

# ACM-ICPC TEAM REFERENCE DOCUMENT

## Izhevsk State Technical University (Gogolev, Kismatov, Lykov)

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## 1 Common

### 1.1 template

```

optional, complex, string, vector, list, map, set, deque, queue,
stack, bitset,
algorithm, sstream, istream, iomanip, cstdio, cstdlib, ctime,
cstring, cmath,
cstdarg, cassert, ctime, tuple, unordered_set, unordered_map,
random, chrono
using namespace std;

#define ln "\n"
#define pb push_back
#define mp make_pair
#define ins insert
#define sz(x) (int)x.size()

#define All(x) (x).begin(), (x).end()
#define fi(a, b) for (auto i = (a); i <= (b); i++)
#define fdi(a, b) for (auto i = (a); i >= (b); i--)
#define fx(A) for (auto &x : (A))

typedef long long ll;
typedef vector<int> vi;
typedef vector<vi> vvi;
typedef pair<int, int> pii;

template<typename T>
ostream& operator<<(ostream& os, vector<T> v) {
    fz(v) {
        os << z << " ";
    }
    return os;
}

#ifdef LOCAL
#define dbg(x) {cerr << __LINE__ << "\t" << #x << ":\n" << (x) << ln;}
#else
#define dbg(x) {}
#endif

#ifdef LOCAL
#define ass(x) if (!x) { cerr << __LINE__ << "\tassertion failed:\n" << #x << ln; abort(); }
#else
#define ass(x) assert(x)
#endif

mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    cout.tie(0);
    #ifdef LOCAL

```

```

    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
#endif
}

```

## 1.2 compile

```

g++ --std=c++20 -DLOCAL -O2 -Wall -Wl,--stack
=67108864 -o !.exe

```

## 1.3 check

```

:start
gen.exe
sol.exe
copy output.txt answer.txt
NAME.exe
fc output.txt answer.txt
if ERRORLEVEL 1 goto end
goto start
:end

```

## 1.4 how to solve problems

1. Мат. формулировка
2. Введение параметров
3. Обращение к теории
4. Способы решения:
  - (a) Фиксация параметра
  - (b) Рассмотреть частные случаи
  - (c) Рассмотреть ответ и его свойства
  - (d) Индукция
  - (e) Выдвижение гипотезы
  - (f) Поиск закономерностей
  - (g) Упрощение
  - (h) Предположение
  - (i) Рассмотрение
  - (j) Сведение к другой задаче
  - (k) Сведение к известной задаче
  - (l) Сведение известной задачи к моей
  - (m) Моделирование
  - (n) Обобщение
  - (o) Нахождение контрпримеров
  - (p) Зарандомить (полностью)
  - (q) Найти инвариант

# 2 Geometry

## 2.1 structures

```

const ld eps = 1e-6;
struct Point {
    ld x;
    ld y;
    Point operator+(Point B) {
        return Point{x + B.x, y + B.y};
    }
    Point operator-(Point B) {
        return Point{x - B.x, y - B.y};
    }
    Point operator*(double t) {
        return Point{x * t, y * t};
    }
}

```

```

    }
    bool operator==(Point B) {
        return abs(x - B.x) <= eps && abs(y - B.y) <= eps;
    }
    Point norm() {
        ld U = sqrt(x * x + y * y);
        return Point{x / U, y / U};
    }
    ld len() {
        return sqrt(x * x + y * y);
    }
}

struct Line{
    ll A;
    ll B;
    ll C;
};

struct Rec{
    Line L;
    Seg sg;
    Point V;
};

struct Seg {
    Point a;
    Point b;
    Point to_vec() {
        return Point{b.x - a.x, b.y - a.y};
    }
};

ld dot(Point p1, Point p2) {
    return p1.x * p2.x + p1.y * p2.y;
}

ld dot(Point p1, Point p2, Point p3) {
    return dot(p2 - p1, p3 - p1);
}

ld cross(Point p1, Point p2) {
    return p1.x * p2.y - p2.x * p1.y;
}

ld cross(Point p1, Point p2, Point p3) {
    return cross(p2 - p1, p3 - p1);
}

```

## 2.2 segment intersection

```

optional<Point> intersect(Seg s1, Seg s2) {
    if (fabs(cross(s1.to_vec(), s2.to_vec())) < eps) {
        return {};
    }
    double t1 = cross((s2.a - s1.a), s2.to_vec()) / cross(s1.to_vec(), s2.to_vec());
    double t2 = cross((s1.a - s2.a), s1.to_vec()) / cross(s2.to_vec(), s1.to_vec());
    if (t1 < -eps || t1 > 1 + eps) {
        return {};
    }
    if (t2 < -eps || t2 > 1 + eps) {
        return {};
    }
    return s1.a + (s1.b - s1.a) * t1;
}

