The Chinese University of Hong Kong
Department of Economics

ECON3011

Intermediate Microeconomic Theory

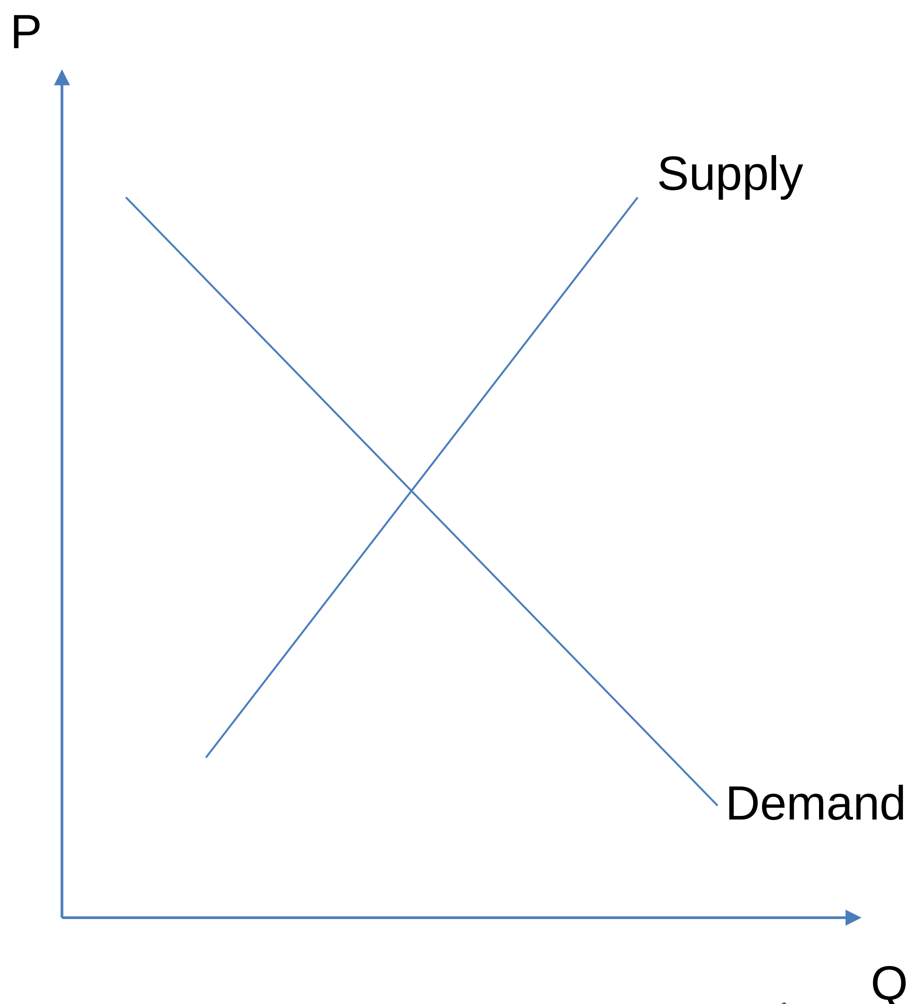
2025-2026 (First Term)

Lecture 6: Classical Theory of Supply

Wallace K C Mok

Acknowledgement: Parts of this lecture slide set were kindly provided by Prof. Shi

The Market



In the first part of ECON3011, the focus was on consumption (demand side).

Now in the second part of ECON3011 we focus on:

- Supply Side (Say's Law in the Classical School)
- The two extreme market structures (Perfect Competition and Monopoly) P → exogenous; P → endogenous.
- Combination of Demand and Supply (i.e. Market)
- Demand and Supply in an overall setting (General Equilibrium) all market → equilibrium.
- Imperfect Competitions (Oligopoly) and Elementary Game Theory few sellers; under imperfect information.
- Note that much of the theory rests on unique solution (equilibrium)

Cobweb theorem ⇒ from the inequilibrium → equilibrium

Objectives of this Lecture

- Elements of Production Theory in the Classical Setting
- Returns to scale, marginal products
- Isoquants and MRTS
- Duality and the Firm's Optimization

Supply

- Production Function
  - Isoquants and MRTS
    - Returns to Scale
      - Elasticity of Substitution
  - Introduction to Duality

Production Function

The production function tells us the *maximum* possible output that can be attained by the firm for any given quantity of inputs.

$$Q = f(L, K)$$

- Q = output that is actually produced
- K = Capital
- L = Labor
- f(L,K) = the maximum that can be produced

The production set is a set of technically feasible combinations of inputs and outputs.

Key Concepts

Observable productive resources, such as labor, capital equipment and land (natural resources) used are called inputs or factors of production.

The amount of goods and services produces by the firm is the firm's output.

Production transforms a set of inputs into a set of outputs

Technology determines the quantity of output that is feasible to attain for a given set of inputs.

Partial (∂) vs Total Derivative (d)

$$y = f(x, z)$$

$\frac{\partial y}{\partial x}$ Partial derivative of y with respect to x (keeping z constant)

$\frac{\partial y}{\partial z}$ Partial derivative of y with respect to z (keeping x constant)

$\frac{dy}{dx}$ Total derivative of y with respect to x (z not constant)

$\frac{dy}{dz}$ Total derivative of y with respect to z (x not constant)

$$dy = \frac{\partial y}{\partial x} dx + \frac{\partial y}{\partial z} dz$$

Total Differential

The Production Function & Technical Efficiency

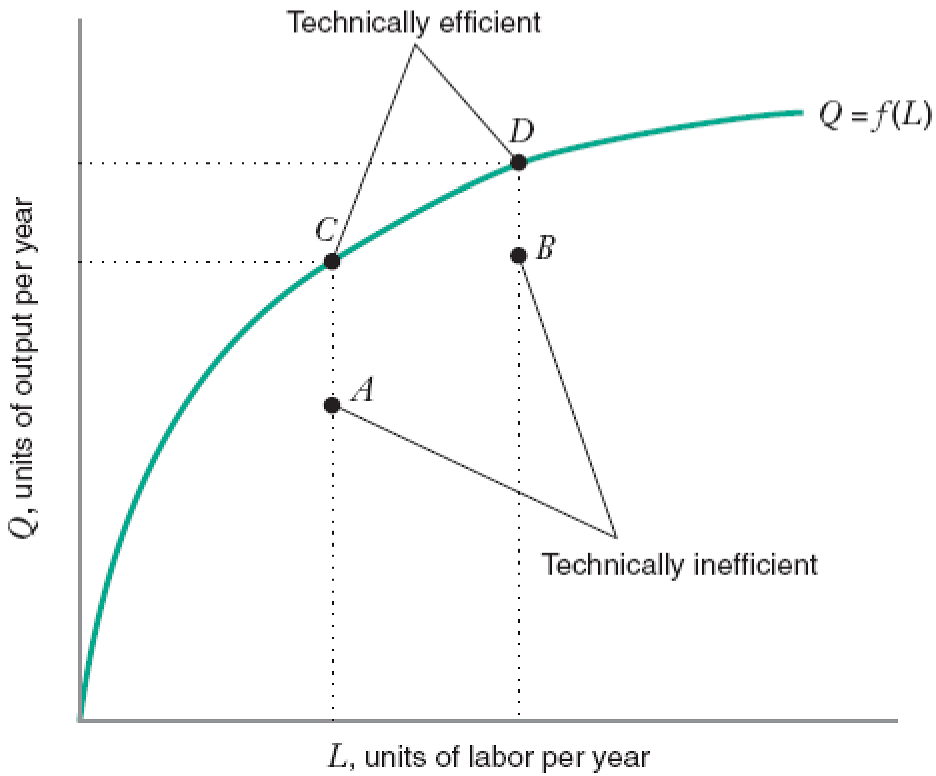
- *Technically efficient*: Sets of points Q in the production function that maximizes output given input

$$Q = f(L,K)$$

- *Technically inefficient*: Sets of points Q that produces less output than possible for a given set of input

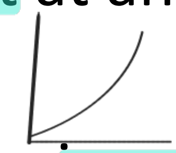
$$Q < f(L,K)$$

The Production Function & Technical Efficiency



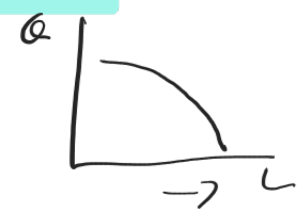
Single Input: Total Product

- **Total Product Function**: A single-input production function. It shows how total output depends on the level of the input
- **Increasing Marginal Returns to Labor**: An increase in the quantity of labor increases total output at an increasing rate.

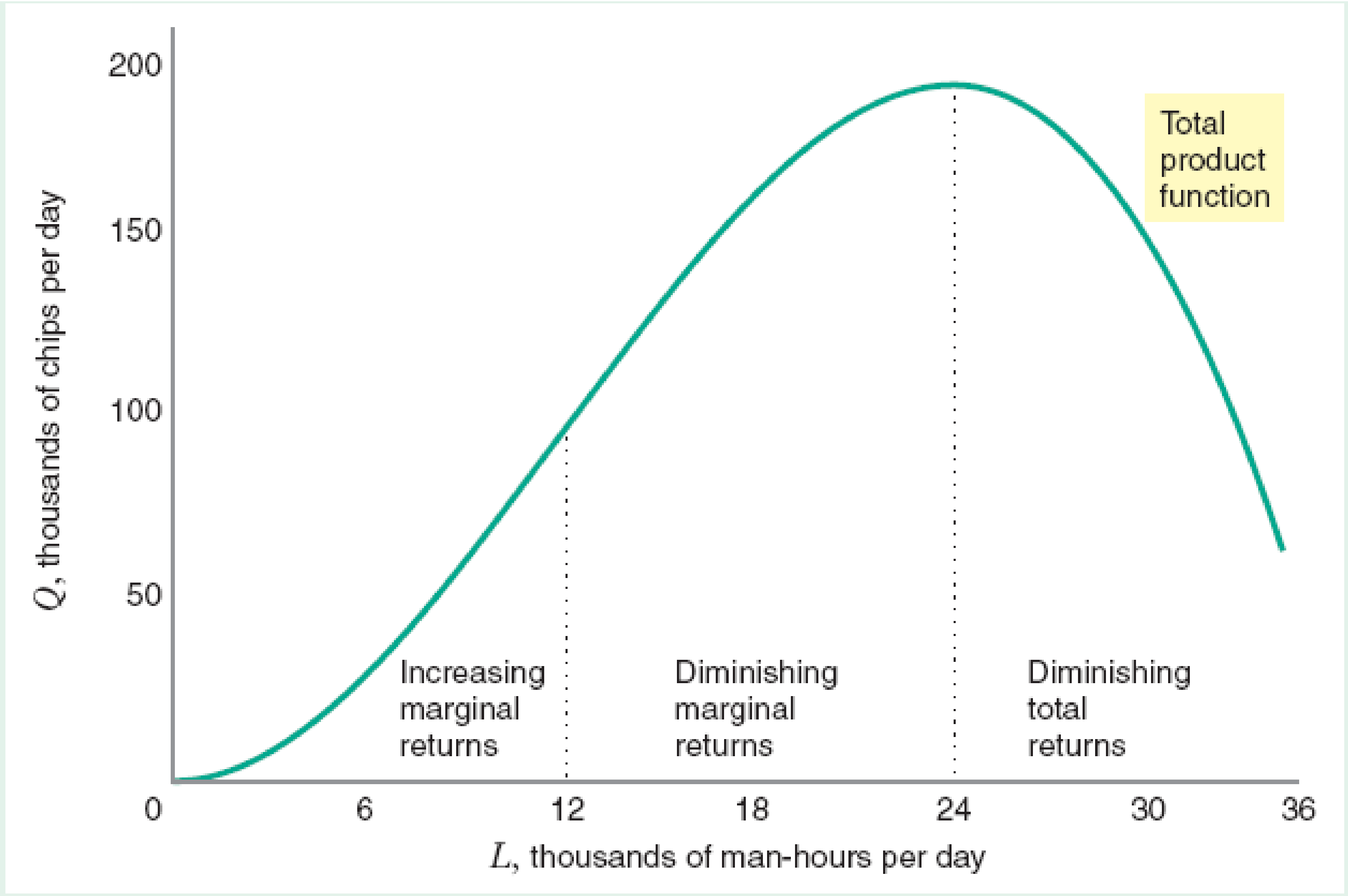


- **Diminishing Marginal Returns to Labor**: An increase in the quantity of labor increases total output **but** at a decreasing rate.

- **Diminishing Total Returns to Labor**: An increase in the quantity of labor decreases total output.



Total Product



The Marginal Product

Definition: The **marginal product** of an input is the change in output that results from a small change in an input *holding the levels of all other inputs constant*.

$$\text{MPL} = \frac{\partial Q}{\partial L} \qquad \text{MPK} = \frac{\partial Q}{\partial K}$$

(holding constant all other inputs)

Example: $Q = K^{0.5} L^{0.5}$ $\Rightarrow MP_L / MP_K$

not be negative in Cobb-Douglas

$$MP_L = 0.5\, K^{0.5} L^{-0.5} \qquad MP_K = 0.5\, K^{-0.5} L^{0.5}$$

The Average Product & Diminishing Returns

Definition: The **average product** of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production:

$$\textbf{Average Product of Labour:} \quad AP_L = Q\,L^{-1}$$

$$\textbf{Average Product of Capital:} \quad AP_K = Q\,K^{-1}$$

$$\textbf{Example (Cobb–Douglas):} \quad Q = K^{0.5}\,L^{0.5}$$

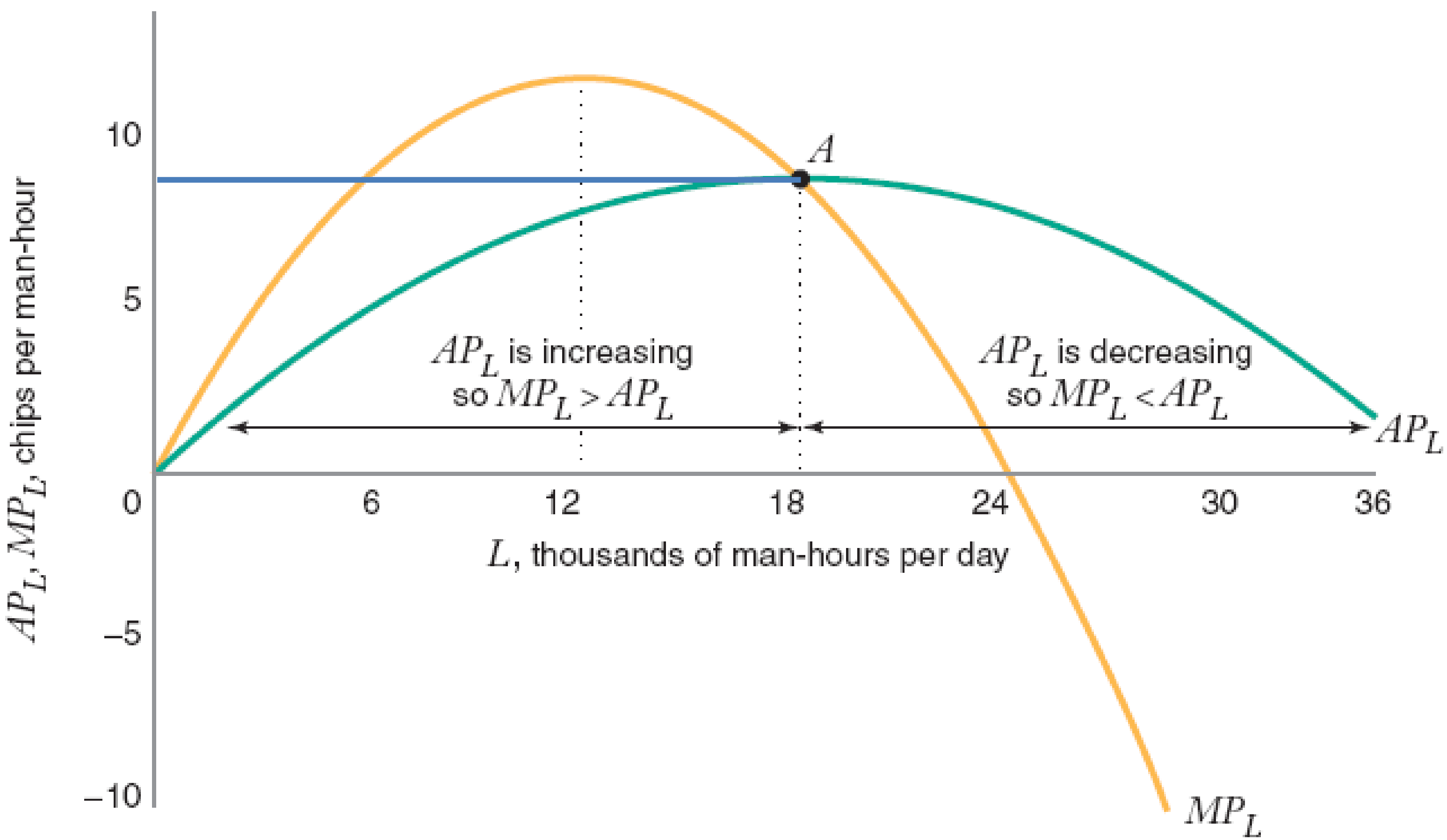
$$AP_L = \frac{Q}{L} = K^{0.5}\,L^{0.5}\,L^{-1} = K^{0.5}\,L^{-0.5}$$

$$AP_K = \frac{Q}{K} = K^{0.5}\,L^{0.5}\,K^{-1} = K^{-0.5}\,L^{0.5}$$

Definition: The **law of diminishing marginal returns** states that marginal products *(eventually)* decline as the quantity used of a single input increases.

doesn't say it will be negative.

Total, Average, and Marginal Products



Total, Average, and Marginal Products

Claim: MP intersects with AP at the maximum of the AP curve.

By definition: $AP_L = Q\,L^{-1}$, $MP_L = \dfrac{dQ}{dL}$

$Q = f(L)$

AP_L **is maximized when:** $\dfrac{d(AP_L)}{dL} = 0$

$\dfrac{dAP_L}{dL} = \dfrac{dQ}{dL} L^{-1} + Q(-1)L^{-2} = 0.$

$\Rightarrow \left(\dfrac{dQ}{dL}\right) L = Q L^{-1}$

$= AP_L$

$$\frac{d(AP_L)}{dL} = \left(\frac{dQ}{dL}\right)L^{-1} + Q\,(-1)L^{-2} = 0$$

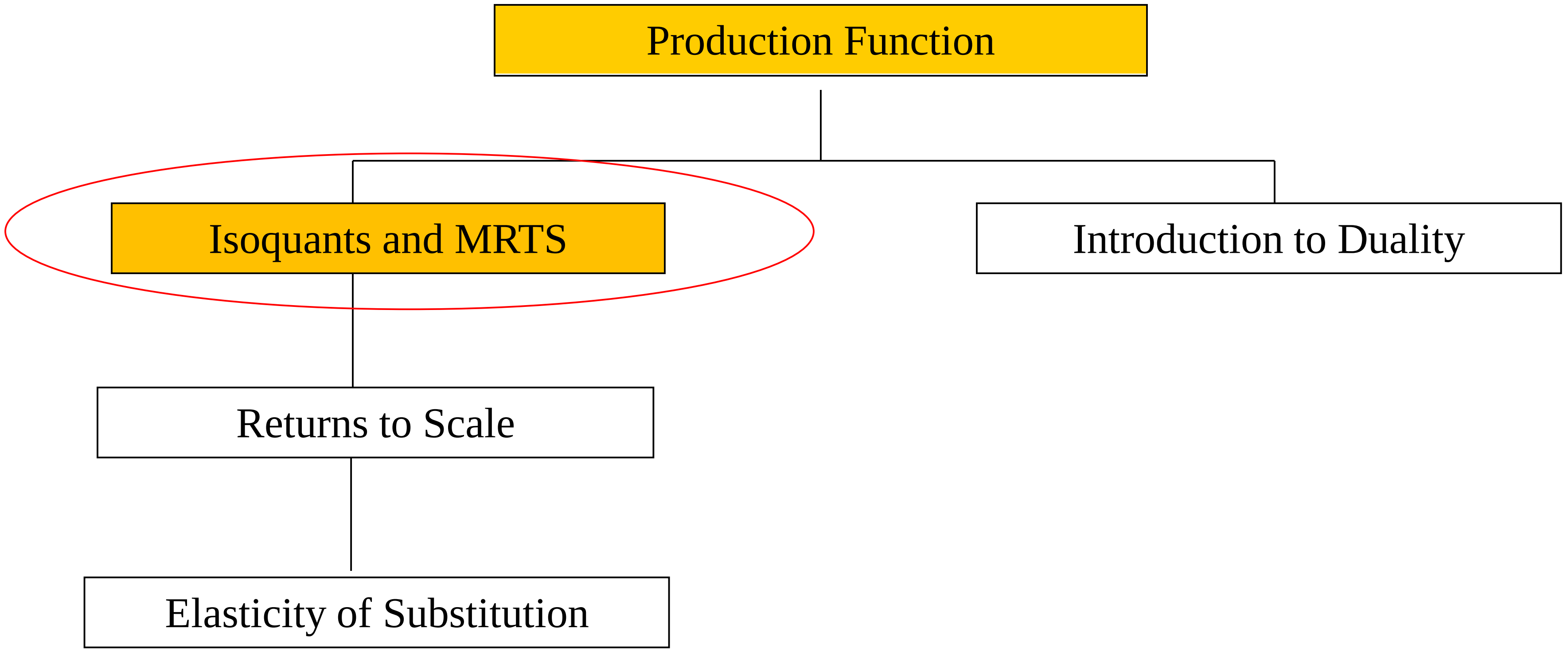
$$\Rightarrow \left(\frac{dQ}{dL}\right)L^{-1} - QL^{-2} = 0$$

$$\Rightarrow \left(\frac{dQ}{dL}\right)L - Q = 0$$

$$\Rightarrow \frac{dQ}{dL} = Q\,L^{-1} = AP_L$$

Therefore, $MP_L = AP_L$ at the maximum of the AP curve.

Supply


Production Function
Isoquants and MRTS
Introduction to Duality
Returns to Scale
Elasticity of Substitution

Isoquants

Definition: An **isoquant** traces out all the combinations of inputs (labor and capital) that allow that firm to produce the same quantity of output

Example: $Q = K^{0.5}L^{0.5}$

What is the equation of the isoquant for $Q = 20$?

$$20 = K^{0.5}L^{0.5}$$
$$\Rightarrow 400 = KL$$
$$\Rightarrow K = 400L^{-1}$$

And…

…and the isoquant for $Q = Q^*$?

$$Q^* = K^{0.5}L^{0.5}$$

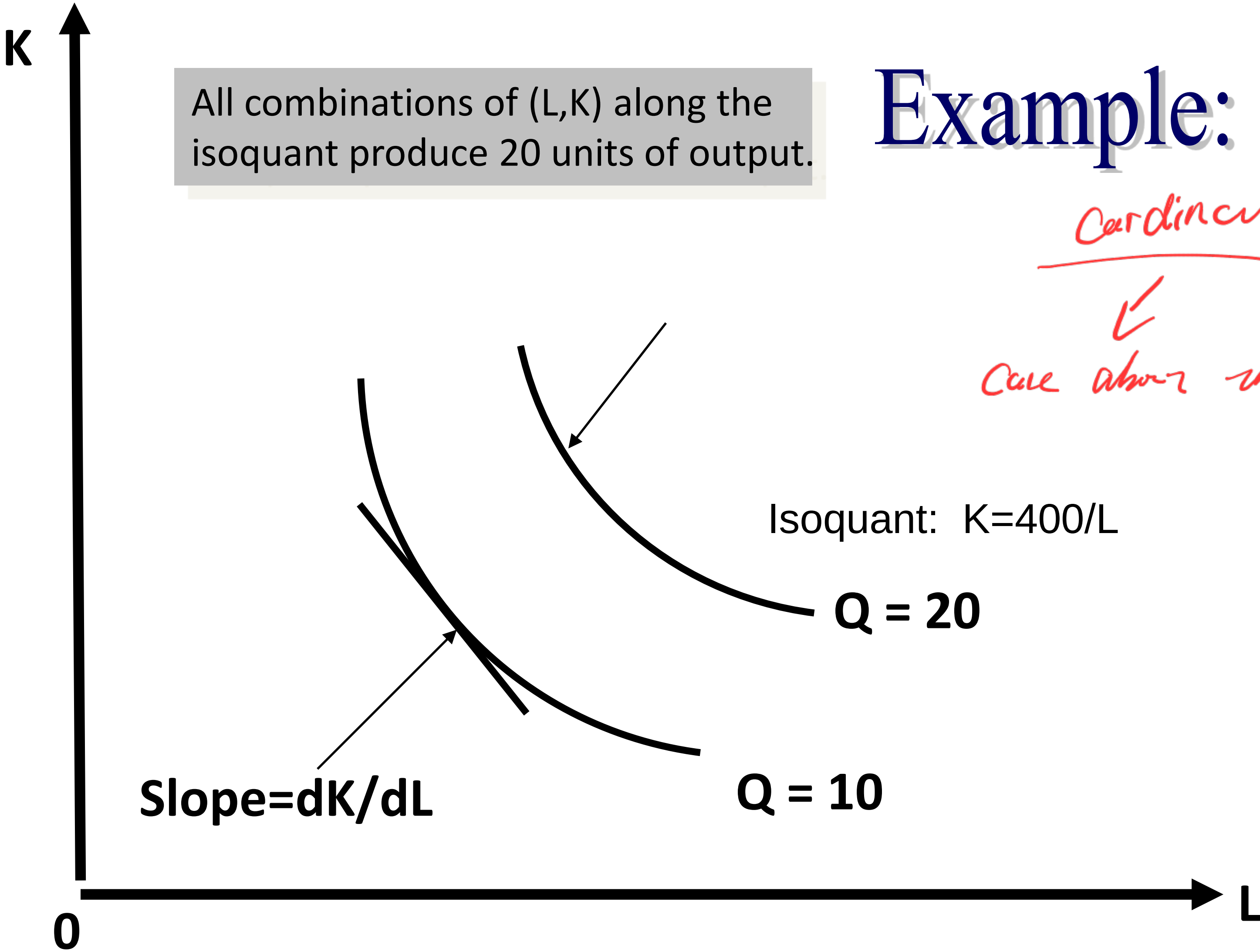
$$\Rightarrow (Q^*)^2 = KL$$
$$\Rightarrow K = (Q^*)^2L^{-1}$$

Isoquants

Example:

Cardinal

care about the rank & number.



Marginal Rate of Technical Substitution

Definition: The **marginal rate of technical substitution** measures the amount of an input, L, the firm would require in exchange for using a little less of another input, K, in order to just be able to produce the same output as before (i.e. on the same isoquant).

$dQ = MP_L \, dL + MP_K \cdot dK = 0$

$$MRTS_{L,K} = -\frac{dK}{dL} \quad \text{(for a constant level of output)}$$

Marginal products and the MRTS are related, totally differentiate Q:

$$MP_L\,(dL) + MP_K\,(dK) = 0$$

$$\Rightarrow \quad \frac{MP_L}{MP_K} = -\frac{dK}{dL} = MRTS_{L,K}$$

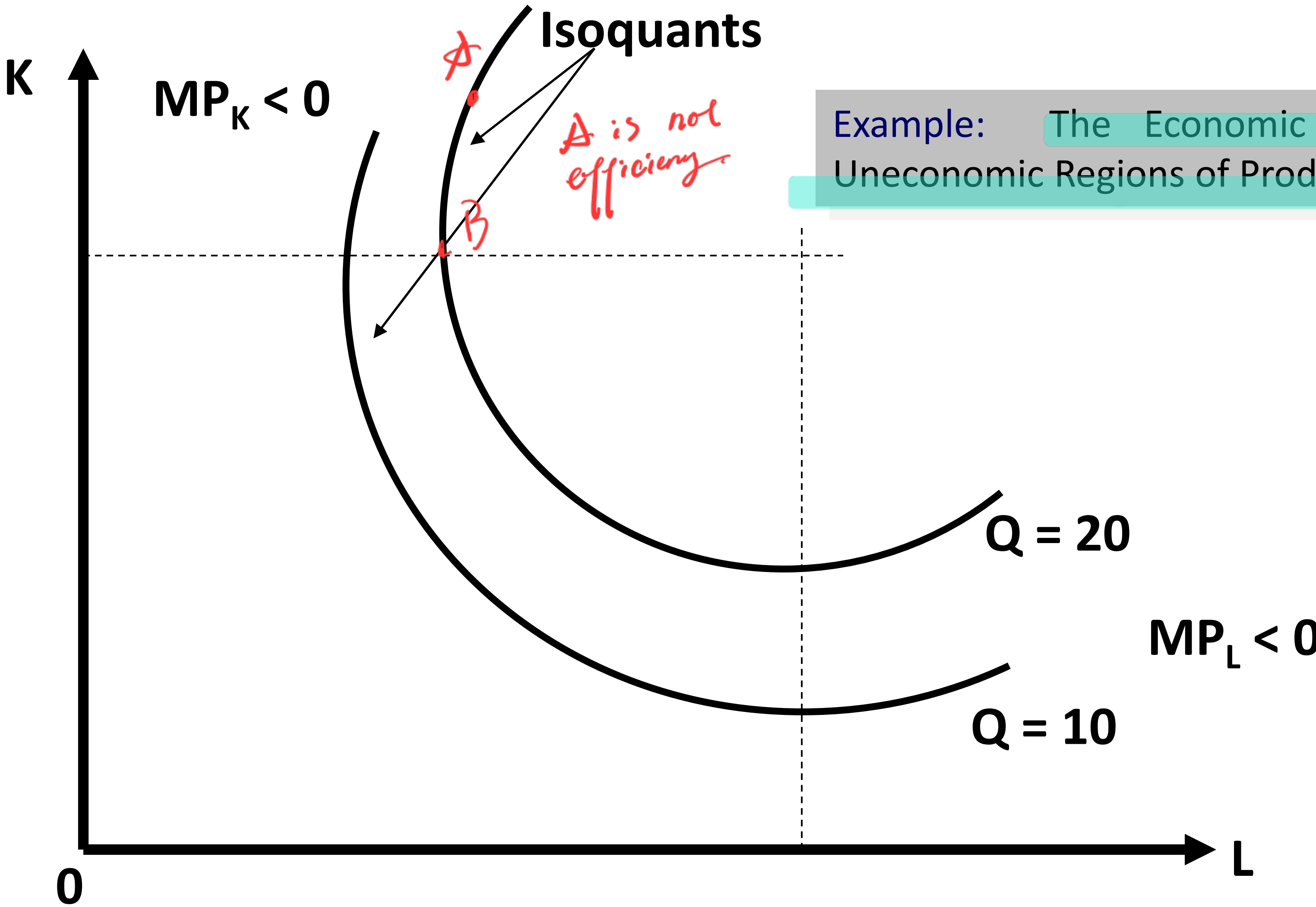
Marginal Rate of Technical Substitution

- If both marginal products are positive, the slope of the isoquant is negative.

- If we have diminishing marginal returns, we also have a diminishing marginal rate of technical substitution - the marginal rate of technical substitution of labor for capital diminishes as the quantity of labor increases, along an isoquant – isoquants are convex to the origin.

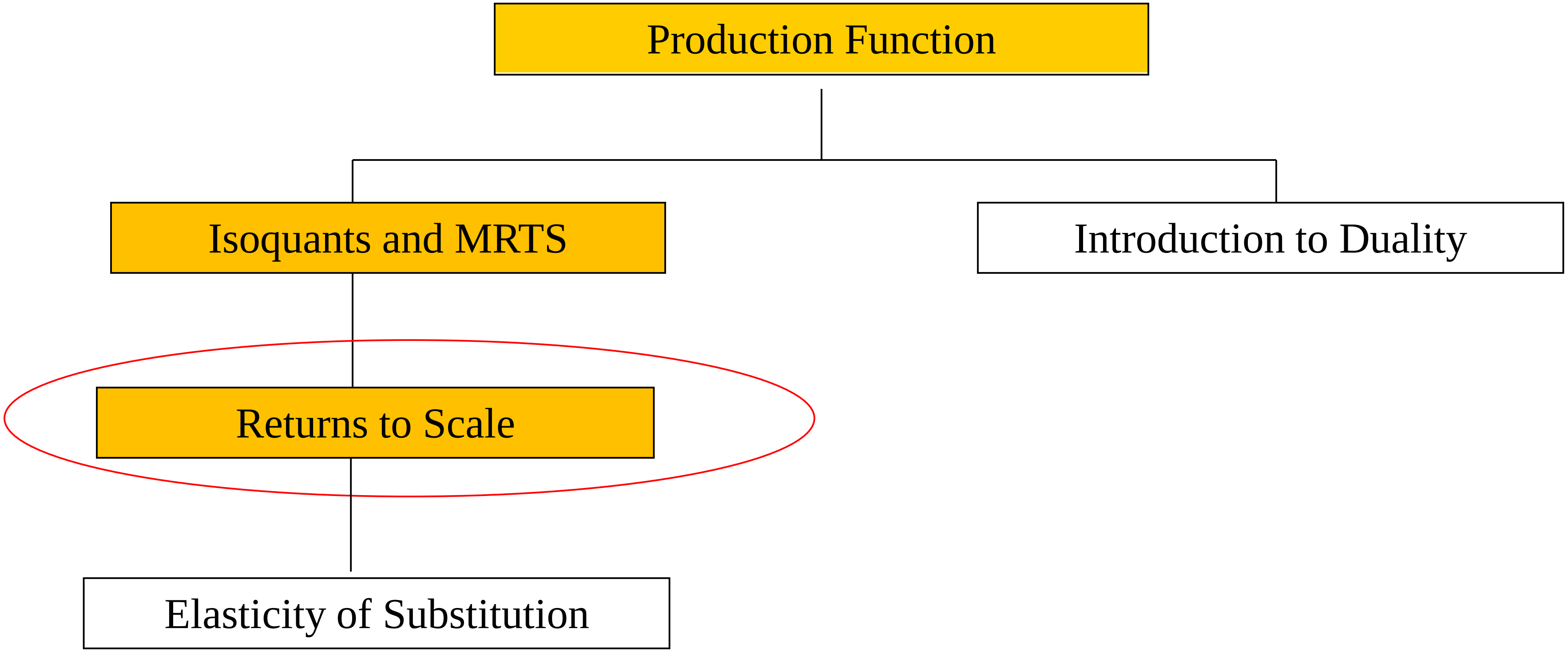
- For many production functions, marginal products eventually become negative. Why don't most graphs of Isoquants include the upwards-sloping portion?

Isoquants



Example: The Economic and the Uneconomic Regions of Production

Supply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Returns to Scale

• How much will output increase when ALL inputs increase by a particular amount?

$$\text{Returns to Scale} = \frac{\%d(\text{quantity of output})}{\%d(\text{quantity of } \textit{all} \text{ inputs})}$$

Returns to Scale

- If a 1% increase in all inputs results in a greater than 1% increase in output, then the production function exhibits increasing returns to scale.

- If a 1% increase in all inputs results in exactly a 1% increase in output, then the production function exhibits constant returns to scale.

- If a 1% increase in all inputs results in a less than 1% increase in output, then the production function exhibits decreasing returns to scale.

Returns to Scale

Let λ represent the amount by which both inputs, labor and capital, increase.

$$\Phi Q = f(\lambda L, \lambda K) \;\; for \; \lambda > 1$$

Let *Φ* represent the resulting proportionate increase in output, *Q*

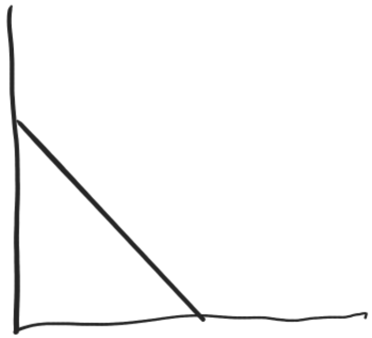
- Increasing returns: $\phi > \lambda$
- Decreasing returns: $\phi < \lambda$
- Constant Returns: $\phi = \lambda$

Checking Returns to Scale

I. *Linear:* Q = aK + bL

where: a, b are positive constants.

perfect substitution of K, L.



$$F(\lambda K, \lambda L) = a(\lambda K) + b(\lambda L)$$

$$= \lambda \left(aK + bL\right)$$

$$= \lambda F(K, L)$$

i.e., Constant Return to Scale.

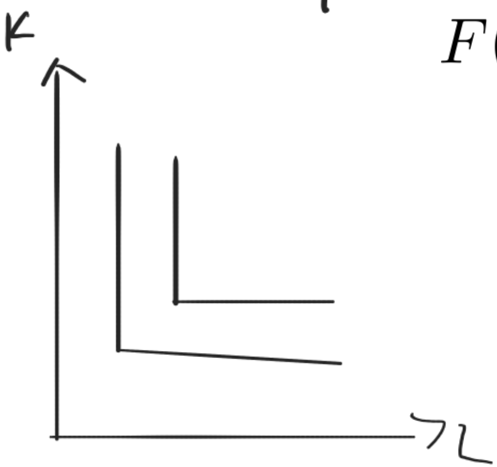
Checking Returns to Scale

II. Leontiff: $Q = A \min(\alpha K, \beta L)$

where: A, α, β are positive constants.

perfect complements

non-differentiable.



$$F(\lambda K, \lambda L) = A \min\big(a(\lambda K),\, b(\lambda L)\big)$$

$$= \lambda \Big(A \min\big(aK,\, bL\big)\Big)$$

$$= \lambda F(K, L)$$

i.e., Constant Return to Scale.

