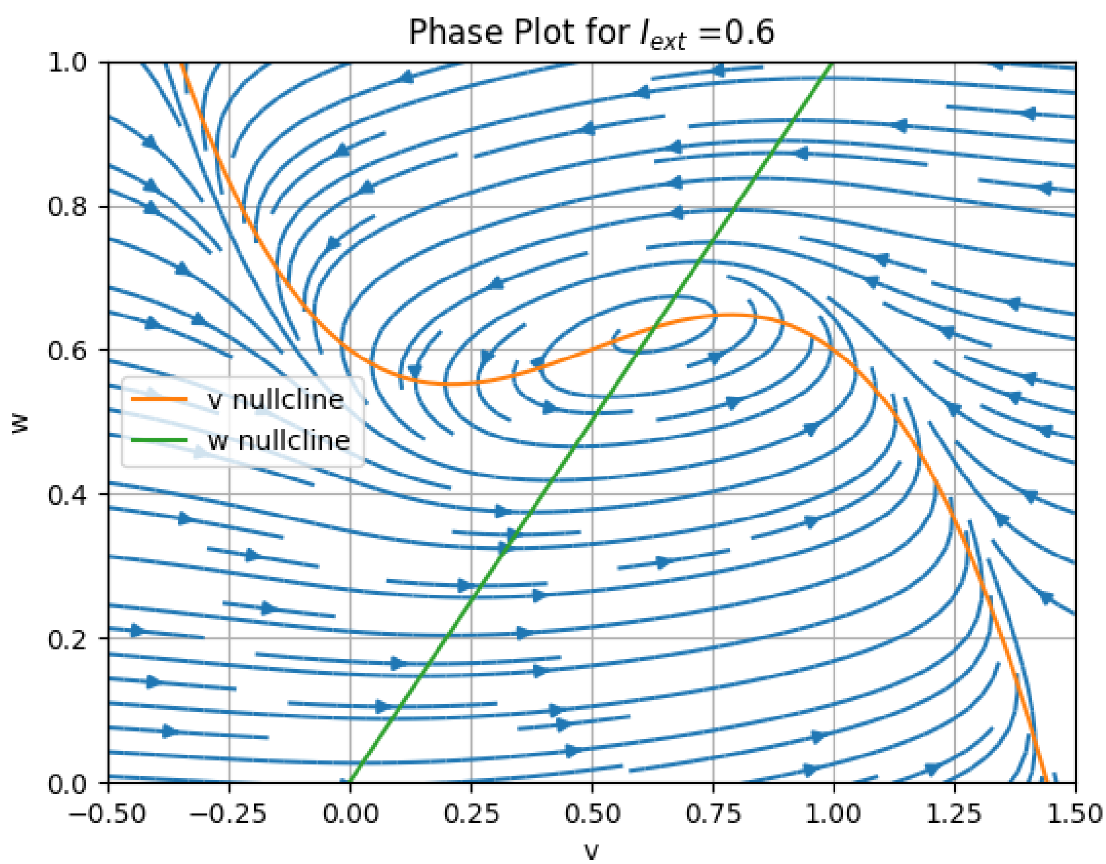
- Case 2 : I_{ext} = 0.6 (Limit cycle Oscillations)

By hit and trial method, varying values of I_{ext} continually, the *approximate* values of I_1 and I_2 obtained are:

I_1 = 0.34 , I_2 = 0.68

Where I_1 and I_2 are the bounds such that for $I_1 < I_{ext} < I_2$, the system exhibits oscillatory behaviour.