```

## 2.3 semiplane intersection

```

ld area(Point a, Point b, Point c) {
    return (ld)1.0 * ((a.x - b.x) * (a.y - c.y) - (a.x - c.x) * (a.y - b.y));
}

bool inside(Point A, Point B, Point C, Point p) {
    if (area(A, B, p) == 0 || area(B, C, p) == 0 || area(A, C, p) == 0)
        return false;
    ld v1 = area(A, B, p);
    ld v2 = area(B, C, p);
    ld v3 = area(A, C, p);
    ld S = area(A, B, C);
    v1 = abs(v1);
    v2 = abs(v2);
}

```

```

    v3 = abs(v3);
    S = abs(S);
    return abs(S - v1 - v2 - v3) <= eps;
}
Point find_bis(Point A, Point B, Point C) {
    auto v = A - B;
    auto v2 = C - B;
    v.norm();
    v2.norm();
    return v + v2;
}
ld get_acos(double x) {
    return acos(max(-1., min(1., x)));
}

Line make_line(Point a, Point b) {
    ll A = round(b.y - a.y);
    ll B = round(a.x - b.x);
    ll C = round(a.y * b.x - a.x * b.y);
    if (A < 0)
    {
        A *= (-1);
        B *= (-1);
        C *= (-1);
    }
    ll g = gcd(A, gcd(abs(B), abs(C)));
    A /= g;
    B /= g;
    C /= g;
    return Line{A, B, C};
}
Point intersect(Line a, Line b) {
    ld d = a.A * b.B - b.A * a.B;
    ld d1 = -a.C * b.B + a.B * b.C;
    ld d2 = -a.A * b.C + a.C * b.A;
    ass(abs(d) > eps);
    Point res = {d1 / d, d2 / d};
    ass(on_line(res, a));
    ass(on_line(res, b));
    return {d1 / d, d2 / d};
}
Point get_ort(Point p, Point p2) {
    return {p2.y - p.y, p.x - p2.x};
}
Point get_ort(Line a) {
    return {a.A, a.B};
}
const ld pi = get_acos(-1.0);
bool good(deque<Rec> &dq) {
    vector<Point> P;
    vector<Rec> v;
    while (sz(dq)) {
        v.pb(dq.back());
        dq.pop_back();
    }
    fi(1, sz(v) - 1) {
        P.pb(intersect(v[i - 1].L, v[i].L));
    }
    P.pb(intersect(v[0].L, v[sz(v) - 1].L));
    ld res = 0;

    P.pb(P[0]);
    ld s = 0, s2 = 0;
    fi(1, sz(P) - 1) {
        s += P[i].y * P[i - 1].x;
        s2 += P[i].x * P[i - 1].y;
    }
    res = abs(s - s2);

    return res > 0 + eps;
}
bool paral(Line a, Line b) {
    return a.A * b.B - a.B * b.A == 0;
}
bool fun(ll T) {
    if (T == 0)
        return true;
    vector<Rec> vec(n + 1);
    set<tuple<ll, ll, ll>> t;
    fi(1, n) {
        Point p = a[i];
        Point p2 = a[go(i, T + 1, n)];
        vec[i] = Rec{make_line(p, p2), Seg{p, p2}, get_ort(p, p2)};
        if (area(p, p2, {p.x + 1'000'000 * vec[i].V.x, p.y + 1'000'000 * vec[i].V.y}) > 0 + eps) {
            vec[i].V = vec[i].V * -1;
        }
    }
}

```

```

        // vec[i].V.norm();
        // t.ins({vec[i].L.A, vec[i].L.B, vec[i].L.C});
    }
    // ass(sz(t) == n);
    vector<Rec> top, bot;
    fi(1, n) {
        if (vec[i].V.y > 0 + eps || (abs(vec[i].V.y) <= eps &&
            vec[i].V.x > 0 + eps)) {
            bot.pb(vec[i]);
        } else {
            top.pb(vec[i]);
        }
    }
    sort(All(top), [](Rec &a, Rec &b)
        { return area(a.V, b.V, {0, 0}) <= eps; });
    sort(All(bot), [](Rec &a, Rec &b)
        { return area(a.V, b.V, {0, 0}) + eps >= 0; });
    vec = vector<Rec>(1);
    fz(bot) vec.pb(z);
    reverse(All(top));
    fz(top) vec.pb(z);
    deque<Rec> dq;
    dbg(vec);
    fi(1, n) {
        while (sz(dq) >= 2) {
            auto rec = dq.back();
            dq.pop_back();
            auto rec2 = dq.back();
            auto P = intersect(rec.L, rec2.L);
            dbg(P);
            if (area(vec[i].sg.a, vec[i].sg.b, P) <= eps) {
                dq.push_back(rec);
                break;
            } else {
                continue;
            }
        }

        while (sz(dq) >= 2) {
            auto rec = dq.front();
            dq.pop_front();
            auto rec2 = dq.front();
            Point P = intersect(rec.L, rec2.L);
            dbg(P);
            dbg(area(vec[i].sg.a, vec[i].sg.b, P));
            if (area(vec[i].sg.a, vec[i].sg.b, P) <= eps) {
                dq.push_front(rec);
                break;
            } else {
                continue;
            }
        }
        dq.pb(vec[i]);
    }

    while (sz(dq) >= 3) {
        auto rec = dq.front();
        dq.pop_front();
        auto rec2 = dq.front();
        auto P = intersect(rec.L, rec2.L);
        dbg(P);
        if (area(dq.back().sg.a, dq.back().sg.b, P) <= eps) {
            dq.push_front(rec);
            break;
        } else {
            continue;
        }
    }

    while (sz(dq) >= 3) {
        auto rec = dq.back();
        dq.pop_back();
        auto rec2 = dq.back();
        auto P = intersect(rec.L, rec2.L);
        dbg(P);
        if (area(dq.front().sg.a, dq.front().sg.b, P) <= eps) {
            dq.push_back(rec);
            break;
        } else {
            continue;
        }
    }

    dbg(T);
    dbg(sz(dq));

    return sz(dq) > 2;
}