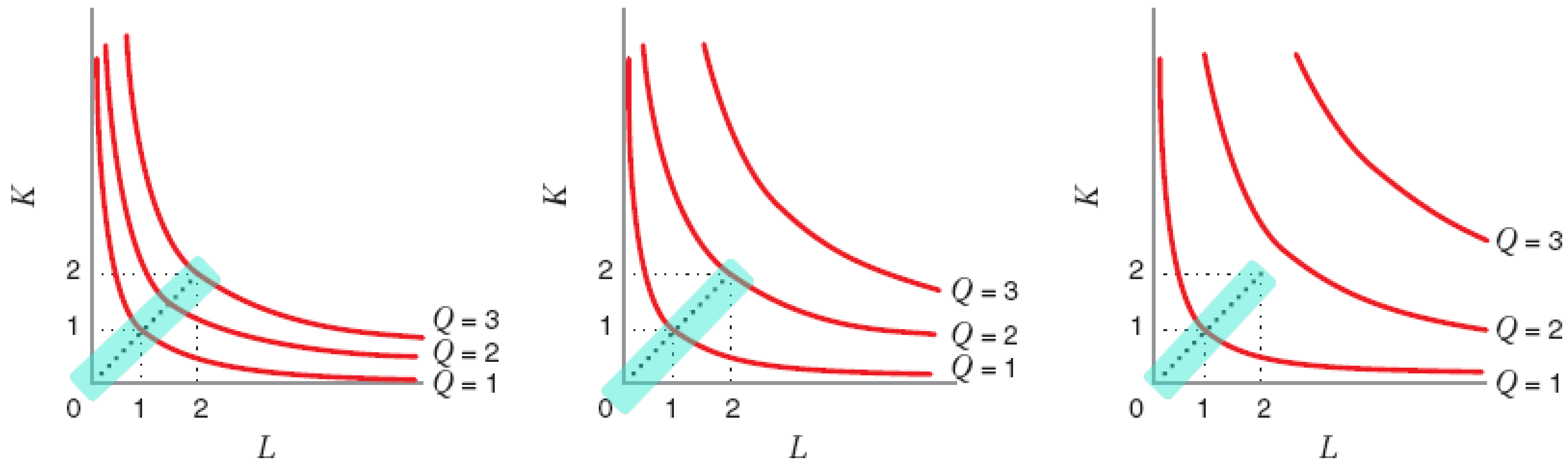
Checking Returns to Scale

III. Cobb-Douglas: $Q = AK^{\alpha}L^{\beta}$

$$
\begin{aligned}
F(\lambda K, \lambda L) &= A\,(\lambda K)^{\alpha}(\lambda L)^{\beta} \\
&= A\,(\lambda^{\alpha} K^{\alpha})(\lambda^{\beta} L^{\beta}) \\
&= \lambda^{\alpha+\beta}\left(A K^{\alpha} L^{\beta}\right) \\
&= \lambda^{\alpha+\beta} F(K, L)
\end{aligned}
$$

If $\alpha+\beta = 1$, Constant Return to Scale;
If $\alpha+\beta > 1$, Increasing Return to Scale;
If $\alpha+\beta < 1$, Decreasing Return to Scale.

Returns to Scale



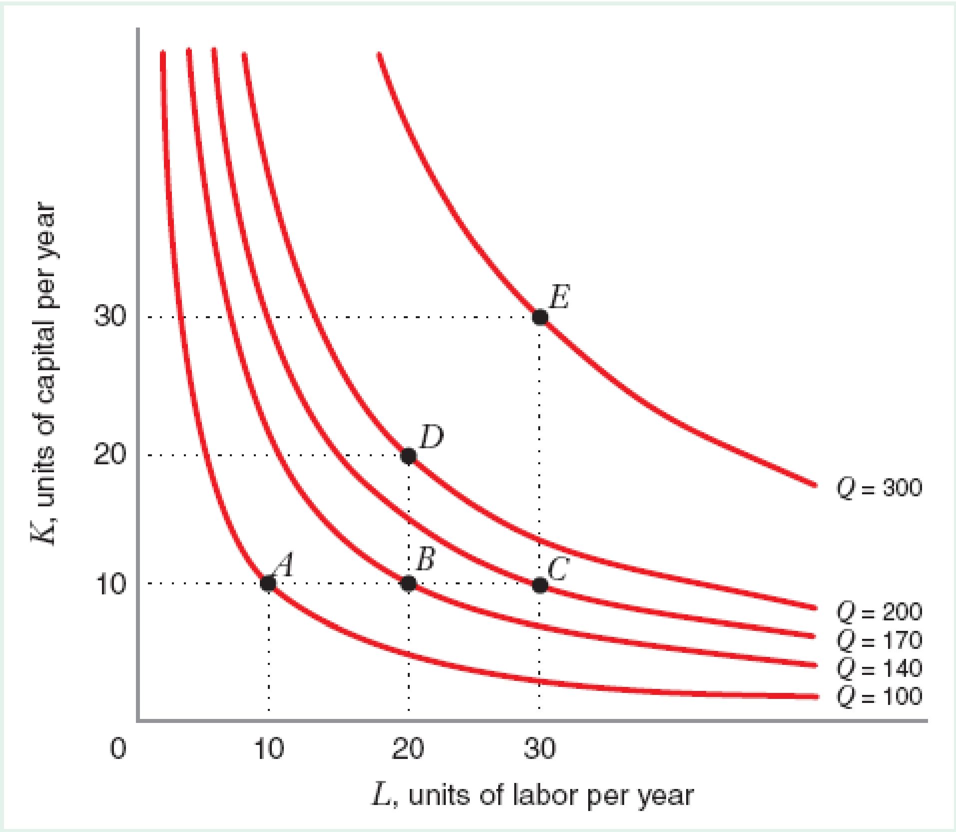
(a) Increasing Returns to Scale (b) Constant Returns to Scale (c) Decreasing Returns to Scale

Returns to Scale vs. Marginal Returns

- Returns to scale: all inputs are increased simultaneously
- Marginal Returns: Increase in the quantity of a single input holding all others constant.

> • The marginal product of a single factor may diminish while the returns to scale do not
>
> • Returns to scale need not be the same at different levels of production

Returns to Scale vs. Marginal Returns



- Production function with CRTS but diminishing marginal returns to labor.

Supply

Production Function

Isoquants and MRTS

Introduction to Duality

Returns to Scale

Elasticity of Substitution

Elasticity of Substitution

- A measure of how easy is it for a firm to substitute labor for capital.

- It is the percentage change in the capital-labor ratio for every one percent change in the $MRTS_{L,K}$ along an isoquant.

Elasticity of Substitution

measure the curvature of the isoquant

Definition: The **elasticity of substitution**, σ, measures how the capital-labor ratio, K/L, changes relative to the change in the $MRTS_{L,K}$. Smaller means harder to substitute K for L (or L for K)

$$\sigma = \frac{\textit{Percentage change} \text{ in capital - labor ratio}}{\textit{Percentage change} \text{ in } MRTS_{L,K}}$$

$$= \frac{\%d\left(\frac{K}{L}\right)}{\%dMRTS_{L,K}}$$

$\frac{d\ln\frac{K}{L}}{d\ln(MRTS)}$

$\%d\frac{K}{L}$

$\frac{dK}{dL}\cdot\frac{L}{K}$

Elasticity of Substitution

Example: (Cobb-Douglas) $Q(L, K) = AL^{\alpha}K^{\beta}$

$$MP_L = \alpha A\, L^{\alpha-1}\, K^{\beta} \qquad MP_K = \beta A\, L^{\alpha}\, K^{\beta-1}$$

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right)$$

$$\ln^{MRTS} = \ln^{(\frac{\alpha}{\beta})} + \ln^{(\frac{K}{L}}$$

$$d\ln^{MRTS} = d\ln^{(\frac{K}{L})}$$

Elasticity of Substitution

Example: (Cobb-Douglas) Q(L, K) = AL$^{\alpha}$K$^{\beta}$

$MRTS_{L,K} = \frac{\alpha}{\beta}\frac{K}{L}$

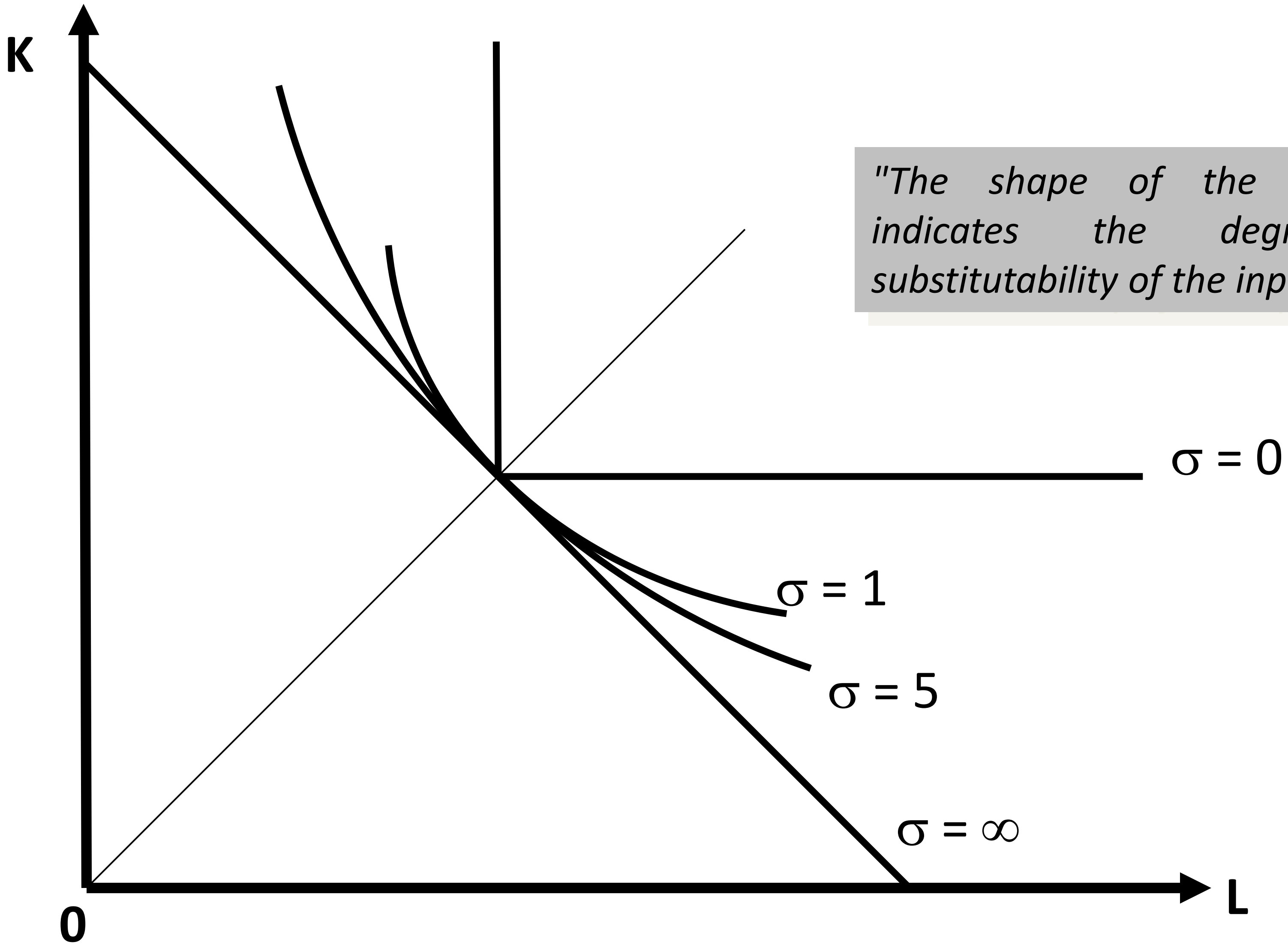
$$\left(\frac{K}{L}\right) = \left(\frac{\beta}{\alpha}\right) MRTS_{L,K}$$

$$\frac{d\frac{K}{L}}{dMRTS} \times \frac{MRTS}{\frac{K}{L}} = \frac{\beta}{\alpha} \times \frac{L}{K} \times \frac{\alpha}{\beta} \times \frac{K}{L}.$$

$$\sigma = \frac{\%\, d\left(\frac{K}{L}\right)}{\%\, d(MRTS_{L,K})^*} = \left(MRTS^*_{L,K}\left(\frac{L}{K}\right)\right)$$

$$= \left(\frac{\beta}{\alpha}\right)\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right)\left(\frac{L}{K}\right) = 1$$

Elasticity of Substitution



"The shape of the isoquant indicates the degree of substitutability of the inputs…"

Outside the Box

The way we think about firms is drastically simplistic…

- In really, firms are much more complex; e.g., organization, contracts, personnel, finance.
- Again, models set good benchmarks for further thinking.
- How should we think about firms economically?
  Ronald Coase
- Firms vs Market Structure

Supply

- Production Function
  - Isoquants and MRTS
    - Returns to Scale
      - Elasticity of Substitution
  - Introduction to Duality

DUALITY

Consider the following:

Consumer chooses x_1, x_2 in order to:

- Maximizing Utility U subject to budget constraint with Expenditure E (*Primal*)
- Minimizing Expenditure E subject to the Utility Constraint U (*Dual*)

Will the optimal x_1, x_2 be the same under these two settings? What does each setting gives you? (Indirect Utility Function u=v(P,M), Expenditure Function E=e(p, u))

given price

given utiliy wanna reach.

Similarly:

Firm chooses K and L:

- Maximizing Profit P*Q subject to Cost Constraint C (*Primal*)
- Minimizing Cost subject to the Production Constraint Q = F(K,L) (*Dual*)

Will the optimal K and L be the same under these two settings?

The Production & Utility Functions

Production Function	Utility Function
Output from inputs	Preference level from purchases
Derived from technologies	Derived from preferences
Cardinal	Ordinal
Marginal Product	Marginal Utility

The Production & Utility Functions

Production Function	Utility Function
Isoquant	Indifference Curve
Marginal Rate of Technical Substitution	Marginal Rate of Substitution

That means a lot of the techniques from Consumer Theory can be carried over to Production Theory, with a change in interpretation

Firm's Primal Problem

Firm's Problem:

$$\max \ \pi \ (\text{profit})$$

$$= p\,Q(L,K) - (wL \ + \ rK)$$

$Q(L,K)$: twice differentiable

$$\text{Profit} = \text{Total Revenue} - \text{Total Cost}$$

exo

$$\text{Profit} = \underline{p}\,Q(L,K) - wL - rK$$

w : wage r : rental rate

primitives: p, w, r.
(perfect competition)

solve for → K, L.

Profit Maximization

$$\text{Profit} = p\,Q(L, K) - wL - rK$$

FOCs:

$$\frac{\partial \pi}{\partial L} = p\,\frac{\partial Q}{\partial L} - w = 0 \qquad (1)$$

$$\frac{\partial \pi}{\partial K} = p\,\frac{\partial Q}{\partial K} - r = 0 \qquad (2)$$

Always ask yourself what are the things we already know (primitives) and what are the things we want to solve for

Profit Maximization

(1) and (2) can be re-written as:

$$(1) \quad \Rightarrow \quad p\,(MP_L) = w$$

$$(2) \quad \Rightarrow \quad p\,(MP_K) = r$$

What do these optimality conditions mean?

Combining (1) and (2):

$$\frac{MP_K}{r} = \frac{MP_L}{w}$$

Profit Maximization

Recall: what is a supply curve?

Using this characterization, we obtain conditions for choosing inputs optimally; From the primary method. just. know π, P.

- They depend on input/factor prices (w and r)
- They depend on price of the output (p)
- <u>The solution does not provide directly a relationship between output level (Q) and price</u>.

We need to characterize the problem differently using the <u>dual</u> approach

=> Cost Minimization.

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Lecture 8: Classical Theory of Supply - II

Wallace K C Mok

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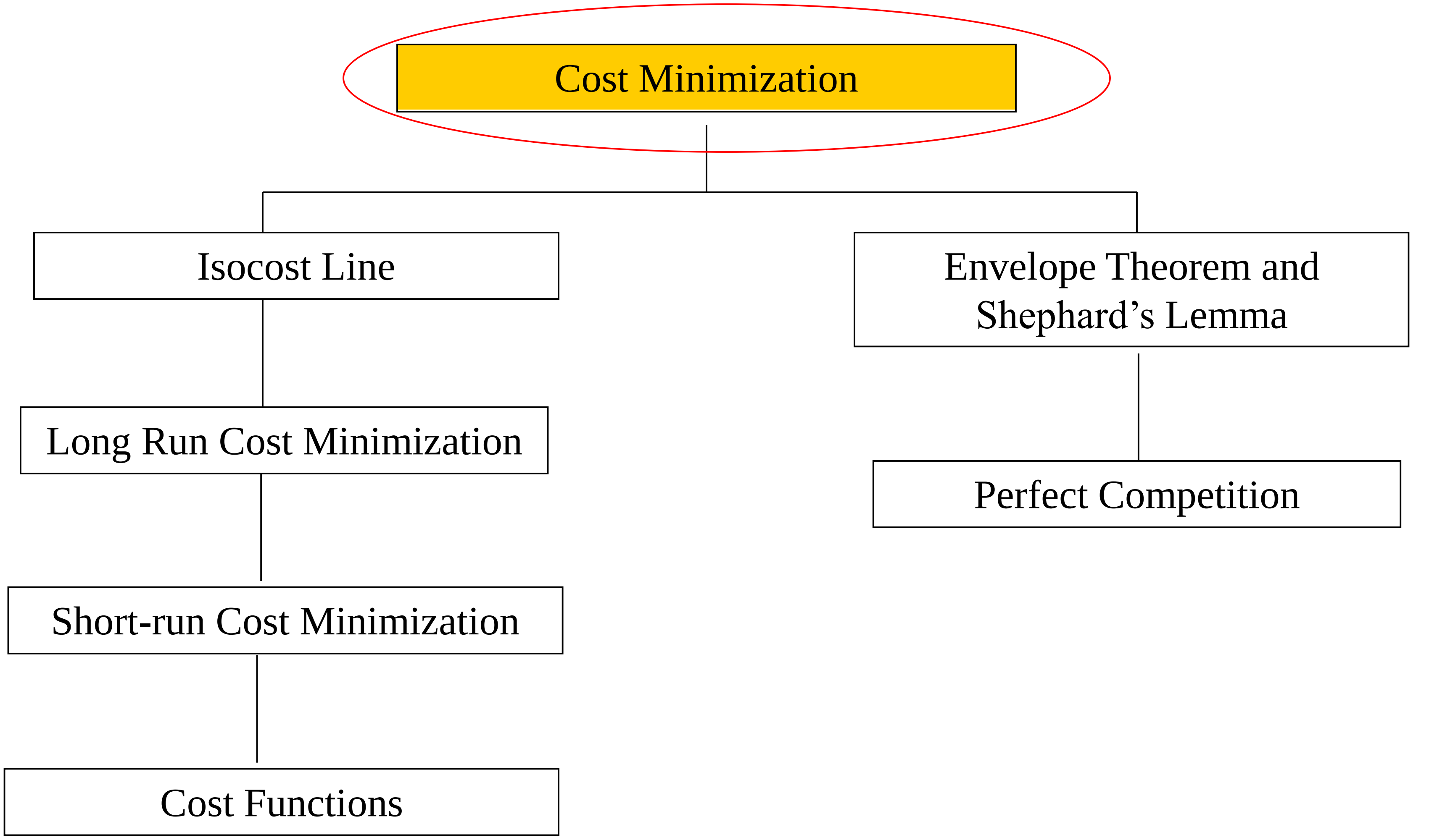
In this lecture, we…..

- Element of Cost Minimization, Isocost and Isoquant
- Short run vs Long Run
- Examples based on different Production Functions
- Average cost, fixed and variable costs in the Short run and Long run
- Duality – Shephard’s Lemma
- The Firm’s Supply Curve
- Shut down decisions under Perfect Competition

Supply II


Cost Minimization
Isocost Line
Envelope Theorem and Shephard's Lemma
Long Run Cost Minimization
Perfect Competition
Short-run Cost Minimization
Cost Functions

Goal: Derive the Supply Curve

Recall: what is a supply curve?

Using the profit max (primal) characterization, we obtain the conditions for choosing inputs optimally, giving the Profit Function (maximum profit given prices of input/output)

- They depend on input/factor prices
- They depend on price of the output
- The solution does not provide **directly** a relationship between output level and price.

We need to characterize the problem differently using the dual

Cost Minimization

Cost minimization problem (Dual): Finding the input combination that minimizes a firm's total cost of producing a particular level of output.

Long run: A period of time when the quantities of all of the firm's input can vary.

Short run: A period of time when **at least one** of its inputs' quantities is fixed.

Focus: ***Cost Function*** $C(Q_o, w, r)$

The minimum cost of producing Q_0 as a function of w, r

Long-Run Cost and Isocost Line

Cost to the Firm:

TC = Total Cost
w = wage rate
L = Quantity of Labor
r = price per unit of capital services
K = Quantity of Capital

$$TC = wL + rK$$

Isocost Line: The set of combinations of labor and capital that yield the same total cost for the firm.

$$TC = rK + wL$$

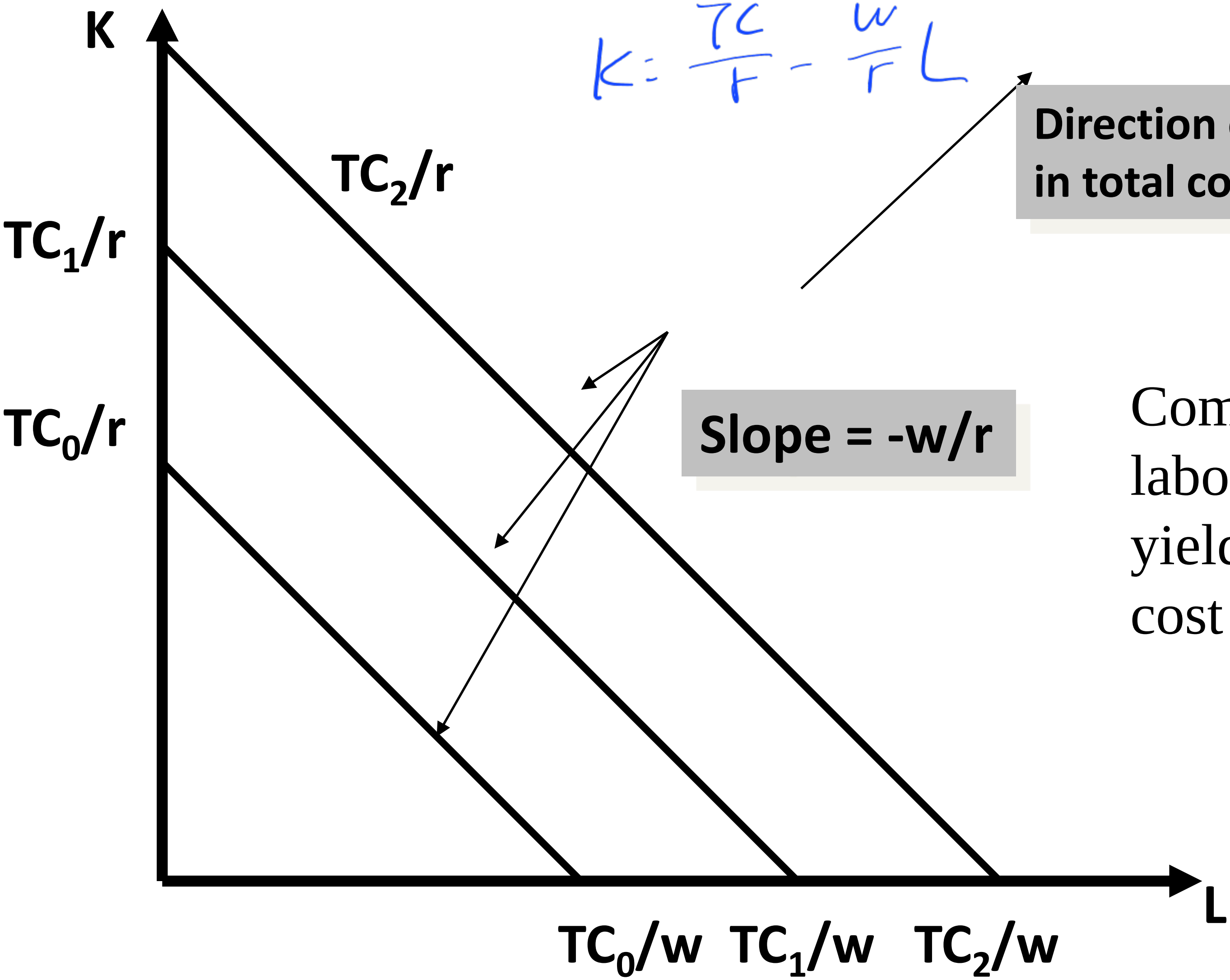
…or…

$$K = \frac{TC}{r} - \left(\frac{w}{r}\right) L$$

is the **isocost line**

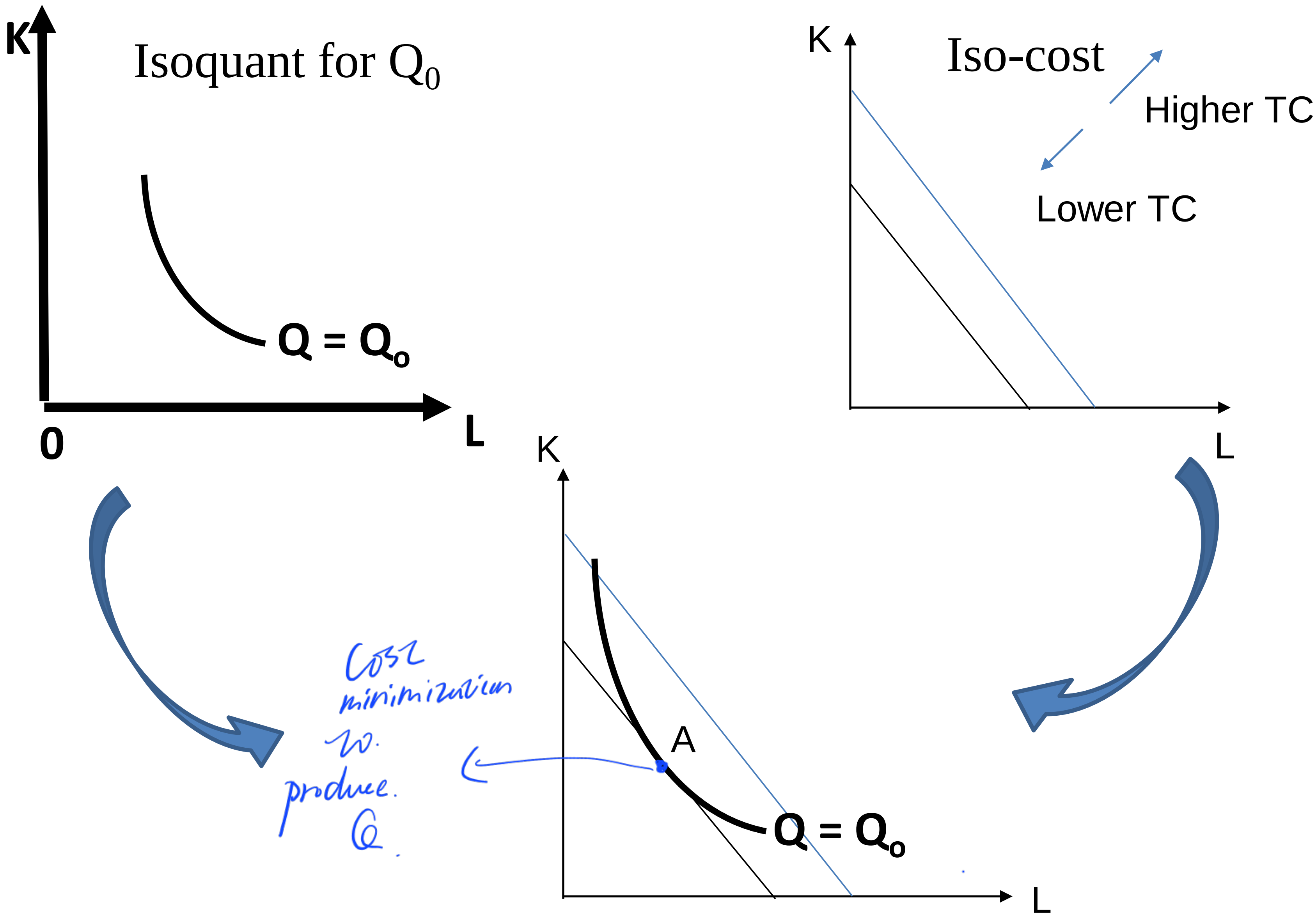
Isocost Lines

$K = \frac{TC}{r} - \frac{w}{r} L$



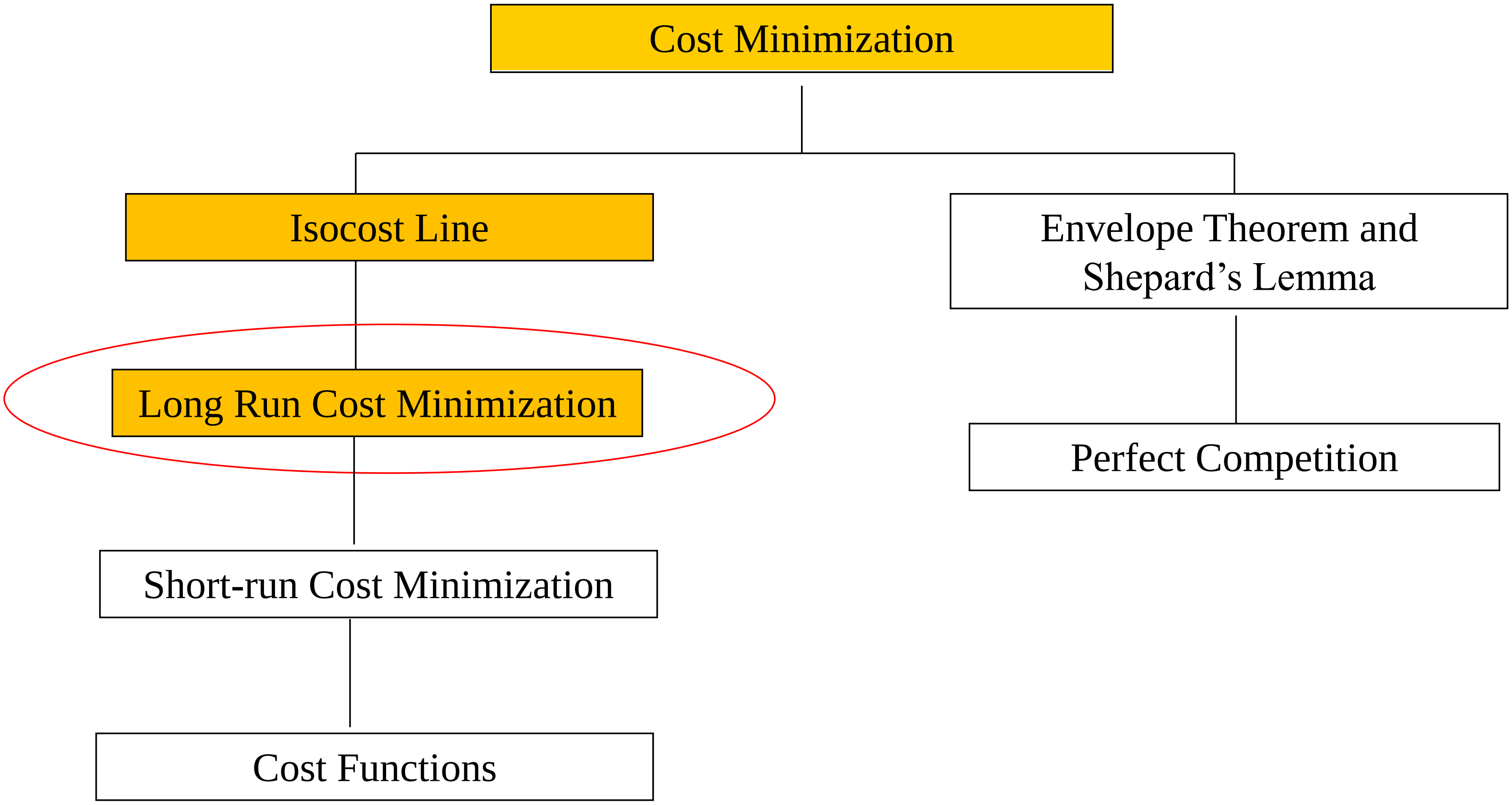
Combinations of labor and capital that yields the same total cost for the firm

Isoquant and Isocost, Cost Min



Cost minimization w. produce Q.

Supply II



Cost Minimization
Isocost Line
Envelope Theorem and Shepard's Lemma
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Long-Run Cost Minimization

Suppose that a firm's owners wish to minimize costs

Let the **desired output be Q_0**

Technology: $Q = f(L,K)$

Firm's problem: min $TC = rK + wL$

- K, L
- Subject to $Q_0 = f(L,K)$

r, w, Q_0

What are the primitives of this problem?

