

Assignment 3: Hello Vectors

Welcome to this week's programming assignment of the specialization. In this assignment we will explore word vectors. In natural language processing, we represent each word as a vector consisting of numbers. The vector encodes the meaning of the word. These numbers (or weights) for each word are learned using various machine learning models, which we will explore in more detail later in this specialization. Rather than make you code the machine learning models from scratch, we will show you how to use them. In the real world, you can always load the trained word vectors, and you will almost never have to train them from scratch. In this assignment you will

- Predict analogies between words.
- Use PCA to reduce the dimensionality of the word embeddings and plot them in two dimensions.
- Compare word embeddings by using a similarity measure (the cosine similarity).
- Understand how these vector space models work.

Important Note on Submission to the AutoGrader

Before submitting your assignment to the AutoGrader, please make sure you are not doing the following:

1. You have not added any *extra* `print` statement(s) in the assignment.
2. You have not added any *extra* code cell(s) in the assignment.
3. You have not changed any of the function parameters.
4. You are not using any global variables inside your graded exercises. Unless specifically instructed to do so, please refrain from it and use the local variables instead.
5. You are not changing the assignment code where it is not required, like creating *extra* variables.

If you do any of the following, you will get something like, `Grader Error: Grader feedback not found` (or similarly unexpected) error upon submitting your assignment. Before asking for help/debugging the errors in your assignment, check for these first. If this is the case, and you don't remember the changes you have made, you can get a fresh copy of the assignment by following these [instructions](#).

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1 - Predict the Countries from Capitals

During the presentation of the module, we have illustrated the word analogies by finding the capital of a country from the country. In this part of the assignment we have changed the problem a bit. You are asked to predict the **countries** that correspond to some **capitals**. You are playing trivia against some second grader who just took their geography test and knows all the capitals by heart. Thanks to NLP, you will be able to answer the questions properly. In other words, you will write a program that can give you the country by its capital. That way you are pretty sure you will win the trivia game. We will start by exploring the data set.



1.1 Importing the Data

As usual, you start by importing some essential Python libraries and the load dataset. The dataset will be loaded as a [Pandas DataFrame](#), which is very a common method in data science. Because of the large size of the data, this may take a few minutes.

```
# Run this cell to import packages.
import pickle
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import w3_unittest

from utils import get_vectors

data = pd.read_csv('./data/capitals.txt', delimiter=' ')
data.columns = ['city1', 'country1', 'city2', 'country2']

# print first five elements in the DataFrame
data.head(5)
```

	city1	country1	city2	country2
0	Athens	Greece	Bangkok	Thailand
1	Athens	Greece	Beijing	China
2	Athens	Greece	Berlin	Germany
3	Athens	Greece	Bern	Switzerland
4	Athens	Greece	Cairo	Egypt

To Run This Code On Your Own Machine:

Note that because the original google news word embedding dataset is about 3.64 gigabytes, the workspace is not able to handle the full file set. So we've downloaded the full dataset, extracted a sample of the words that we're going to analyze in this assignment, and saved it in a pickle file called word_embeddings_capitals.p

If you want to download the full dataset on your own and choose your own set of word embeddings, please see the instructions and some helper code.

- Download the dataset from this [page](#).
- Search in the page for 'GoogleNews-vectors-negative300.bin.gz' and click the link to download.
- You'll need to unzip the file.

Copy-paste the code below and run it on your local machine after downloading the dataset to the same directory as the notebook.

```
import nltk
from gensim.models import KeyedVectors

embeddings = KeyedVectors.load_word2vec_format('./GoogleNews-vectors-negative300.bin', binary = True)
f = open('capitals.txt', 'r').read()
set_words = set(nltk.word_tokenize(f))
select_words = words = ['king', 'queen', 'oil', 'gas', 'happy', 'sad', 'city', 'town', 'village', 'country', 'continent', 'petroleum', 'joyful']
for w in select_words:
    set_words.add(w)

def get_word_embeddings(embeddings):

    word_embeddings = {}
    for word in embeddings.vocab:
        if word in set_words:
            word_embeddings[word] = embeddings[word]
    return word_embeddings

# Testing your function
word_embeddings = get_word_embeddings(embeddings)
print(len(word_embeddings))
pickle.dump( word_embeddings, open( "word_embeddings_subset.p", "wb" ) )
```

Now we will load the word embeddings as a [Python dictionary](#). As stated, these have already been obtained through a machine learning algorithm.

```
word_embeddings = pickle.load(open("./data/word_embeddings_subset.p", "rb"))
len(word_embeddings) # there should be 243 words that will be used in this assignment
```