```

## 2.4 Line and circle

We say first circle center at point  $(0,0)$ . Line is  $(A, B, C)$ . Find  $x_0 = \frac{-AC}{A^2+B^2}$

$$y_0 = \frac{-BC}{A^2+B^2}$$

If dist from  $(x_0, y_0)$  to  $(0,0)$  is bigger than  $r$  then answer is zero. Otherwise if it equals to  $r$  then answer is  $(x_0, y_0)$ . Otherwise we have 2 solutions:

$$d = \sqrt{r^2 - \frac{C^2}{A^2+B^2}} \quad V = \sqrt{\frac{d^2}{A^2+B^2}} \quad a_x = x_0 + BV$$

$$a_y = y_0 - AV \quad b_x = x_0 - BV \quad b_y = y_0 + AV$$

## 2.5 Two circles

We say first circle center at point  $(0,0)$ .  $x^2 + y^2 = r_1^2$   
 $(x - x_2)^2 + (y - y_2)^2 = r_2^2$

$x^2 + y^2 = r_1^2$   $x(-2x_2) + y(-2y_2) + (x_2^2 + y_2^2 + r_1^2 - r_2^2) = 0$  Now we have new task to find intersection of line and circle:  $Ax + By + C = 0$ ,  $A = -2x_2$ ,  $B = -2y_2$ ,  $C = x_2^2 + y_2^2 + r_1^2 - r_2^2$

## 2.6 Two common tangents to circles

We have to find all tangent lines to two circles. First circle is at  $(0,0)$ .

We say first circle center at point  $(0,0)$ .  $x^2 + y^2 = r_1^2$   
 $(x - x_2)^2 + (y - y_2)^2 = r_2^2$

$x^2 + y^2 = r_1^2$   $x(-2x_2) + y(-2y_2) + (x_2^2 + y_2^2 + r_1^2 - r_2^2) = 0$  Now we have new task to find intersection of line and circle:  $Ax + By + C = 0$ ,  $A = -2x_2$ ,  $B = -2y_2$ ,  $C = x_2^2 + y_2^2 + r_1^2 - r_2^2$

## 3 Matrix

### 3.1 Multiply

```
Matrix mult_mtr(Matrix A, Matrix B, ll MOD) {
    vector<vll> res;
    ll n2, m2;
    n2 = A.n;
    m2 = B.m;
    res = vector<vll>(n2+1, vll(m2+1));
    fi(1, n2) {
        fj(1, m2) {
            ll cur = 0;
            fo(1, A.m) {
                cur = cur + A.mtr[i][o] * B.mtr[o][j];
            }
            res[i][j] = cur;
        }
    }
    return Matrix{n2, m2, res};
}
```

### 3.2 Gauss SLAU

```
int gauss (vector < vector<double> > a, vector<double> & ans)
{
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        fi(row, n - 1) {
```

```
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        }
        if (abs (a[sel][col]) < EPS)
            continue;
        fi(col, m) {
            swap (a[sel][i], a[row][i]);
        }
        where[col] = row;
        fi(0, n - 1) {
            if (i != row) {
                double c = a[i][col] / a[row][col];
                fj(col, m) {
                    a[i][j] -= a[row][j] * c;
                }
            }
        }
        ++row;
    }
    ans.assign (m, 0);
    fi(0, m - 1) {
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    }
    fi(0, n - 1) {
        double sum = 0;
        fj(0, m - 1) {
            sum += ans[j] * a[i][j];
        }
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    fi(0, m - 1) {
        if (where[i] == -1) return INF;
    }
    return 1;
}
```