Long-Run Cost Minimization

Cost minimization subject to satisfaction of the isoquant equation:

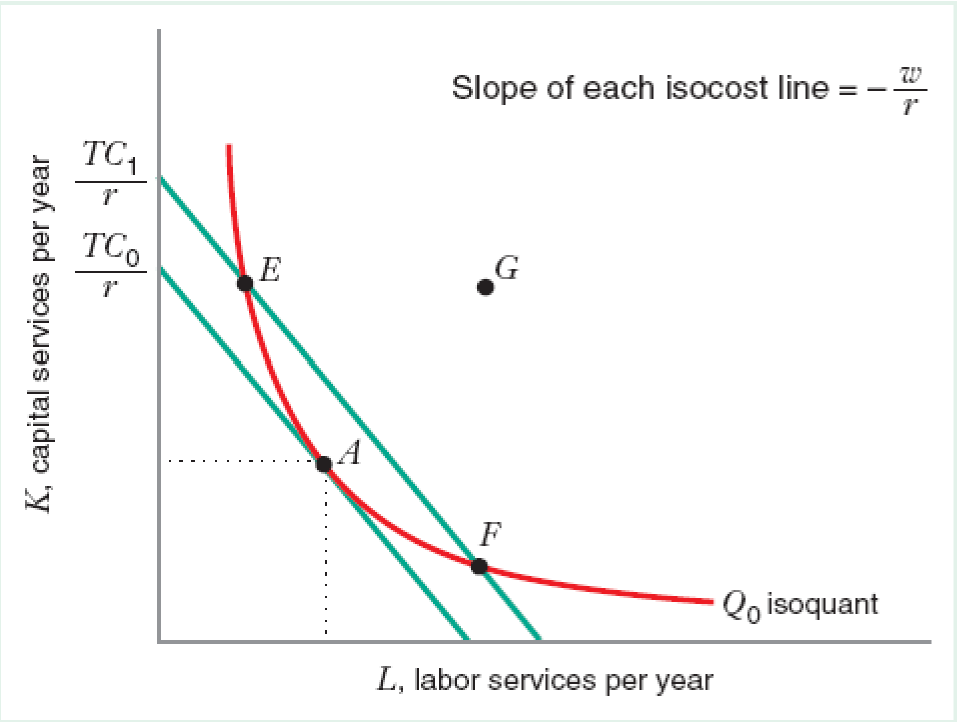
$$Q_0 = f(L, K)$$

Tangency Condition:

$$MRTS_{L,K} = -\frac{MP_L}{MP_K} = -\frac{w}{r} \text{ (or) } \quad \frac{MP_L}{w} = \frac{MP_K}{r}$$

Constraint: $Q_0 = f(K, L)$

Long-Run Cost Minimization



Solution to cost minimization:

- Point where isoquant is just tangent to isocost line (A)
- **G** – Technically Inefficient
- **E & F** – Technically Efficient but do not minimize cost

Long-Run Cost Minimization

Solution to cost minimization:

- Slope of isoquant = slope of isocost line

$$-MRTS_{L,K} = \frac{w}{r} \qquad \text{(or)} \qquad \frac{MP_L}{MP_K} = \frac{w}{r}$$

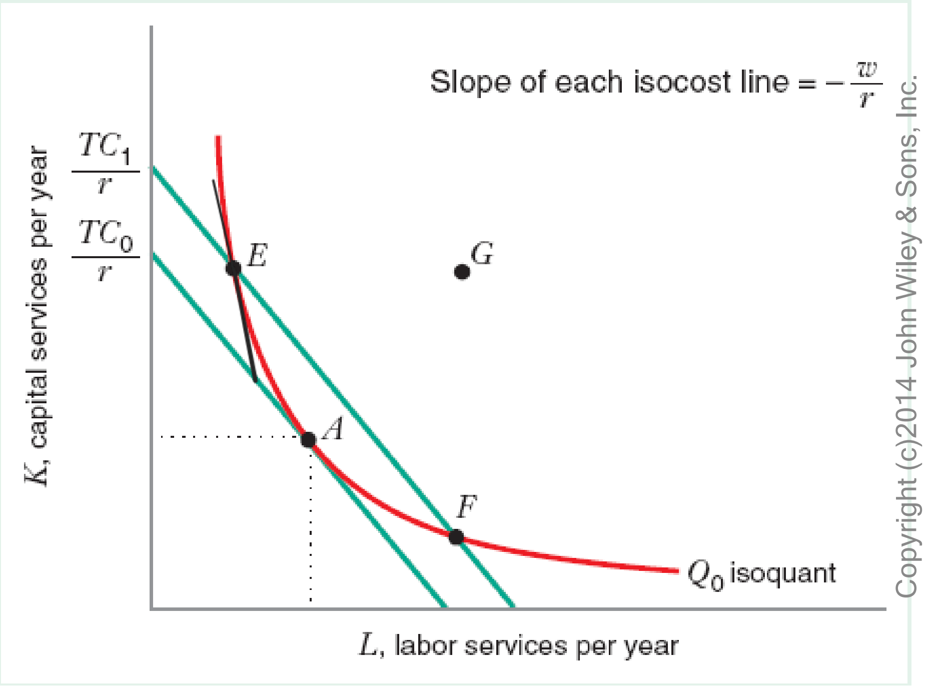
- Ratio of marginal products = ratio of input prices

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Long-Run Cost Minimization

- At point E $\quad \dfrac{MP_L}{MP_K} > \dfrac{w}{r} \quad$ (or) $\quad \dfrac{MP_L}{w} > \dfrac{MP_K}{r}$

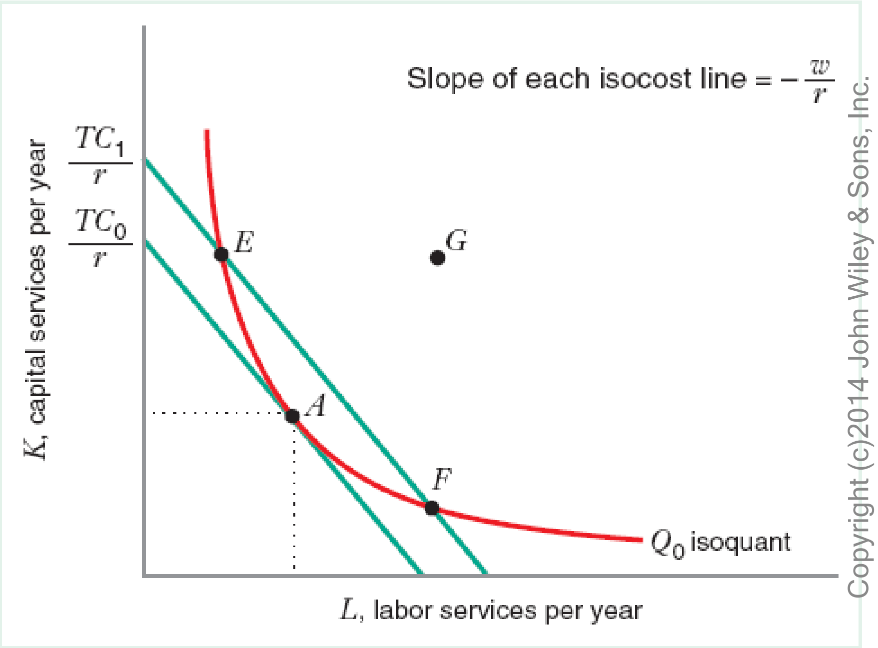
- This implies the firm could spend an additional dollar on labor and save more than a dollar by reducing its employment of capital and keep output constant



Long-Run Cost Minimization

- At point F $\dfrac{MP_L}{MP_K} < \dfrac{w}{r}$ (or) $\dfrac{MP_L}{w} < \dfrac{MP_K}{r}$

- This implies the firm could spend an additional dollar on capital and save more than a dollar by reducing its employment of labor and keep output constant



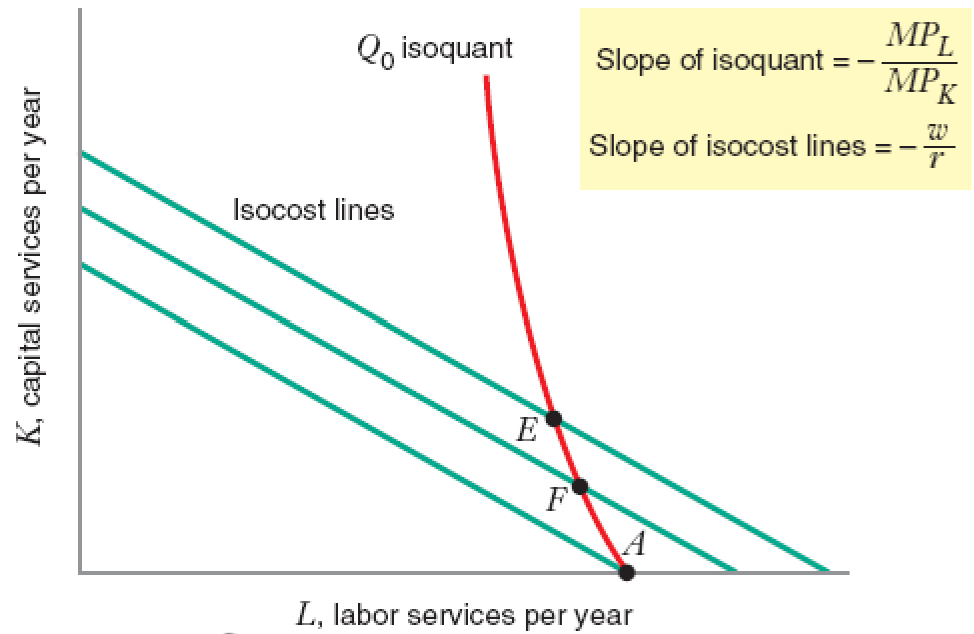
Corner Solution

The cost-minimizing input combination for producing Q_0 units of output occurs at point A where the firms uses no capital. At this corner point the isocost line is flatter than the isoquant.

$$-\left(\frac{MP_L}{MP_K}\right) < -\left(\frac{w}{r}\right)$$

$$\Rightarrow \quad \frac{MP_L}{w} > \frac{MP_K}{r}$$

First check the isoquant's shape.



$$\underbrace{\frac{MP_L}{MP_K} > \frac{w}{r}}_{\text{always here.}}$$

Cost Minimization: Example I

I. Cobb - Douglas: $Q = AL^{\alpha}K^{\beta}$

$MP_L = \alpha A L^{\alpha-1} K^{\beta}$

$MP_K = \beta A L^{\alpha} K^{\beta-1}$

$\Rightarrow MRTS = \frac{\alpha}{\beta} \times \frac{K}{L} = \frac{w}{r}$

Tangency condition: $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{\alpha}{\beta}\left(\frac{K}{L}\right) = \frac{w}{r} \Rightarrow K = \left(\frac{w}{r}\right)\left(\frac{\beta}{\alpha}\right) L$

$\Downarrow$

$K = \frac{w}{r}\frac{\beta}{\alpha} \times L$

Production constraint: $Q_0 = A\,L^{\alpha}K^{\beta} \Rightarrow Q_0 = A\,L^{\alpha}\left[\left(\frac{w}{r}\right)\left(\frac{\beta}{\alpha}\right)L\right]^{\beta} \Rightarrow Q_0 = A\,L^{\alpha+\beta}\left(\frac{w}{r}\frac{\beta}{\alpha}\right)^{\beta}$

$Q_0 = A L^{\alpha} K^{\beta}$

$= A L^{\alpha} \left(\frac{w}{r}\frac{\beta}{\alpha} L\right)^{\beta}$

$= A L^{\alpha+\beta} \left(\frac{w\beta}{r\alpha}\right)^{\beta}$

$K = L\left(\frac{r\alpha}{w\beta}\right)^{-1}$

$\Rightarrow L^{\alpha+\beta} = \frac{Q_0}{A}\left(\frac{r}{w}\frac{\alpha}{\beta}\right)^{\beta}$

$\frac{\beta}{\alpha+\beta} - \frac{\alpha+\beta}{\alpha+\beta} = \frac{-\alpha}{\alpha+\beta}$

$\Rightarrow \mathbf{L^*(Q_0, w, r)} = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{r}{w}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

$\underline{L = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\alpha+\beta}}}$

Similarly, $\mathbf{K^*(Q_0, w, r)} = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{w}{r}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$

$K = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$

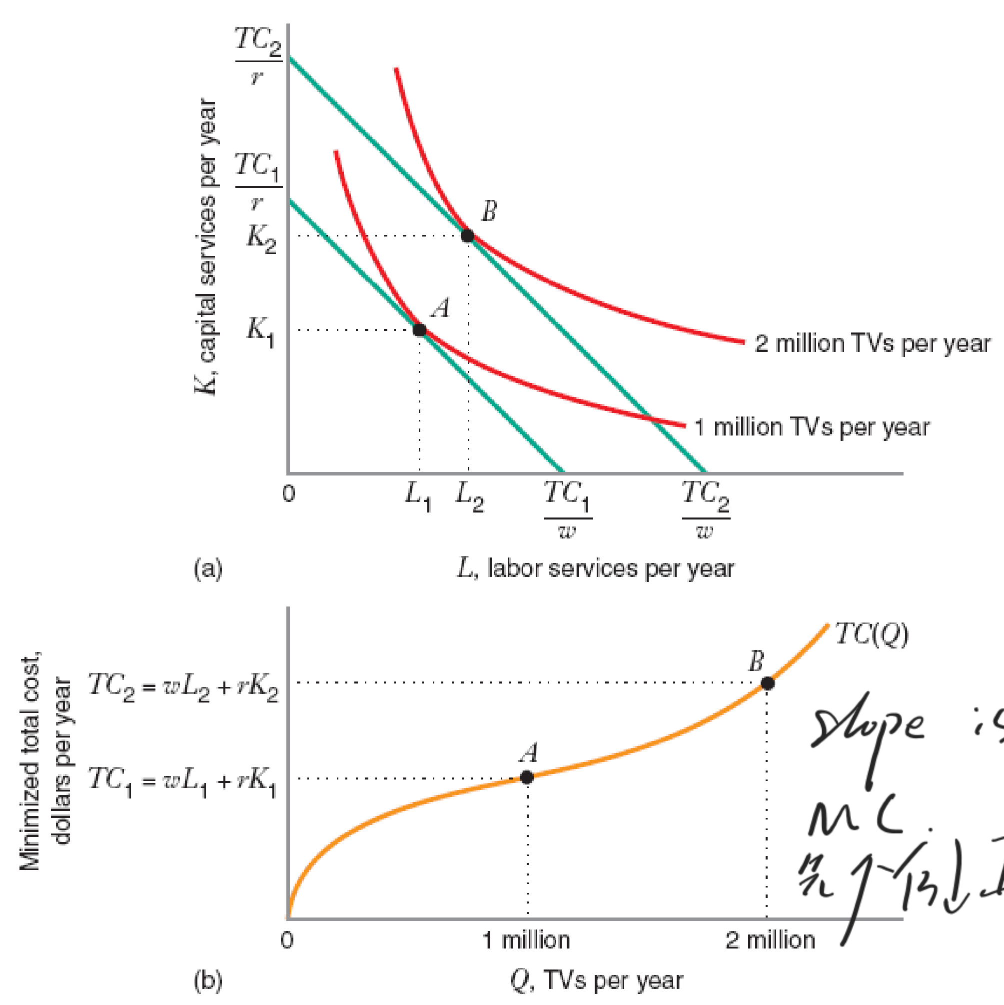
Long Run Cost Functions

The **Long Run total cost function** relates minimized total cost to output, Q, and to the factor prices (w and r).

$$TC(Q, w, r) = w\,L^*(Q, w, r) + r\,K^*(Q, w, r)$$

Where: L* and K* are the long run input demand functions

Long Run Cost Functions



As Quantity of output increases from 1 million to 2 million, with input prices(w, r) constant, cost minimizing input combination moves from TC_1 to TC_2 which gives the TC(Q) curve.

slope is MC. 先下降再升

Long Run Total Cost Function: Example I

I. Cobb - Douglas: $Q = AL^{\alpha}K^{\beta}$

Factor/input demand functions: $L^*(Q_0, w, r) = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{r}{w}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

$$K^*(Q_0, w, r) = \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$$

Total cost function: $TC(Q_0, w, r) = wL^*(Q_0, w, r) + rK^*(Q_0, w, r)$

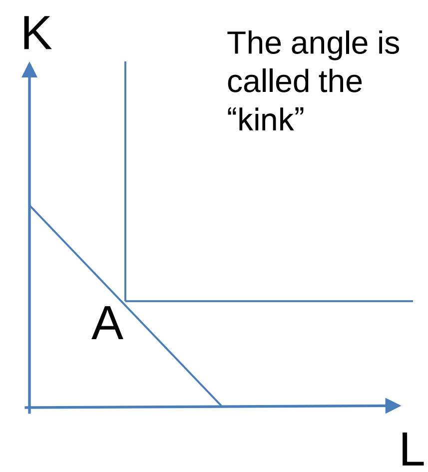
$$= \left(\frac{Q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left[w\left(\frac{r}{w}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + r\left(\frac{w}{r}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right]$$

Total Cost Function: Examples II

II. Leontief: $Q = A\min(aK, bL)$

Does the tangency condition work? not differentiable.

not work.



The angle is called the "kink"

Instead, optimally at kink:

$$aK^* = bL^*$$

Production constraint:

$$Q_0 = A\min(aK^*, bL^*) \quad \Rightarrow \frac{Q_0}{A} = aK^* = bL^*$$

$$\Rightarrow L^*(Q_0, w, r) = \frac{Q_0}{Ab}$$

$$K^*(Q_0, w, r) = \frac{Q_0}{Aa}$$

What do you notice about the input demand functions?

Total Cost Function: Examples II

II. Leontief: $Q = A \min\left(aK, bL\right)$

Factor demand functions:

$$\Rightarrow \; L^*(Q_0, w, r) = \frac{Q_0}{Ab} \qquad\qquad K^*(Q_0, w, r) = \frac{Q_0}{Aa}$$

So, Total cost function:

$$TC(Q_0, w, r) = wL^*(Q_0, w, r) + rK^*(Q_0, w, r)$$

$$= w\frac{Q_0}{Ab} + r\frac{Q_0}{Aa} = \left(\frac{w}{Ab} + \frac{r}{Aa}\right) Q_0$$

Total Cost Function: Examples III

III. Linear: $Q = aK + bL$

Does the tangency condition work?
Sometimes (when)?

may work
when.
infinite. solutions

$$\textbf{If } MRTS_{L,K} = \frac{b}{a} > \frac{w}{r} \text{ (or } \frac{b}{w} > \frac{a}{r}\text{):}$$

(labor's marginal product per dollar (b/w) exceeds capital's): Use only *Labour*

$$L^*(Q_0, w, r) = \frac{Q_0}{b}, \quad K^*(Q_0, w, r) = 0$$

$$\textbf{If } MRTS_{L,K} = \frac{b}{a} < \frac{w}{r} \text{ (or } \frac{b}{w} < \frac{a}{r}\text{):}$$

(Otherwise): Use only *Capital*

$$L^*(Q_0, w, r) = 0, \quad K^*(Q_0, w, r) = \frac{Q_0}{a}$$

Total Cost Function: Example I

III. Linear: $Q = aK + bL$

If $MRTS_{L,K} = \frac{b}{a} > \frac{w}{r}$ (or $\frac{b}{w} > \frac{a}{r}, \frac{w}{b} < \frac{r}{a}$):

$$TC(Q_0, w, r) = wL^*(Q_0, w, r) + rK^*(Q_0, w, r) = w\frac{Q_0}{b} + r \cdot 0 = \frac{wQ_0}{b}$$

If $MRTS_{L,K} = \frac{b}{a} < \frac{w}{r}$ (or $\frac{b}{w} < \frac{a}{r}, \frac{w}{b} > \frac{r}{a}$):

$$TC(Q_0, w, r) = wL^*(Q_0, w, r) + rK^*(Q_0, w, r) = w \cdot 0 + r\frac{Q_0}{a} = \frac{rQ_0}{a}$$

In summary: $TC(Q_0, w, r) = Q_0 \min\left(\frac{w}{b}, \frac{r}{a}\right)$

Supply II

Cost Minimization

Isocost Line

Long Run Cost Minimization

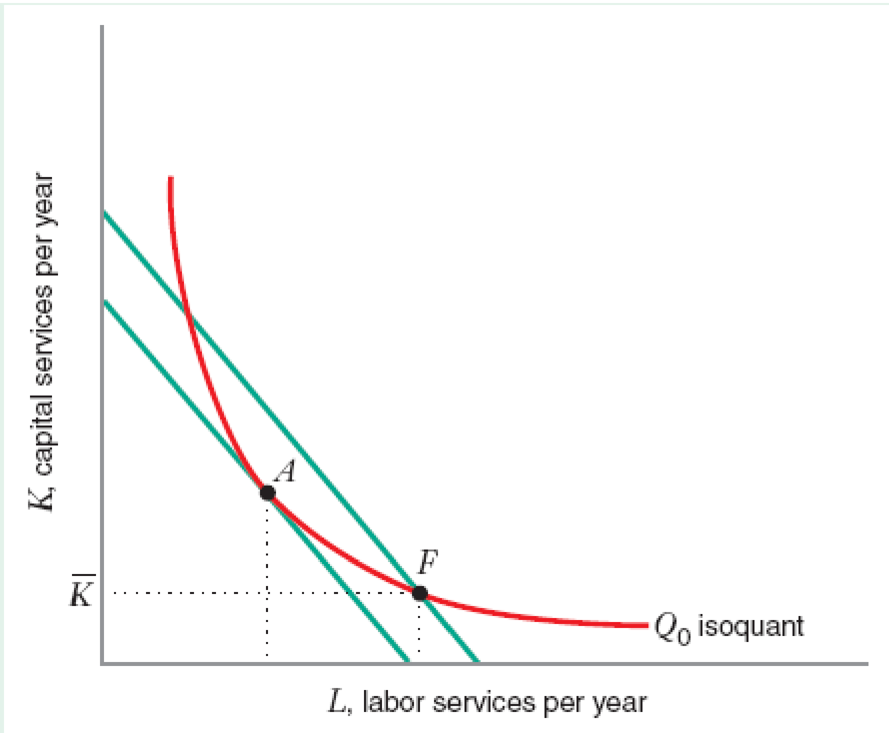
Short-run Cost Minimization

Cost Functions

Envelope Theorem and Shepard's Lemma

Perfect Competition

Short-Run Cost Minimization



$TC_s > TC_L$

One **fixed** Input – Capital $\overline{K}$

- Desired output is Q_0
- Short run combination is point F
- If the firm were free to adjust all of its inputs, the cost-minimizing combination is at Point A

Short-Run Cost Minimization

Example:

$$Q = 50\sqrt{LK}$$

- Now Capital is fixed $\bar{K}$

$$L = \frac{Q^2}{2500\,\bar{K}}$$

unrelated with wage

This is an example of a *short-run factor/input demand function.*

Short-Run Cost Minimization

Total Variable Costs – the sum of total expenditures on variable inputs, such as labor and materials, at the short-run cost-minimizing input combination

Total Fixed Costs – the cost of fixed inputs; it does not vary with output

Short-Run Cost Minimization

- Short run: One input is fixed, capital $\bar{K}$. Firm can vary the other input, i.e. labor.

So demand for labor will be independent of price of the product (see the previous example)

- Short run demand for labor will also depend on quantity produced. As quantity increased, labor used increases holding capital fixed.

Example: Cobb-Douglas

What is the long run total cost function for production function ?

$$Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$L^*(Q,w,r) = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} \qquad K^*(Q,w,r) = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

$$\textbf{Total cost: } TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$$

$$= w\frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} + r\frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

$$= \frac{Q}{50}(wr)^{\frac{1}{2}} + \frac{Q}{50}(wr)^{\frac{1}{2}} = \frac{Q}{25}(wr)^{\frac{1}{2}}$$

Example: Short-Run Cost Minimization

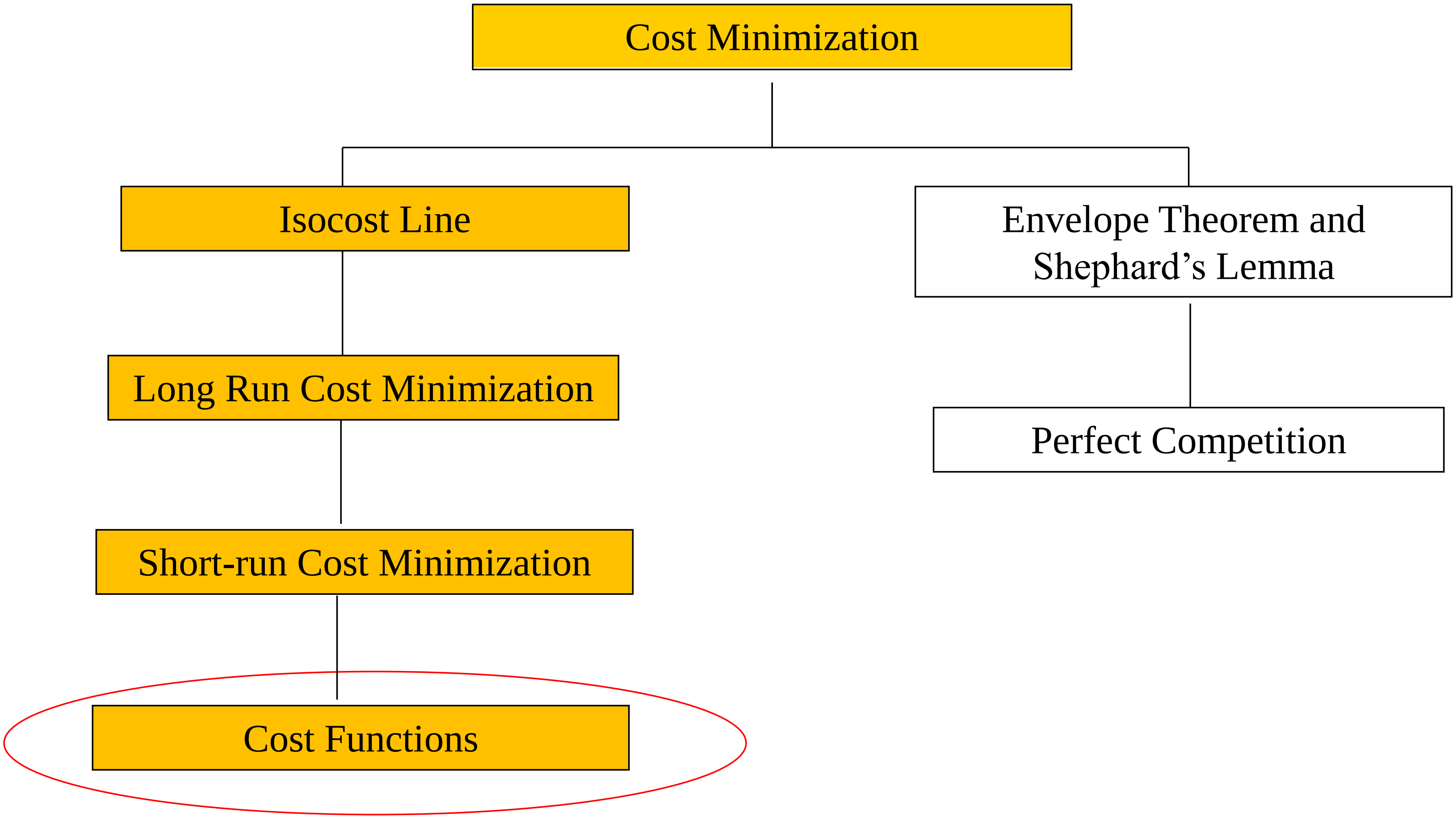
What is the short-run total cost function for production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$ *if K is fixed at* $\overline{K}$ *?*

$$L^*(Q, w, r, \overline{K}) = \frac{Q^2}{2500\overline{K}}$$

$$TC(Q, w, r) = wL^*(Q, w, r, \overline{K}) + r\overline{K}$$

$$= w\frac{Q^2}{2500\overline{K}} + r\overline{K}$$

Supply II


Cost Minimization
Isocost Line
Envelope Theorem and Shephard’s Lemma
Long Run Cost Minimization
Perfect Competition
Short-run Cost Minimization
Cost Functions

Example

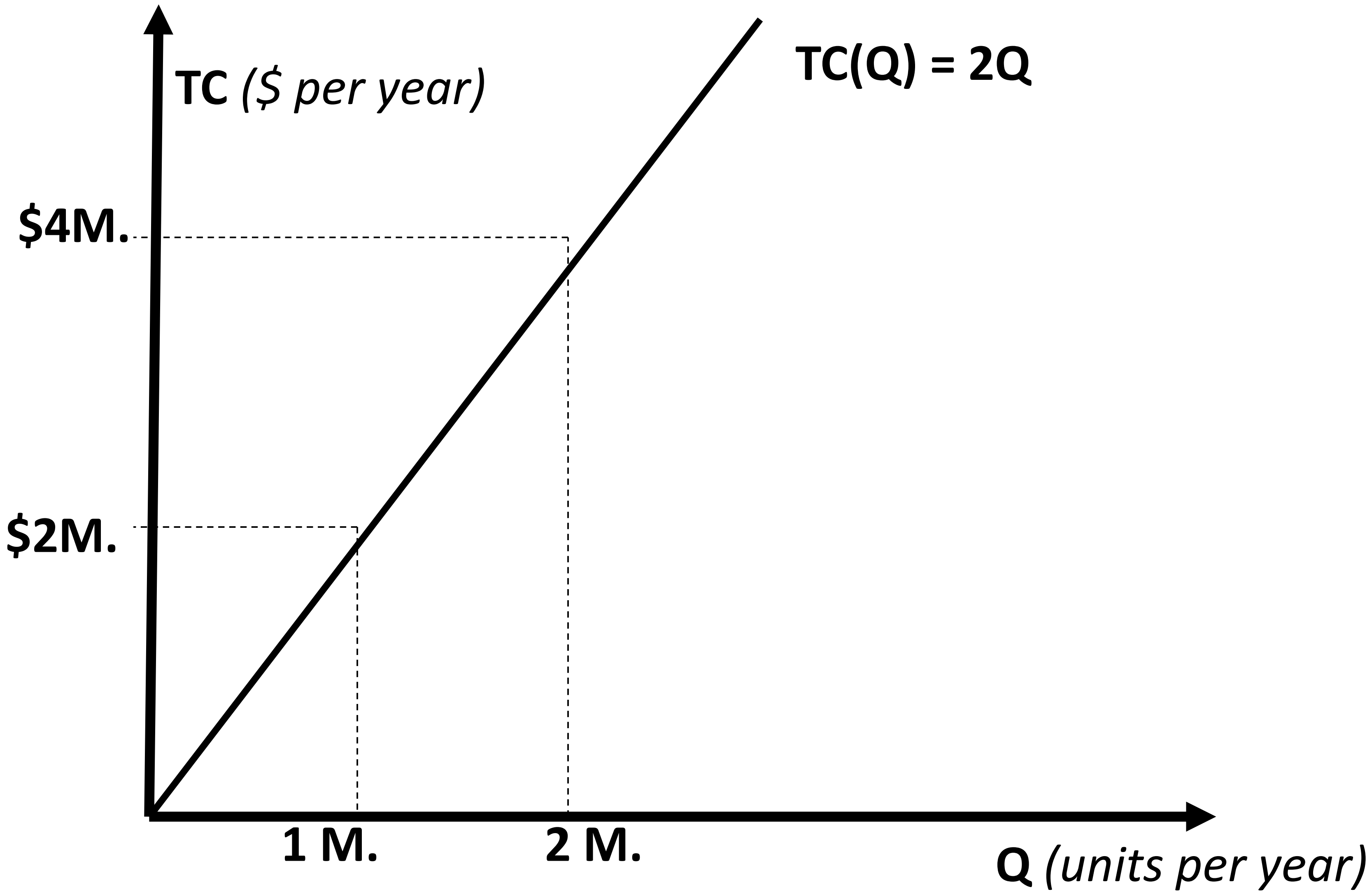
What is the long run total cost function for production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$ *?*

$$TC(Q,w,r) = \frac{Q}{25}(wr)^{\frac{1}{2}}$$

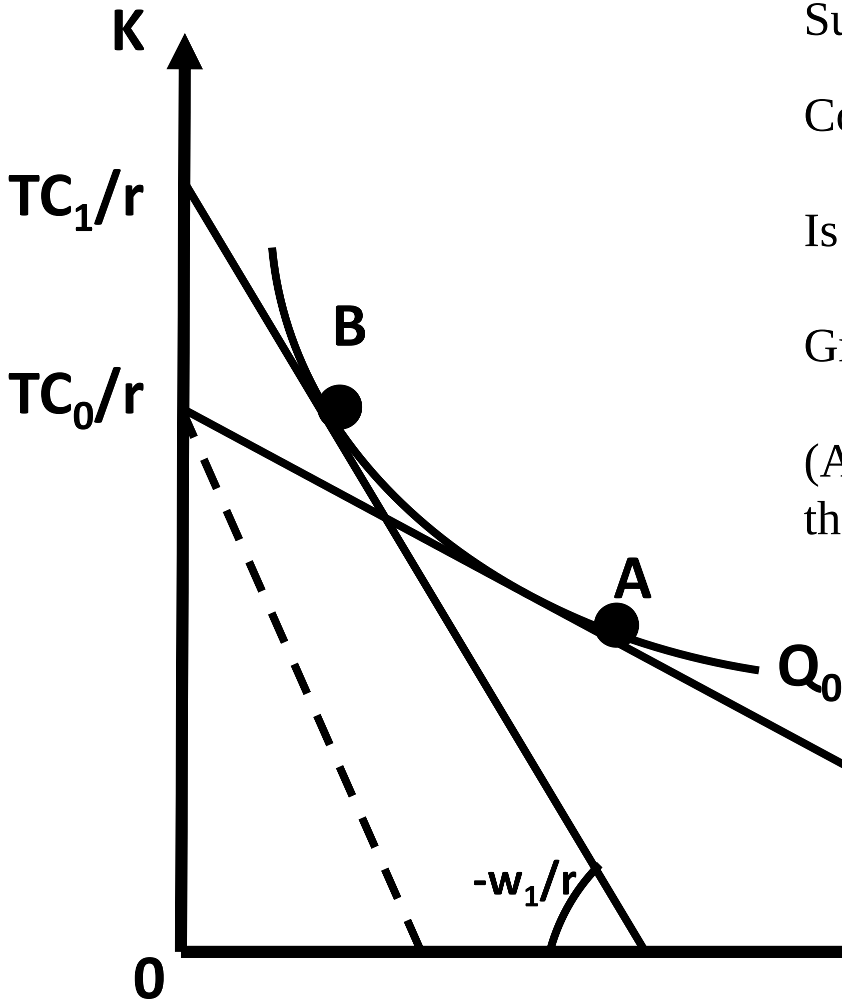
What is the graph of the total cost curve when w = 25 and r = 100?

$$TC(Q) = 2Q$$

A Total Cost Curve



A Change in Input Prices



Suppose wage increases from w_0 to w_1

Cost minimization: A > B

Is the cost at A higher or B higher?

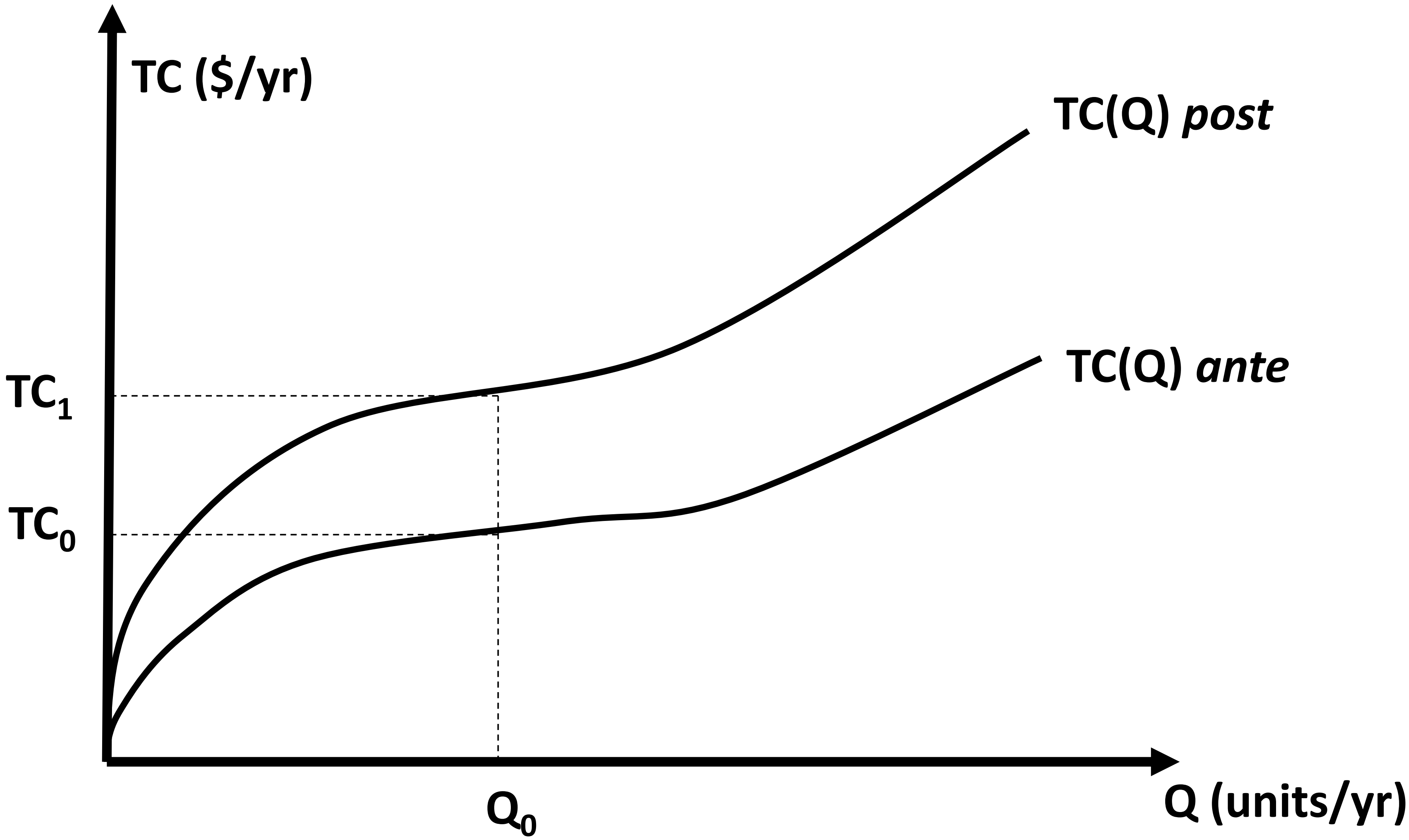
Graphically how do we see it?

(Algebraically calculate the derivative of the cost function)

$TC = \frac{Q}{25}(wr)^{\frac{1}{2}}$

$w \uparrow \Rightarrow TC \uparrow$

A Shift in the Total Cost Curve



Long Run Average Cost and marginal cost Functions

The ***long run average cost function*** is the long run total cost function divided by output, Q.

That is, the LRAC function tells us the firm's cost per unit of output…

$$AC(Q,w,r) = \frac{TC(Q,w,r)}{Q}$$

The ***long run <u>marginal</u> cost function*** is the long run total cost for producing an extra unit of output, Q

$$MC(Q,w,r) = \frac{dTC(Q,w,r)}{dQ}$$

Long Run Marginal Cost Function

a. What are the long run average and marginal cost functions for this production function?

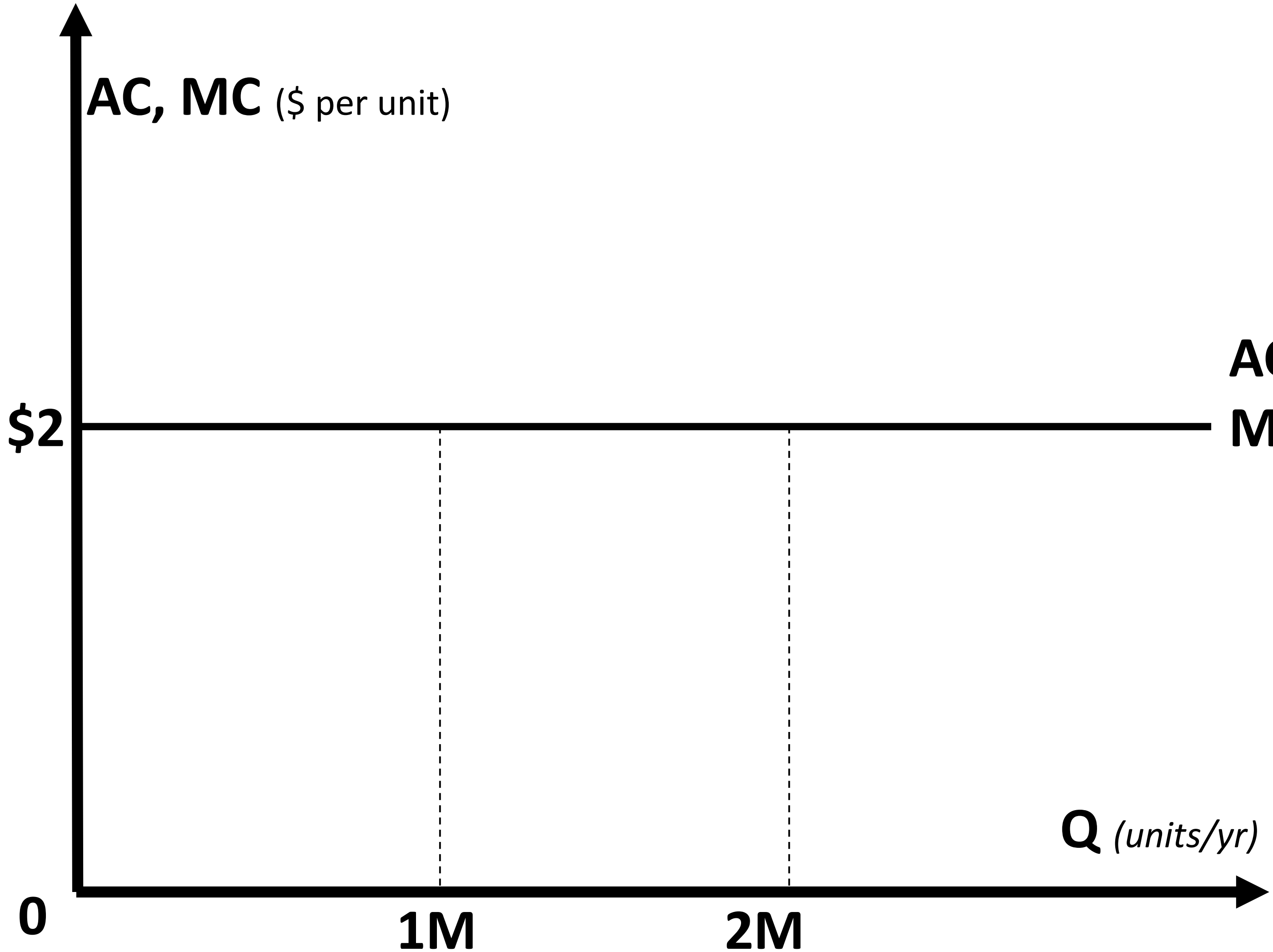
$$AC(Q,w,r) = \frac{(wr)^{\frac{1}{2}}}{25}, \quad MC(Q,w,r) = \frac{(wr)^{\frac{1}{2}}}{25}$$

b. What are the long run average and marginal cost curves when w = 25 and r = 100?

$$\mathrm{AC(Q) = 2Q/Q = 2,\ MC(Q) = d(2Q)/dQ = 2}$$

AC = MC.