243

Each of the word embedding is a 300-dimensional vector.

```
print("Dimension: {}".format(word_embeddings['king'].shape[0]))
```

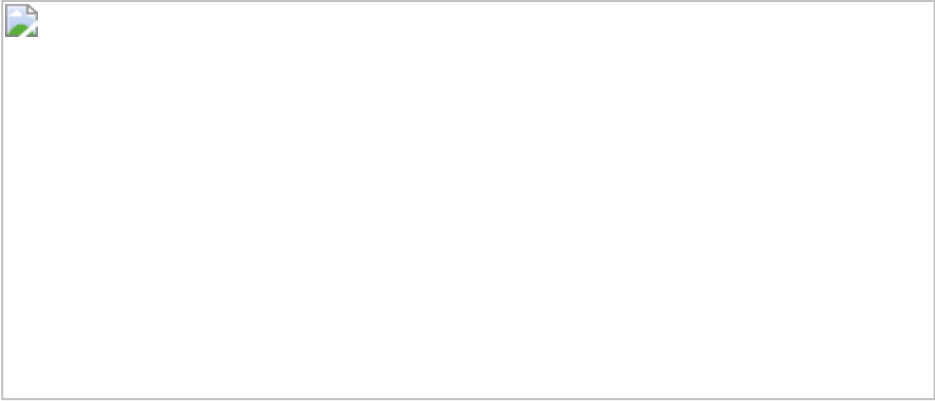
Predict relationships among words

Now you will write a function that will use the word embeddings to predict relationships among words.

- The function will take as input three words.
- The first two are related to each other.

- It will predict a 4th word which is related to the third word in a similar manner as the two first words are related to each other.
- As an example, "Athens is to Greece as Bangkok is to ___"?
- You will write a program that is capable of finding the fourth word.
- We will give you a hint to show you how to compute this.

A similar analogy would be the following:



You will implement a function that can tell you the capital of a country. You should use the same methodology shown in the figure above. To do this, you'll first compute the cosine similarity metric or the Euclidean distance.

1.2 Cosine Similarity

The cosine similarity function is:

$$\cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum_{i=1}^n A_i B_i}{\sqrt{\sum_{i=1}^n A_i^2} \sqrt{\sum_{i=1}^n B_i^2}}$$

A_i and B_i represent the word vectors and A_i or B_i represent index i of that vector. Note that if A and B are identical, you will get $\cos(\theta) = 1$.

- Otherwise, if they are the total opposite, meaning, $A = -B$, then you would get $\cos(\theta) = -1$.
- If you get $\cos(\theta) = 0$, that means that they are orthogonal (or perpendicular).
- Numbers between 0 and 1 indicate a similarity score.
- Numbers between -1 and 0 indicate a dissimilarity score.

Exercise 1 - cosine_similarity

Implement a function that takes in two word vectors and computes the cosine distance.

Hints

```
# UNQ_C1 GRADED FUNCTION: cosine_similarity
import numpy as np

def cosine_similarity(A, B):
    """
    Input:
    A: a numpy array which corresponds to a word vector
    B: a numpy array which corresponds to a word vector
    Output:
    cos: numerical number representing the cosine similarity between A and B.
    """
    ### START CODE HERE ###
    dot = np.dot(A, B)                # dot product
    norma = np.linalg.norm(A)         # norm of A
    normb = np.linalg.norm(B)         # norm of B
    cos = dot / (norma * normb)        # cosine similarity
    ### END CODE HERE ###

    return cos
```

```
# feel free to try different words
king = word_embeddings['king']
queen = word_embeddings['queen']

cosine_similarity(king, queen)
```

0.6510956

Expected Output:

Approx 0.651095

```
# Test your function
unittest.test_cosine_similarity(cosine_similarity)

All tests passed
```

1.3 Euclidean Distance

You will now implement a function that computes the similarity between two vectors using the Euclidean distance. Euclidean distance is defined as:

$$d(\mathbf{A}, \mathbf{B}) = \sqrt{(\mathbf{A}_1 - \mathbf{B}_1)^2 + (\mathbf{A}_2 - \mathbf{B}_2)^2 + \dots + (\mathbf{A}_n - \mathbf{B}_n)^2} = \sqrt{\sum_{i=1}^n (\mathbf{A}_i - \mathbf{B}_i)^2}$$

- n is the number of elements in the vector
- A and B are the corresponding word vectors.
- The more similar the words, the more likely the Euclidean distance will be close to 0.