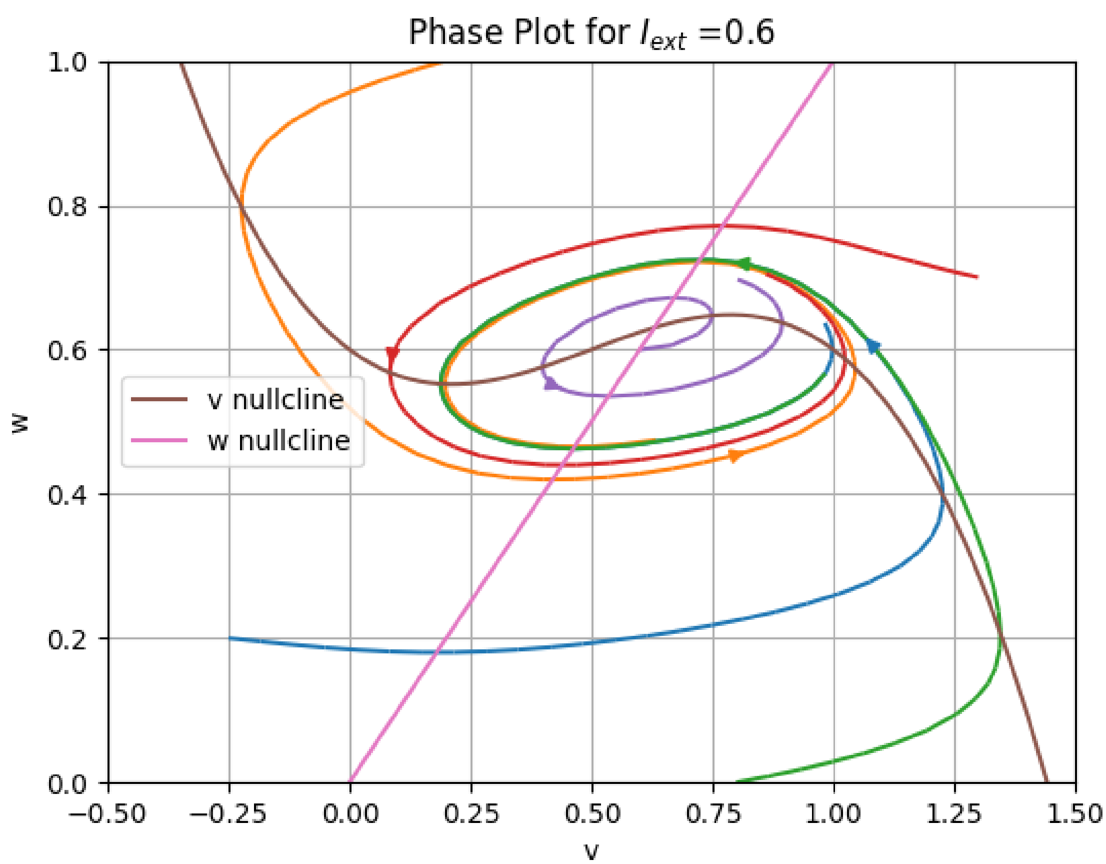
(a) Phase Portrait plots



The fixed point is clearly unstable, as a slight perturbation from the fixed point leads to trajectories that lead to a limit cycle behaviour as shown by the circulating fields around the fixed point.

Since, after the perturbations, the system moves away from the fixed point, it is an unstable fixed point.

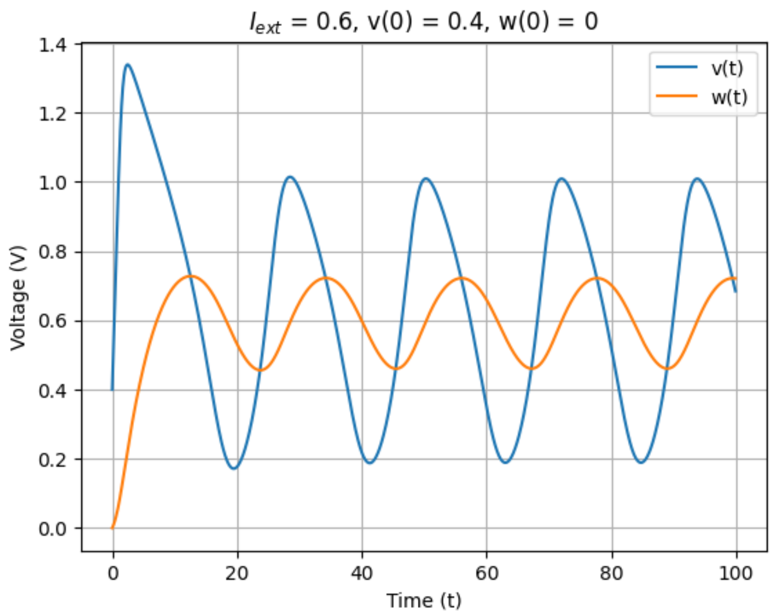
This can be further confirmed, by plotting the trajectories for various starting points as shown below. The following plot shows that trajectories starting from all four regions (differentiated by the two nullclines), end up along the limit cycle and hence the system exhibits oscillatory behaviour.