## 4 Strings

### 4.1 Suf LCP

```
string s;
vi p, c, rc;
int x = 1;
int gett(int a, int x) {
    if (a > x) return a - x;
    return sz(s) - 1 - (x - a) % (sz(s) - 1);
}
int gett2(int a, int x) {
    if (a + x < sz(s))
        return a + x;
    return 1 + (a + x) % sz(s);
}
bool comp1(int a, int b) {
    return (s[a] < s[b]);
}
int n, S;
vvi e;
void Sort(vector<int> &p) {
    fi(1, n) {
        int t = gett(p[i], x);
        e[rc[t]].pb(t);
    }
    p.clear();
    p.pb(0);
    fi(0, n) {
        fz(e[i]) {
            p.pb(z);
        }
        e[i].clear();
    }
}
void init() {
    n = sz(s) + 1;
    s = "#" + s + (char)31;
    c = vi(n + 1);
    rc = vi(n + 1);
    fi(0, n) p.pb(i);
    sort(Allf(p), comp1);
    c[1] = 0;
```

```

fi(2, n) {
    if (s[p[i]] == s[p[i - 1]]) c[i] = c[i - 1];
    else c[i] = c[i - 1] + 1;
}
fi(1, n) {
    rc[p[i]] = c[i];
}
e = vvi(n + 1);
x = 1;
while (x < n) {
    Sort(p);
    c[1] = 0;
    fi(2, n) {
        if (rc[p[i]] == rc[p[i - 1]] && rc[gett2(p[i], x)] == rc[
            [gett2(p[i - 1], x)]) {
            c[i] = c[i - 1];
        } else {
            c[i] = c[i - 1] + 1;
        }
    }
    fi(1, n) {
        rc[p[i]] = c[i];
    }
    x *= 2;
}
}

```

## 4.2 LCP

```

vll lcm(n + 1);
ll k = 0;
fi(1, n) {
    ll x = rev[i] - 1;
    ll cnt = max(k - 1, (ll)0);
    fj(cnt, 1e5 + 10) {
        if (s[i + j] == s[arr[x] + j]) cnt++;
        else break;
    }
    if (x == 0) cnt = 0;
    lcm[x] = cnt;
    k = cnt;
}

```

## 4.3 Suf automaton

```

struct state {
    int len, link;
    map<char, int> next;
};
const int MAXLEN = 1'000'000;
state st[MAXLEN*2];
int siz, last;
void sa_init() {
    siz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    siz++;
}
void sa_extend(char c) {
    int cur = siz++;
    st[cur].len = st[last].len + 1;
    int p;
    for (p = last; p != -1 && !st[p].next.count(c); p = st[p].
        link) {
        st[p].next[c] = cur;
    }
    if (p == -1) {
        st[cur].link = 0;
    } else {
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len) {
            st[cur].link = q;
        } else {
            int clone = siz++;
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            for (; p != -1 && st[p].next[c] == q; p =
                st[p].link)
                st[p].next[c] = clone;
        }
    }
}

```

```

        st[q].link = st[cur].link = clone;
    }
    last = cur;
}

```

## 4.4 pref function

```

vi prefix_function(string s) {
    int n = sz(s);
    vi pi(n);
    fi(1, n - 1) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j]) {
            j = pi[j - 1];
        }
        if (s[i] == s[j]) j++;
        pi[i] = j;
    }
    return pi;
}

```

## 4.5 Z function

```

vi z_function(string s) {
    int n = sz(s);
    vi z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] - 1 > r) {
            l = i;
            r = i + z[i] - 1;
        }
    }
    return z;
}

```

## 4.6 Aho Corasick

```

struct Node {
    int leaf = -1;
    char c = '\0';
    Node* par = nullptr;
    map<char, Node*> nxt;
    Node* suf = nullptr;
    map<char, Node*> sufs;
    Node(char t, Node* p) {
        c = t;
        par = p;
    }
};
Node* root;
void add(string& s, int j) {
    Node* cur = root;
    fi(0, sz(s) - 1) {
        if (cur->nxt.count(s[i])) {
            cur = cur->nxt[s[i]];
        } else {
            Node* t = new Node(s[i], cur);
            cur->nxt[s[i]] = t;
            cur = t;
        }
    }
    cur->leaf = j;
}
Node* get_link(Node* a) {
    if (a->suf) return a->suf;
    if (a == root) {
        a->suf = root;
        return a->suf;
    }
    if (a->par == root) {
        return root;
    }
}

```

```

    }
    char t = a->c;
    Node* cur = get_link(a->par);
    while(cur != root && !cur->nxt.count(t)) {
        cur = get_link(cur);
    }
    if(cur->nxt.count(t)) {
        cur = cur->nxt[t];
    }
    return a->suf = cur;
}
Node* go(Node* a, char c) {
    Node* cur = a;
    while(cur != root && !cur->nxt.count(c)) {
        cur = get_link(cur);
    }
    if(cur->nxt.count(c)) {
        cur = cur->nxt[c];
    }
    return cur;
}

string s;
ll n;
vector<string> a;
void init() {
    root = new Node('\0', nullptr);
    fi(1, n) {
        add(a[i], i);
    }
}