Average & Marginal Cost Curves



Short Run Cost Functions

a. What are the short-run average and marginal cost functions for this production function?

$$SRAC(Q,w,r,\overline{K}) = w\left(\frac{Q}{2500\overline{K}}\right) + r\frac{\overline{K}}{Q}$$

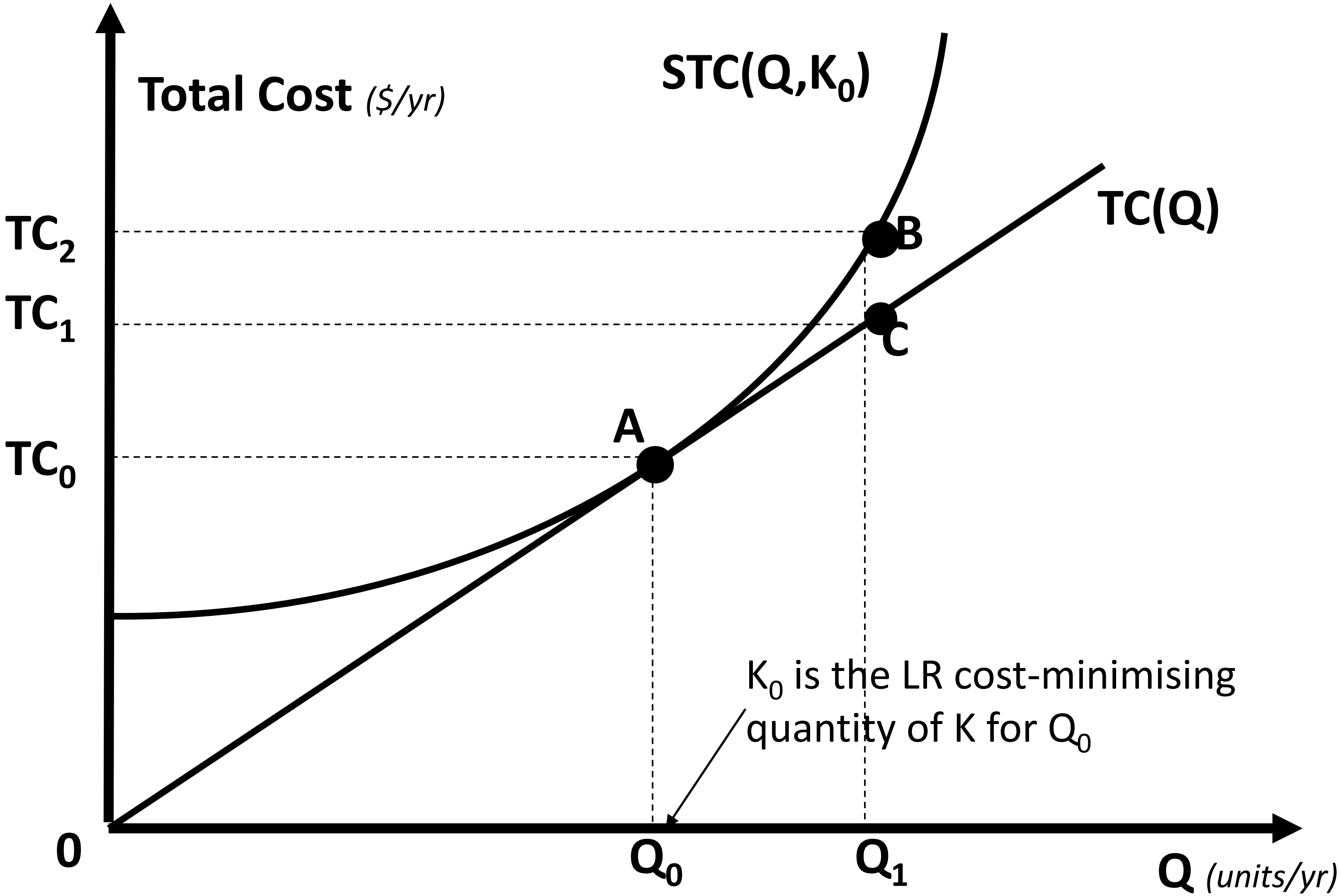
$$SRMC(Q,w,r,\overline{K}) = w\left(\frac{Q}{1250\overline{K}}\right)$$

b. What are the short-run average and marginal cost curves when w = 25 and r = 100?

$$SRAC(Q,w,r,\overline{K}) = \frac{Q}{100\overline{K}} + \frac{100\overline{K}}{Q}$$

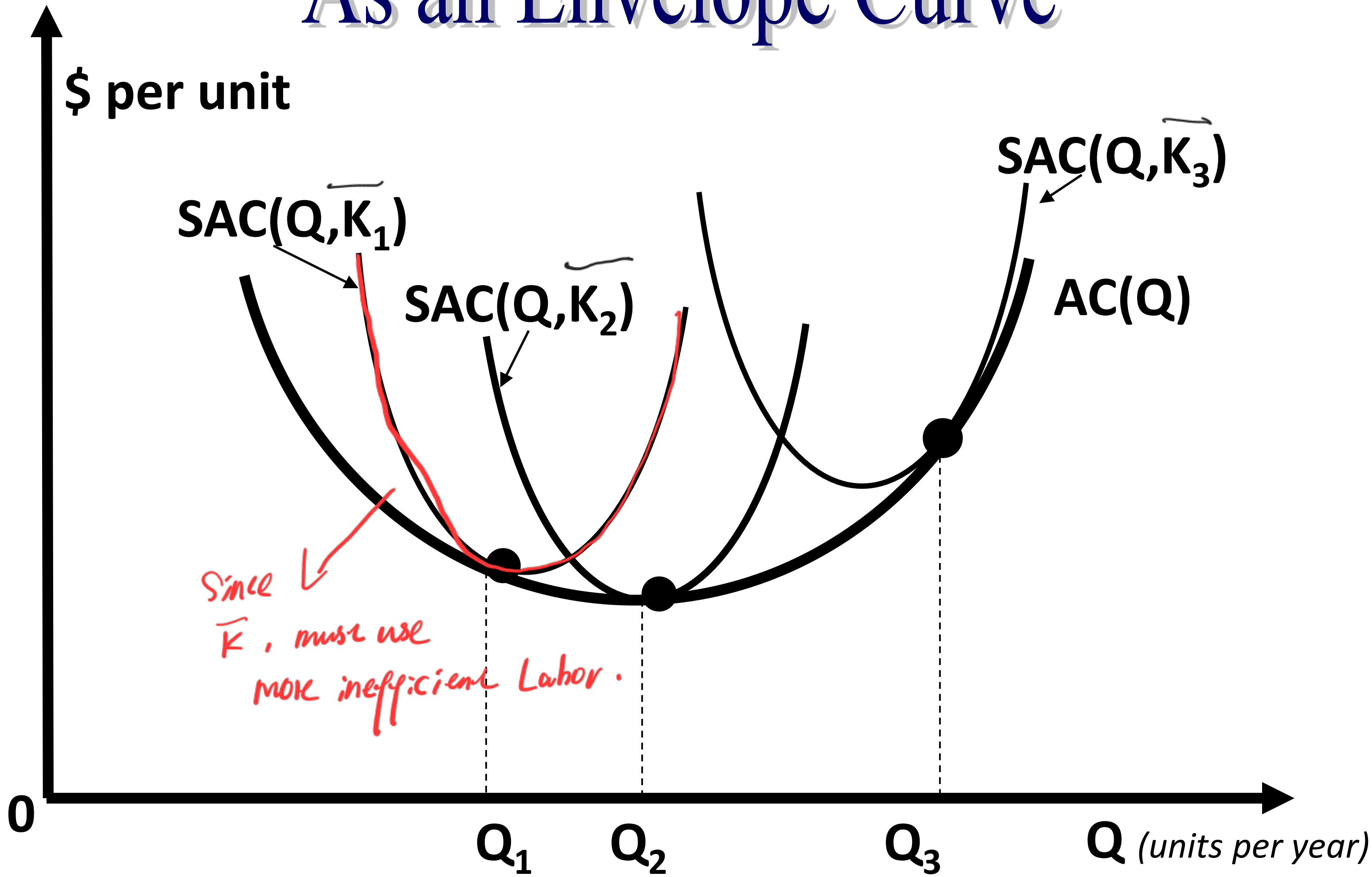
$$SRMC(Q,w,r,\overline{K}) = \frac{Q}{50\overline{K}}$$

Long and Short Run Total Cost Functions



In General.... LRAC ≤ SRAC

As an Envelope Curve



Supply II

Cost Minimization

Isocost Line

Long Run Cost Minimization

Short-run Cost Minimization

Cost Functions

Envelope Theorem and Shephard's Lemma

Perfect Competition

$Q = KL$

$C = wL + rK.$

Duality of Production and Cost

If we want to derive cost functions from production …

Step One: Cost Minimization Problem

=> Factor demand functions

$$L^*(Q, w, r), K^*(Q, w, r)$$

Step Two: $TC(Q, w, r) = wL^*(Q, w, r) + rK^*(Q, w, r)$

Now consider the following: If I want to see how a change in w affects TC, should I use partial or total Derivative?

Envelope Theorem

- Definition:

- Value function (or maximum/minimum function)

- Examples: Indirect Utility Function $v=v(p,M)$
  Profit Function
  Cost Function
  Lagrangian

Envelope Theorem

- *The Envelope Theorem*:

Suppose that value function *m(a)* is defined as following, where a is a parameter:

$$m(a) = \max_x f(x(a), a)$$

(*NB: to min, it is max - f(x(a), a)*)

Then the Total Derivative of m(a) with respect to a equals the Partial derivative of f(x(a), a) with respect to a, when f(x(a), a) is evaluated at x=x(a):

After getting the min C^*

$$\frac{dm(a)}{da} = \frac{\partial f(x(a), a)}{\partial a}$$

$\frac{dC^*}{dw}$ \\ equal

So for: TC = wL + rK $\frac{dC}{dw} = L^*$

Duality of Production and Cost

For $\pi = P \cdot F(K, L) - TC$. (profit)

$$\frac{d\pi}{dP} = \left.\frac{\partial \pi}{\partial P}\right|_{\text{at optimum}} = F^*(K, L)$$

$Shephard's Lemma :$

$$\frac{\partial TC(Q, w, r)}{\partial w} = L^*(Q, w, r)$$

$$\frac{\partial TC(Q, w, r)}{\partial r} = K^*(Q, w, r)$$

If you use Primal Profit Maximization $\pi = P * f(K, L) - wL - rK$, then:

$$\frac{d\pi}{dP} = \frac{\partial \pi}{\partial P} = f(K^*, L^*)$$ (Hotelling's Lemma)

Example of using Shephard's Lemma

Can we characterize or "recover" production function if we are only given cost function?   Yes.

$$\text{Given } TC(Q,w,r) = \frac{Q}{25}(wr)^{\frac{1}{2}}, \text{ want } Q = f(K,L)$$

Step One: Shephard's Lemma

$$L^*(Q,w,r) = \frac{\partial TC(Q,w,r)}{\partial w} = \frac{1}{2}\frac{Q}{25}w^{-\frac{1}{2}}r^{\frac{1}{2}} = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}}$$

$$K^*(Q,w,r) = \frac{\partial TC(Q,w,r)}{\partial r} = \frac{1}{2}\frac{Q}{25}w^{\frac{1}{2}}r^{-\frac{1}{2}} = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

Example

Step Two: Eliminate r, w

$$L^*(Q,w,r) = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}}, \quad K^*(Q,w,r) = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

$$\text{From } L^*\text{: } \left(\frac{r}{w}\right)^{\frac{1}{2}} = \frac{L^*\cdot 50}{Q} \Rightarrow \boxed{\left(\frac{w}{r}\right)^{\frac{1}{2}} = \frac{Q}{L^*\cdot 50}}$$

$$\text{Plug into } K^*\text{: } K^* = \frac{Q}{50}\cdot\frac{Q}{L^*\cdot 50} = \frac{Q^2}{2500L^*}$$

$$\Rightarrow Q^2 = 2500K^*L^*, \quad Q = 50\sqrt{K^*L^*}$$

Verifying the answer

Recall: Production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$?

$$L^*(Q,w,r) = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} \qquad K^*(Q,w,r) = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

Total cost: $TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$

$$= w\frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} + r\frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

$$= \frac{Q}{50}(wr)^{\frac{1}{2}} + \frac{Q}{50}(wr)^{\frac{1}{2}} = \frac{Q}{25}(wr)^{\frac{1}{2}}$$

Supply II

- Cost Minimization
  - Isocost Line
    - Long Run Cost Minimization
      - Short-run Cost Minimization
        - Cost Functions
  - Envelope Theorem and Shepard’s Lemma
    - Perfect Competition

Perfectly Competitive Markets

Perfect Competition *(No market power)*

efficient.

Oligopolies
Monopolistic Competition

Monopoly *(Full market power)*

Price discrimination
1st degree => efficient
2nd degree
3rd degree.

A **perfectly competitive market** consists of firms that produce **identical products** that sell at the **same price**.

Each firm's volume of output is so **small** in comparison to the overall market demand that **no single firm has an impact on the market price**.

Perfectly Competitive Markets - Conditions

A. Firms produce **undifferentiated products** in the sense that consumers perceive them to be identical;

B. Consumers have **perfect information** about the prices all sellers in the market charge;

C. Each buyer's purchases are so **small** that he/she has an imperceptible effect on market price;

D. Each seller's sales are so **small** that he/she has an imperceptible effect on market price. Each seller's input purchases are so **small** that he/she perceives no effect on input prices;

E. All firms *(industry participants and new entrants)* have **equal access to resources** *(technology, inputs).*

Implications of Conditions

The Law of One Price: Conditions (a) and (b) imply that there is a single price at which transactions occur.

Price Takers: Conditions (c) and (d) imply that buyers and sellers take the price of the product as given when making their purchase and output decisions.

Free Entry: Condition (e) implies that all firms have identical long run cost functions

The Profit Maximization Condition

- Assuming the firm sells output Q, its economic profit is:

$$\pi = TR(Q) - TC(Q)$$

$\frac{d\pi}{dQ} = MR - MC = 0$

MR=MC $\Rightarrow$ P=MC

(In competitive case MR = P)

- TR(Q) = Total revenue from selling the quantity Q

$$\Rightarrow \quad TR(Q) = P \times Q$$

NB: P is given here

- TC(Q) = Total economic cost of producing the quantity Q

The Profit Maximization Condition

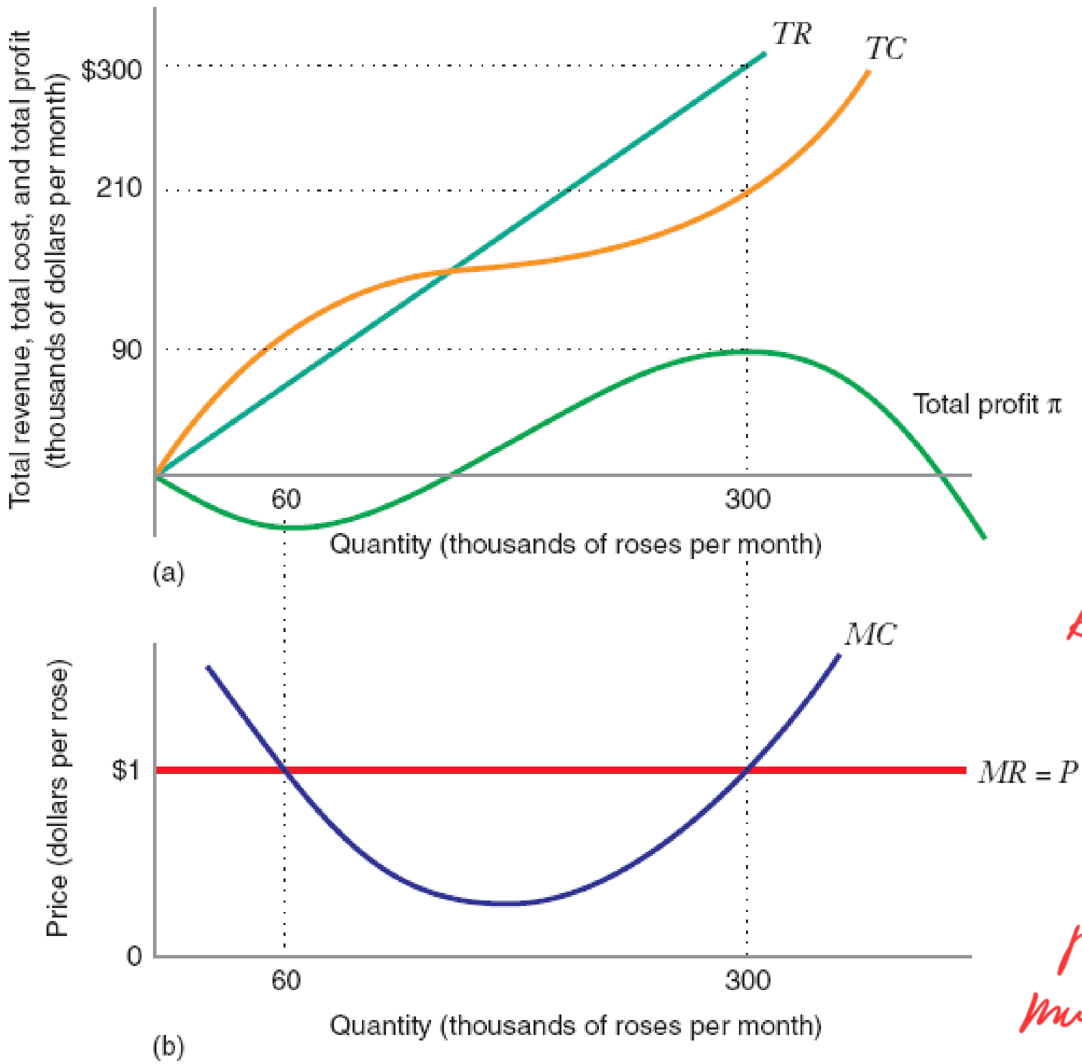
- Since P is taken as given, firm chooses Q to maximize profit.
- Marginal Revenue: The rate which TR change with output.

$$MR = \frac{dTR}{dQ}$$

- Since firm is a **price taker**, increase in TR from 1 unit change in Q is equal to P

$$MR = \frac{d(TR)}{dQ} = \frac{d(P*Q)}{dQ} = P$$

The Profit Maximization Condition



For Cobb-Douglas constant return to scale. MC is constant

At competitive case.

MC = MR = P

⇓

not sure how much Q to produce.

The Profit Maximization Condition

At profit maximizing point:

1. P = MC = MR

2. MC rising

"firm demand" = P *(sells as much as likes at P)*
"firm supply" ?

Short Run Equilibrium

Short run: the period of time in which the firm's plant size is fixed and the number of firms in the industry is fixed.

Assumption: All fixed costs are *sunk*.

Short-run total cost:

$$STC(q) = \begin{cases} FC + VC(q) & q > 0 \\ FC & q = 0 \end{cases}$$

Shut Down Price

The firm will choose to produce a positive output q only if:

$$\pi(q) > \pi(0), \text{ or}$$

$$PQ - VC(q) - FC > -FC \Rightarrow PQ - VC(q) > 0$$

$$\Rightarrow \ PQ > VC(q) \Rightarrow \ P > AVC(q)$$

Definition: The price below which the firm would opt to produce zero is called the **shut down price Ps**. In this case, Ps is the minimum point on the AVC curve.

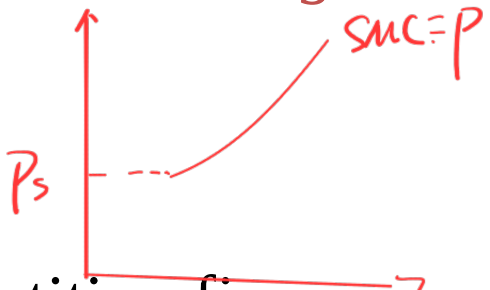
Short Run Supply Function

Therefore, the firm's short run supply function is defined by:

1. P=SMC, where SMC slopes upward as long as P $\geq$ Ps

P=SMC>AVC.

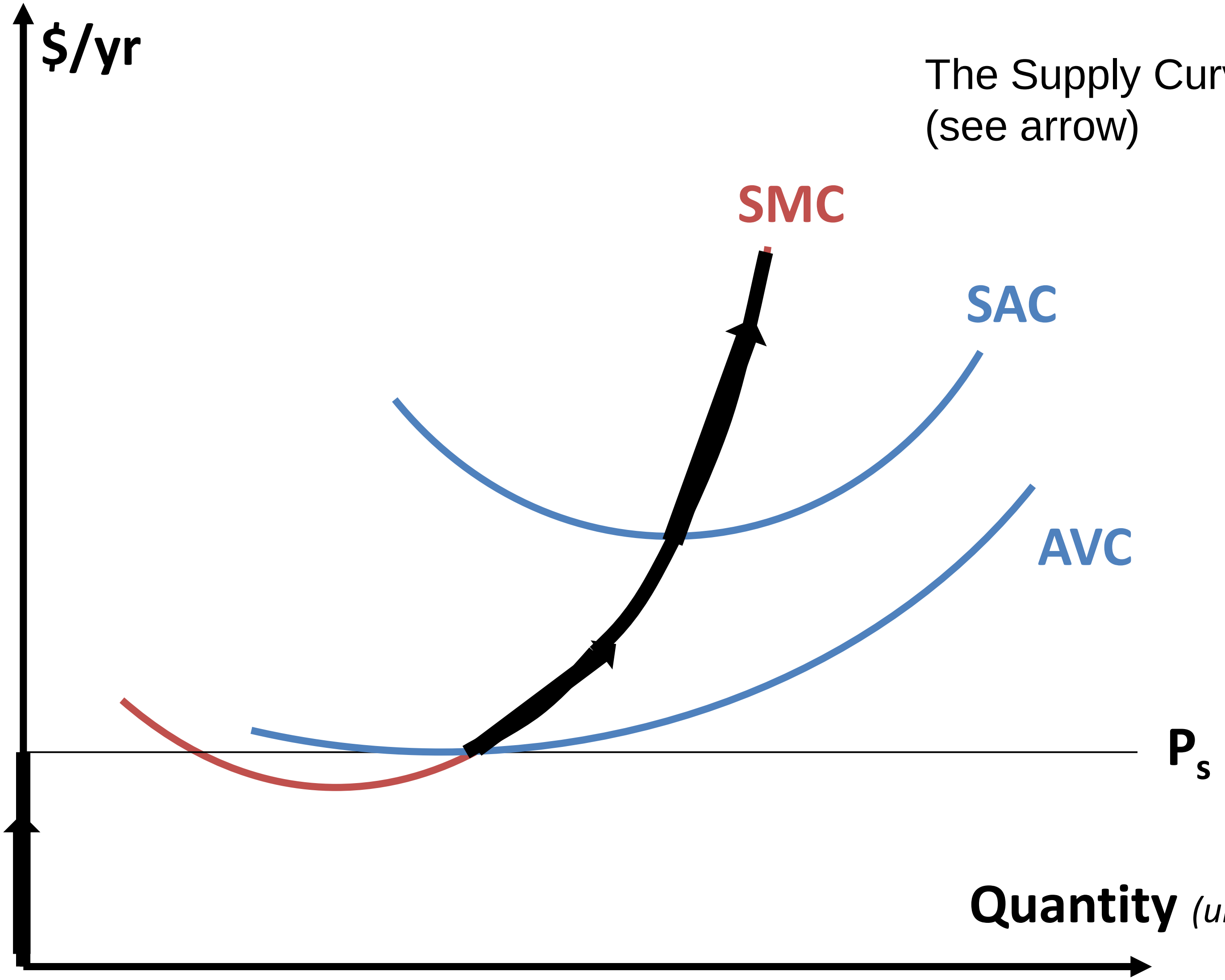
2. 0 where P < Ps



This means that a perfectly competitive firm may choose to operate in the short run even if economic profit is negative.

Short Run Supply Curve

The Supply Curve is the **thick** line (see arrow)



Next Lecture….

Now that “we” have covered the economic behaviour of a “consumer” and a “firm”

1. **Market Supply** and Aggregation

2. Combining with Market Demand – *General Equilibrium Theory*

The Chinese University of Hong Kong
Department of Economics

ECON3011

Intermediate Microeconomic Theory

2025-2026 (First Term)

Lecture 8: Aggregation and General Equilibrium

Wallace K C Mok

Acknowledgement: Parts of this lecture slide set were kindly provided by Prof. Shi

In this Lecture

- Aggregation – How do we add up firms’ supply to form the market “Supply Curve” in the Short and Long Run

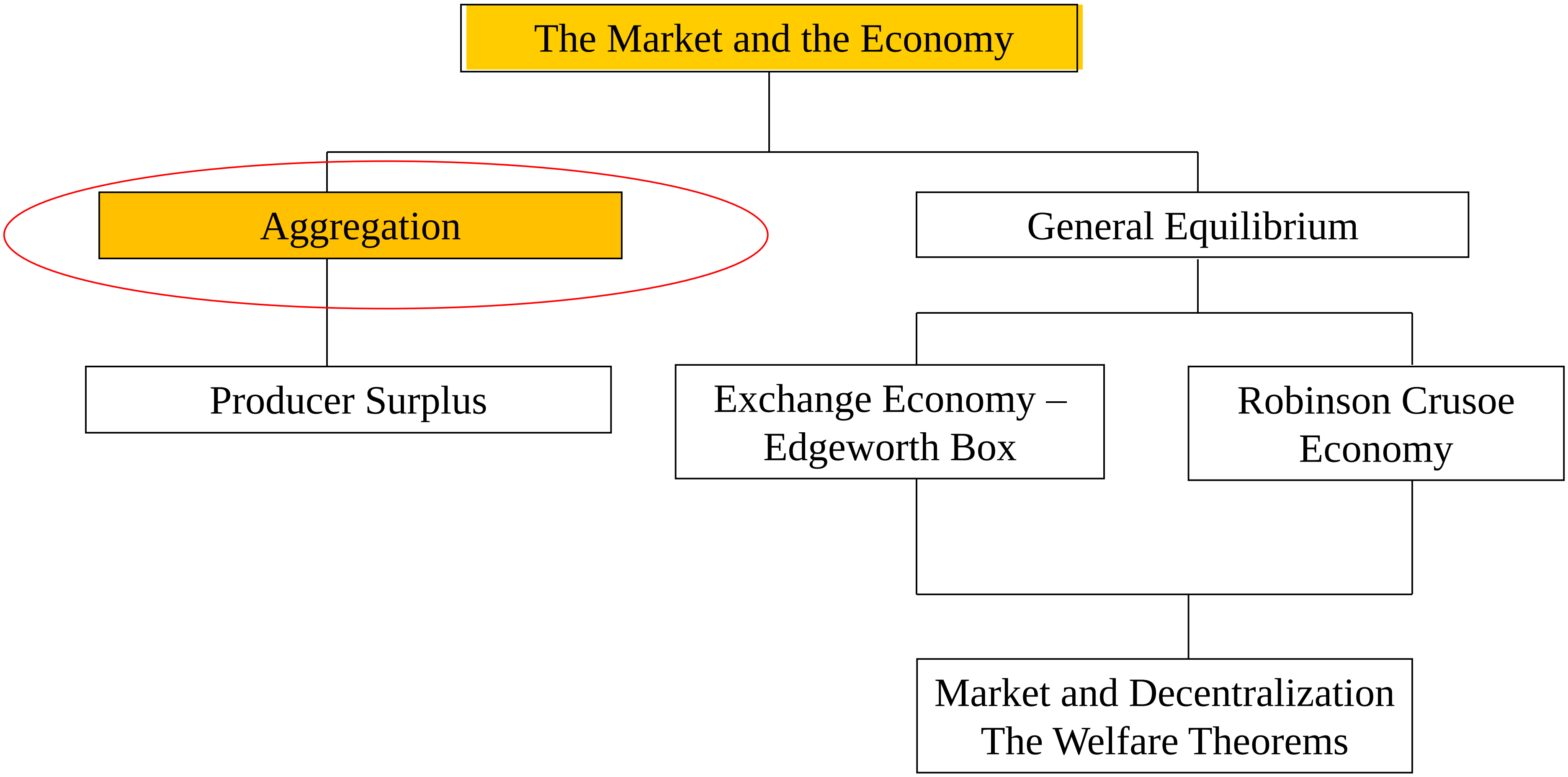
- Does equilibrium in one market implies equilibrium in other markets? Partial vs General Equilibrium

- fixed resources
  Pure Exchange Economy vs Robinson Crusoe one individual both consumer / producer

- Market vs Command Economy

- Welfare Theorems

The Market and the Economy

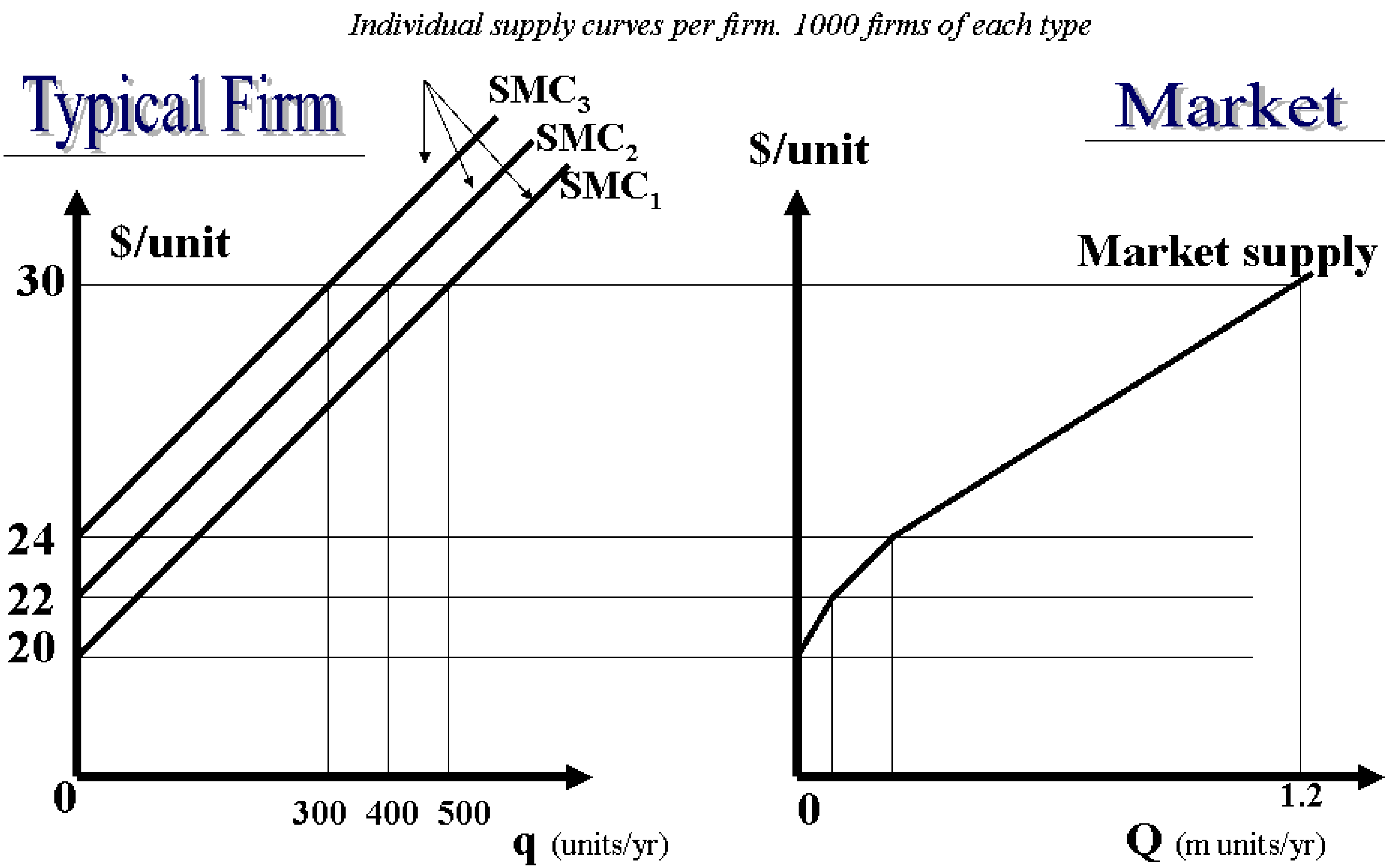


Market Supply and Equilibrium

The **market supply** at any price is the sum of the quantities each firm supplies at that price.

The ***short run market supply curve*** is the horizontal sum of the individual firm supply curves.

Short Run market & Supply Curves



Short Run Perfectly Competitive Equilibrium

A ***short run perfectly competitive equilibrium*** occurs when the market quantity demanded equals the market quantity supplied.

a set of price,
If producer & consumer
is quantity decision
happens to
be same.

$$\sum_{i=1}^{n} Q_s^i(P) = Q_d(P)$$

And $Q_s^i(P)$ is determined by the firm's individual profit maximization condition.

Example: Short run Equilibrium

300 Identical Firms

$$\textbf{Demand: } Q^d(P) = 60 - P$$

$P = 60 - Q$

TC VC

$$\textbf{Short-run total cost: } STC(q) = 0.1 + 150q^2$$

$\frac{dSTC}{dQ} \Rightarrow$

$$\textbf{Short-run marginal cost: } SMC(q) = 300q = P$$

$$\textbf{Non-sunk fixed cost: } NSFC = 0$$

$\frac{TVC}{Q} = 150q$

$$\textbf{Average variable cost: } AVC(q) = 150q$$

Minimum AVC = 0 so as long as price is positive, firm will produce

Deriving a Short Run Market Equilibrium

Short-run equilibrium

Profit maximization condition: $P = SMC(q) = 300q$

300 firms

$$\Rightarrow \; q^s(P) = \frac{P}{300} \text{ and } Q^s(P) = \underbrace{300 \cdot \frac{P}{300} = P}$$

$$\underbrace{Q^s(P) = Q^d(P)} \Rightarrow P = 60 - P \Rightarrow P = 30$$

$$q^* = \frac{30}{300} = \underbrace{0.1}_{\text{each firm}}, \quad Q^* = 30$$

Deriving a Short Run Market Equilibrium

Do firms make positive profits at the market equilibrium?

$$SAC = \frac{STC}{q} = \frac{0.1}{q} + 150q$$

When each firm produces $q = 0.1$, SAC per firm is:

$$\frac{0.1}{0.1} + 150(0.1) = 1 + 15 = 16$$

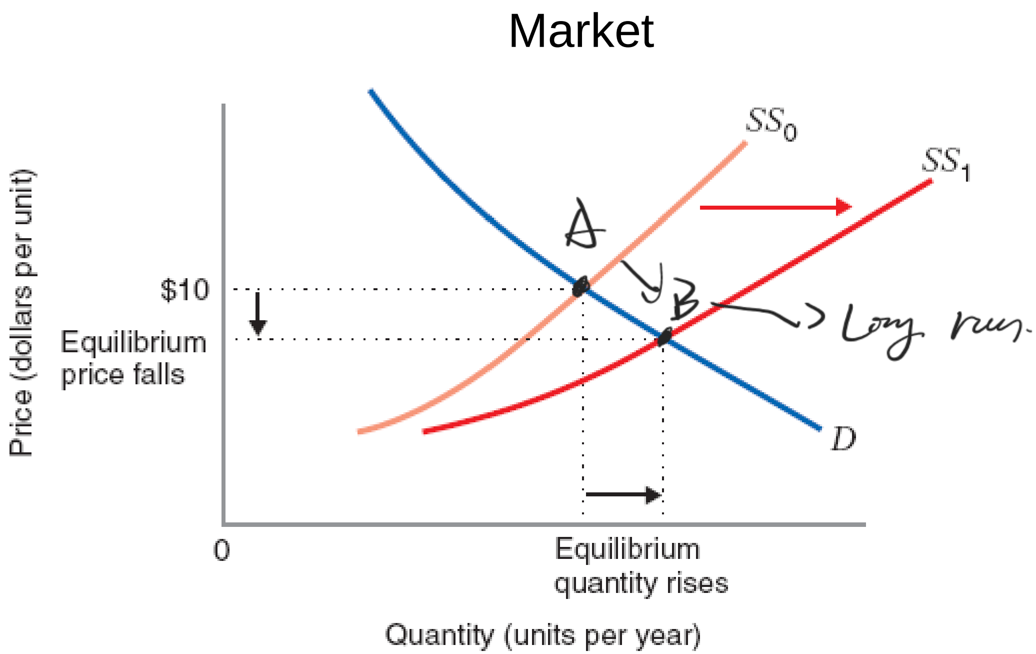
therefore $P^* > SAC$ so profits are positive

Long Run Market Equilibrium

Long run: the period of time in which all the firm's inputs can be adjusted. The number of firms in the industry can change as well.