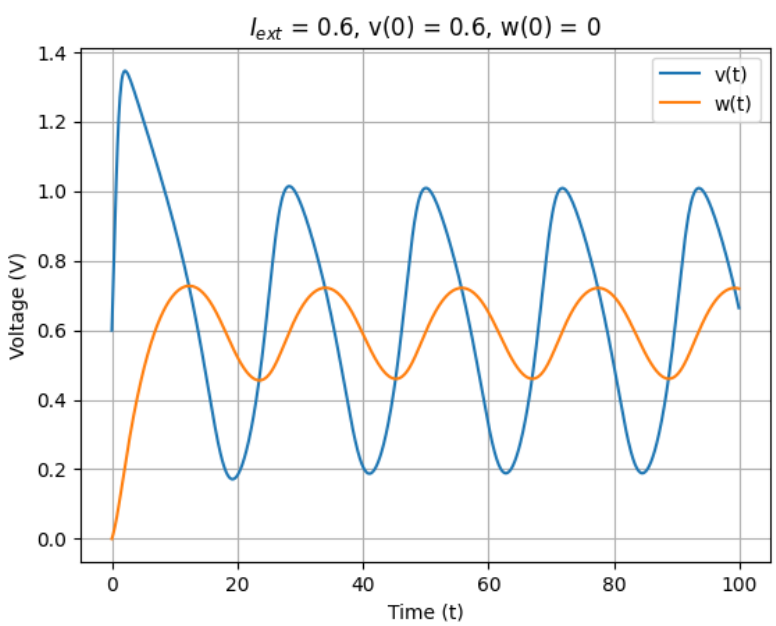
The plots of v(t) and w(t) vs time for the same conditions, show the previously described oscillatory behaviour as shown below.