```

## 5 Graphs

### 5.1 cutpoints

```

void dfs(ll x, ll p = -1) {
    if (color[x]) return;
    color[x] = 1;
    _time++;
    tin[x] = _time;
    tup[x] = _time;
    ll q = 0;
    bool f = 0;
    fy(e[x]) {
        if (y == p) continue;
        if (!color[y]) {
            q++;
            dfs(y, x);
            if (tup[y] >= tin[x]) {
                f = 1;
            }
            tup[x] = min(tup[x], tup[y]);
        } else {
            tup[x] = min(tup[x], tin[y]);
        }
    }
    if (p == -1) {
        if (q > 1) ans.pb(x);
    } else {
        if (f) ans.pb(x);
    }
}

```

### 5.2 bridges

```

void dfs(ll x, ll p = -1) {
    if (color[x]) return;
    color[x] = 1;
    tin[x] = _time++;
    tup[x] = tin[x];
    for (auto &to : e[x]) {
        if (to == p) continue;
        if (color[to]) {
            tup[x] = min(tup[x], tin[to]);
            continue;
        } else {
            dfs(to, x);
        }
    }
}

```

```

        tup[x] = min(tup[x], tup[to]);
        if (tup[to] > tin[x]) {
            ans.pb(t[{to, x}]);
            dbg(mp(to, x));
        }
    }
}

```

### 5.3 min cost max flow

```

bool deikstr() {
    vll d(_k + 1, INF);
    set<pll> t;
    d[S] = 0;
    t.ins({d[S], S});
    vector<Edge*> p(_k + 1, NULL);
    while (sz(t)) {
        auto [_, x] = *t.begin();
        t.erase(t.begin());
        fy(e[x]) {
            if (y->f + 1 <= y->c && d[y->y] > d[x] + y->w) {
                t.erase({d[y->y], y->y});
                d[y->y] = d[x] + y->w;
                t.ins({d[y->y], y->y});
                p[y->y] = y;
            }
        }
    }
    if (d[T] == INF) return false;
    int x = T;
    while (x != S) {
        p[x]->f++;
        p[x]->rev->f--;
        x = p[x]->rev->y;
    }
    fi(1, _k) {
        fy(e[i]) {
            y->w = y->w + d[i] - d[y->y];
        }
    }
    return true;
}

```

### 5.4 flow

```

struct Edge{
    int y;
    int f;
    int c;
    Edge* rev;
};
int _num[MAX];
int n, m;
int S, T;
int K = 0;
int num(int x) {
    if (_num[x] == 0) {
        K++;
        _num[x] = K;
    }
    return _num[x];
}
vector<vector<Edge*>>e;
void add_edge(int x, int y, int c) {
    Edge* v = new Edge{y, 0, c, NULL};
    Edge* v2 = new Edge{x, 0, c, v};
    v->rev = v2;
    e[x].pb(v);
    e[y].pb(v2);
}
vi color;
int C = 1;
bool dfs(int x) {
    if (x == T) return true;
    if (color[x]) return false;
    color[x] = 1;
    fy(e[x]) {
        if (y->f + C <= y->c && dfs(y->y)) {

```

```

        y->f += C;
        y->rev->f -= C;
        return true;
    }
    return false;
}

while(C >= 1){
    C = (1 << 30);
    while(C >= 1) {
        color = vi(n+1);
        while(dfs(S)) {
            color = vi(n+1);
        }
        C/=2;
    }
}

```

## 5.5 Kuhn algorithm

```

vi u;
vi p;
int CUR = 1;
bool dfs(int v){
    if(u[v] == CUR)return false;
    u[v] = CUR;
    fz(e3[v]){
        if(p[z] == -1 || dfs(p[z])){
            p[z] = v;
            return true;
        }
    }
    return false;
}

```

## 5.6 Dinic flow

```

const int MAXN = ...;
const int INF = 1000000000;

struct edge {
    int a, b, cap, flow;
};

int n, s, t, d[MAXN], ptr[MAXN], q[MAXN];
vector<edge> e;
vector<int> g[MAXN];

void add_edge (int a, int b, int cap) {
    edge e1 = { a, b, cap, 0 };
    edge e2 = { b, a, 0, 0 };
    g[a].push_back ((int) e.size());
    e.push_back (e1);
    g[b].push_back ((int) e.size());
    e.push_back (e2);
}

bool bfs() {
    int qh=0, qt=0;
    q[qt++] = s;
    memset (d, -1, n * sizeof d[0]);
    d[s] = 0;
    while (qh < qt && d[t] == -1) {
        int v = q[qh++];
        for (size_t i=0; i<g[v].size(); ++i) {
            int id = g[v][i],
                to = e[id].b;
            if (d[to] == -1 && e[id].flow < e[id].cap) {
                q[qt++] = to;
                d[to] = d[v] + 1;
            }
        }
    }
    return d[t] != -1;
}

int dfs (int v, int flow) {
    if (!flow) return 0;