The firm should use long run cost functions for evaluating the cost of outputs it might produce in this longer term period…i.e., decisions to modify plant size, enter or exit, change production process and so on would all be based on long term analysis

Comparative Statics from Short run to Long run

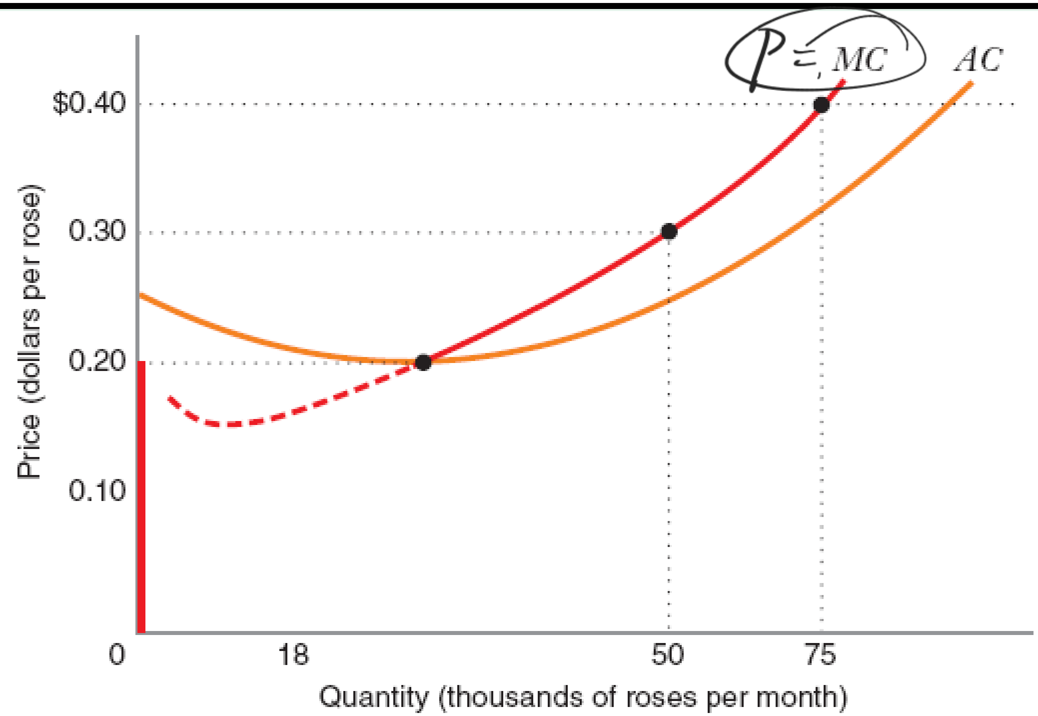


Supply shifts when number of firms increase

Firm's Long Run Supply Curve

The firm's long run supply curve:

$$q^s(P) = \begin{cases} MC(q) & P \geq \min(AC) \\ 0 & P < \min(AC) -> (exits) \end{cases}$$



Long Run Market Equilibrium

A long run perfectly competitive equilibrium occurs at a market price, P*, a number of firms, n*, and an output per firm, q* that satisfies:

Long run profit maximization with respect to output and plant size:

$$P^* = MC(q^*)$$

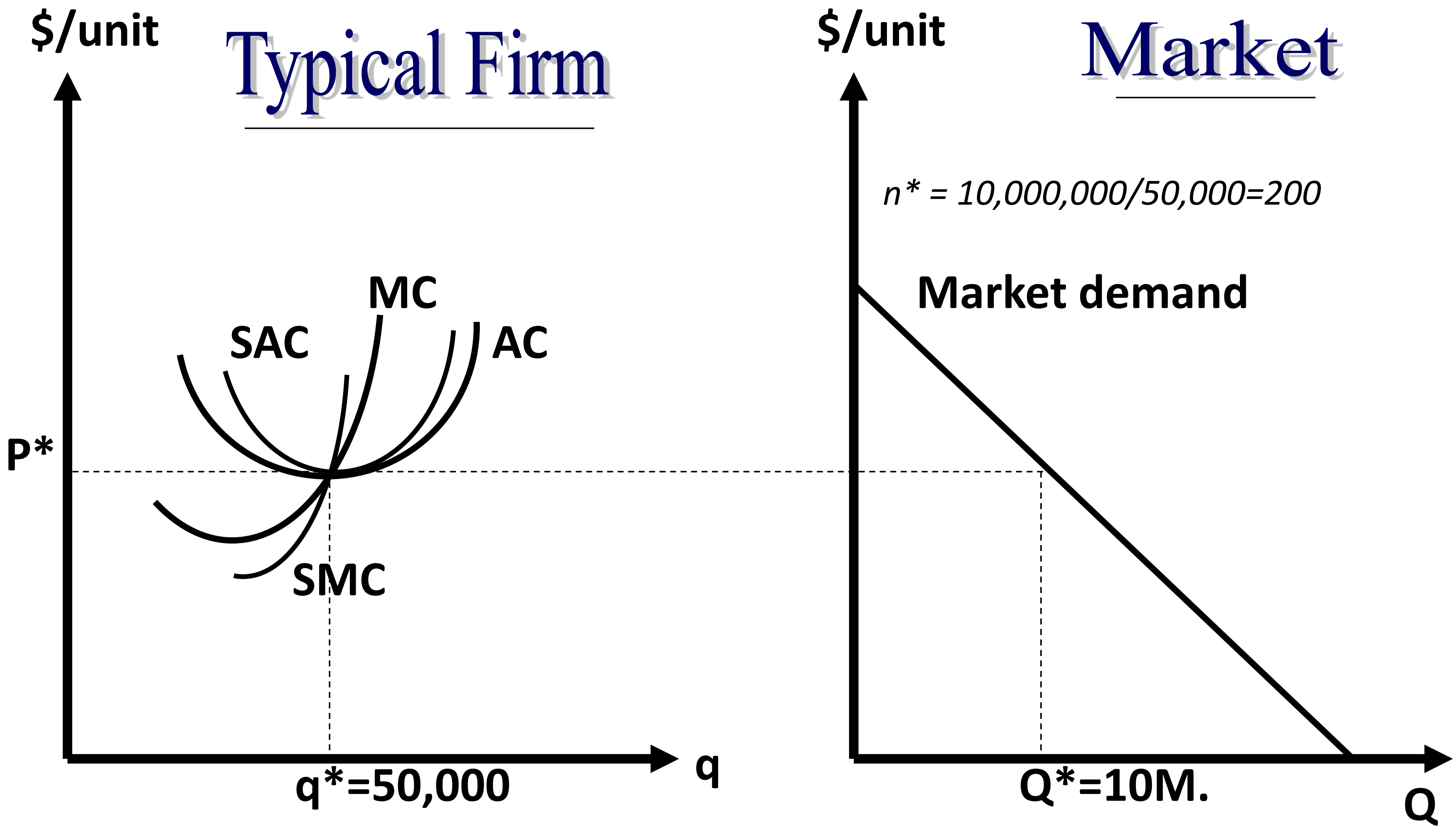
Zero economic profit

$$P^* = AC(q^*)$$

Demand equals supply

$$Q^d(P^*) = n^* q^*$$

Long Run Perfectly Competitive



Calculating Long Run Equilibrium

Costs :

$$TC(q) = 40q - q^2 + 0.01q^3$$

$$AC(q) = 40 - q + 0.01q^2$$

$$MC(q) = 40 - 2q + 0.03q^2$$

Demand :

$$Q^d(P) = 25000 - 1000P$$

$P = MC = AC$ In long run competitive.

The long-run equilibrium satisfies the following:

a. $P^* = 40 - 2q^* - 0.03(q^*)^2$

b. $P^* = 40 - q^* + 0.01(q^*)^2$

c. $25000 - 1000P^* = n^*q^*$

Calculating Long Run Equilibrium

Using (a) and (b), we have:

$$40 - 2q^* + 0.03(q^*)^2 = 40 - q^* + 0.01(q^*)^2$$

$$\Rightarrow \quad q^* = 50, \quad P^* = 15, \quad Q^d(P^*) = 10000$$

Using (c), we have: $n^* = \dfrac{10000}{50} = 200$

Calculating Long Run Equilibrium

"If anyone can do it, you can't make money at it" Or if the firm's strategy is based on skills that can be easily imitated or resources that can be easily acquired, in the long run your economic profit will be competed away.

The Long Run Market Supply Curve tells us the total quantity of output that will be supplied at various market prices, assuming that all long run adjustments (plant, entry) take place.

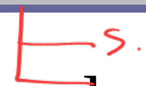
Long Run Market Supply Curve

Since new entry can occur in the long run, we *cannot* obtain the long run market supply curve by summing the long run supplies of current market participants

Instead, we must construct the long run market supply curve.

If $P > \min(AC)$, entry would occur, driving price back to $\min(AC)$
If $P < \min(AC)$, firms would earn negative profits and would supply nothing

Long Run Market Supply Curve



If higher overall production does not change input costs, i.e. we are in a **constant-cost industry**, then the Long Run Market Supply Curve is **horizontal** at P=MC=AC

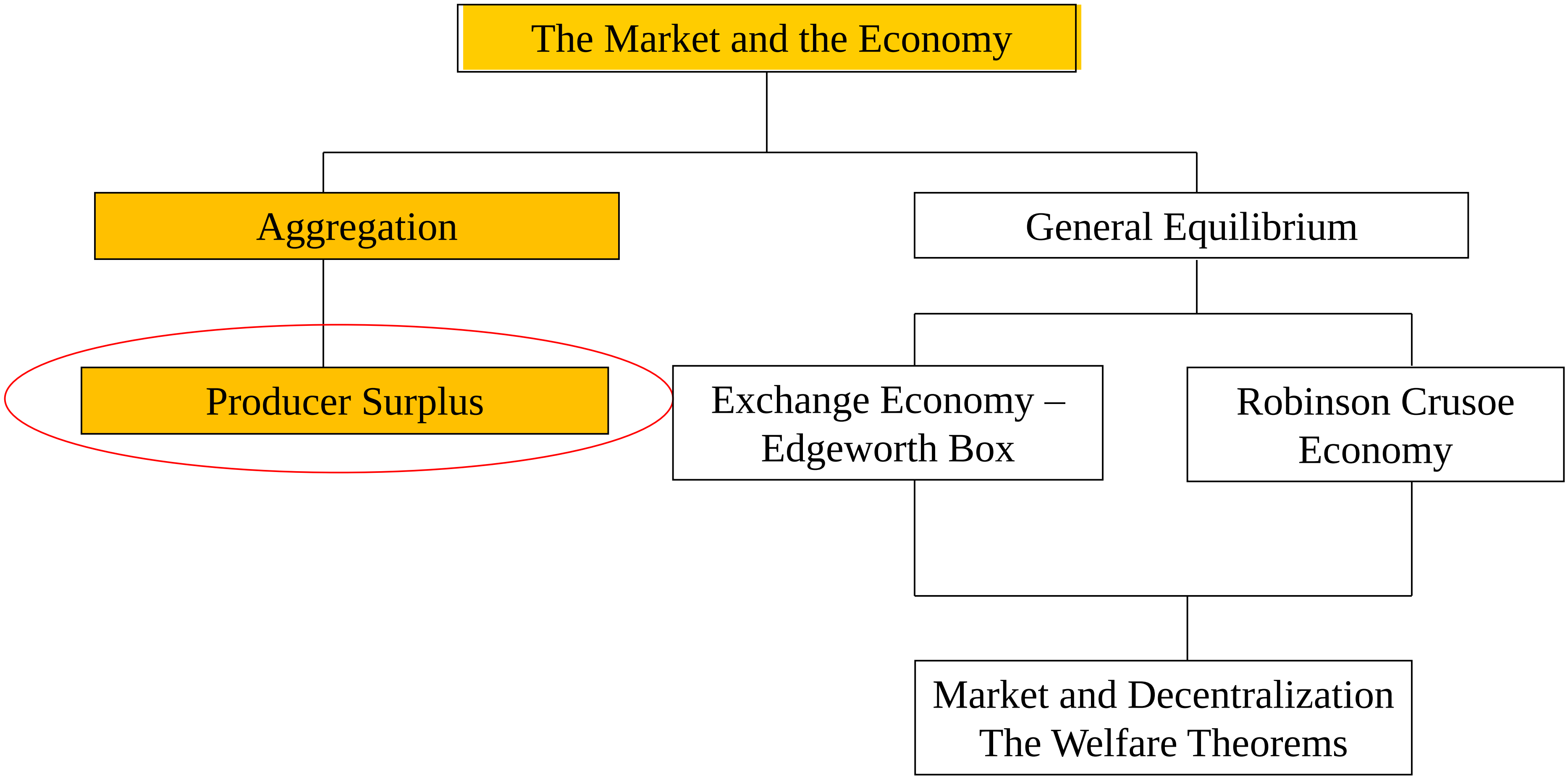
If more firm entry ↑ => L, k demand ↑ => w, r ↑ => P=MC=AC ↑

If higher overall production increases input costs, i.e. we are in a **increasing-cost industry**, then the Long Run Market Supply Curve is **upward sloping**. This is likely be the case due to scarce resources

If higher overall production reduces input costs, i.e. we are in a **decreasing-cost industry**, then the Long Run Market Supply Curve is **downward sloping**

hard to occur..

The Market and the Economy



Producer Surplus

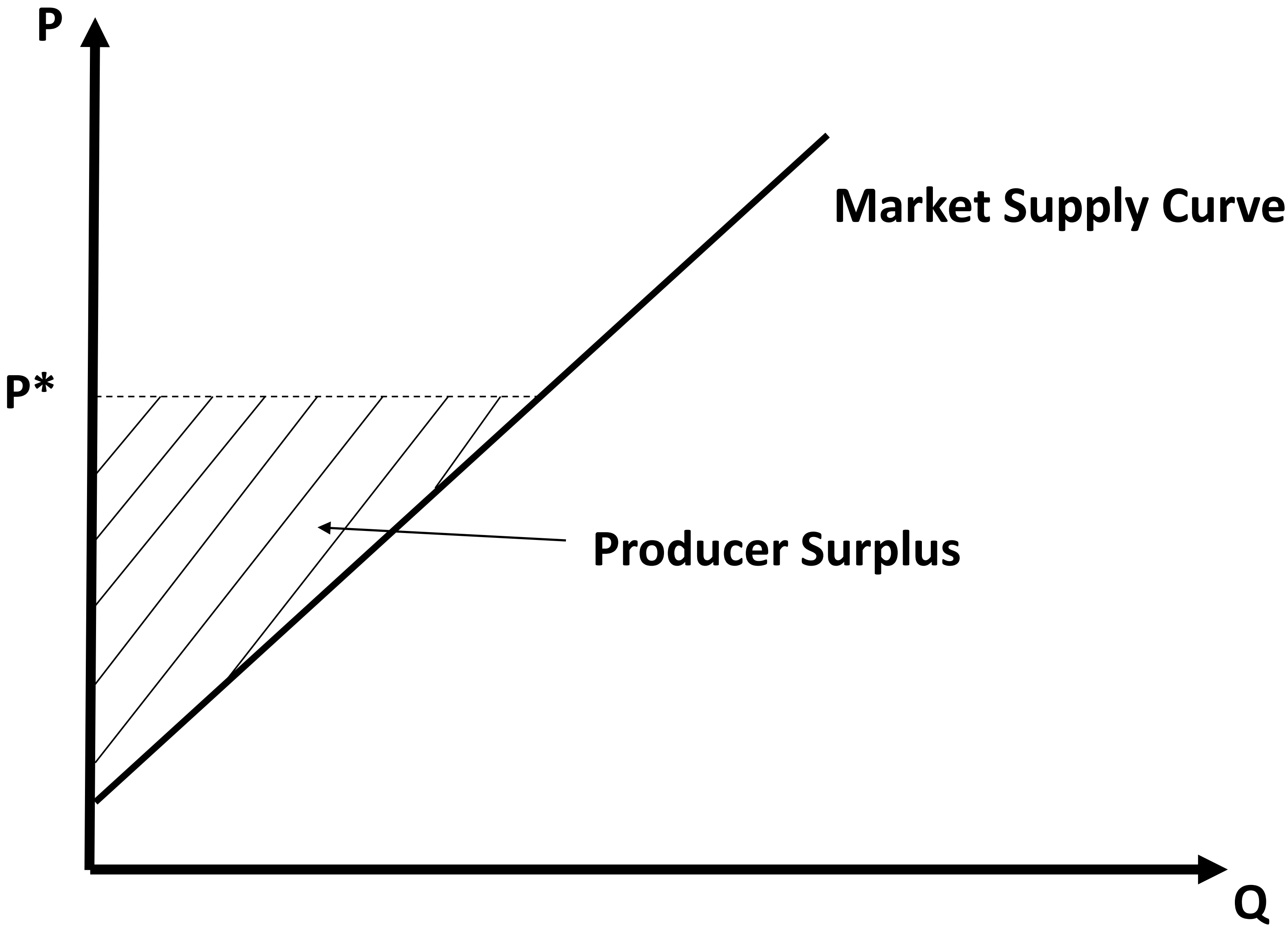
Definition: ***Producer Surplus*** is the area above the market supply curve and below the market price. It is a monetary measure of the benefit that producers derive from producing a good at a particular price.

Producer Surplus

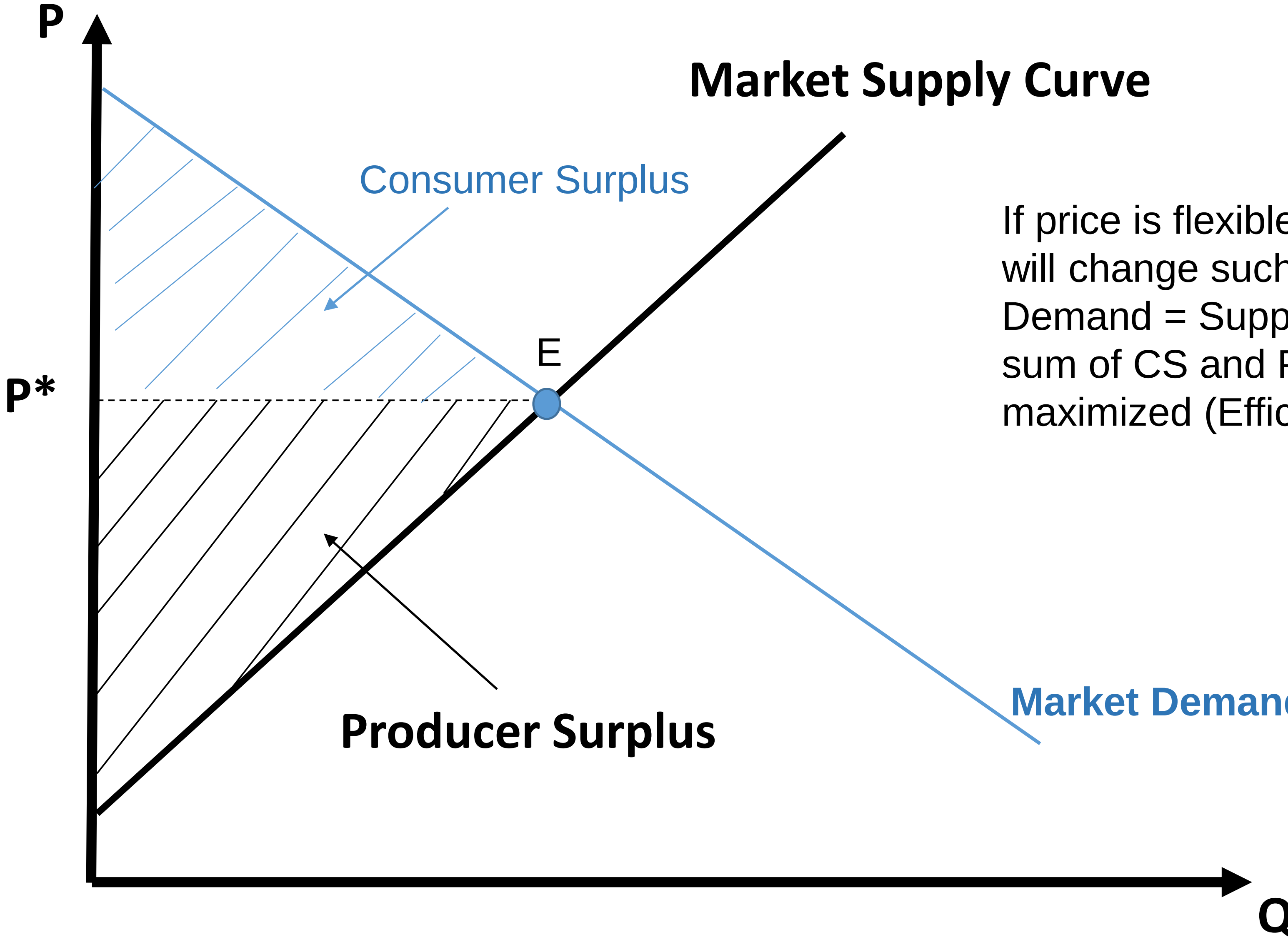
In Short run, since the market supply curve is simply the sum of the individual supply curves…which equal the marginal cost curves the difference between price and the market supply curve measures the surplus of all producers in the market.

Producer's surplus does not deduct fixed costs, so it does not equal profit.

Producer Surplus

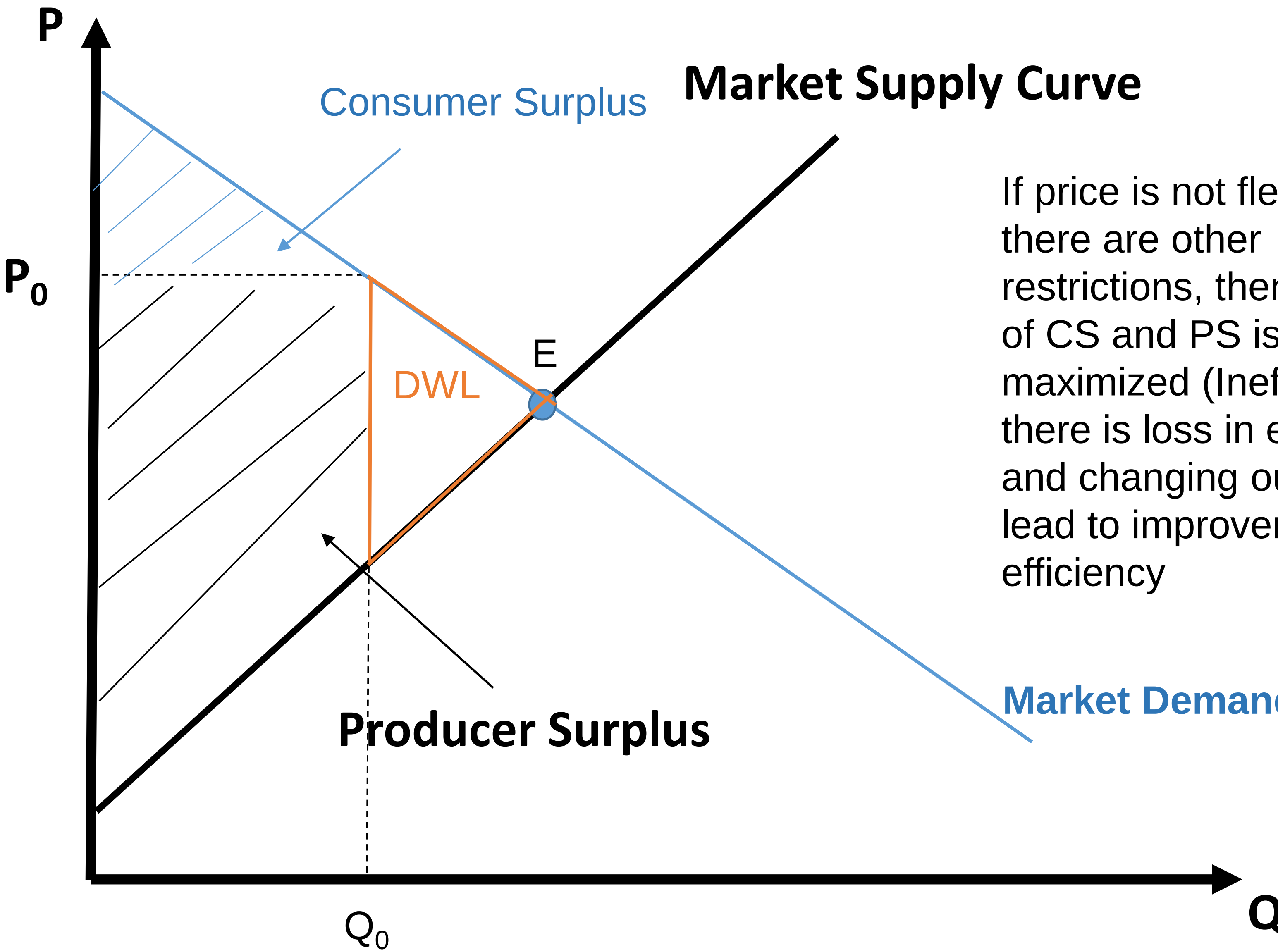


Market Efficiency



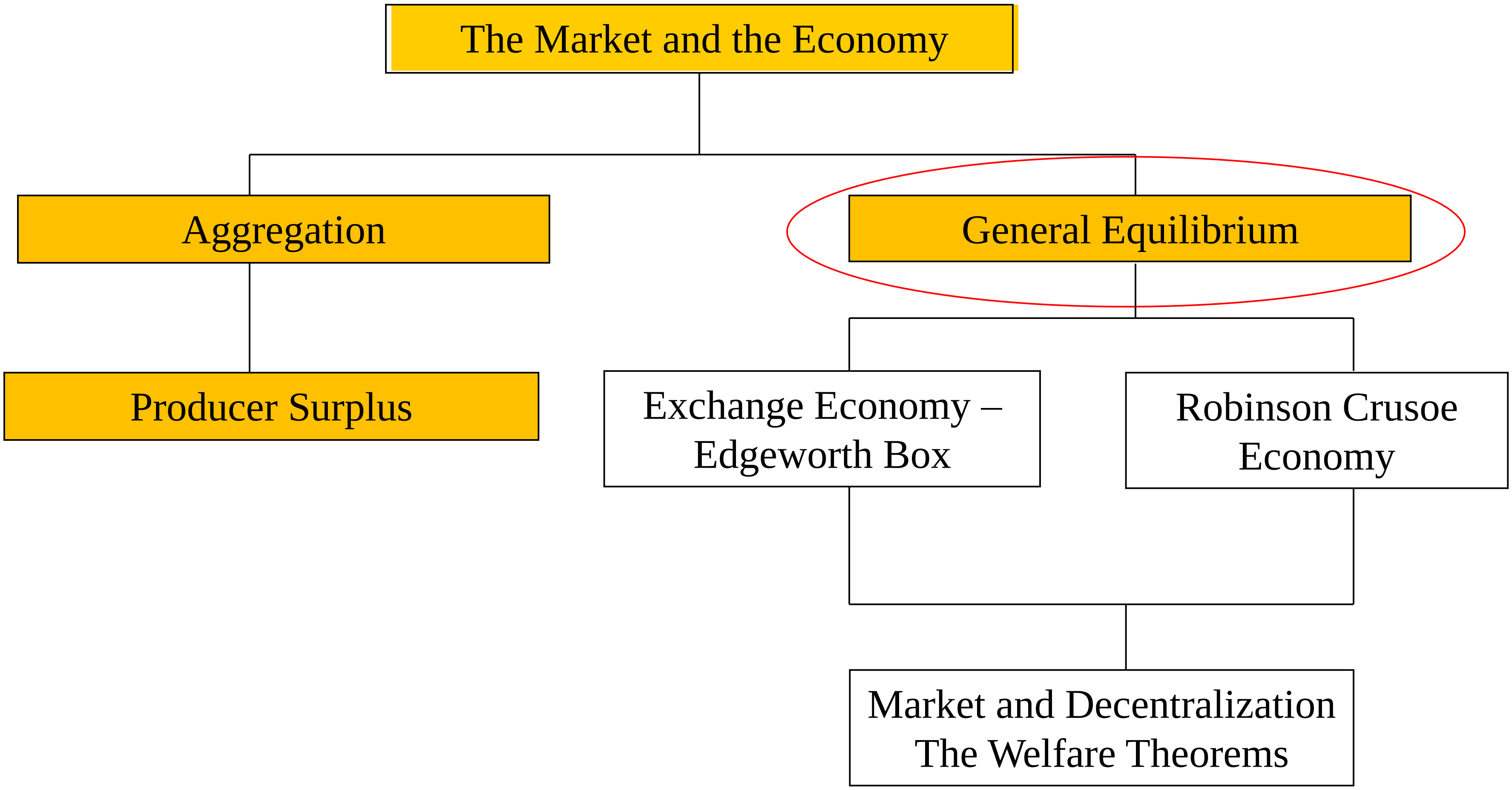
If price is flexible, then it will change such that Demand = Supply, the sum of CS and PS is maximized (Efficiency)

Market Failure



If price is not flexible or there are other restrictions, then the sum of CS and PS is not maximized (Inefficiency), there is loss in efficiency and changing output can lead to improvement in efficiency

The Market and the Economy



Partial vs. General Equilibrium

Partial Equilibrium analysis is the study of how equilibrium is determined in only a single market (e.g. a single product market).

General Equilibrium analysis is the study of how equilibrium is determined in all markets simultaneously (e.g. product markets and labor markets).

Economic Efficiency

In the **partial equilibrium** framework, perfect competitive markets yield equilibrium prices and quantities that are *efficient*:
- Maximizing social surplus.

In the **general equilibrium** framework, perfect competitive markets yield the *efficient* outcome.
-*What is the criterion?*

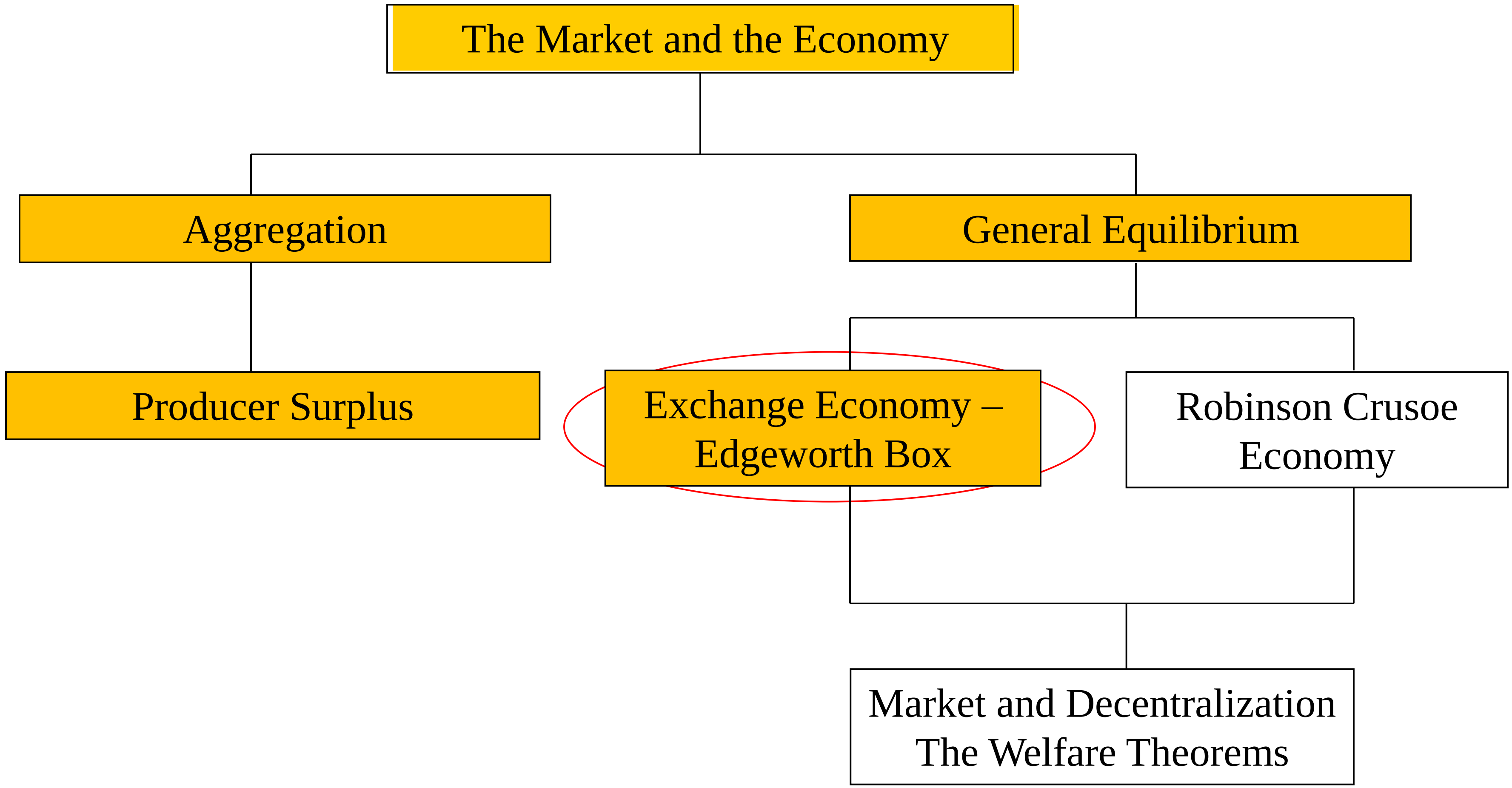
Economic Efficiency

Definition: An economic situation is ***Economically Efficient*** or ***Pareto Efficient*** if there is no other feasible allocation of goods and inputs that would make any person better off without hurting somebody else.

Demand = Supply.

Definition: An economic situation is ***Economically Inefficient*** or ***Pareto Inefficient*** if there is an alternative feasible allocation of goods and inputs that would make all consumers better off than the initial allocation does.

The Market and the Economy



Edgeworth Box Diagram

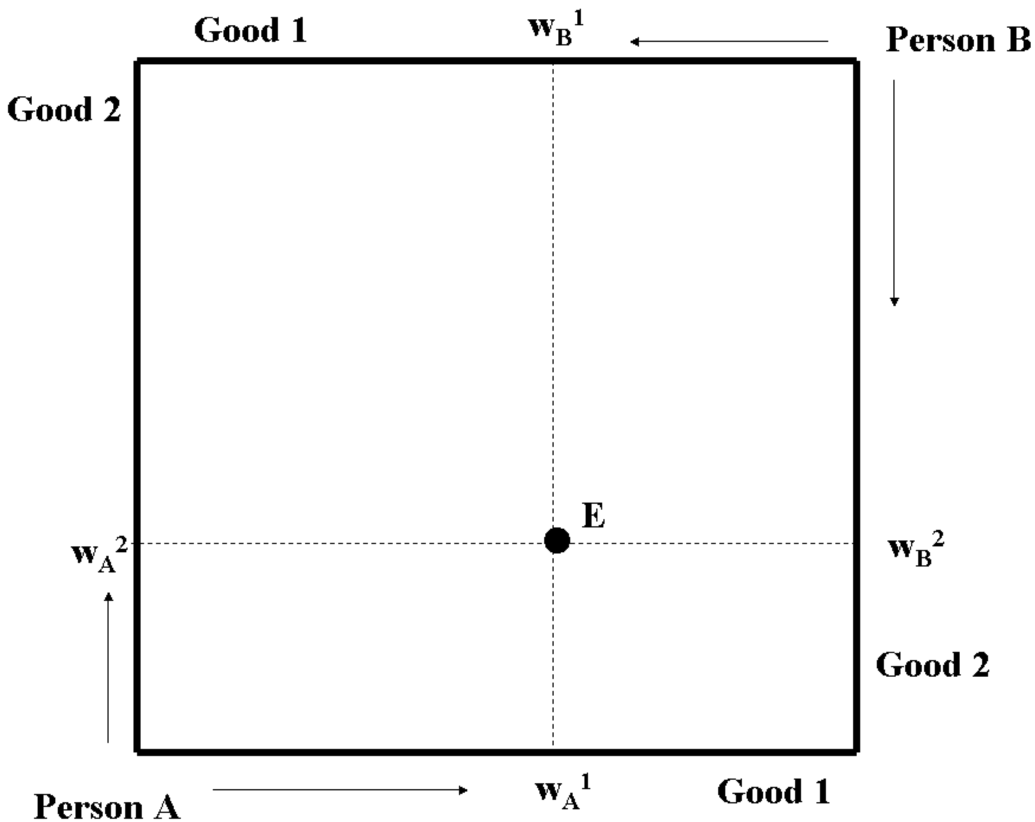
Edgeworth Box: A graph showing all the possible allocations of goods in a two-good economy, given the total available supply of each good.

Exchange economy.

A(G1, G2)

B(G1, G2)

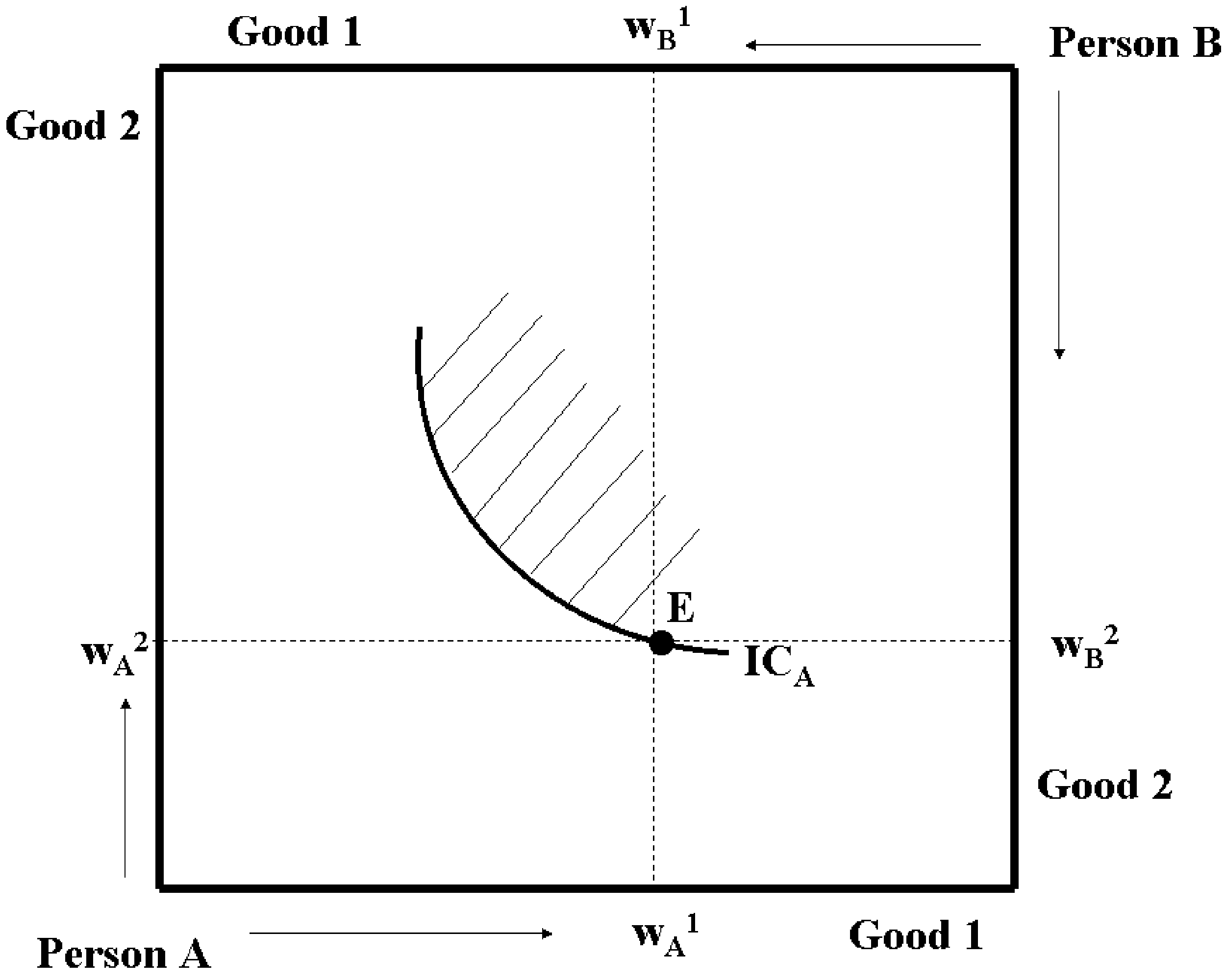
given.