(b) v(t) and w(t) vs time plots

(iii) $v(0) = 0.4 < a$, $w(0) = 0$

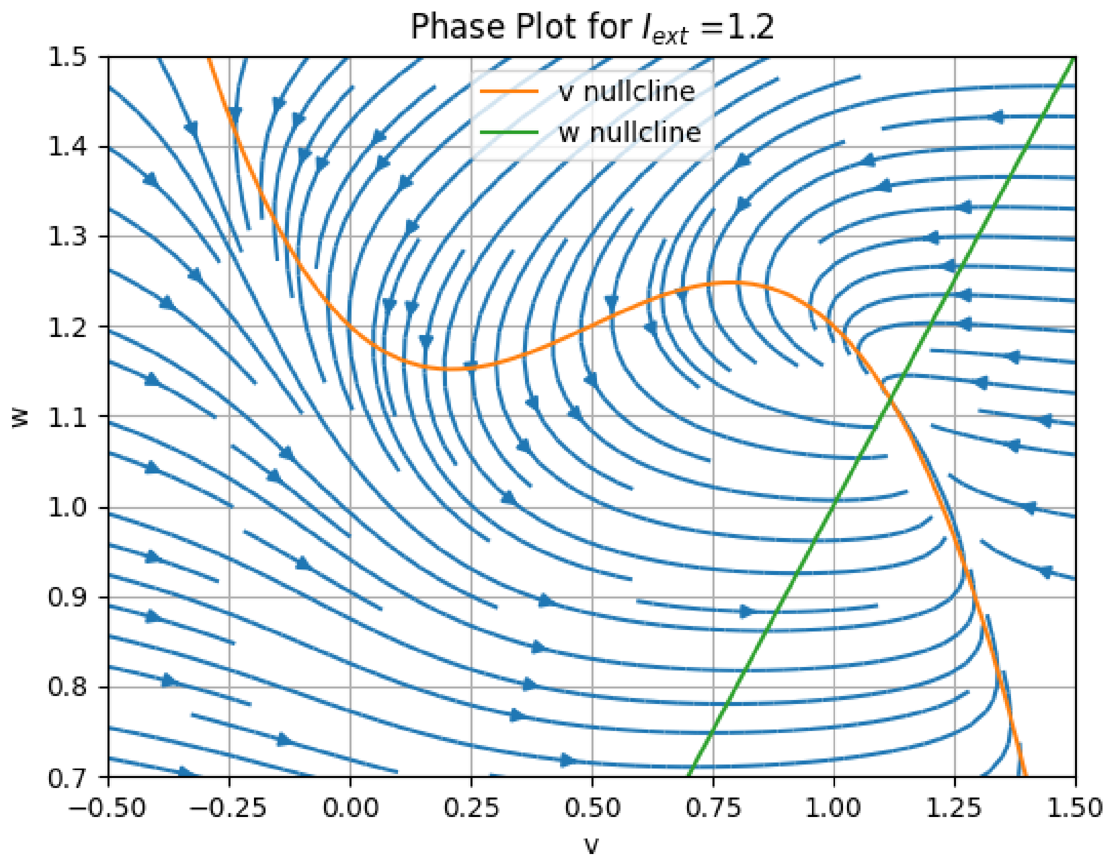


(iv) $v(0) = 0.6 > a$, $w(0) = 0$



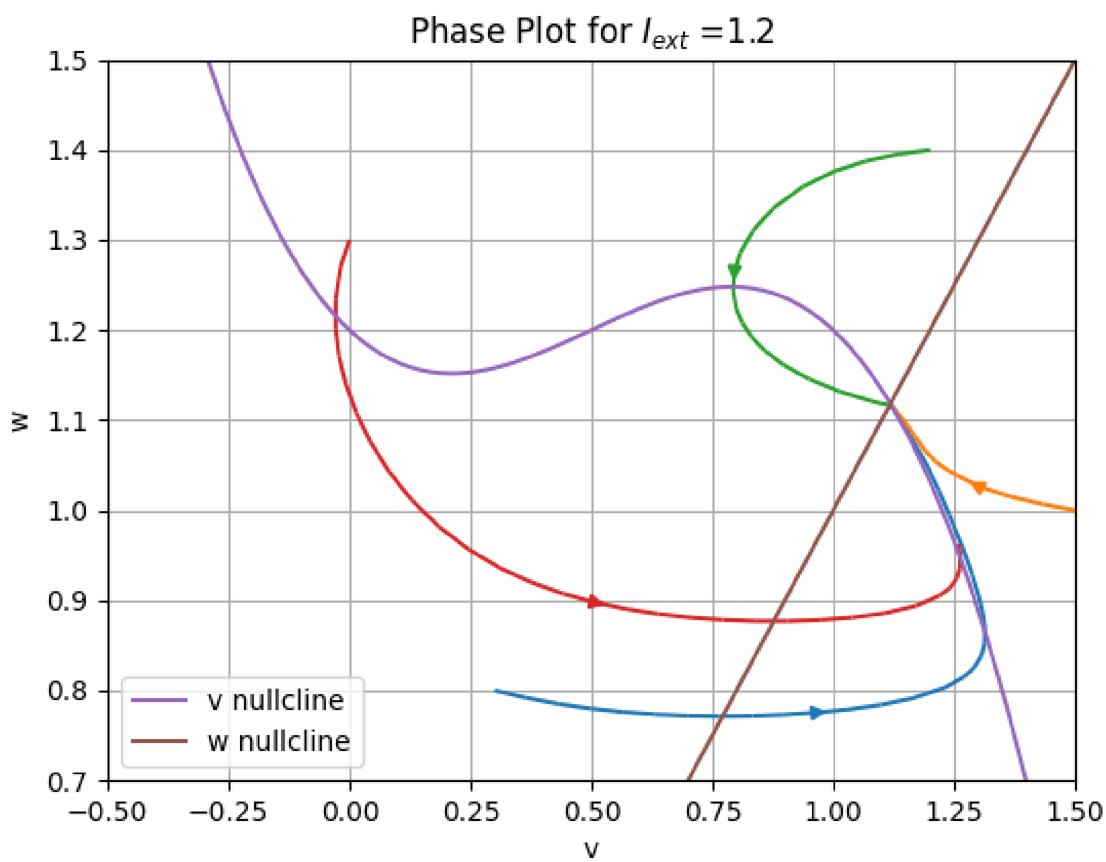
- Case 3 : I_{ext} = 1.2 (Depolarization)

(a) Phase Portrait plots



The fixed point (ie. Point of intersection of the two nullclines), as seen in the above phase portrait is stable, since any small perturbations from this point, move back towards the same fixed point.