```

```

    if (v == t) return flow;
    for (; ptr[v]<(int)g[v].size(); ++ptr[v]) {
        int id = g[v][ptr[v]],
            to = e[id].b;
        if (d[to] != d[v] + 1) continue;
        int pushed = dfs (to, min (flow, e[id].cap - e[id].flow));
        if (pushed) {
            e[id].flow += pushed;
            e[id^1].flow -= pushed;
            return pushed;
        }
    }
    return 0;
}

int dinic() {
    int flow = 0;
    while(1) {
        if (!bfs()) break;
        memset (ptr, 0, n * sizeof ptr[0]);
        while (int pushed = dfs (s, INF))
            flow += pushed;
    }
    return flow;
}

```

## 5.7 dijkstra

```

vector<int> dijkstra(int s){
    vector<int> d(n + 1, INF);
    d[s] = 0;
    set<pair<int, int>> t;
    t.insert(mp(d[s], s));
    vector<int> color(n + 1, 0);
    while(sz(t)) {
        auto[_ , x] = *t.begin();
        t.erase(t.begin());
        if(color[x]) continue;
        color[x] = true;
        for(auto &[y, w] : e[x]) {
            if(d[y] > d[x] + w) {
                d[y] = d[x] + w;
                t.insert(mp(d[y], y));
            }
        }
    }
    return d;
}

```

# 6 Data Structures

## 6.1 fenwick Tree

```

// Linear
vll fen;
void add(int p, ll val) {
    for(int i = p; i <= n; i = (i | (i + 1))) {
        fen[i] += val;
    }
}

ll sum(int p) {
    ll res = 0;
    for(int i = p; i >= 0; i = (i & (i + 1)) - 1) {
        res += fen[i];
    }
    return res;
}

// Matrix
vvll fen;
void add(int x, int y, ll val) {
    for(int i = x; i <= n; i = (i | (i + 1))) {
        for(int j = y; j <= n; j = (j | (j + 1))) {
            fen[i][j] += val;
        }
    }
}

```

```

ll sum(int x, int y) {
    ll res = 0;
    for(int i = x; i >= 0; i = (i & (i + 1)) - 1) {
        for (int j = y; j >= 0; j = (j & (j + 1)) - 1) {
            res += fen[i][j];
        }
    }
    return res;
}

```

## 6.2 treap

```

struct Node {
    ll val;
    ll cnt;
    ll y;
    Node *l;
    Node *r;
};

void upd(Node *a) {
    a->cnt = 1;
    if (a->l)
        a->cnt += a->l->cnt;
    if (a->r)
        a->cnt += a->r->cnt;
}

Node *merge(Node *a, Node *b) {
    if (!a)
        return b;
    if (!b)
        return a;
    if (a->y > b->y) {
        a->r = merge(a->r, b);
        upd(a);
        return a;
    }
    b->l = merge(a, b->l);
    upd(b);
    return b;
}

pair<Node *, Node *> split(Node *a, ll q) {
    if (!a)
        return {0, 0};
    ll left = 0;
    if (a->l)
        left += a->l->cnt;
    if (left >= q) {
        auto [l, r] = split(a->l, q);
        a->l = r;
        upd(a);
        return {l, a};
    }
    auto [l, r] = split(a->r, q - left - 1);
    a->r = l;
    upd(a);
    return {a, r};
}

```

## 6.3 Long

```

struct Long {
    int base = 10000;
    int len = 4;
    vi num = {1, 0};
    int operator[](int x) const {
        if (x > num[0]) return 0;
        return num[x];
    }
    int& operator[](int x) {
        if (x > num[0]) {
            num.resize(x + 1, 0);
            num[0] = x;
        }
        return num[x];
    }
};

Long operator+(const Long& A, const Long& B) {
    Long res;

    int n = max(A[0], B[0]);