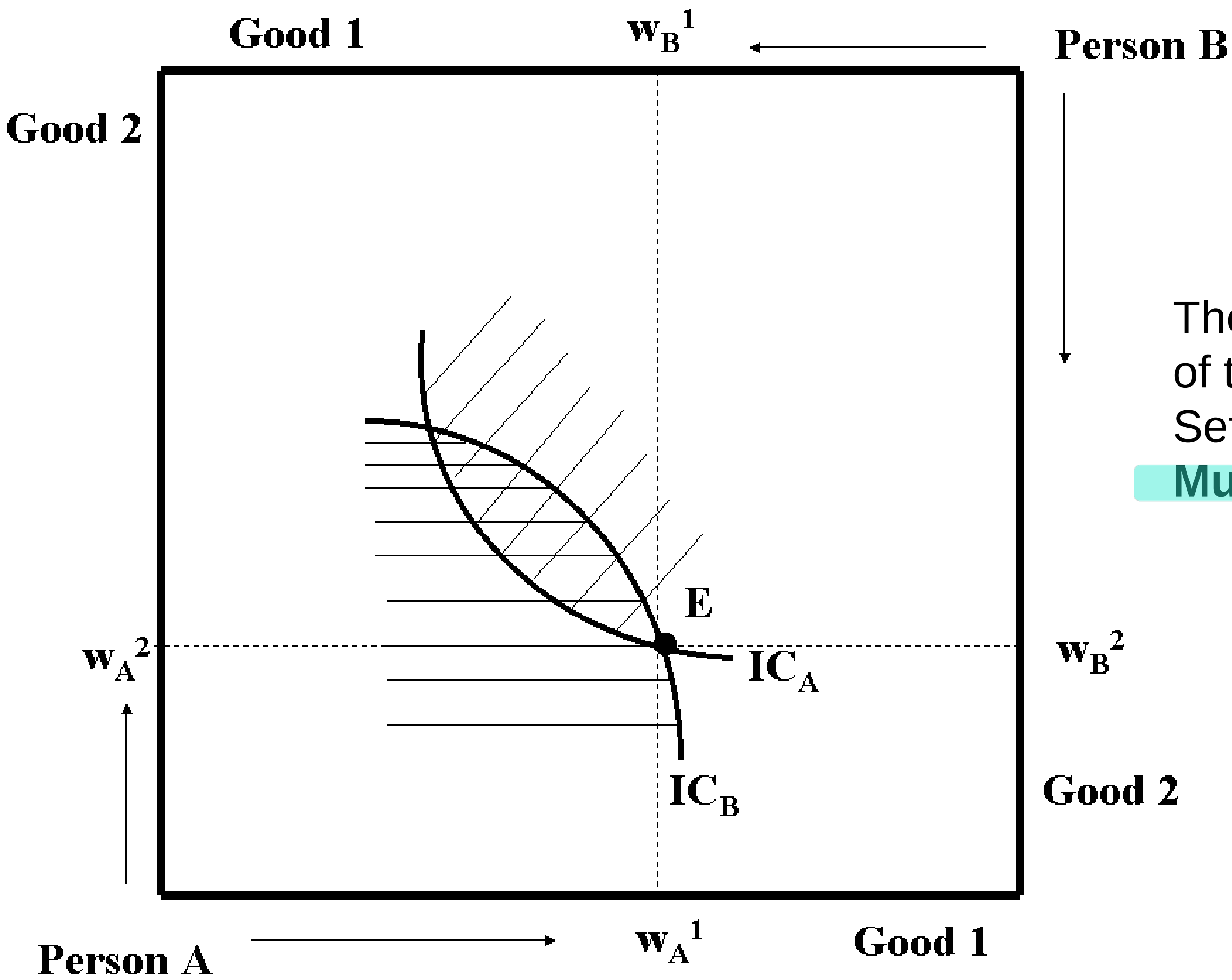
E=Endowment

Good 1 20
Good 2 30

Upper Contour Set of Person A



Upper Contour Sets of A and B



The overlapping area of the Upper contour Sets is the **Region of Mutual Advantage**

Edgeworth Box Diagram

1. The length of the side of the box measures the total amount of the good available.

2. Person A's consumption choices are measured from the lower left hand corner, Person B's consumption choices are measured from the upper right hand corner.

3. We can represent an initial endowment, (w_A^1, w_A^2), (w_B^1, w_B^2) as a point in the box. This is the allocation that consumers have before any exchange occurs.

Edgeworth Box Diagram

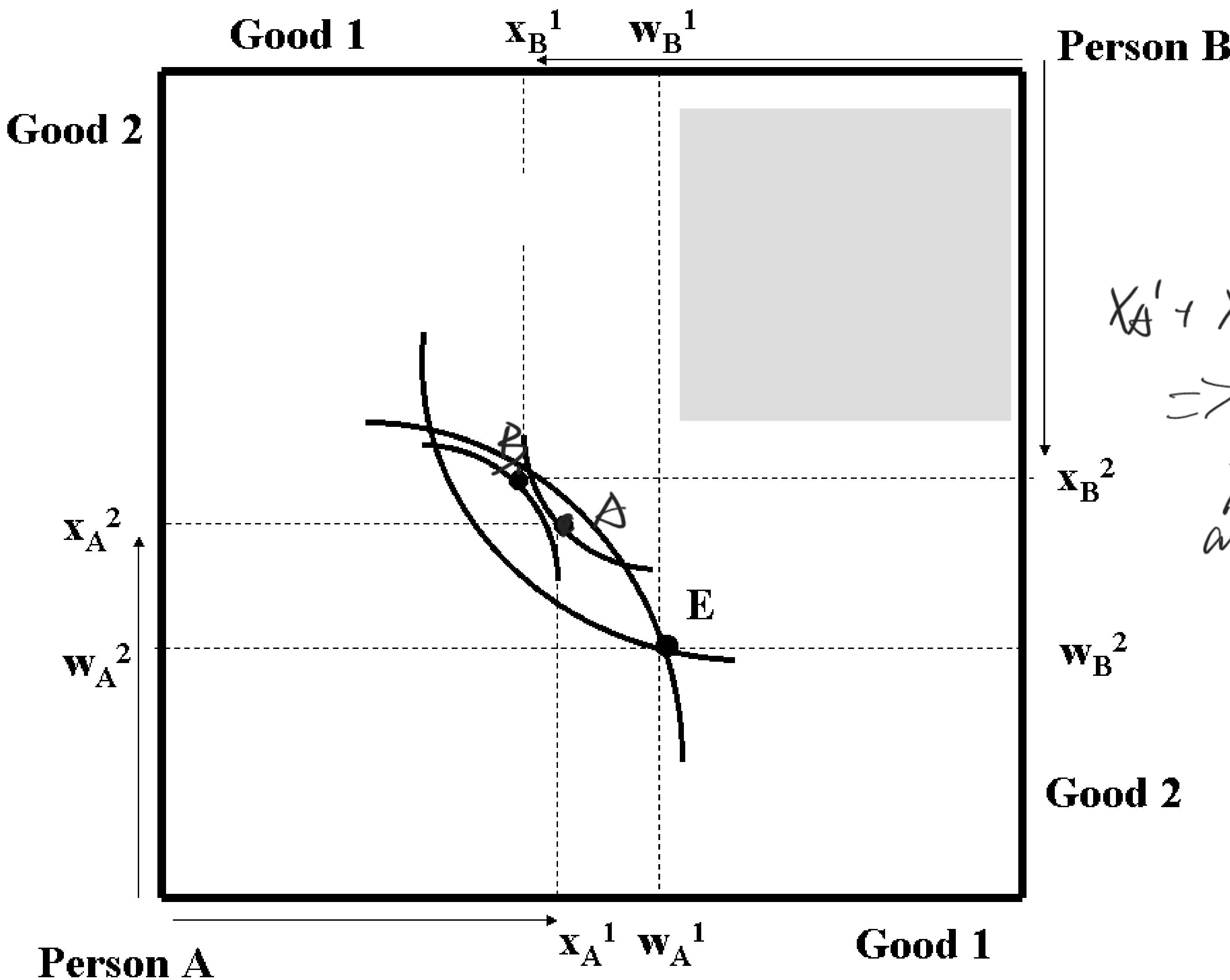
4. Any other feasible consumption allocation is a point in the box such that, for each individual:

"final demand" $\leq$ *"initial supply"*

- $x_A^1 + x_B^1 \leq w_A^1 + w_B^1$
- $x_A^2 + x_B^2 \leq w_A^2 + w_B^2$

5. We can represent indifference curves of the individuals between the goods in the standard way measured from the appropriate corners.

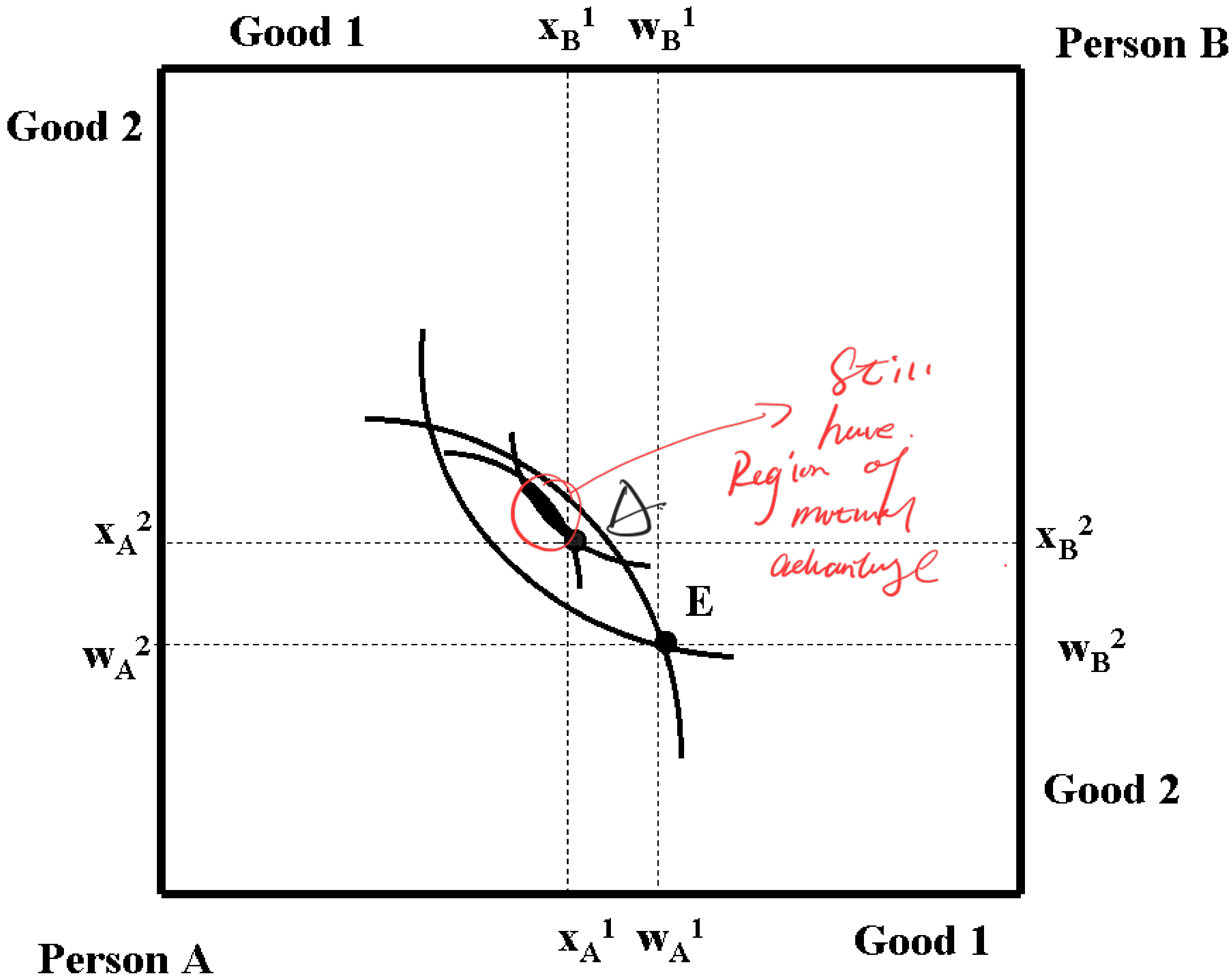
Edgeworth Box – Infeasible Allocation



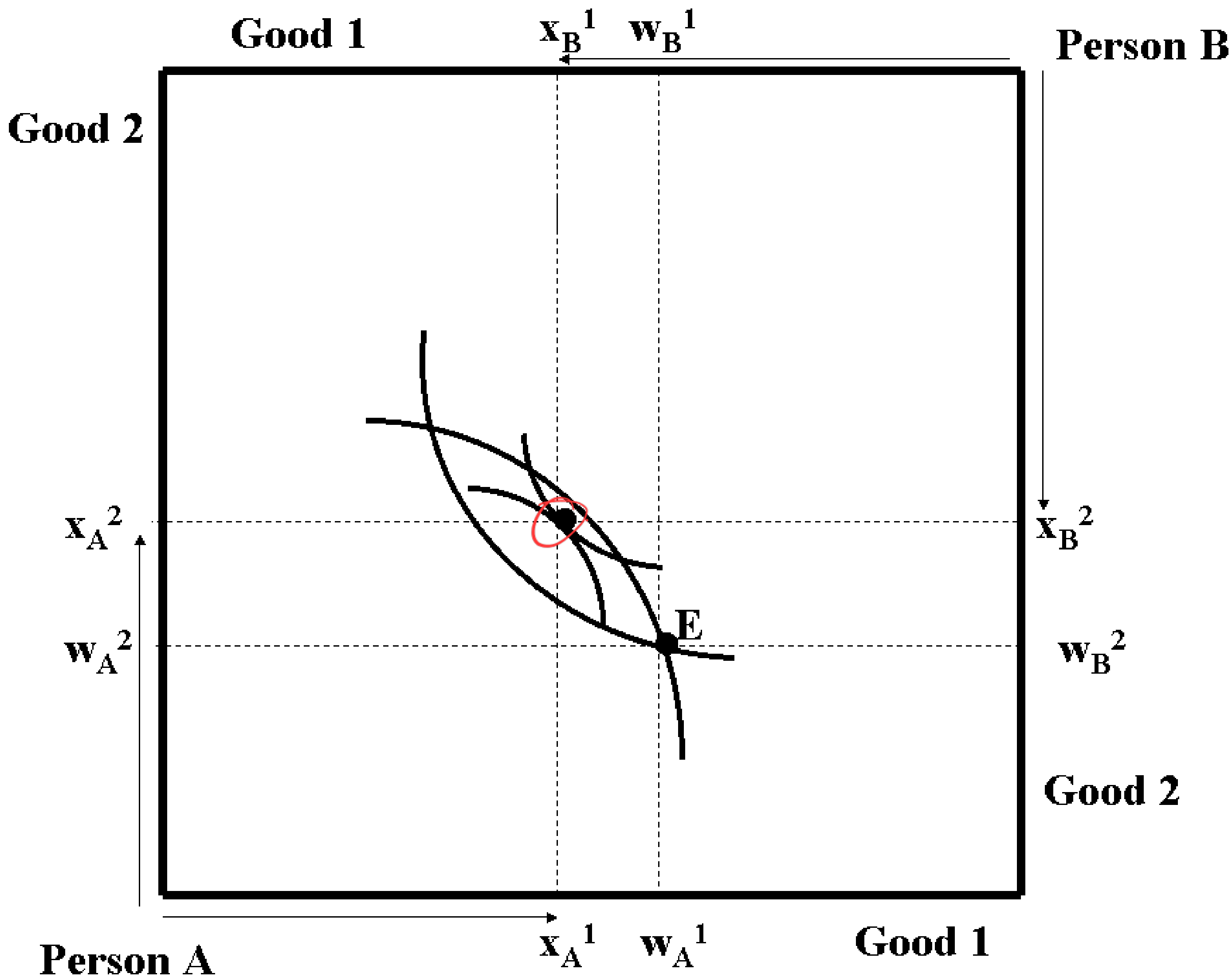
$X_A^1 + X_B^1 > X^1$

=> not feasible allocation.

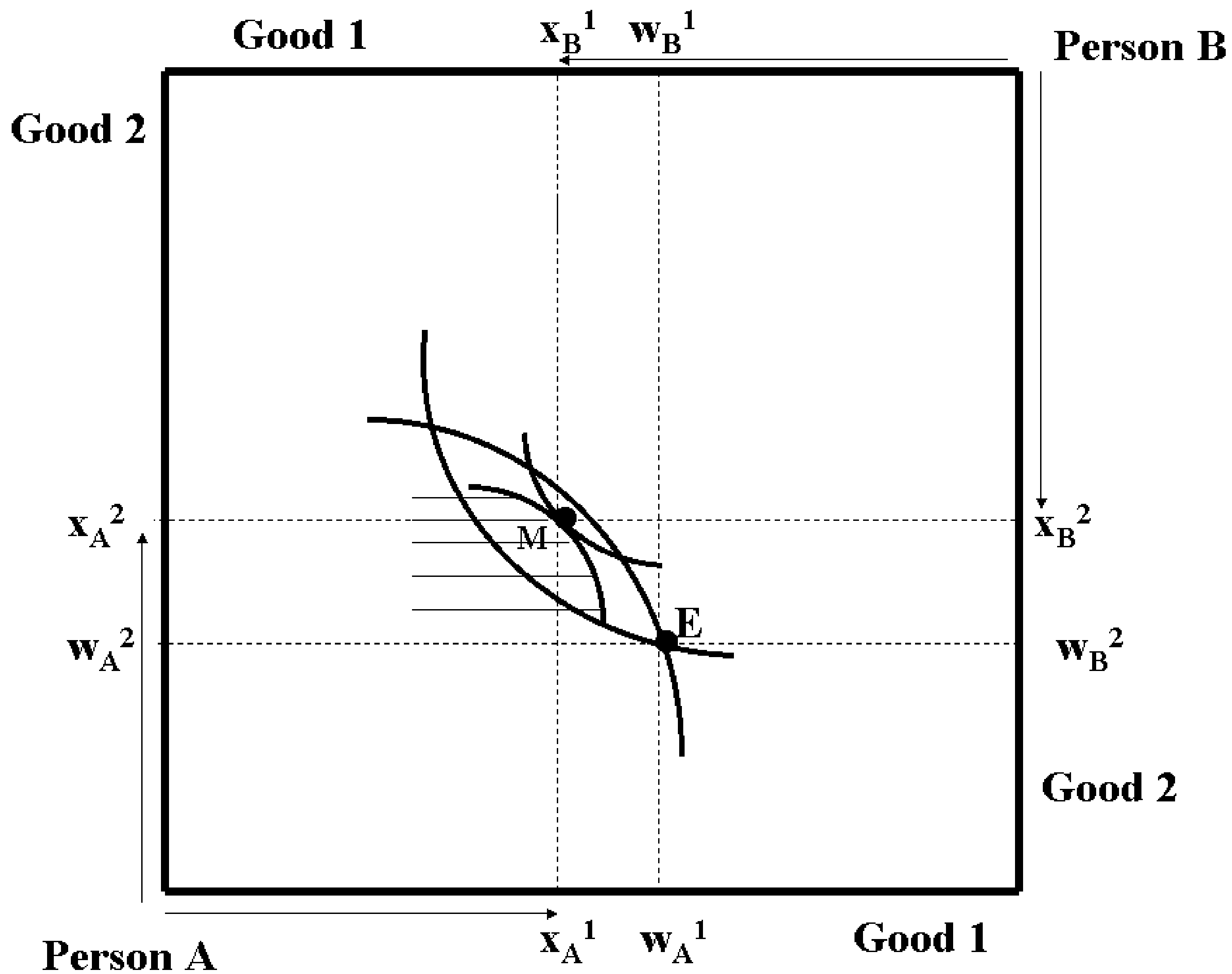
Edgeworth Box –Allocation Can be Improved



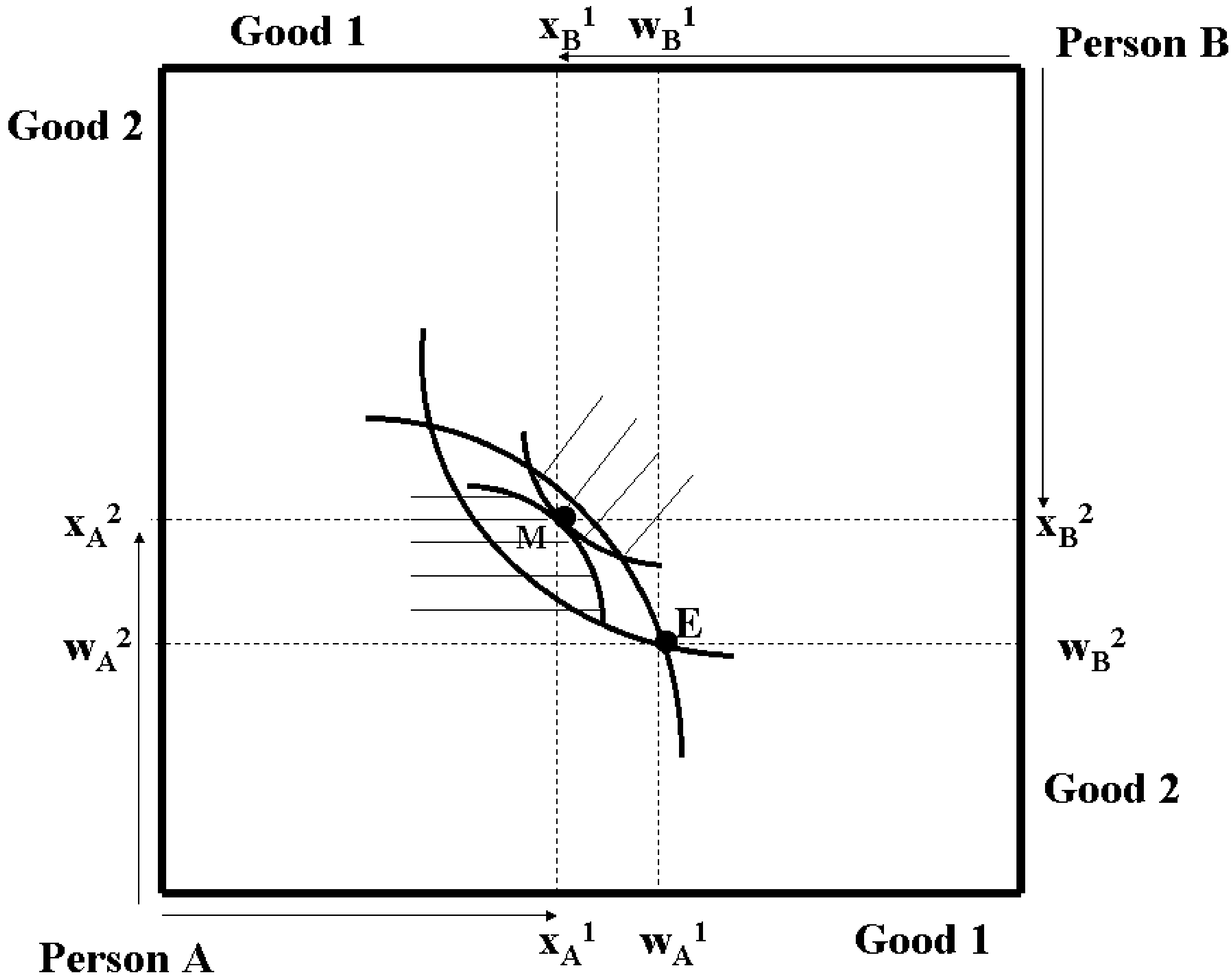
Edgeworth Box – Economically Efficient Allocation



Edgeworth Box – Economically Efficient Allocation



Edgeworth Box – Economically Efficient Allocation



Pareto Set / Contract Curves

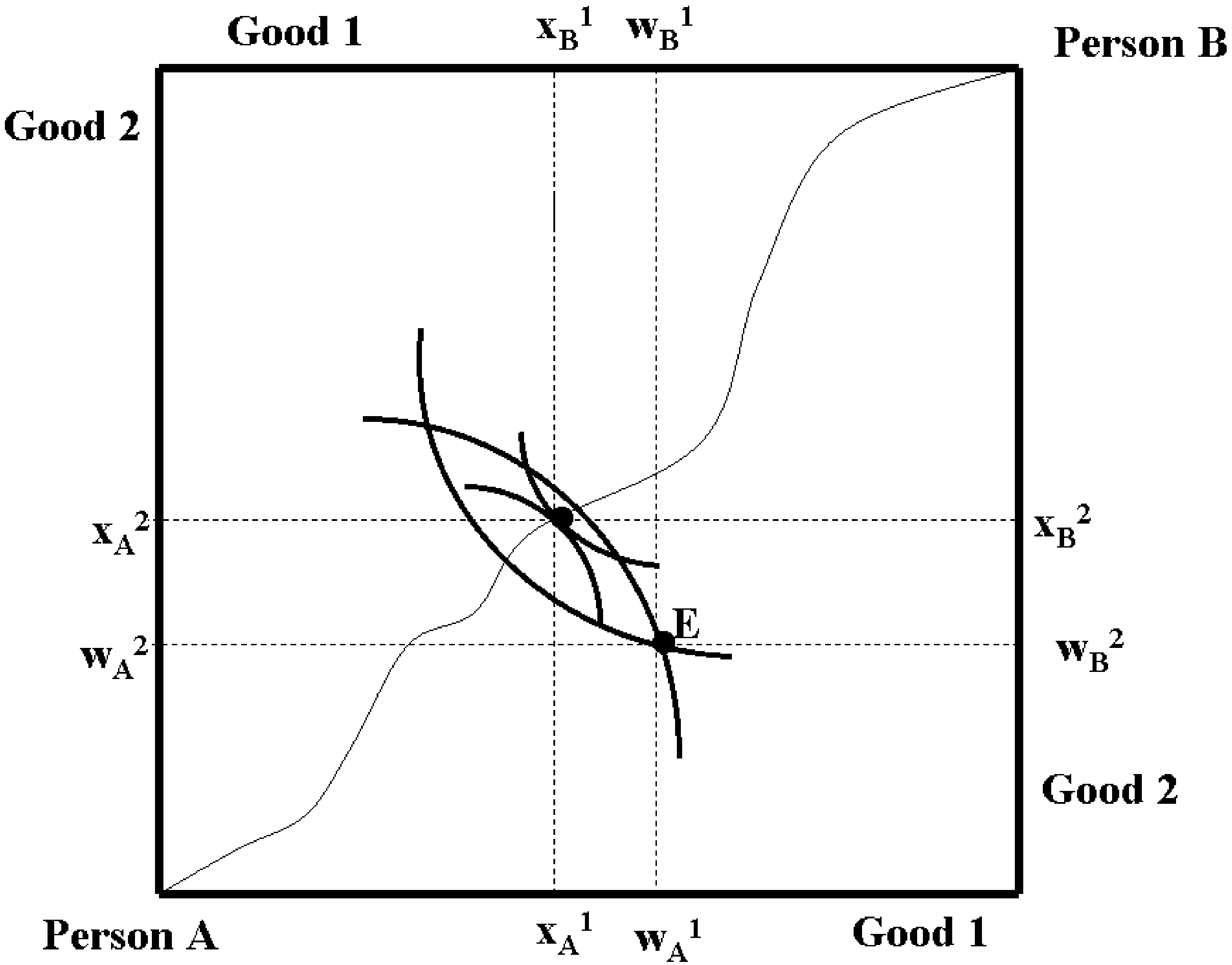
1. M is **Pareto efficient**

2. M is at a tangency point of the two individuals' indifference curves

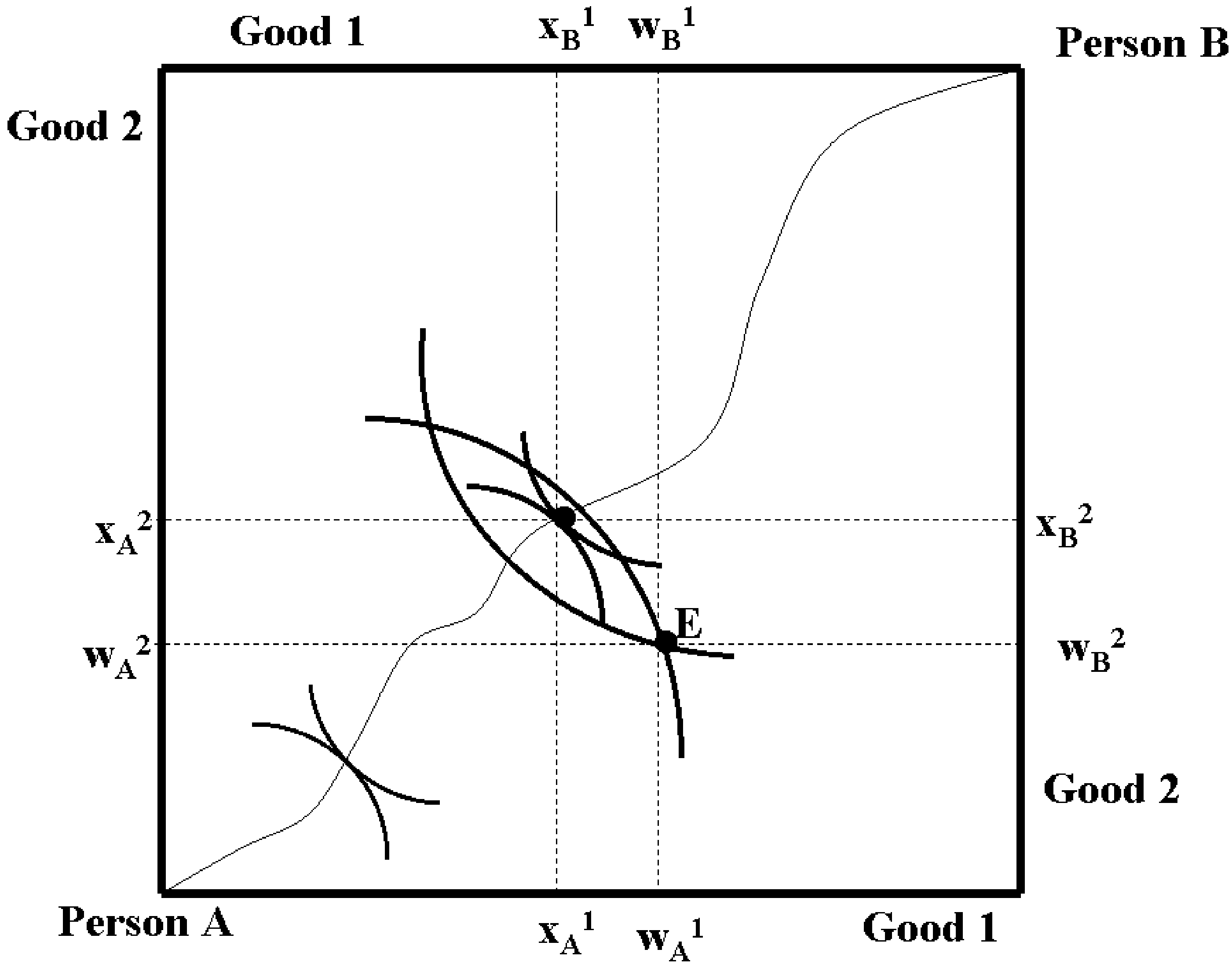
$$MRS^{A}_{1,2} = MRS^{B}_{1,2}$$

3. *Definition:* The set of all Pareto efficient points in the Edgeworth box is known as the ***Pareto set*** or the ***Contract Curve***.

The Contract Curve



The Contract Curve



The Role of Prices

Suppose that agents are **presented with prices**, (p_1,p_2) that they take as **given** and can use to value their initial endowment of goods

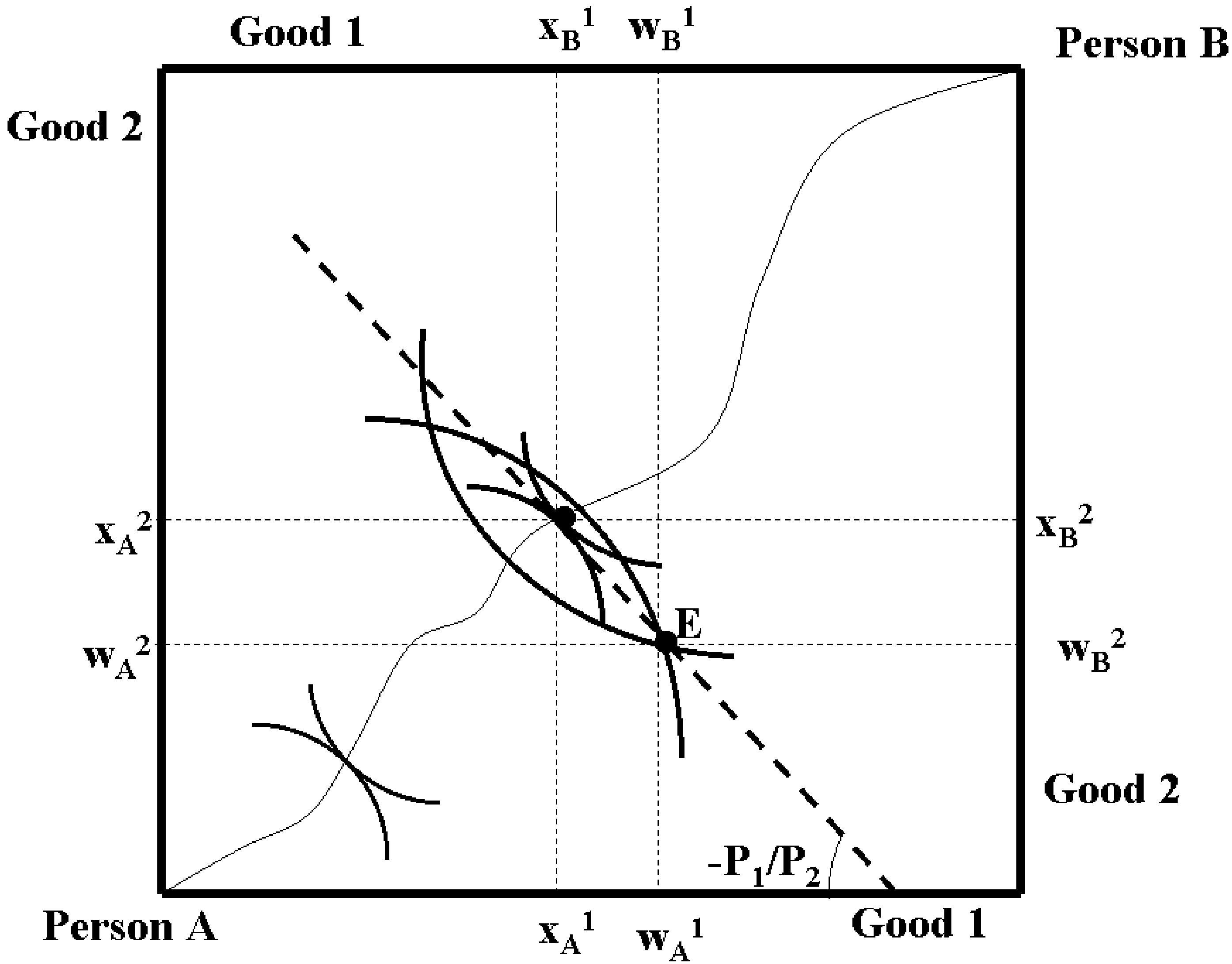
$$\textbf{Budget constraint: } p_1 w_1 + p_2 w_2 = I$$

$$\textbf{Optimality condition: } MRS = \frac{p_1}{p_2}$$

So that in the General Equilibrium –

$$MRS^A_{1,2} = \frac{p_1}{p_2} = MRS^B_{1,2}$$

Economically Efficient Price Allocation



How these Prices are formed?