This can be further confirmed, by plotting the trajectories of various initial conditions as shown below.

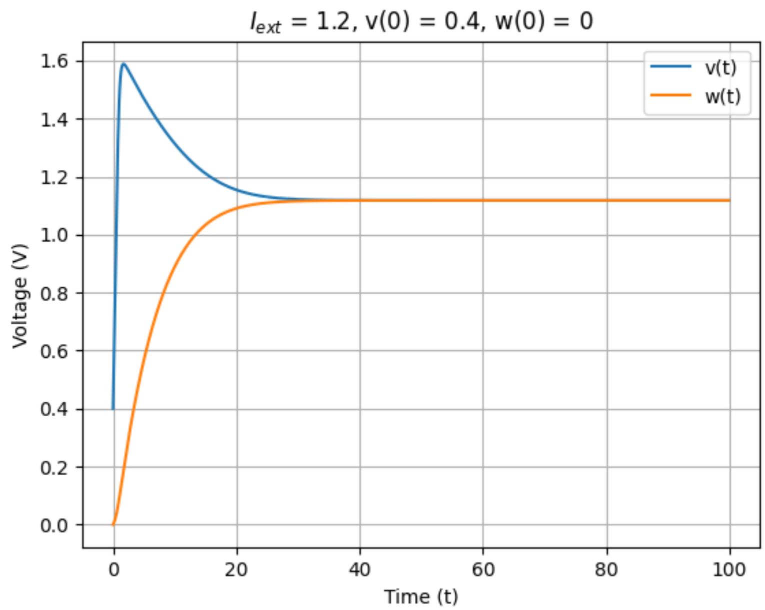


In the plots of v(t) and w(t) vs time, we can again see that they both converge to this same non-zero equilibrium point, which is also an indication of the depolarization in the membrane's potential.

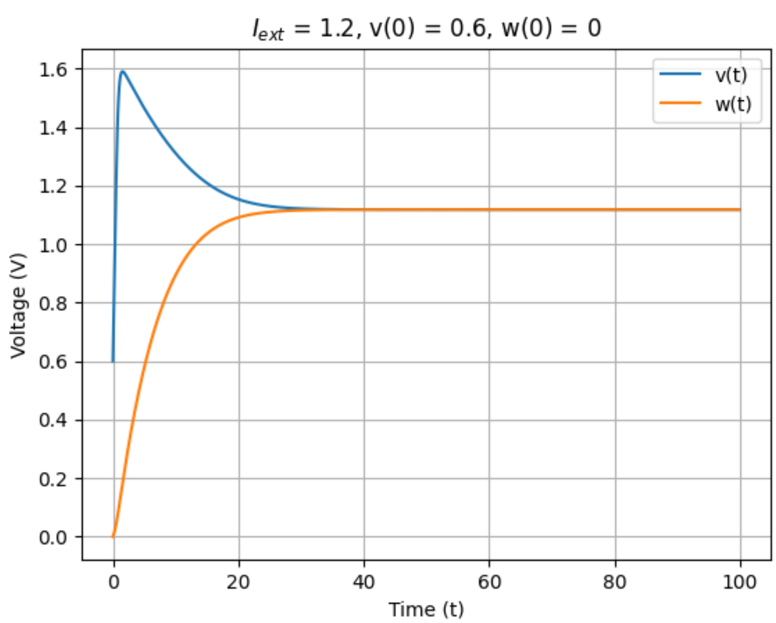
This depolarization in the output can be seen in the below plots for two different initial conditions.