Exercise 2 - euclidean

Implement a function that computes the Euclidean distance between two vectors.

Hints

```
# UNQ_C2 GRADED FUNCTION: euclidean
import numpy as np

def euclidean(A, B):
    """
    Input:
        A: a numpy array which corresponds to a word vector
        B: A numpy array which corresponds to a word vector
    Output:
        d: numerical number representing the Euclidean distance between A and B.
    """
    ### START CODE HERE ###
    d = np.linalg.norm(A - B) # directly compute using numpy norm
    ### END CODE HERE ###

    return d
```

```
# Test your function
euclidean(king, queen)

2.4796925
```

Expected Output:

2.4796925

```
# Test your function
unittest.test_euclidean(euclidean)

All tests passed
```

1.4 Finding the Country of each Capital

Now, you will use the previous functions to compute similarities between vectors, and use these to find the capital cities of countries. You will write a function that takes in three words, and the embeddings dictionary. Your task is to find the capital cities. For example, given the following words:

- 1: Athens 2: Greece 3: Baghdad,

your task is to predict the country 4: Iraq.

Exercise 3 - get_country

Instructions:

1. To predict the capital you might want to look at the *King - Man + Woman = Queen* example above, and implement that scheme into a mathematical function, using the word embeddings and a similarity function.
2. Iterate over the embeddings dictionary and compute the cosine similarity score between your vector and the current word embedding.
3. You should add a check to make sure that the word you return is not any of the words that you fed into your function. Return the one with the highest score.

```
# UNQ_C3 GRADED FUNCTION: get_country

def get_country(city1, country1, city2, word_embeddings, cosine_similarity=cosine_similarity):
    """
    Input:
        city1: a string (the capital city of country1)
        country1: a string (the country of capital1)
        city2: a string (the capital city of country2)
        word_embeddings: a dictionary where the keys are words and values are their embeddings
        cosine_similarity: function to compute cosine similarity
    Output:
        country: a tuple (best matching country, similarity score)
    """
    ### START CODE HERE ###

    # store the words that must be excluded
    group = {city1, country1, city2}

    # get embeddings
    city1_emb = word_embeddings[city1]
    country1_emb = word_embeddings[country1]
    city2_emb = word_embeddings[city2]

    # vector analogy: country2 ≈ country1 - city1 + city2
    vec = country1_emb - city1_emb + city2_emb

    # initialize
    similarity = -1
    country = ('', -1)

    # Loop through all words in embeddings dictionary
    for word, word_emb in word_embeddings.items():

        if word not in group: # exclude input words
            # compute cosine similarity
            cur_similarity = cosine_similarity(vec, word_emb)

            # keep the best match
            if cur_similarity > similarity:
                similarity = cur_similarity
                country = (word, similarity)

    ### END CODE HERE ###

    return country
```

```
# Testing your function, note to make it more robust you can return the 5 most similar words.
get_country('Athens', 'Greece', 'Cairo', word_embeddings)
('Egypt', 0.7626822)
```

Expected Output: (Approximately)

('Egypt', 0.7626821)

```
# Test your function
w2 unittest test get_country(get_country)
All tests passed
```

1.5 Model Accuracy

Now you will test your new function on the dataset and check the accuracy of the model:

$$\text{Accuracy} = \frac{\text{Correct \# of predictions}}{\text{Total \# of predictions}}$$

Exercise 4 - get_accuracy

Instructions: Implement a program that can compute the accuracy on the dataset provided for you. You have to iterate over every row to get the corresponding words and feed them into you `get_country` function above.

Hints

```
# UNQ_C4 GRADED FUNCTION: get_accuracy

def get_accuracy(word_embeddings, data, get_country=get_country):
    """
    Input:
        word_embeddings: a dictionary where the key is a word and the value is its embedding
        data: a pandas DataFrame containing all the country and capital city pairs
    Output:
        accuracy: float, percentage of correct predictions
    """
    ### START CODE HERE ###
    # initialize correct count
    num_correct = 0

    # Loop through all rows in dataset
    for i, row in data.iterrows():
        # unpack city-country pairs
        city1 = row['city1']
        country1 = row['country1']
        city2 = row['city2']
        country2 = row['country2']

        # predict using analogy function
        predicted_country2, _ = get_country(city1, country1, city2, word_embeddings)

        # check correctness
        if predicted_country2 == country2:
            num_correct += 1

    # total samples
    m = len(data)

    # compute accuracy
    accuracy = num_correct / m
    ### END CODE HERE ###

    return accuracy
```