- Market gives prices, "Free market" gives price flexibility that will ensure Demand = Supply

- Suppose there exists a "**Walrasian Auctioneer**", he quotes prices and look at demand of both persons, and adjust these prices until there is no excess demand

- *Excess Demand (p) = Demand of A (p) + Demand of B (p) – Supply of the Good*

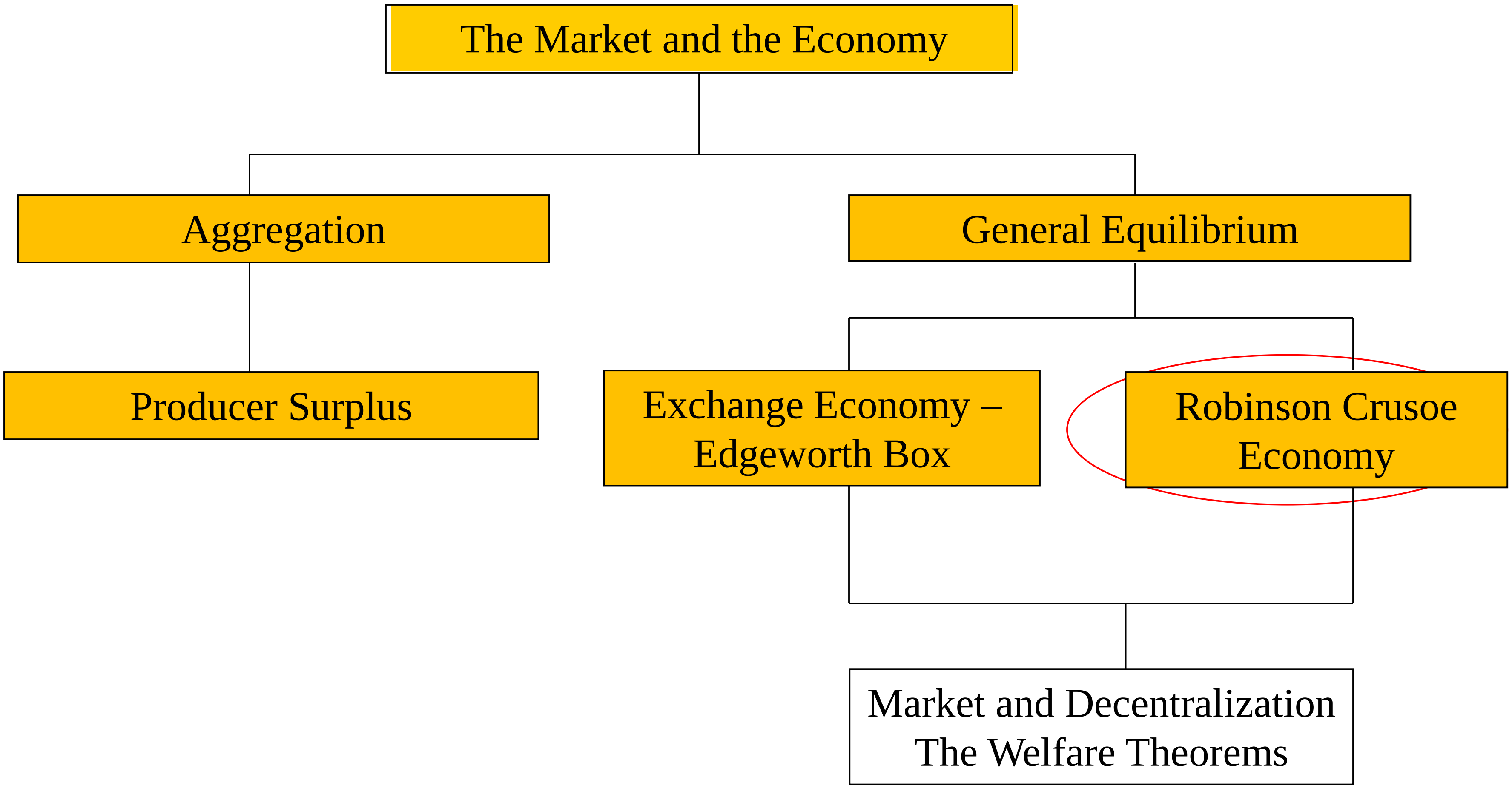
- A **competitive equilibrium** is reached, if the Walrasian Auctioneer can quote a set of prices such that: there is **no Excess Demand** in **both Good 1 and Good 2 (material balance)**

- In this competitive equilibrium, both Person A and Person B have maximized their utilities subject to their budget constraints, and that material balance is satisfied

- Note that here the ***Competitive Equilibrium is Pareto Efficient***

- ***Note also Pareto Efficiency does not imply Equity***

The Market and the Economy



Substitution Efficiency

***Imagine a one-man economy,* he provides input and there are two goods can be made**

Labor

A B

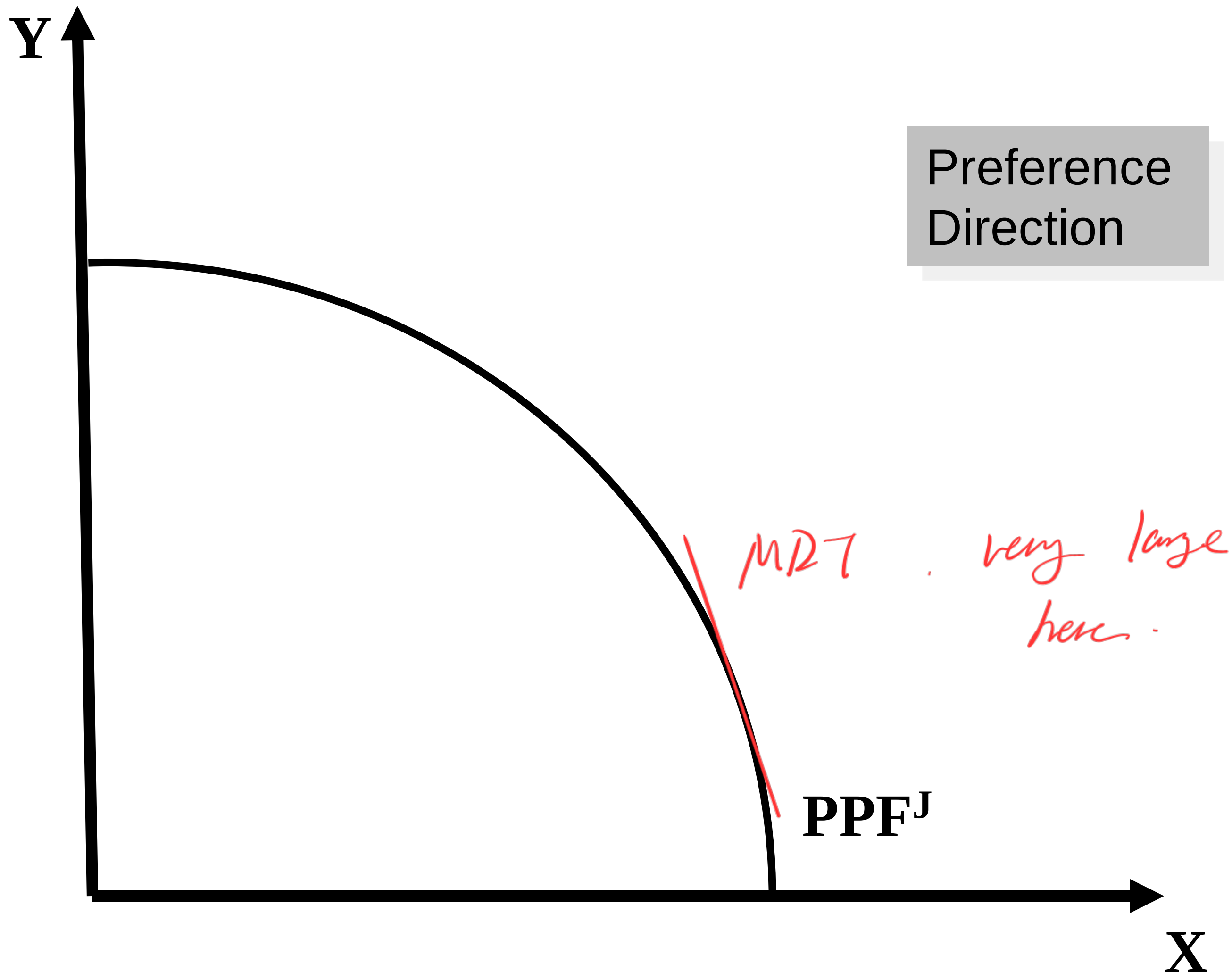
<u>*Definition:*</u> The **production possibility frontier** (PPF) of an individual is the maximum combinations of goods A and B that can be produced with the individual's input (e.g., labor) per unit of time.

<u>*Definition:*</u> An individual achieves **efficiency in production** if s/he produces combinations of goods on the PPF

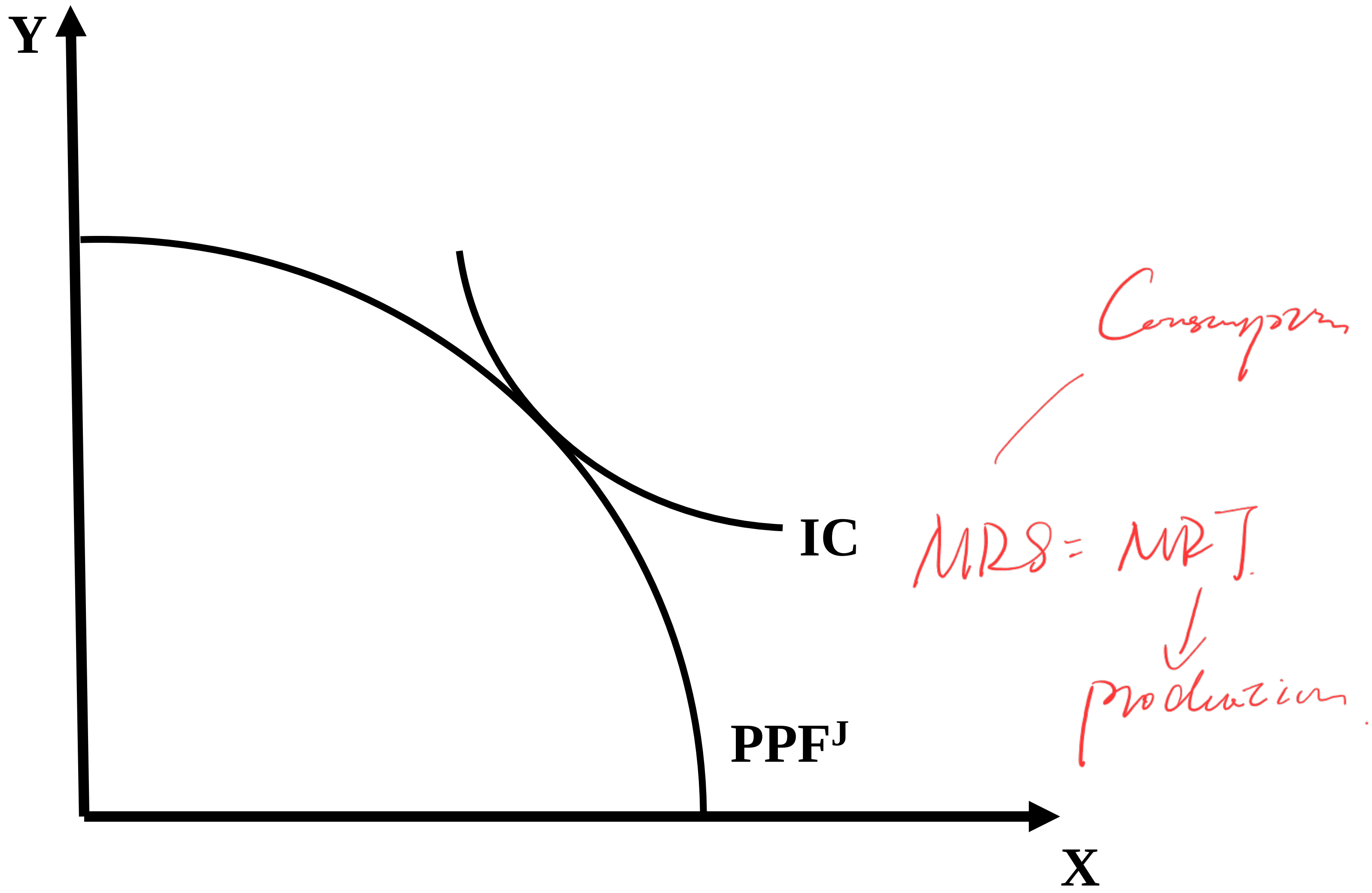
<u>*Definition:*</u> The slope of the production possibility frontier is the **marginal rate of transformation** (MRT).

The MRT tells us how much more of good Y can be produced if the production of good X is reduced by a small amount.

The Efficient Product Mix



The Efficient Product Mix



The Efficient Product Mix

At this point, the consumer's willingness to give up good X in order to get good Y just equals the rate at which a producer has to give up good X in order to produce more of good Y.

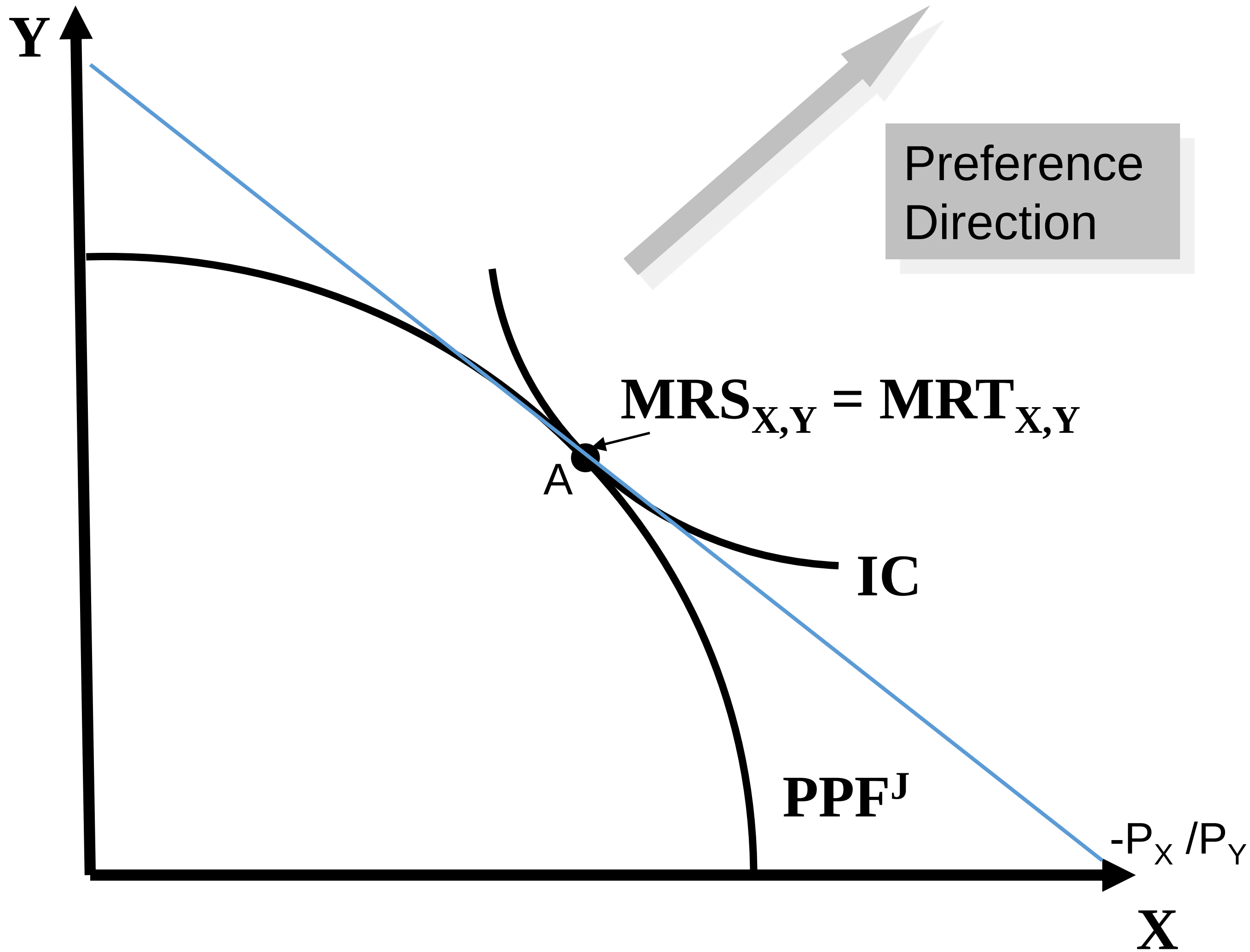
$$MRT_{X,Y} = MRS_{X,Y}$$

Can this person's production decision and consumption decision be decentralized (separated)?

Imagine there is a "Walrasian Auctioneer", setting P_x and P_y
Profit is maximized by maximising $I=P_xX+P_yY$ and the man maximises his utility subject to the budget constraint

$$MRT_{X,Y} = P_X/P_Y = MRS_{X,Y} = P_X/P_Y$$

Decentralization



Market gives Prices and these Prices drive efficiency

Equilibrium in Many Markets

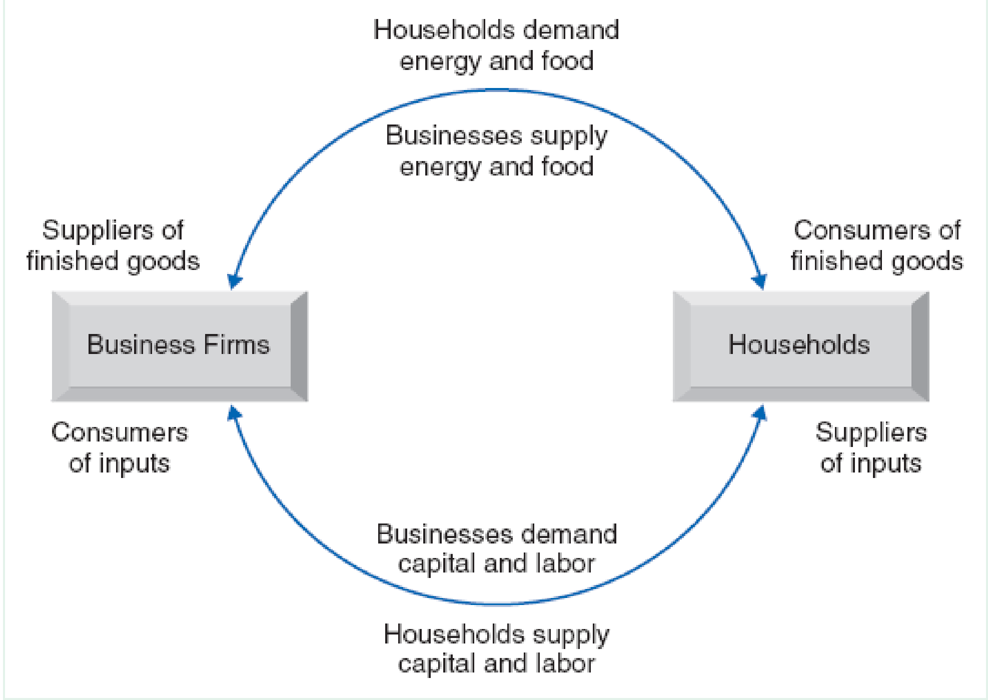
Consider an economy with:

2 types of households – white-collar households and blue-collar households

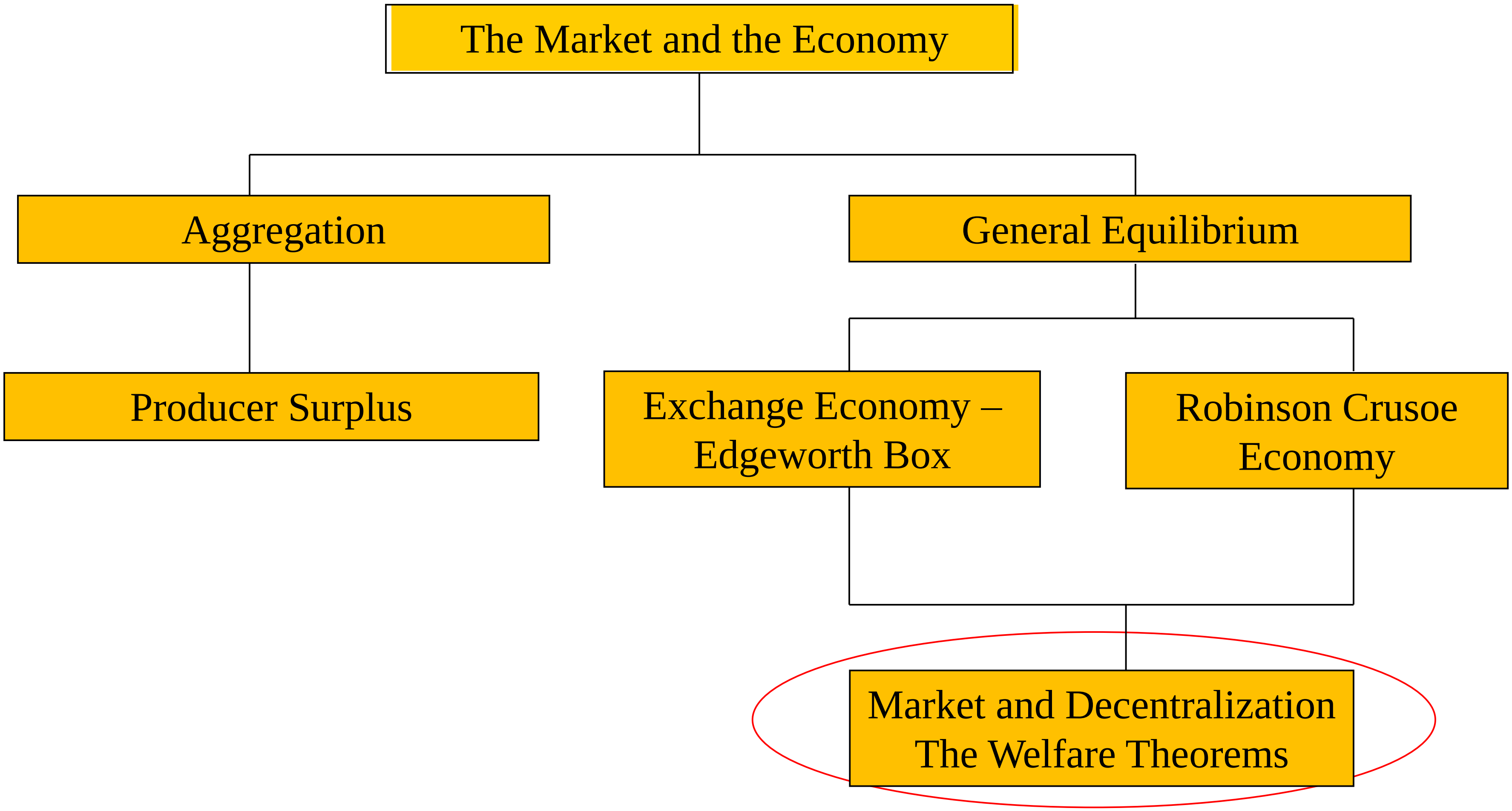
purchasing
2 goods – energy and food – each of which is produced with

2 input services – labor and capital

The analysis is similar



The Market and the Economy



Market vs Command Economy

<u>The Market Economy and Decentralization</u>

Note that in both the Exchange Economy and the Robinson Crusoe Economy, the existence of a free market will offer prices that lead to Efficiency

This is an example of Decentralization – prices are given, consumers maximize their utilities based on these prices, and firms maximize their profit based on these prices, they do so **independently**. The set of prices given by the market will ensure Demand = Supply

<u>The Command Economy</u>

However, it is also possible that a policymaker will dictate and tell consumers how much to consume and tell firms how much to produce, such that the outcome is Efficient, this is an example of a Command Economy. No market (hence no prices) will be needed

Fundamental Theorems of Welfare Economics

First Theorem of Welfare Economics (*Invisible Hand Theorem*)
The allocation of goods and inputs that arises in a general competitive equilibrium is economically efficient. That is, given the resources available to the economy, there is no other feasible allocation of goods and inputs that could simultaneously make all consumers better off.

greeding, more is better
free market.
perfect information
Completeness information.

Second Theorem of Welfare Economics (*Normative "Invisible Hand"*) Any economically efficient allocation of goods and inputs can be attained as a general competitive equilibrium through a judicious allocation of the economy's scarce supplies of resources.

Free Market and the Government

- If a free market with perfect competition is socially desirable, why do we need government?
- For **social efficiency**, we probably ***don't*** (when there is perfect competition).
- In addition, government-led policies often create distortions.
- *Normatively*, the necessity of government often relies on criticisms of free market.

Criticism of Free Market

- Market Failures (loss of efficiency)

1. Market power: monopoly, oligopoly
2. Externalities: e.g., pollution, education, etc.
3. Public goods: e.g., fire station, national defense, etc.
4. Information asymmetry: e.g., used cars, insurance, etc.

- Coordination Problem: e.g., electricity

Other topics in General Equilibrium Theory (for further study in microeconomics)

- In the exchange economy: suppose there are N replicates of individuals A and B, can they do better amongst themselves without trading by forming groups (Coalitions)

- Mathematical Proof of the First Welfare Theorem and the Second Welfare Theorem

- Incorporating Uncertainty and Market Completeness (Arrow-Debreu Securities)

The Chinese University of Hong Kong
Department of Economics

ECON3011

Intermediate Microeconomic Theory

2025-2026 (First Term)

Lecture 9: Imperfect Competition and Elementary Game Theory

Wallace K C Mok

Acknowledgement: Parts of this lecture slide set were kindly provided by Prof. Shi

In this Lecture

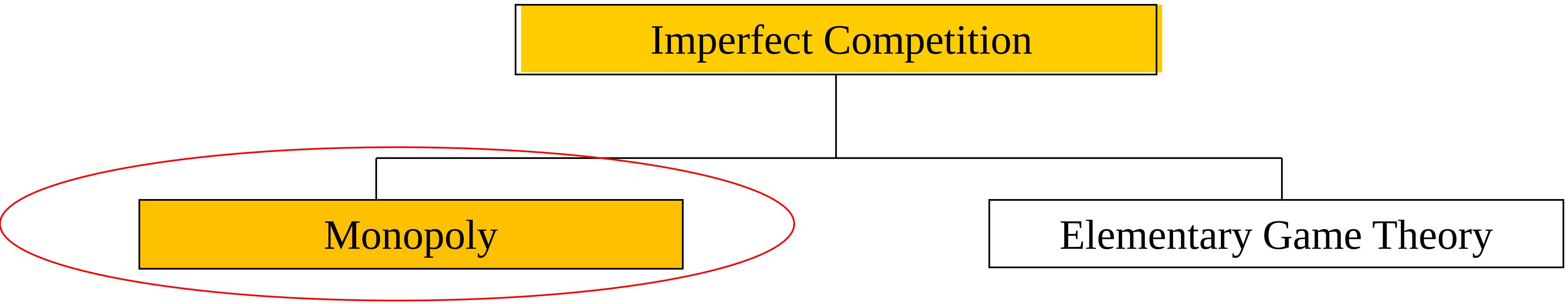
- Monopoly as the opposite of Perfect Competition, and its welfare consequences

- When decisions are inter-dependent, and that there is imperfect information, a different approach is needed – Game Theory

- Nash Equilibrium (Pure Strategy/Mixed Strategies)

- Dominant Strategy and Dominated Strategy

Imperfect Competition



Criticism of Free Market

- Market Failures (loss of efficiency)

1. Market power: monopoly, oligopoly
2. Externalities: e.g., pollution, education, etc.
3. Public goods: e.g., fire station, national defense, etc.
4. Information asymmetry: e.g., used cars, insurance, etc.

- Natural Monopoly
- Coordination Problem: e.g., electricity

Market Structures

Four Key Dimensions

- The number of sellers
- The number of buyers
- Entry conditions
- The degree of product differentiation

Product Differentiation

Definition: **Product Differentiation** between two or more products exists when the products possess attributes that, in the minds of consumers, set the products apart from one another and make them less than perfect substitutes.

Examples: Pepsi is sweeter than Coke, Brand Name batteries last longer than "generic" batteries.

- ***Vertical Differentiation***

i.e. one product is viewed as **unambiguously better** than another so that, at the **same price**, all consumers would buy the better product

- ***Horizontal Differentiation***

i.e. at the same price, some consumers would prefer the **characteristics** of product A while other consumers would prefer the **characteristics** of product B.

Types of Market Structures

Degree of Product Differentiation	Number of Firms		
	Many	Few	One
Firms produce identical products	Perfect Competition	Oligopoly with homogeneous products	Monopoly
Firms produce differentiated products	Monopolistic Competition	Oligopoly with differentiated products	------------

A Monopoly

<u>*Definition:*</u> A **Monopoly Market** consists of a single seller facing many buyers.

Can not adjust P, Q seperately.

The monopolist's profit maximization problem:

$$\max_{Q} \pi(Q) = TR(Q) - TC(Q)$$

The monopolist's profit maximization condition:

$$\frac{dTR(Q)}{dQ} - \frac{dTC(Q)}{dQ} = 0$$

$$\Rightarrow \quad MR(Q) = MC(Q)$$

Marginal Revenue Curve and Demand

As the Monopoly controls the entire market, its Total Revenue TR = P(Q)*Q not P*Q with P given!

$$MR = \frac{dTR}{dQ} == P + Q\frac{dP}{dQ}$$

+ −

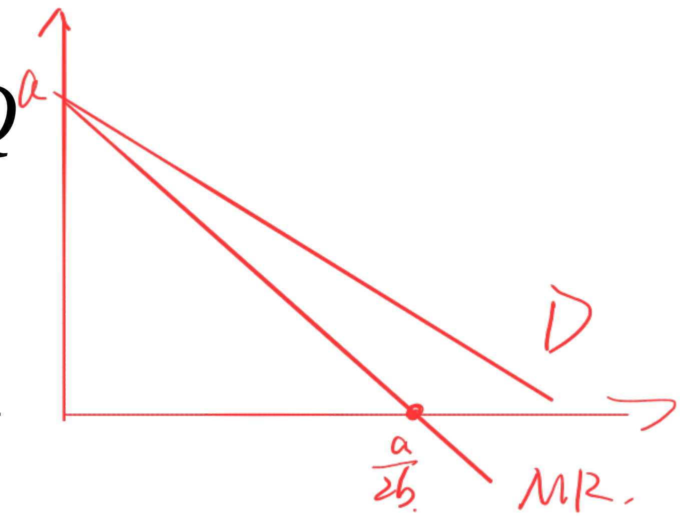
Example: Linear Demand

- Given the demand curve, what is the marginal revenue curves?

$$P = a - bQ$$

$$TR(Q) = P(Q) * Q = aQ - bQ^2$$

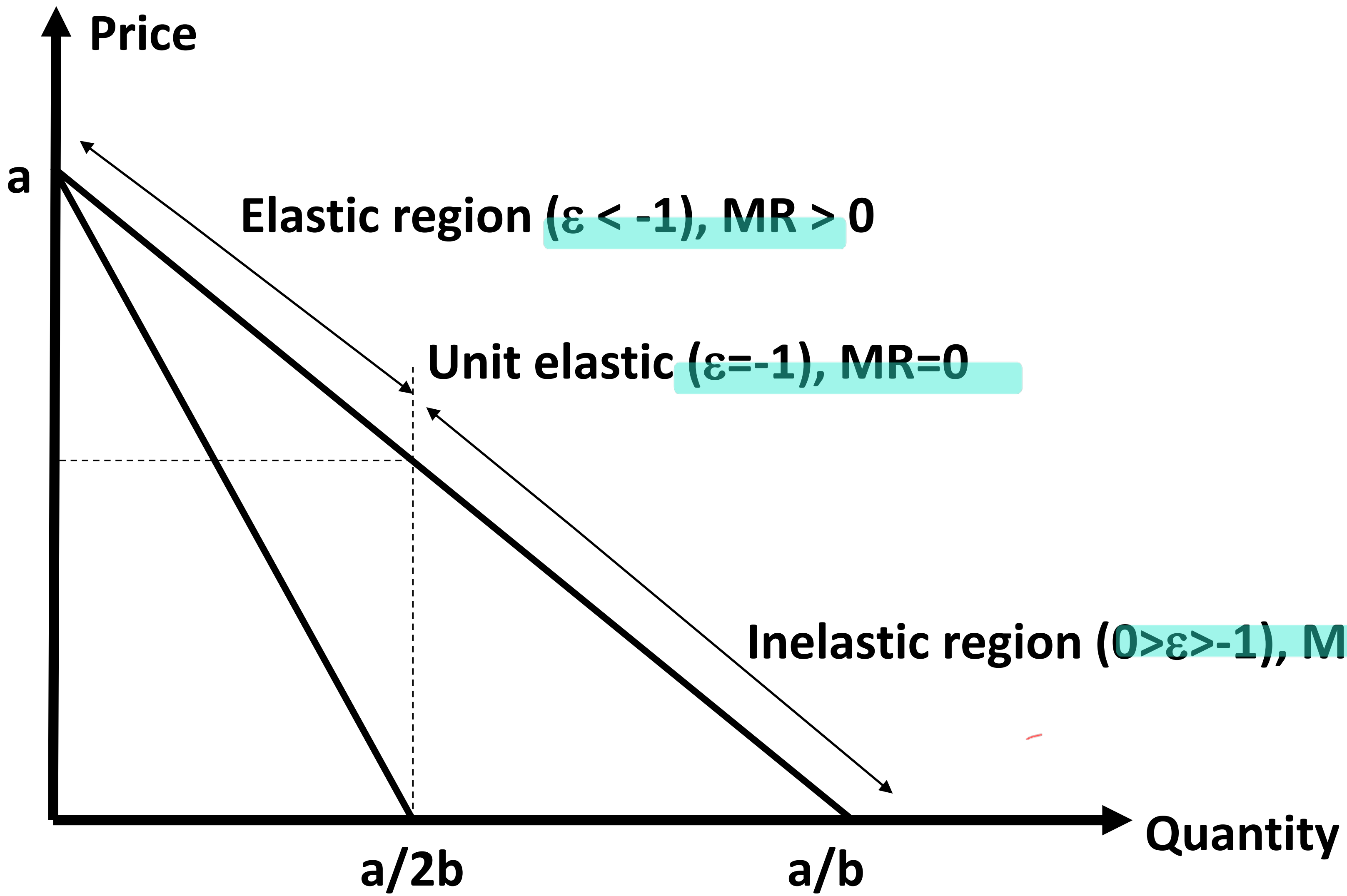
$$MR = a - 2bQ$$



Vertical intercept is: a

Horizontal intercept is $Q = \dfrac{a}{2b}$

Example: Linear Demand



Inverse Elasticity Pricing Rule

We can rewrite the MR curve as follows:

$$MR = P + Q\left(\frac{dP}{dQ}\right)$$

$$= P\left(1 + \left(\frac{Q}{P}\right)\left(\frac{dP}{dQ}\right)\right)$$

$$= P\left(1 + \frac{1}{\epsilon}\right)$$

Market Power and the Lerner Index

Definition: An agent has **Market Power** if s/he can affect, through his/her own actions, the price that prevails in the market. Sometimes this is thought of as the degree to which a firm can raise price above marginal cost.

(0, 1)

Definition: the ***Lerner Index of market power*** is the price-cost margin, (P*-MC)/P*. This index ranges between 0 (for the competitive firm) and 1, for a monopolist facing a unit elastic demand.

The Lerner Index of Market Power

Restating the monopolist's profit maximization condition, we have:

MR = MC.

$$P^* \left(1 + \frac{1}{\epsilon}\right) = MC(Q^*) \ldots \text{or} \ldots$$

$$\frac{P^* - MC(Q^*)}{P^*} = -\frac{1}{\epsilon}$$

In words, the monopolist's ability to price above marginal cost depends on the elasticity of demand.

Example: Linear Demand

- Given P and MC what is the profit maximizing P and Q?

$$P = a - bQ \qquad MC = c$$

$$MR = a - 2bQ$$

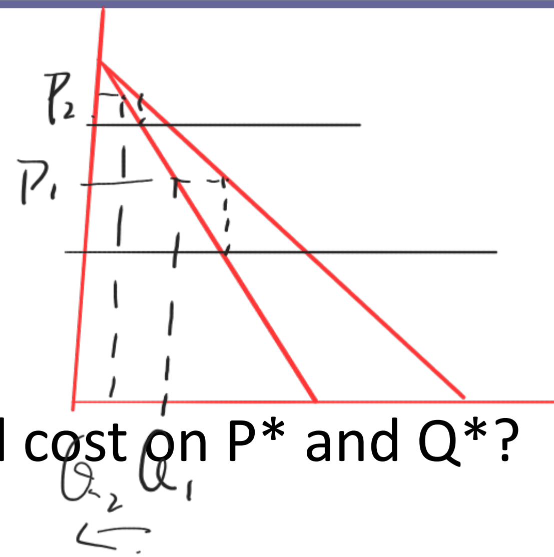
MR = MC.

$$a - 2bQ^* = c \qquad Q^* = \frac{a-c}{2b}$$

$$P^* = a - b\left(\frac{a-c}{2b}\right) = a - \frac{1}{2}a + \frac{1}{2}c = \frac{a+c}{2}$$

Example: Linear Demand

- Comparative statics



$$P^* = \frac{a+c}{2} \qquad Q^* = \frac{a-c}{2b}$$

What is the effect of a higher marginal cost on P* and Q*?

$$\frac{dP^*}{dc} = \frac{1}{2} > 0 \qquad \frac{dQ^*}{dc} = -\frac{1}{2} < 0$$

What is the economic interpretation?

In addition,

$$\frac{dP^*}{da} = \frac{1}{2} > 0 \qquad \frac{dQ^*}{da} = \frac{1}{2} > 0$$

What is the economic meaning of a?

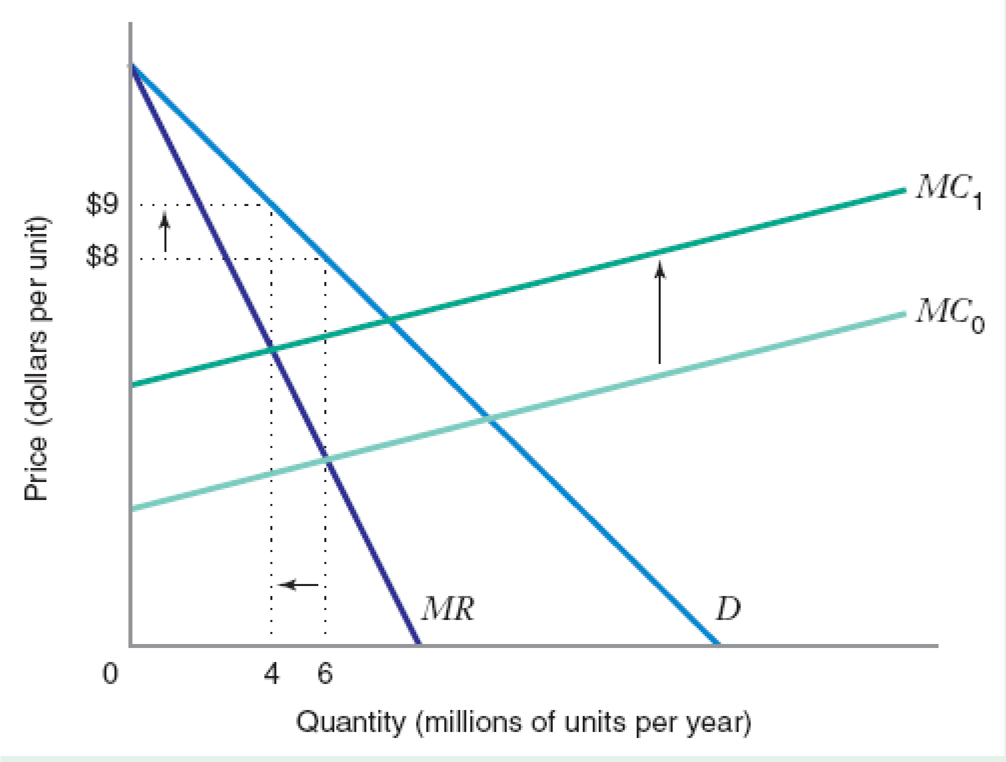
a is the concept

$a \uparrow \Rightarrow$ market demand shift to right.