(b) v(t) and w(t) vs time plots

(i) v(0) = 0.4 < a, w(0) = 0



(ii) v(0) = 0.6 > a, w(0) = 0

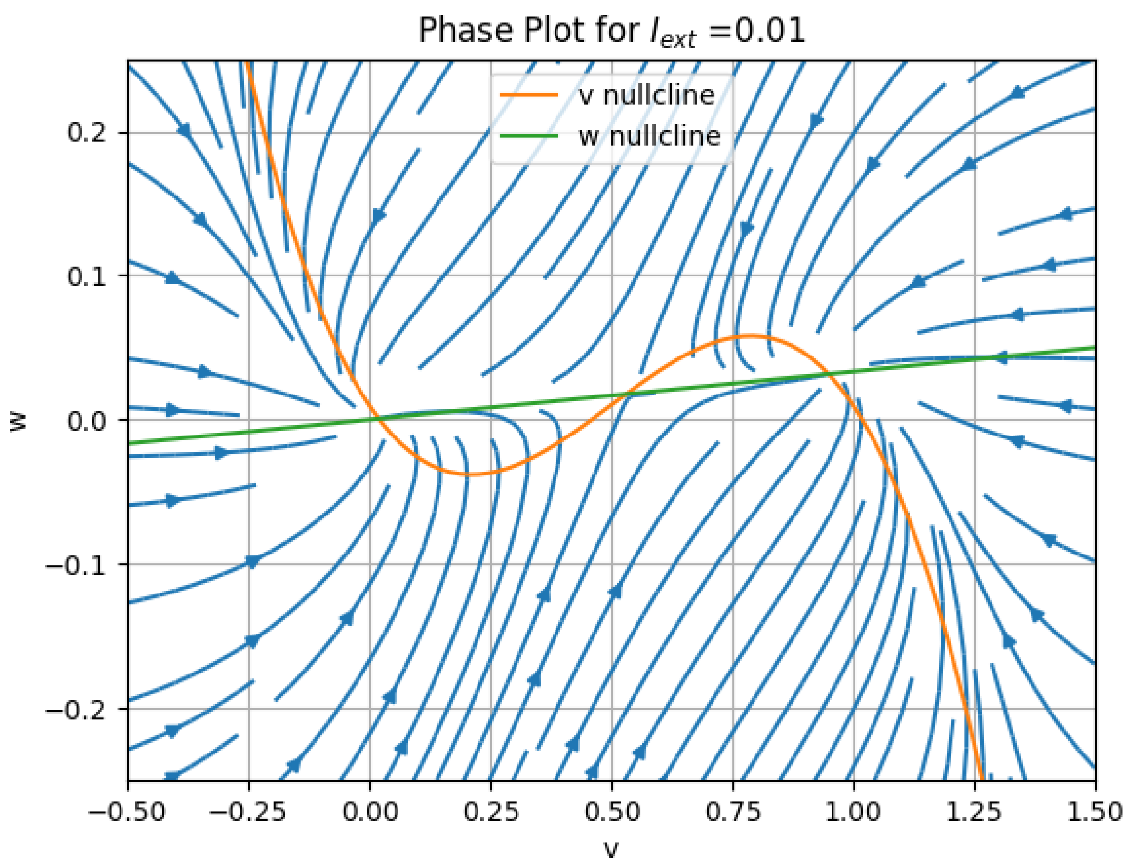


- Case 4 : I_{ext} = 0.01 (Bistability)

(For this case, the Parameter values are changed: a = 0.5, b = 0.02, r = 0.6)