```

```

    fi(1, n) {
        res[i] = A[i] + B[i];
        if (i > 1) {
            res[i] += res[i - 1] / res.base;
            res[i - 1] %= res.base;
        }
    }
    while(res[res[0]] >= res.base) {
        int m = res[0];
        res[m + 1] = res[m] / res.base;
        res[m] %= res.base;
    }
    return res;
}

Long operator*(const Long& A, const int& B) {
    Long res;
    int n = A[0];
    fi(1, n) {
        res[i] = A[i] * B;
        if (i > 1) {
            res[i] += res[i - 1] / res.base;
            res[i - 1] %= res.base;
        }
    }
    while(res[res[0]] >= res.base) {
        int m = res[0];
        res[m + 1] = res[m] / res.base;
        res[m] %= res.base;
    }
    return res;
}

Long operator*(const Long& A, const Long& B) {
    Long res;
    fi(1, A[0]) {
        if (A[i] == 0) continue;
        fj(1, B[0]) {
            if (A[i] * B[j] == 0) continue;
            int x = i + j - 1;
            res[x] += A[i] * B[j];
            while(x > 1 && res[x - 1] >= res.base) {
                res[x] += res[x - 1] / res.base;
                res[x - 1] %= res.base;
            }
        }
        while(res[res[0]] >= res.base) {
            int m = res[0];
            res[m + 1] += res[m] / res.base;
            res[m] %= res.base;
        }
    }
    return res;
}

pair<Long, int> operator/(const Long& A, const int B) {
    Long res;
    ll x = 1;
    ll d = 0, m = 0;
    fdi(A[0], 1) {
        ll val = m * res.base + A[i];
        d = val / B;
        m = val % B;
        if (x == 1 && d == 0) continue;
        res[x] = d;
        x++;
    }
    reverse(Allf(res.num));
    return mp(res, m);
}

void print(const Long& A) {
    printf("%d", A[A[0]]);
    fdi(A[0] - 1, 1) {
        printf("%04d", A[i]);
    }
    printf("\n");
}

Long str_to_Long(const string& s) {
    int n = sz(s);
    Long res;
    int x = 1;
    for(int i = n - 1; i >= 0; i -= 4) {
        int val = 0;
        for(int j = max(0, i - 3); j <= i; j++) {
            val = val * 10 + (s[j] - '0');
        }
        res[x] = val;
        x++;
    }
    return res;
}

```



```

Long int _to_Long(int num) {
    Long res;
    int x = 1;
    while(num) {
        int val = num % res.base;
        num /= res.base;
        res[x] = val;
        x++;
    }
    return res;
}

```

## 6.4 persistent treap

```

int next(int l,int r){
    return l + rnd()%(r - l + 1);
}
const int MAX = 1e7;

int siz = 1;
vi Left(MAX,-1),Right(MAX,-1),Siz(MAX,0),Val(MAX,0);

void upd(int v){
    if(v == -1)return;
    Siz[v] = 1;
    if(Left[v] != -1)Siz[v] += Siz[Left[v]];
    if(Right[v] != -1)Siz[v] += Siz[Right[v]];
}

bool INIT;
int cop(int v){
    if(INIT)return v;
    int x = siz;
    siz++;
    Val[x] = Val[v];
    Left[x] = Left[v];
    Right[x] = Right[v];
    Siz[x] = Siz[v];
    return x;
}

int merg(int L,int R){
    if(L == -1)return R;
    if(R == -1)return L;

    if(next( 1, (Siz[L] + Siz[R])*100 ) <= Siz[L] * 100){
        L = cop(L);
        Right[L] = merg(Right[L], R);
        upd(L);
        return L;
    }else{
        R = cop(R);
        Left[R] = merg(L,Left[R]);
        upd(R);
        return R;
    }
}

pair<int,int> split(int v,int k){
    if(Siz[v] <= k){
        return mp(v,-1);
    }
    if(k == 0){
        return mp(-1,v);
    }
    v = cop(v);
    int siz = 0;
    if(Left[v] != -1)siz = Siz[Left[v]];
    if(siz >= k){
        auto t = split(Left[v],k);
        Left[v] = t.second;
        upd(v);
        return mp(t.first,v);
    }else{
        auto t = split(Right[v], k - siz - 1);
        Right[v] = t.first;
        upd(v);
        return mp(v,t.second);
    }
}

int root = -1;
void seg_copy(int i,int j,int k){

```

```

        auto t = split(root,j - 1);
        auto g = split(root,i - 1);
        auto h = split(g.second,k);
        auto f = split(t.second,k);
        root = merg(t.first,merg(h.first,f.second));
    }

    int n,m;
    vi cnt,from,to;
    vi res;

    void solve() {
        INIT = true;
        fi(1, n){
            int v = siz++;
            upd(v);
            Val[v] = i;

            root = merg(root,v);
        }
        INIT = false;
    }
}

```

## 7 DP

### 7.1 fft

```

using cd = complex<double>;
const double PI = acos(-1);
int reverse(int num, int lg_n) {
    int res = 0;
    for (int i = 0; i < lg_n; i++) {
        if (num & (1 << i))
            res |= 1 << (lg_n - 1 - i);
    }
    return res;
}

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    int lg_n = 0;
    while ((1 << lg_n) < n)
        lg_n++;

    for (int i = 0; i < n; i++) {
        if (i < reverse(i, lg_n))
            swap(a[i], a[reverse(i, lg_n)]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd & x : a) {
            x /= n;
        }
    }
}

vector<int> multiply(vector<int>const& a, vector<int>const&
    b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < sz(a) + sz(b)) {
        n <<= 1;
    }
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
}

```

```

vector<int> result(n);
for (int i = 0; i < n; i++)
    result[i] = round(fa[i].real());
for( int i = 0; i < n; i++) {
    result[i] = min(result[i], 5);
}
while(result.back() == 0) result.pop_back();
return result;
}

