Tutorial Sheet 4

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1- $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant & $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here

$\boxed{T(n) = \Theta(n^2)}$

2- $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are const. & $f(n)$ is a positive function.

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2.$

$\because n^c = n^2 \quad \therefore n^c = f(n)$

$\therefore$ case 2 is applied here

$\boxed{T(n) = \Theta(n^2 \log n)}$

3- $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$\because$ a & b are const. & $f(n)$ is a +ve function. $\therefore$ M.T. applicable

$c = \log_b a = \log_2 1 = 0 \Rightarrow n^c = n^0 = 1$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\boxed{T(n) = \Theta(2^n)}$.

4- $T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^n$

$\because$ a is not constant, its value depends on n.

$\therefore$ Master's theorem is not applicable here.

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5- $T(n) = 16\,T(n/4) + n$

$a = 16$, $b = 4$ $f(n) = n$

$\because$ a & b are const. and $f(n)$ is a +ve function

$c = \log_b a = \log_4 16 = 2\log_4 4 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here

$\boxed{T(n) = \Theta(n^2)}$

6- $T(n) = 2\,T(n/2) + n\log n$

$a = 2$, $b = 2$, $f(n) = n\log n$

$\because$ a & b are constant and $f(n)$ is a +ve function.

$\because c = \log_b a = \log_2 2 = 1$; $n^c = n^1 = n$

$f(n) > n^c$

$\therefore$ case 3 is applied.

$\boxed{T(n) = \Theta(n\log n)}$

7- $T(n) = 2T(n/2) + n/\log n$

$a = 2$, $b = 2$, $f(n) = n/\log n$

$\because$ a & b are constant and $f(n)$ is a +ve function

$c = \log_b a = \log_2 2 = 1$; $n^c = n^1 = n$

$\because$ non-polynomial difference b/w $f(n)$ and n^c

$\therefore$ Master's theorem is not applicable.

8- $T(n) = 2\,T(n/4) + n^{0.51}$; a & b are constants

$a = 2$, $b = 4$ $f(n) = n^{0.51}$; $f(n)$ is +ve function

$\therefore$ Master's theorem applicable

$c = \log_b a = \log_4 2 = 0.50$; $n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ case 3 applied

$\boxed{T(n) = \Theta(n^{0.51})}$

9- $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5$, $b = 2$, $f(n) = 1/n$

As $a < 1$ $\therefore$ Master's theorem is not applicable.

10- $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem applicable.

$c = \log_b a = \log_4 16 = 2\log_4 4 = 2$. ; $n^c = n^2$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applied here.

$\boxed{T(n) = \Theta(n!)}$

11- $T(n) = 4T(n/2) + \log n$

$a = 4$ $b = 2$ $f(n) = \log n$. a&b are const. f(n) is +ve.

$\therefore$ Master's theorem applicable.

$c = \log_b a = \log_2 2^2 = 2$; $n^c = n^2$

$f(n) < n^c$

$\therefore$ Case 1 is applied

$\boxed{T(n) = \Theta(n^2)}$

12- $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n}$ $b = 2$, $f(n) = \log n$.

As a is not constant $\therefore$ Master's Theorem not applicable.

13- $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$ a&b are constant, f(n) is +ve

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 3 = 1.58$; $n^c = n^{1.58}$

$f(n) < n^c$

$\therefore$ case 1 is applied here

$\boxed{T(n) = \Theta(n^{1.58})}$

14- $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$, $b = 3$, ~~f(n)~~ (constants) $f(n) = \sqrt{n}$ (+ve)

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_3 3 = 1$; $n^c = n^1 = n$

$\therefore f(n) < n^c$

$\therefore$ case 1 is applicable.

$\boxed{T(n) = \Theta(n)}$

15- $T(n) = 4T(n/2) + cn$

$a=4$ $b=2$ (constants) $f(n) = c.n$ (+ve)

$\therefore$ master's theorem is applicable

$c = \log_b a = \log_2 2^2 = 2$; $n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applicable here

$\boxed{T(n) = \Theta(n^2)}$

16- $T(n) = 3T(n/4) + n\log n$

$a = 3$ $b = 4$ (constants) $f(n) = n\log n$ (+ve)

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79$; $n^c = n^{0.79}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable here

$\boxed{T(n) = \Theta(n\log n)}$

17- $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$ (constants), $f(n) = n/2$ (+ve)

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\therefore f(n) < n^c$

$\therefore$ case 1 is applied here

$\boxed{T(n) = \Theta(n)}$

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18- $T(n) = 6T(n/3) + n^2 \log n$

$a = 6$, $b = 3$ (constants) $f(n) = n^2 \log n$ (+ve)

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 6 = 1.63$; $n^c = n^{1.63}$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here. $\boxed{T(n) = \Theta(n^2 \log n)}$

19- $T(n) = 4T(n/2) + n/\log n$

$a = 4$, $b = 2$ (constants). $f(n) = n/\log n$. (+ve)

$\therefore$ M.T. applicable

$c = \log_b a = \log_2 2^2 = 2$; $n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here $\boxed{T(n) = \Theta(n^2)}$

20- $T(n) = 64T(n/8) - n^2 \log n$

$\because$ a & b are constants but $f(n)$ is a -ve function.

$\therefore$ Master's theorem is not applicable here.

21- $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$ (constants), $f(n) = n^2$ (+ve)

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_3 7 = 1.77$; $n^c = n^{1.77}$.

$\because f(n) > n^c$

$\therefore$ Case 3 is applicable here. $\boxed{T(n) = \Theta(n^2)}$

22- $T(n) = T(n/2) + n(2 - \cos n)$

As $f(n)$ is not a regular function $\therefore$ Master's theorem not applied here.