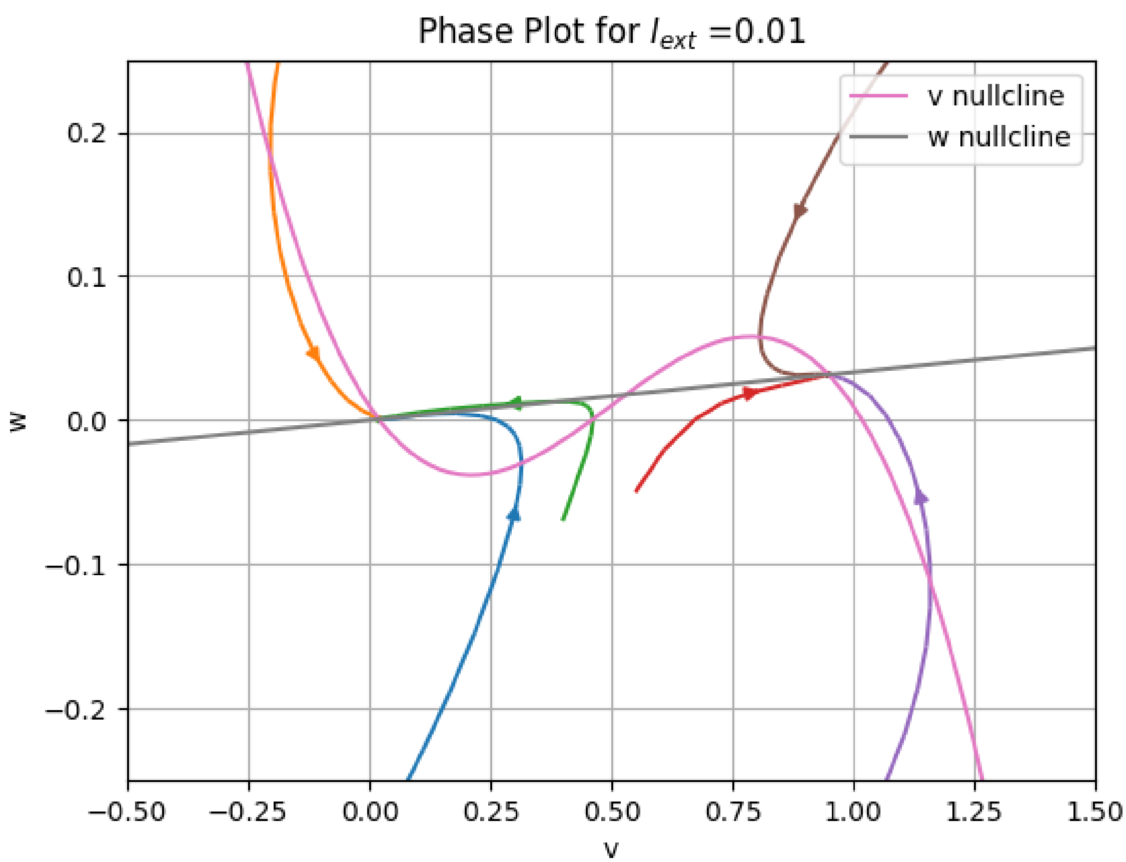
These values (I_{ext} = 0.01 and b/r = 0.03) are chosen such that the two nullclines intersect each other at three different points (instead of just a single point, like in all the three previous cases). Do note that this is just one possible set of values out of many to achieve these conditions.

(a) Phase Portrait plots



Let the three fixed points be P1, P2, P3 (from left to right).
From the above phase portrait plot, we can see that P1 and P3 are stable fixed points whereas P2 is an unstable fixed point (Saddle node to be more specific).

As shown in the below plot, slight perturbations lead back to points P1 and P3. But for point P2, the trajectories move away from it, hence confirming the stability of P1, P3 and the instability of P2.

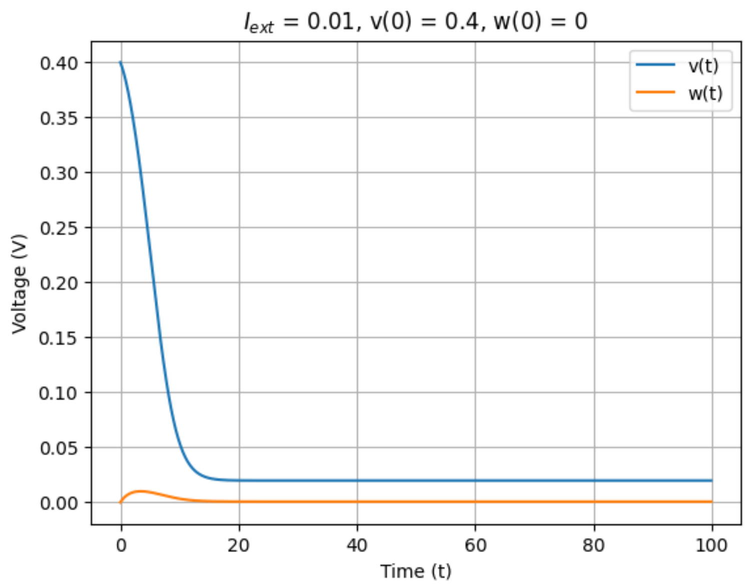


This condition is called bistability because the system eventually settles to one of two possible states, ie. fixed points P1 or P3.

This bistable behaviour based on different initial conditions is clearly demonstrated in the v(t) and w(t) plots vs time, shown below.

(b) v(t) and w(t) vs time plots

(i) v(0) = 0.4 < a, w(0) = 0



(ii) v(0) = 0.6 > a, w(0) = 0