```

## 7.2 convex hull trick

```

struct Line {
    mutable ll k, m, p;
    bool operator<(const Line &o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = INF;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end())
            return x->p = inf, 0;
        if (x->k == y->k)
            x->p = x->m > y->m ? inf : -inf;
        else
            x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z))
            z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

```

## 8 Comb

### 8.1 basic

$$\begin{aligned}
 Cat(n, k) &= \frac{1}{n+1} C(2n, n) \\
 N(n, k) &= \frac{1}{n} C(n, k) C(n, k-1) \\
 \sum_{n=0}^{\infty} \sum_{k=1}^n N(n, k) z^n t^k &= \frac{1+z(t-1) - \sqrt{1-2z(t+1)+z^2(t-1)^2}}{2z} \\
 S(n, k) &= \sum_{j=0}^{k-1} (-1)^j C(k, j) \cdot (m - j)^n O(k \log n)
 \end{aligned}$$

## 9 Math

### 9.1 Simpson

```

double a, b;
const int n = 1000*1000;
double s = 0;
double h = (b - a) / N;
fi(0, n) {
    double x = a + h * i;
    s += f(x) * ((i==0 || i==N) ? 1 : ((i&1)==0) ? 2 : 4);
}

```

```

}
s *= h / 3;

```

### 9.2 2 SAT

```

ll n;
vvi e;
vi color;
vvi e2;
vvi _num;
int id = 1;

int num(int v, int t) {
    if(!_num[v][t]) _num[v][t] = id++;
    return _num[v][t];
}

vvi e2r;
vi u;
vi ord;
void dfs2(int v) {
    if(u[v]) return;
    u[v] = 1;
    fz(e2[v]) {
        dfs2(z);
    }
    ord.pb(v);
}

int comp = 0;
vi component;
void dfs3(int v) {
    if(component[v]) return;
    component[v] = comp;
    fz(e2r[v]) {
        dfs3(z);
    }
}

vi ans;
bool can() {
    ord.clear();
    ans.clear();
    e2r = vvi(4*n + 1);
    comp = 1;
    fi(1, 4*n) {
        fz(e2[i]) {
            e2r[z].pb(i);
        }
    }

    u = vi(4*n + 1);
    fi(1, 4*n) {
        dfs2(i);
    }
    dbg("S");
    component = vi(4*n + 1);

    fdi(sz(ord) - 1, 0) {
        if(!component[ord[i]]) {
            dfs3(ord[i]);
            comp++;
        }
    }
    dbg("D");
    fi(1, 2*n) {
        if(component[num(i, 1)] == component[num(i, 2)])
            return false;
        if(component[num(i, 1)] > component[num(i, 2)])
            ans.pb(i);
    }
    return true;
}

```

### 9.3 gcd

```

int gcd (int a, int b, int & x, int & y) {
    if (a == 0) {
        x = 0; y = 1;
        return b;
    }
    int x1, y1;
    int d = gcd (b%a, a, x1, y1);
}

```

```

    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}

```

## 9.4 Diofant

```

void shift_solution (int & x, int & y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}

int find_all_solutions (int a, int b, int c, int minx, int maxx,
    int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution (a, b, c, x, y, g))
        return 0;
    a /= g; b /= g;

    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;

    shift_solution (x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution (x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    int lx1 = x;

    shift_solution (x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution (x, y, a, b, -sign_b);
    int rx1 = x;

    shift_solution (x, y, a, b, - (miny - y) / a);
    if (y < miny)
        shift_solution (x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    int lx2 = x;

    shift_solution (x, y, a, b, - (maxy - y) / a);
    if (y > maxy)
        shift_solution (x, y, a, b, sign_a);
    int rx2 = x;

    if (lx2 > rx2)
        swap (lx2, rx2);
    int lx = max (lx1, lx2);
    int rx = min (rx1, rx2);

    return (rx - lx) / abs(b) + 1;
}

```

## 9.5 chinese theorem

$$\sum_{i=1}^n a_i \left( \frac{M}{m_i} \right) \varphi(m_i)$$

## 9.6 joseph

```

int joseph (int n, int k) {
    if (n == 1) return 0;
    if (k == 1) return n-1;
    if (k > n) return (joseph (n-1, k) + k) % n;
    int cnt = n / k;
    int res = joseph (n - cnt, k);
    res -= n % k;
    if (res < 0) res += n;
    else res += res / (k - 1);
    return res;
}

```

## 9.7 discret log

```

int solve (int a, int b, int m) {
    int n = (int) sqrt (m + .0) + 1;
    int cur;
    int an = 1;
    fi(0, n - 1) {
        an = (an * a) % m;
    }

    map<int,int> vals;

    cur = an;
    fi(1, n) {
        if (!vals.count(cur))
            vals[cur] = i;
        cur = (cur * an) % m;
    }
    cur = b;
    fi(0, n) {
        if (vals.count(cur)) {
            int ans = vals[cur] * n - i;
            if (ans < m) return ans;
        }
        cur = (cur * a) % m;
    }
    return -1;
}

```