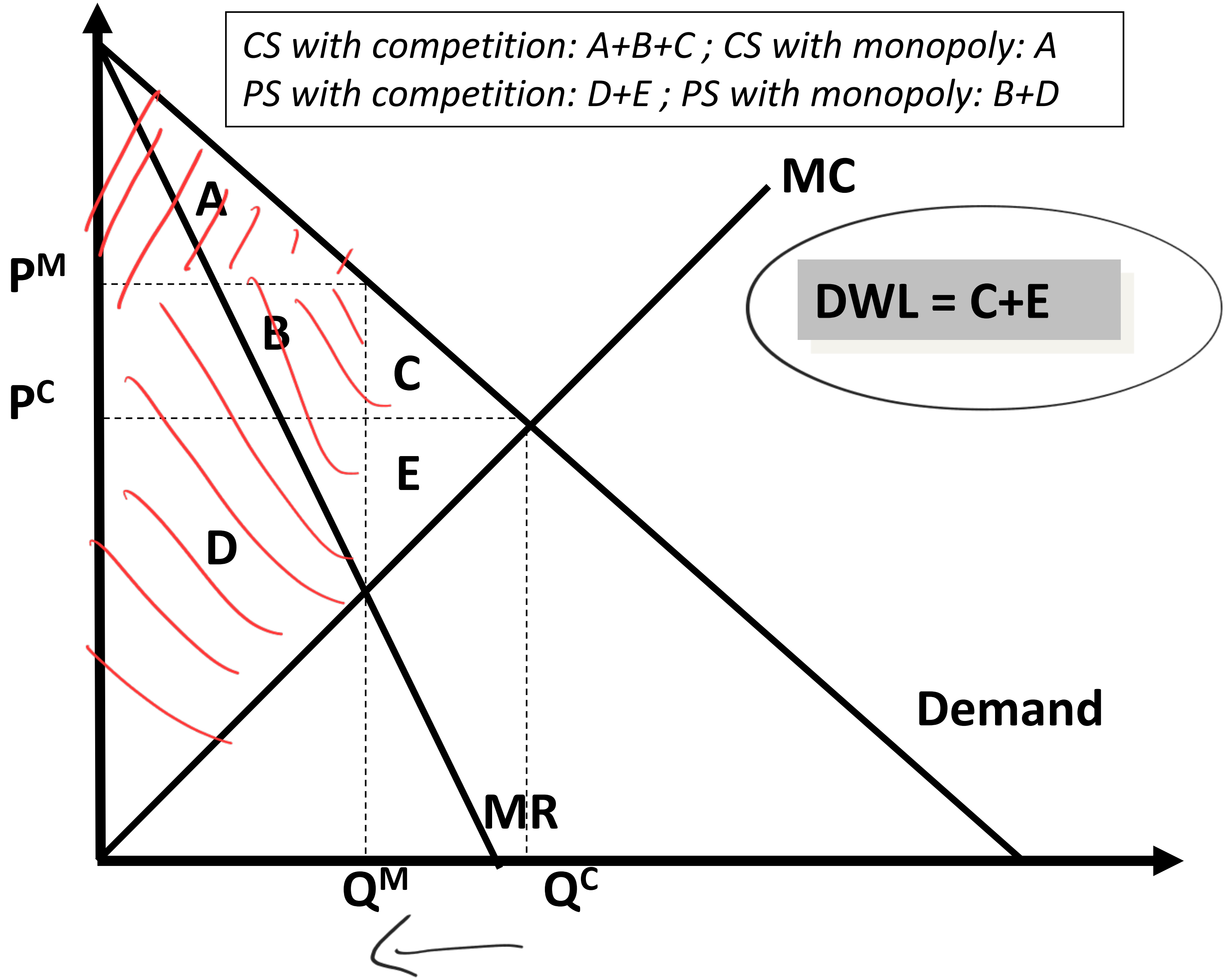
can not determined by monopolist..

Comparative Statics – Shifts in Marginal Cost

- When MC shifts up, Q falls and P increases.



The Welfare Economies of Monopoly



Normative Views of Monopoly

Why might monopolies be bad?

- They create deadweight loss
- Monopolies cause potential monopolists to waste resources trying to become monopolies
  - Rent-seeking activities
- Distributional issue: monopolies transfer income from consumers to monopolists

Normative Views of Monopoly

Why might monopolies be good?

- Innovation usually involves high research and development (R&D) cost (as set-up cost) – Monopoly creates incentive to innovate

e.g., Patents in pharmaceuticals, high-tech, etc.

- Existence of natural monopoly

e.g., Telecommunication, railroads, etc.

Oligopoly

Assumptions:

- Many Buyers and Few Sellers

- Each firm faces downward-sloping demand because each is a large producer compared to the total market size

- Outcomes (e.g. profits) depend on joint decisions, which are affected by strategies, i.e. decisions are interdependent.

- Presence of Imperfect /Incomplete Information. The need to incorporate Game Theory

- There is no one dominant model of oligopoly. We will study 3 models.

Imperfect Competition

- Imperfect Competition
  - Monopoly
  - Elementary Game Theory

Usage of Game Theory

<u>**Single-agent decisions**:</u>

e.g., classical consumer theory, firm decisions in **perfect competition, monopoly**, and monopolistic competition

⇒ Optimization techniques (differential calculus, etc.), usually no Game Theory needed

<u>**Interdependent/strategic decisions**:</u>

e.g., Oligopoly (the focus of this part of the course)

Principal-agent problems (insurer vs insured, boss vs staff, price discrimination)

Mechanism Design and Contract Theory

Economics of Information

=> **Game theory** + optimization

Elements of a Game

Game Elements

Players: agents participating in the game *(Toyota, Honda)*

Strategies: Actions that each player may take under any possible circumstance *(Build, Don't build)*

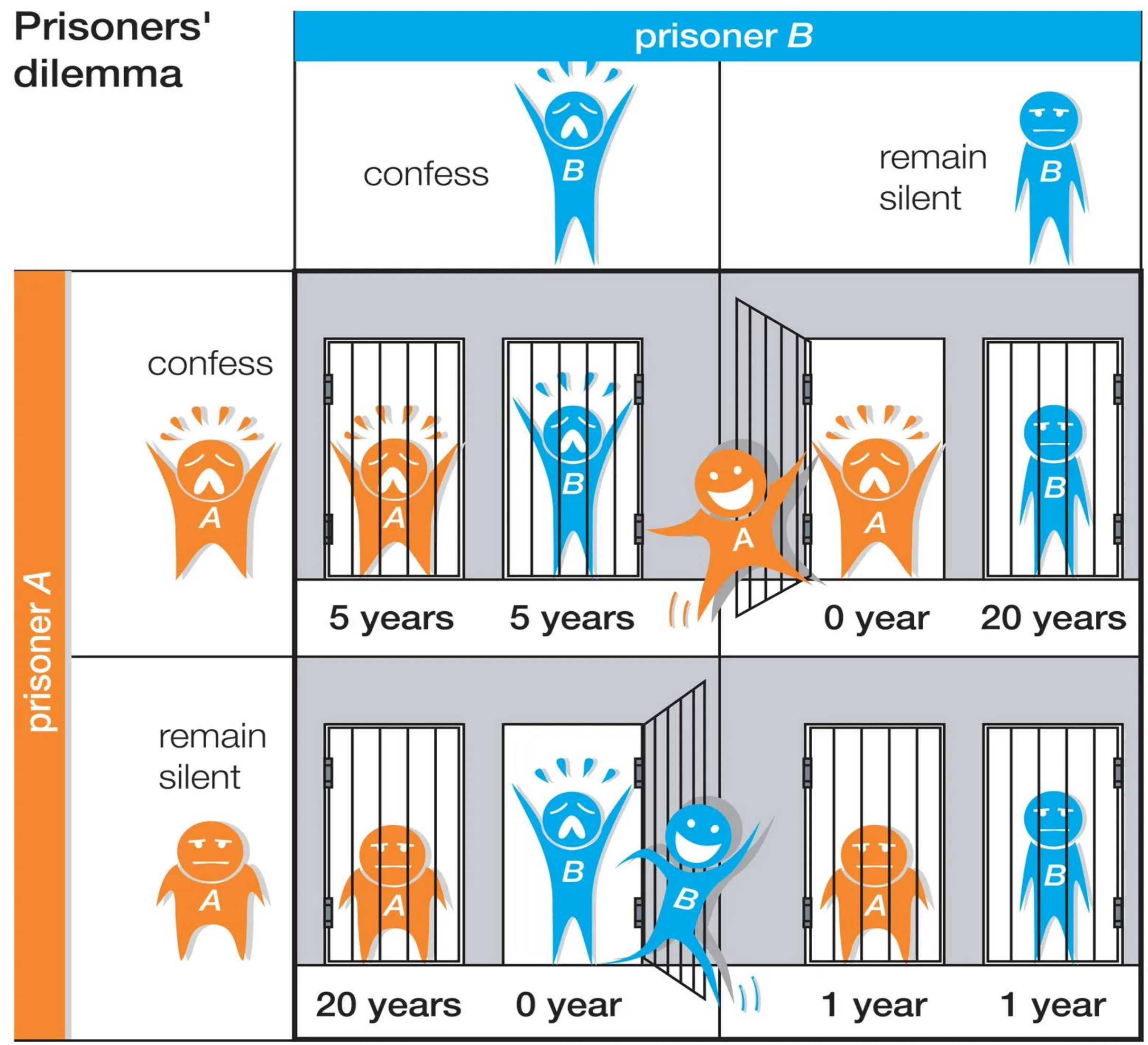
Outcomes: The various possible results of the game

Payoffs: The benefit that each player gets from each possible outcome of the game *(the profits entered in each cell of the matrix)*

Information: A full specification of who knows what when (full or imperfect information)

Timing: Who can take what decision when and how often the game is repeated
Simultaneous one-shot (usually represented as a normal form (**matrix**))
Sequential / dynamic n-period (usually in extensive form (**tree**))

Prisoners' Dilemma



Full information. First. Best. Solution.

Prisoners' Dilemma

Game Elements

Players: 2 people, A and B

Strategies: Confess or Remain Silent

Outcomes: The various possible results of the game

Payoffs: Years detained for A and B

Information: *Imperfect (A and B cannot Communicate)*

Timing: One-Shot Simultaneous Move

Efficient Outcome vs Nash Equilibrium

An Efficient Outcome occurs when One cannot be better off without the other person worse off.

Could be economically inefficient.

A **Nash Equilibrium** occurs when each player chooses a strategy that gives him/her the highest payoff, given the strategy chosen by the other player(s) in the game.

In a Nash Equilibrium, no player could be better off by switching to a different strategy (i.e. no incentive to deviate).

There can be multiple Nash Equilibria in a game.

Game 1: Capacity Expansion

Toyota

Honda

	Build a new plant	Do not Build
Build a new plant	16,16	20,15
Do not Build	15,20	18,18

Payoff : (Honda, Toyota)

The Nash Equilibrium is a Pure Strategy for both players

Dominant and Dominated Strategy

A *dominant strategy* is a strategy that is better than any other strategy that a player might choose, *no matter what strategy* the other player follows.

When a player has a dominant strategy, that strategy will be the player's Nash Equilibrium strategy.

A player has a **dominated strategy** when the player has another strategy that gives it a higher payoff no matter what the other player does.

Dominated Strategy

A player has a **dominated strategy** when the player has another strategy that gives it a higher payoff no matter what the other player does.

dominant.

dominated.

Toyota

Honda

	Build a New Plant	Don't Build
Build a New Plant	12,4	20,3
Don't Build	15,6	18,5

Dominant or Dominated Strategy

Why look for dominant or dominated strategies?

A dominant strategy equilibrium is particularly compelling as a "likely" outcome

Similarly, because dominated strategies are unlikely to be played, these strategies *can be eliminated* from consideration in more complex games. This can make solving the game easier.

Dominated Strategy

Game III: Dominated Strategies

Toyota

Honda

	Build Large	Build Small	Do Not Build
Build Large	0,0	12,8	18,9
Build Small	8,12	16,16	20,15
Do Not Build	9,18	15,20	18,18

Payoff : (Honda, Toyota)

Example IV: Coordination Game

Example: Bank Runs

Depositor 2

Depositor 1

	Withdraw	Don't Withdraw
Withdraw	25,25	50,0
Don't Withdraw	0,50	110,110

Payoff : (D1, D2)

Which Nash Equilibrium will be the outcome depends on signal/expectation

Example V: Matching Pennies

Player 2

Player 1

	Heads	Tails
Heads	1,-1	-1,1
Tails	-1,1	1,-1

Players will pursue a Mixed Strategy – 50% probability head and 50% probability tail

Extensions covered in Advanced Courses

One-Shot Simultaneous

- **If two or more Nash Equilibrium: Mixed Strategies**

- **Cooperative Games, Trust, Agreements**

like prison dillem

Dynamic Games

person know itself in which place of decision tree.

- **Information Set (Singleton vs Non-Singleton)**

- **Repeating Games : Punishing Strategy, Trigger Strategy**

- **Bayesian Games and Beliefs Updates (very advanced)**

- **Classic Reference: Gibbons (*A primer in Game Theory, Applied Game Theory for Economists*)**

Next Class: Oligopoly

There is no single model for oligopoly

- **Bertrand model**: identical or differentiated products; price competition
- **Cournot model**: identical products; quantity competition
- **Stackelberg model**: firms act sequentially; can be Cournot or Bertrand

The Chinese University of Hong Kong
Department of Economics

ECON3011

Intermediate Microeconomic Theory

2025-2026 (First Term)

Lecture 10: Oligopoly

Wallace K C Mok

Acknowledgement: Parts of this lecture slide set were kindly provided by Prof. Shi

In this Lecture

- We study Oligopoly decisions, requiring the use of Game Theory

- Static and Dynamic/Sequential Oligopoly Games

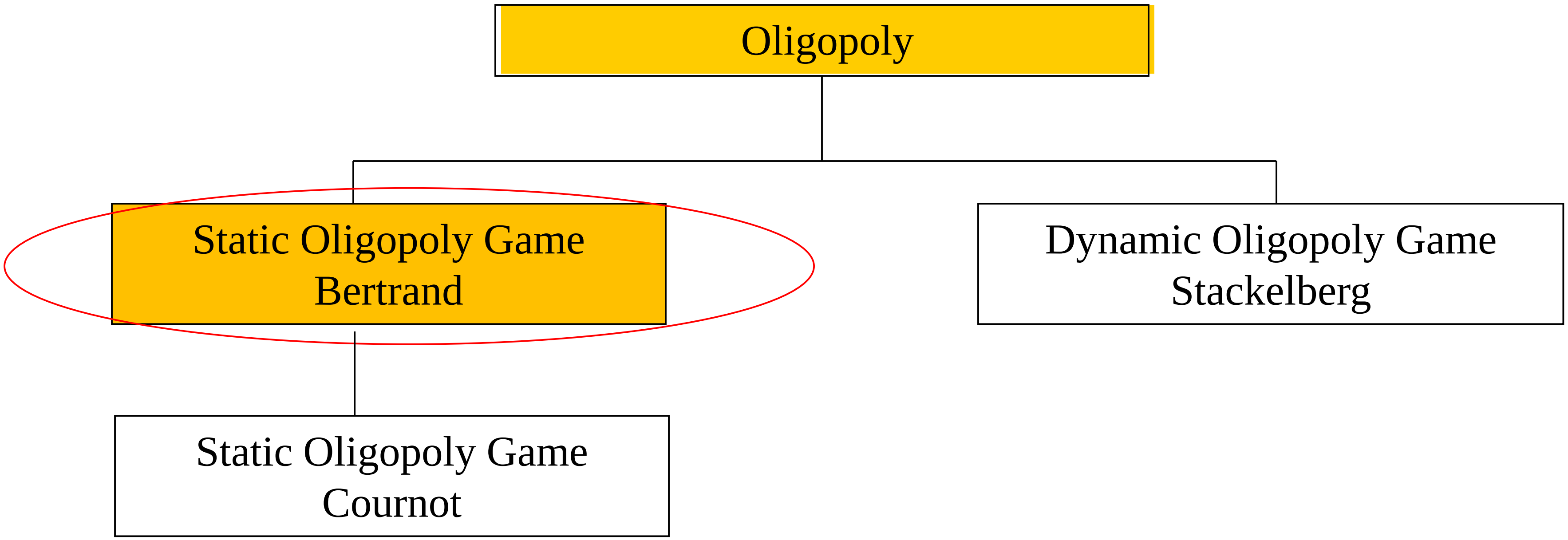
Types of Oligopoly Competition

- Bertrand (Static – Price)

- Cournot (Static – Quantity)

- Stackelberg (Dynamic, Leader -> Followers)

Oligopoly


Oligopoly
Static Oligopoly Game
Bertrand
Dynamic Oligopoly Game
Stackelberg
Static Oligopoly Game
Cournot

Bertrand Competition w/ Homogenous Product

Bertrand Oligopoly: Set Up

• Firms set price, taking as given the price(s) set by other firm(s), to maximize profits

• Homogeneous product (Consumer will go for the lower price firm)

• Simultaneous

• Non-cooperative

• Identical (constant) MCs

•Any firm charging a lower price than its rivals will obtain the entire market demand.

Bertrand Competition w/ Homogenous Product

The situation where every firm sets P = MC is the unique Nash Equilibrium of this game.

• Why is it a Nash equilibrium?

(**Existence**) Suppose all firms set P = MC. Consider an arbitrary firm *i*:

- If $P_i > MC$, profit is still zero => not better off;
- If $P_i < MC$, negative profit => worse off.

Indeed, the situation when every firm sets P = MC is a Nash equilibrium.

Bertrand Competition w/ Homogenous Product

The situation where every firm sets $P = MC$ is the unique Nash Equilibrium of this game.

•Why is it *the only* Nash equilibrium?
(**Uniqueness**)
(1) Indeed, for any firm, the strategy $P < MC$ is (weakly) dominated by $P = MC$; (why?)

taking all market, but lose money.

(2) Given that other firms $P > MC$, one has incentive to set $P - \varepsilon \geq MC$ (ε is some small number);

=> there exist no N.E. involving such strategies!

Bertrand Competition w/ Homogenous Product

"***Bertrand paradox***": with homogenous products, if firms compete by choosing prices, they will optimally choose P = MC;

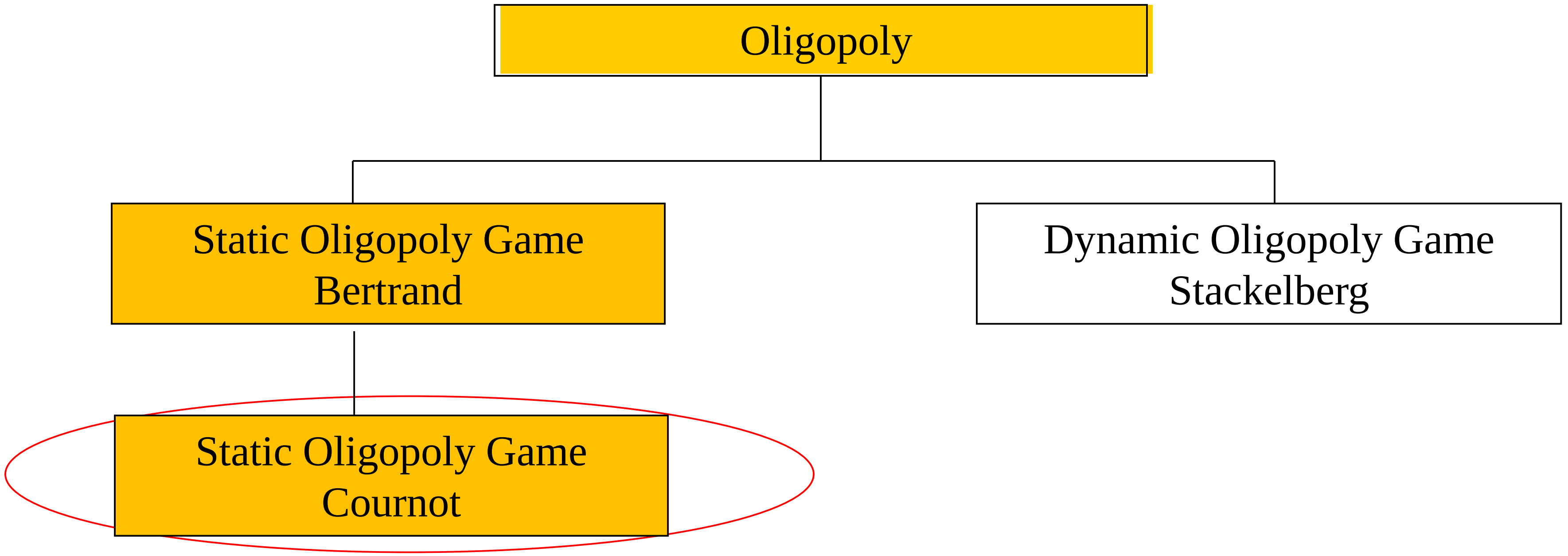
"Choosing prices" : market power;

"P = MC" : no market power.

How can we revise the Bertrand model?

- Quantity competition: *Cournot*
- Bertrand with product differentiation
- Sequential move (*Stackelberg* - Bertrand)

Oligopoly


Oligopoly
Static Oligopoly Game
Bertrand
Dynamic Oligopoly Game
Stackelberg
Static Oligopoly Game
Cournot

Cournot Competition

Set-Up

- Firms set outputs (quantities)
- Homogeneous Products
- Simultaneous move
- Non-cooperative

In a Cournot game, each firm sets its output (quantity) <u>taking as given the output level</u> of its competitor(s), so as to maximize profits.

Price adjusts according to demand.

Cournot Competition

Set-Up

Given:

- an inverse demand function, P(Q)

- $Q = Q_1 + Q_2$ *(where Q_1 is supply by firm 1, Q_2 is for firm 2)*

- Identical constant marginal costs: c

<u>Firm 1's profit maximization problem:</u>

$$\max_{Q_1} \pi_1 = P(Q)Q_1 - cQ_1$$

Cournot Competition

Firm 1's profit maximization problem:

$$\text{Max } \pi_1 = P(Q)Q_1 - cQ_1$$

$\frac{dP}{dQ} Q_1 + P(Q) - c = 0$

FOC implies:

$$\frac{dP}{dQ}Q_1 + P(Q_1 + Q_2) - c = 0$$

When we solve for $Q_1(Q_2)$, (i.e Q_1 as a function of Q_2), this is called a *best-response function (or reaction function)* for firm 1.

Similarly, we solve for $Q_2(Q_1)$, i.e. Q_2 as a function of Q_1;

Nash equilibrium is the **intersection of the best response (reaction) functions.**

Cournot Competition: Example

ASSUME $P(Q) = 100 - Q,\ C = 10$

$100 - Q_1 - Q_2.$

Firm 1's profit maximization problem:

$$\max_{Q_1} \pi_1 = P(Q)Q_1 - cQ_1$$

$$= (100 - Q_1 - Q_2)Q_1 - 10Q_1$$

FOC implies:

$$100 - 2Q_1 - Q_2 - 10 = 0$$

$$\Rightarrow \quad Q_1 = 45 - \frac{1}{2}Q_2$$

Best-response/reaction function for firm 1

Cournot Competition: Example

Similarly, Firm 2's reaction function: $Q_2 = 45 - \frac{1}{2}Q_1$

Best-response reaction function for firm 2

Solving for Q_1, Q_2**:**

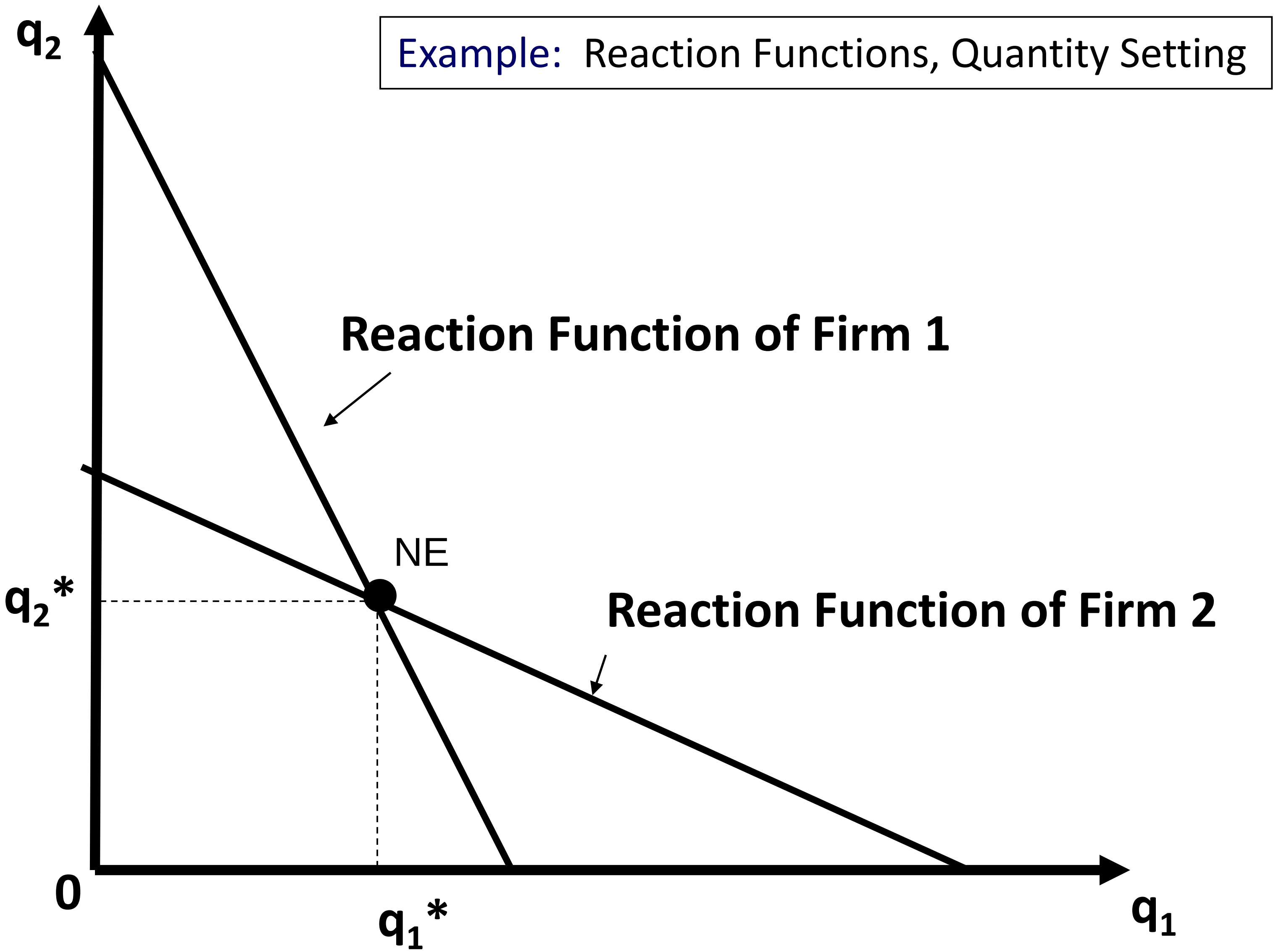
$Q_1^* = 30,$

$Q_2^* = 30,$

$P^* = 100 - 30 - 30 = 40;$

$\pi_1^* = \pi_2^* = (40 - 10)(30) = 900.$

Profit Maximization



Cournot Competition

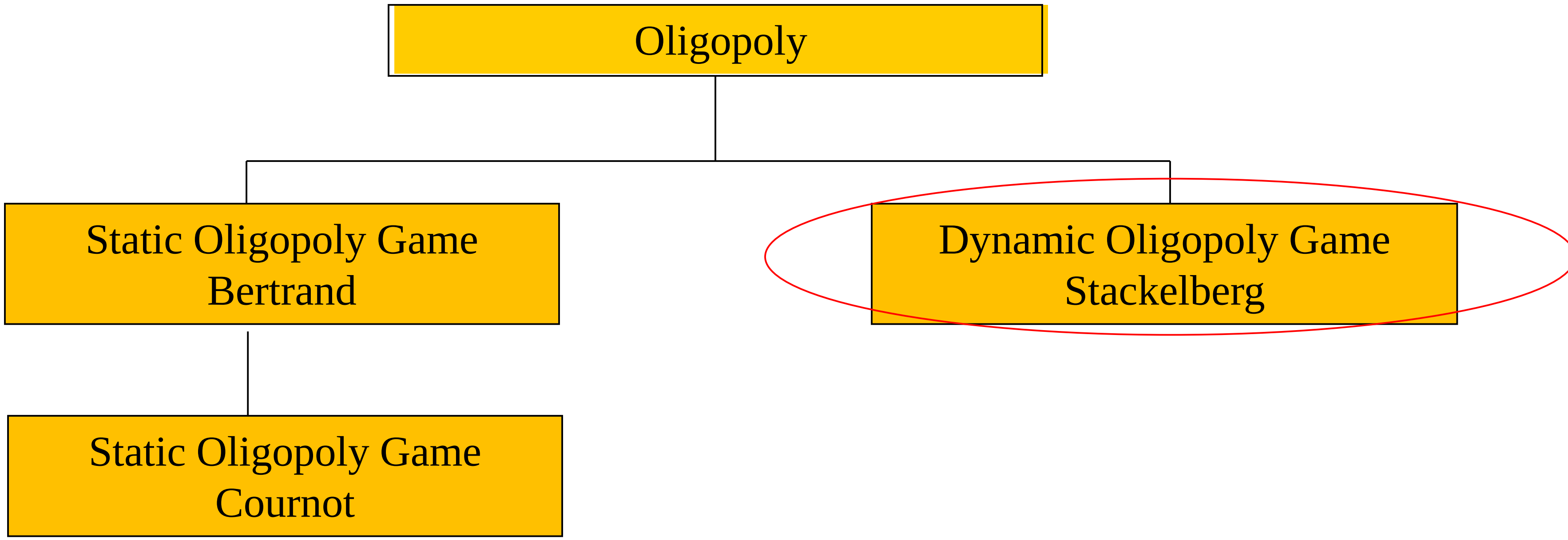
Quantity choices in a Cournot model (with a linear demand) are called *Strategic Substitutes*.

In practice, the Cournot model is often used to examine empirically industries, in which:

(1) Products are highly homogenous;
(2) Subjective production differentiation, such as advertising, is not present;
(3) Production capacity is limited.

e.g., sugar, salt, etc.

Oligopoly



Oligopoly
Static Oligopoly Game
Bertrand
Dynamic Oligopoly Game
Stackelberg
Static Oligopoly Game
Cournot

Static Game: Dominated Strategy

Game I: Dominated Strategies

Toyota

Honda

	Build Large	Build Small	Do Not Build
Build Large	0,0	12,8	18,9
Build Small	8,12	16,16	20,15
Do Not Build	9,18	15,20	18,18

(Two-Player) Sequential Move Games

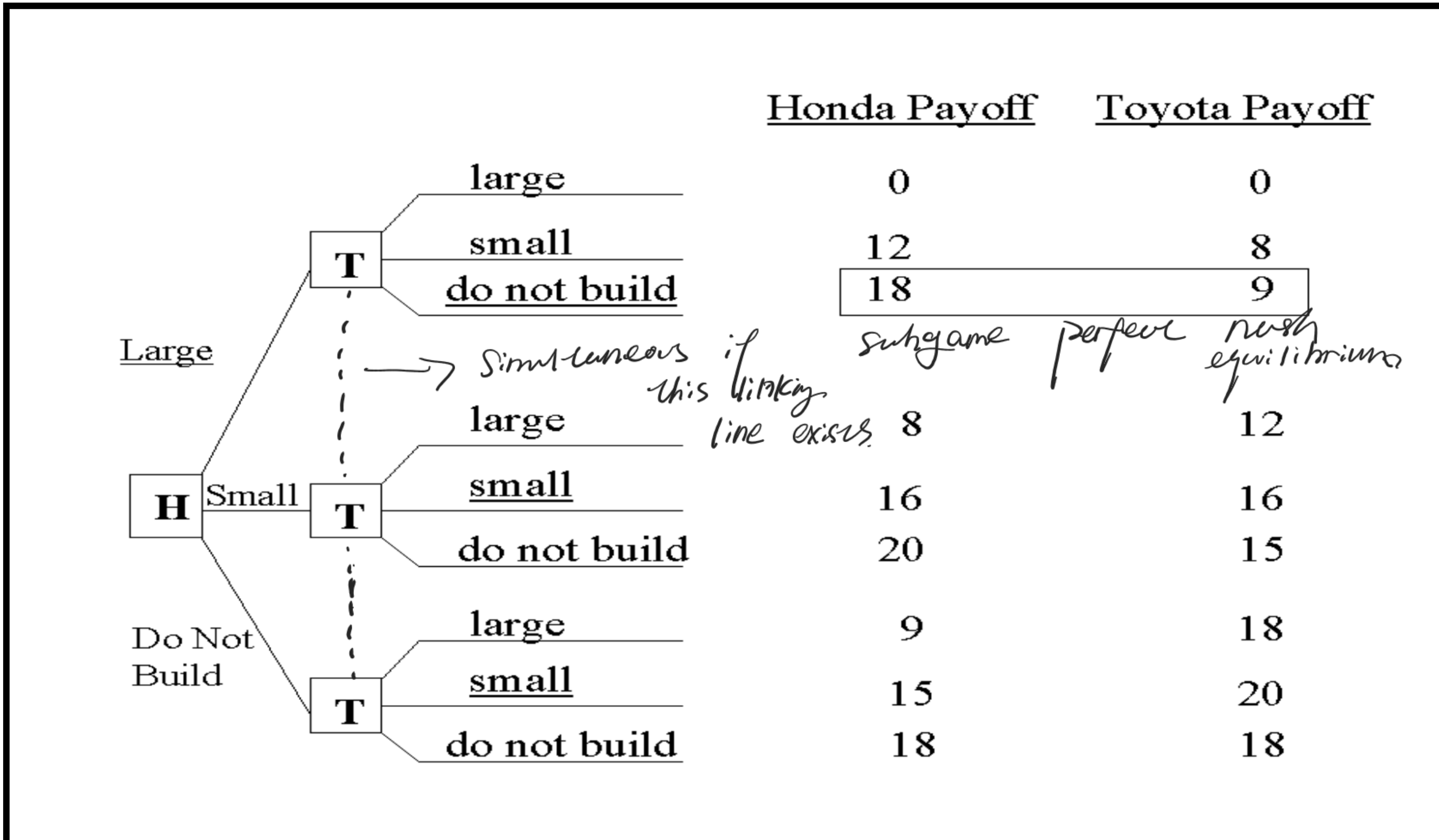
Games in which one player (the first mover) takes an action before another player (the second mover). The second mover observes the action taken by the first mover before deciding what action it should take.

At the turn of the second player, he knows which action the first player implemented – i.e. <u>A Singleton Information Set</u>

Backward induction is a procedure for solving a sequential-move game by starting at the end of the game tree and finding the optimal decision for the player at each decision point.

Sequential Move Game – Game Tree

Game Tree 1: Toyota and Honda, Revisited



Stackelberg Oligopoly

Stackelberg model of oligopoly (with quantity competition) is a situation in which one firm acts as a quantity **leader**, choosing its quantity first, with all other firms acting as **followers**.

Call the first mover the "*leader*" and the second mover the "*follower*".

The second firm is in the same situation as a Cournot firm: it takes the leader's output as **given** and maximizes profits accordingly, using its residual demand.

The second firm's behavior can, then, be summarized by a Cournot reaction function.

The leader has a "*First Mover Advantage*"

Stackelberg Competition: Example

ASSUME $P(Q) = 100 - Q,\ C = 10$

Backward induction:

In stage two, firm 2 (the follower) moves given Q_1^*,

$$\max_{Q_2} \pi_2 = P(Q)Q_2 - cQ_2$$
$$= (100 - Q_1^* - Q_2)Q_2 - 10Q_2$$

FOC (of firm 2):

$$100 - Q_1^* - 2Q_2 - 10 = 0$$
$$\Rightarrow \quad Q_2 = 45 - \frac{1}{2}Q_1^*$$

Stackelberg Competition: Example

Backward induction:

In stage 1, firm 1 (the leader) moves, anticipating

2's strategy: $Q_2 = 45 - \frac{1}{2}Q_1$

直接将 firm 2's $Q_2 \sim Q_1$ 直接代入 Q 中

$$\max_{Q_1} \pi_1 = P(Q)Q_1 - cQ_1$$
$$= (100 - Q_1 - Q_2(Q_1))Q_1 - 10Q_1$$
$$= (100 - Q_1 - (45 - \frac{1}{2}Q_1))Q_1 - 10Q_1$$

having extra information than Cournot . .

FOC (of firm 1):

$45 - Q_1 = 0 \Rightarrow \quad Q_1^* \quad = 45$

Stackelberg Competition: Example

Since $Q_1^* = 45$,

$$Q_2 = 45 - \frac{1}{2}(45)$$
$$= 27.5$$
$$P = 37.5$$

P C. — Cournot. — Stackelberg. — Monopoly.

no profit.

Cournot. Total π 1800 each 900 < Stackelberg. Total π more market power.

π.

π_1 1237.5

π_2 756.25

Cournot vs. Stackelberg

Example: Reaction Functions, Quantity Setting