

1)

$$\frac{2}{N} < 37 < \sqrt{N} < N < N \log(\log(N)) < N \log N < N \log(N^2) \\ < N \log^2(N) < N^{1.5} < N^2 < N^4 < 2^{N/2} < 2^N$$

2)

a) $O(N) \Rightarrow \frac{2000}{500} = \frac{N}{10}$
 $N = 40 \text{ seconds}$

b) $O(N \log(N)) \Rightarrow \frac{2000 \log(2000)}{500 \log(500)} = \frac{N}{10}$

$$N = 40 \cdot \frac{\log(2000)}{\log(500)}$$

$$N = 48.9228 \text{ seconds}$$

c) $O(N^2) \Rightarrow \frac{(2000)^2}{(500)^2} = \frac{N}{10}$

$$N = 64 \times 10$$

$$N = 640 \text{ seconds}$$

3)

$$T(n) = 1 + T(n-1)$$

$$T(n-1) = 1 + T(n-2)$$

$$T(n-2) = 1 + T(n-3)$$

... and so on

This recurrence relation goes until $T(0) = 0$.

and so, this recurrence relation executes in the runtime " $O(n)$ " because each iteration takes " n " elements.

Therefore, the pseudocode for sorting takes about " $O(n)$ ".

4)

a)

Run time of

$f()$: $O(n)$: it is recursive program that runs n times for $n = 0$ to n .

$g()$: $O(n)$: it has a loop that runs n times $i = 0$ to $n-1$.

b) Space complexity:

$f()$: $O(n)$: since it is recursive calling n times, it takes n system stack.

$g()$: $O(1)$: constant.

c)

```
int h(int n) {
    return n * (n-1) / 2;
}
```

it takes $O(1)$ time and $O(1)$ Space.

5)

Recurrence relation of $f(n)$

$$T(n) = T(n/2) + O(1)$$

So, its time complexity is $O(\log n)$

Complexity of $g(n)$

loop iterates $\log(n)$ times

complexity of $f(n)$ is also $\log(n)$

hence time complexity is $\log(n) \times \log(n) = \log^2(n)$

$$\therefore \boxed{O(\log^2(n))}$$

6)

- 1) Read the value of n
- 2) Initialize a variable $k = 0$
- 3) Create a boolean array of size 10 and initialize the boolean array with false values
- 4) Repeat the loop until all the digits 0-9 are found. In other word, repeat the loop until the boolean array's values are all true.
- 5) As the loop-repeat, increment k by 1.
- 6) Multiply n with k and store the result in another variable x .
- 7) Now take each digit in x by $\text{mod}(\% 10)$ and update values in boolean array corresponding to digit as true.
- 8) Exit loop
- 9) Return k .

7)

- ① In the regular battle ship board its shape is square and we have to use two loops in order to scan the complete board.

thus efficiency $O(n \times n)$

- ② Same goes with the normal square boards too. For a regular square boards of $N \times N$ dimensions efficiency of the algorithm to scan the battle field will be $O(n \times n)$.

- ③ $N \times M$ the efficiency will be $O(n \times m)$.

8)

- [A] $O(1)$ is enough because it happens once per unit.

- [B] $O(n)$ because you have to loop through the list n times and compare each number in the list with the given number - whether the two numbers are equals.

- [C] case 1: if the list is sorted, then it is $O(1)$ because the first element is the smallest number.

case 2: if the list is not sorted; then it is $O(n)$ because you need to go through the list n times.

- [D] $O(n^2)$ because you have to check each elements.
→ loop (loop) .

- [E] $O(n)$ because you only need to go through the list once.