NOTE: The cell below takes about 30 SECONDS to run.

```
accuracy = get_accuracy(word_embeddings, data)
print(f"Accuracy is {accuracy:.2f}")

Accuracy is 0.92
```

Expected Output:

\$\approx\$ 0.92

```
# Test your function
w2 unittest test get_accuracy(get_accuracy, data)

All tests passed
```

2 - Plotting the vectors using PCA

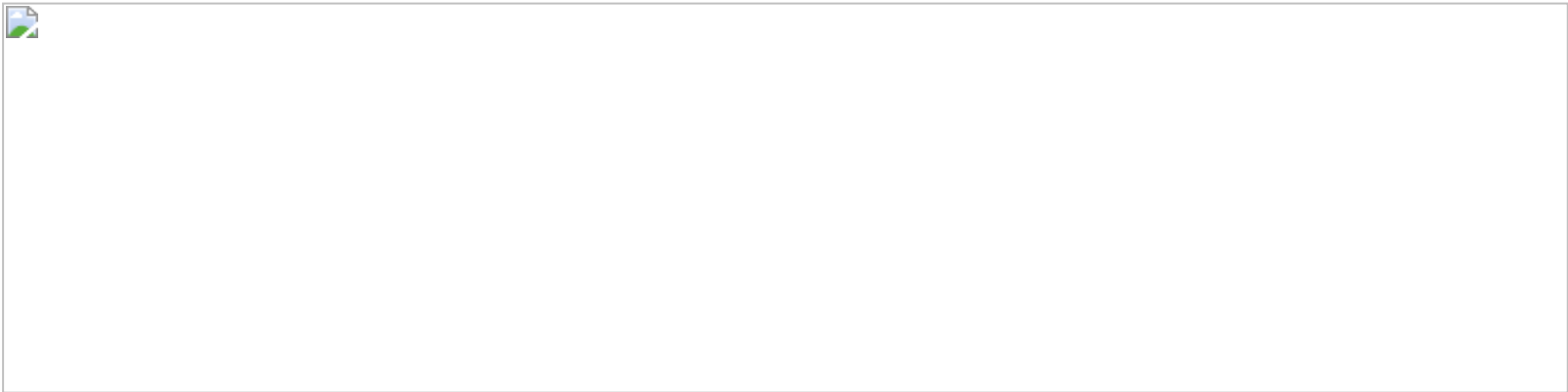
Now you will explore the distance between word vectors after reducing their dimension. The technique we will employ is known as [principal component analysis \(PCA\)](#). As we saw, we are working in a 300-dimensional space in this case. Although from a computational perspective we were able to perform a good job, it is impossible to visualize results in such high dimensional spaces.

You can think of PCA as a method that projects our vectors in a space of reduced dimension, while keeping the maximum information about the original vectors in their reduced counterparts. In this case, by *maximum infomation* we mean that the Euclidean distance between the original vectors and their projected siblings is minimal. Hence vectors that were originally close in the embeddings dictionary, will produce lower dimensional vectors that are still close to each other.

You will see that when you map out the words, similar words will be clustered next to each other. For example, the words 'sad', 'happy', 'joyful' all describe emotion and are supposed to be near each other when plotted. The words: 'oil', 'gas', and 'petroleum' all describe natural resources. Words like 'city', 'village', 'town' could be seen as synonyms and describe a similar thing.

Before plotting the words, you need to first be able to reduce each word vector with PCA into 2 dimensions and then plot it. The steps to compute PCA are as follows:

1. Mean normalize the data
2. Compute the covariance matrix of your data (Σ).
3. Compute the eigenvectors and the eigenvalues of your covariance matrix
4. Multiply the first K eigenvectors by your normalized data. The transformation should look something as follows:



Exercise 5 - compute_pca

Instructions:

Implement a program that takes in a data set where each row corresponds to a word vector.

- The word vectors are of dimension 300.
- Use PCA to change the 300 dimensions to `n_components` dimensions.
- The new matrix should be of dimension `m, n_components` .
- First de-mean the data
- Get the eigenvalues using `linalg.eigh` . Use 'eigh' rather than 'eig' since R is symmetric. The performance gain when using eigh instead of eig is substantial.
- Sort the eigenvectors and eigenvalues by decreasing order of the eigenvalues.
- Get a subset of the eigenvectors (choose how many principle components you want to use using `n_components`).
- Return the new transformation of the data by multiplying the eigenvectors with the original data.

