$$\boldsymbol{u} = \frac{3/4}{\rho\gamma^2}\boldsymbol{S} \tag{1}$$

1 $u_{i,j}$

$$u_{i,j} = \frac{3/4}{\rho\gamma^2}\, S_{i,j} + \left(\frac{3/4}{\rho\gamma^2}\right)_{,j} S_i \tag{2}$$

2 $\left(\frac{3/4}{\rho\gamma^2}\right)_{,j}$

$$\left(\frac{3/4}{\rho\gamma^2}\right)_{,j} = \left(\frac{3/4}{\rho\gamma^2}\right)\left(-\nabla_j \ln\rho - \nabla_j \ln\gamma^2\right) \tag{3}$$

3 $\boldsymbol{\nabla} \ln\gamma^2$

Eq.(11) of BEO96:

$$\gamma^2 = \tfrac{1}{2} + \sqrt{\tfrac{1}{4} + \tfrac{9}{16}\,\frac{\boldsymbol{S}^2}{\rho^2}} \tag{4}$$

Take gradient:

$$\boldsymbol{\nabla}\gamma^2 = \frac{1}{2\sqrt{\frac{1}{4} + \frac{9}{16}\,\frac{\boldsymbol{S}^2}{\rho^2}}}\,\frac{9}{16}\left(\frac{\boldsymbol{\nabla}\boldsymbol{S}^2}{\rho^2} - \frac{\boldsymbol{S}^2}{\rho^2}\boldsymbol{\nabla}\ln\rho\right) \tag{5}$$

where

$$(\boldsymbol{\nabla}\boldsymbol{S}^2)_i = 2S_j S_{j,i} \tag{6}$$

Write as

$$\boldsymbol{\nabla}\gamma^2 = \frac{1}{2\sqrt{\frac{1}{4} + \frac{9}{16}\,\frac{\boldsymbol{S}^2}{\rho^2}}}\,\frac{9/16}{\rho^2}\,\left(\boldsymbol{\nabla}\boldsymbol{S}^2 - \boldsymbol{S}^2\boldsymbol{\nabla}\ln\rho\right) \tag{7}$$

so $\boldsymbol{\nabla}\ln\gamma^2 = \gamma^{-2}\boldsymbol{\nabla}\gamma^2$.