Hints

```
# UNQ_C5 GRADED FUNCTION: compute_pca

import numpy as np

def compute_pca(X, n_components=2):
    """
    Input:
        X: of dimension (m,n) where each row corresponds to a word vector
        n_components: Number of components you want to keep.
    Output:
        X_reduced: data transformed in n_components dims
    """
    ### START CODE HERE ###
    # 1. mean center the data
    X_demeaned = X - np.mean(X, axis=0)

    # 2. calculate the covariance matrix
    covariance_matrix = np.cov(X_demeaned, rowvar=False)

    # 3. eigen decomposition
    eigen_vals, eigen_vecs = np.linalg.eigh(covariance_matrix)

    # 4. sort eigenvalues (and eigenvectors) in decreasing order
    idx_sorted = np.argsort(eigen_vals)          # ascending order
    idx_sorted_decreasing = idx_sorted[::-1]     # reverse -> descending

    # 5. sort eigenvectors
    eigen_vecs_sorted = eigen_vecs[:, idx_sorted_decreasing]

    # 6. select top n_components eigenvectors
    eigen_vecs_subset = eigen_vecs_sorted[:, :n_components]

    # 7. project data onto new subspace
    X_reduced = np.dot(X_demeaned, eigen_vecs_subset)
    ### END CODE HERE ###

    return X_reduced
```

```
# Testing your function
np.random.seed(1)
X = np.random.rand(3, 10)
X_reduced = compute_pca(X, n_components=2)
print("Your original matrix was " + str(X.shape) + " and it became:")
print(X_reduced)
```

Your original matrix was (3, 10) and it became:

[[0.43437323	0.49820384
[0.42077249	-0.50351448
[-0.85514571	0.00531064

```
# Test your function
import unittest
def test_compute_pca():
    # All tests passed
```

Expected Output:

Your original matrix was: (3,10) and it became:

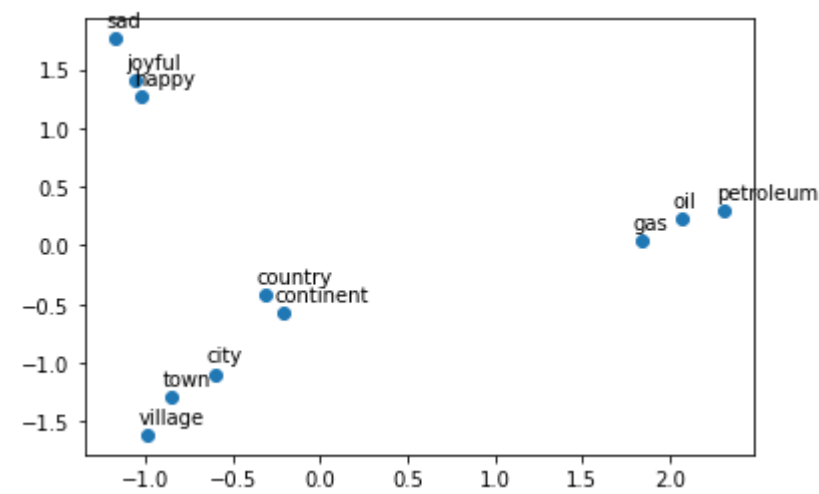
0.43437323	0.49820384
0.42077249	-0.50351448
-0.85514571	0.00531064

Now you will use your `pca` function to plot a few words we have chosen for you. You will see that similar words tend to be clustered near each other. Sometimes, even antonyms tend to be clustered near each other. Antonyms describe the same thing but just tend to be on the other end of the scale They are usually found in the same location of a sentence, have the same parts of speech, and thus when learning the word vectors, you end up getting similar weights. In the next week we will go over how you learn them, but for now let's just enjoy using them.

```
words = ['oil', 'gas', 'happy', 'sad', 'city', 'town',  
         'village', 'country', 'continent', 'petroleum', 'joyful']  
  
# given a list of words and the embeddings, it returns a matrix with all the embeddings  
X = get_vectors(word_embeddings, words)  
  
print('You have 11 words each of 300 dimensions thus X.shape is:', X.shape)
```

You have 11 words each of 300 dimensions thus X.shape is: (11, 300)

```
# We have done the plotting for you. Just run this cell.  
result = compute_pca(X, 2)  
plt.scatter(result[:, 0], result[:, 1])  
for i, word in enumerate(words):  
    plt.annotate(word, xy=(result[i, 0] - 0.05, result[i, 1] + 0.1))  
  
plt.show()
```



What do you notice?

The word vectors for gas, oil and petroleum appear related to each other, because their vectors are close to each other. Similarly, sad, joyful and happy all express emotions, and are also near each other.