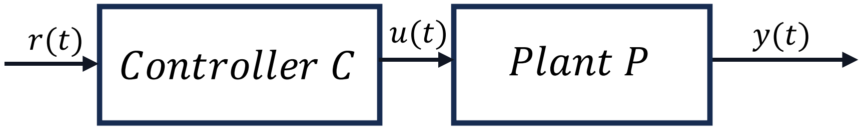
CS1 Week 2 Annotated

Modeling, Model examples

1. Recap Last week

Open-Loop System

- System Σ is split into controller C and plant P.
- A ***controller*** is a device or algorithm that determines the necessary inputs to achieve a desired system behavior, while a ***plant*** is the physical system or process being controlled.



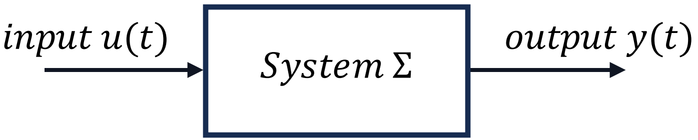
Open Loop vs. Closed Loop

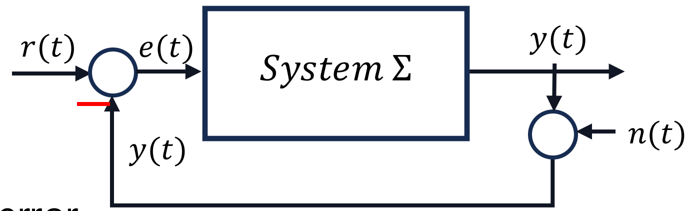
Open Loop:

- no feedback / input doesn't depend on output
- simple, but imprecise – controller follows input and doesn't know output

Closed Loop:

- feedback / input may depend on output
- powerful, but also complicated.
- difficult to control





control gain k: strength of controller due to experienced error
disturbance and noise: unwanted signals that interfere with input/output and hence our system → **can lead to instability of system**

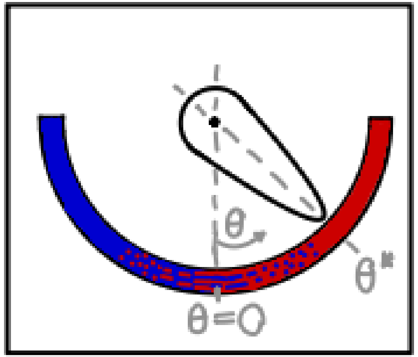
Example Shower Temperature

open loop approach

- goal: set shower to optimal Temp T^*
- we know: $T(\Theta) = k * \Theta + T_0$
- solve for angle Θ^* that corresponds to T^*:

$$\Theta^* = \frac{1}{k(T^* - T_0)}$$

- In principle we just need to set the angle to Θ^* and the system works



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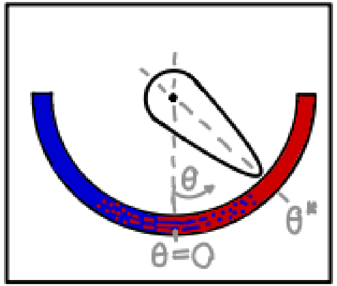

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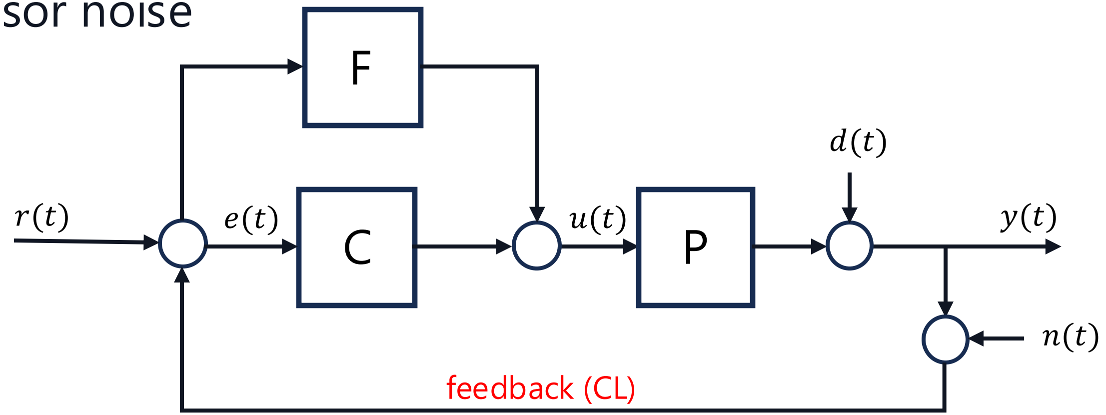
as you can see feedforward control works, but let's say the Temp. function wasn't quite right and the factor k was wrong, we'll set our angle Θ^* to and realise it's not our desired T^*. What now? With OL control our hands are tied and there is nothing we can do to compensate the **error**

Real System: Combination of CL & OL

- 2 degrees of freedoch F,C (Feedforward & Feedback)
- combine OL & CL to have fast and robust control
- disturbance d(t) directly affects output
- noise n(t) is sensor noise



Exam Question 2018WA6

You are designing a speed controller for a car. You can measure the current speed v of the car and can input the position of the gas pedal p_{pedal}. Which type of controller solution would ***not*** work in this scenario?

A. Feedforward controller

B. Feedforward and feedback controller

C. All other mentioned options would not work

D. Feedback controller

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- A. Feedforward controller
- B. Feedforward and feedback controller
- C. All other mentioned options would not work
- D. Feedback controller

with only feed forward we'd need a law that gives us for a desired speed the exact position of the gas pedal. If that law isn't entirely correct there's nothing we can do to correct it. Even if the law is correct what if suddenly the wind or the slope changes we wouldn't be able to react to that.

Quiz 1

What does SISO stand for?

A. multiple input single output

B. single input single output

C. super important system operation

D. simple input spaghetti output

Quiz 1: Solution

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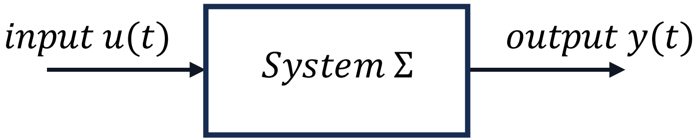

Quiz 2

What does MIMO stand for?

- **A.** multiple input single output
- **B.** mostly incoherent mathematical operations
- **C.** multiple input multiple output
- **D.** music in music out

Quiz 2: Solution

What does MIMO stand for?

A. multiple input single output

B. mostly incoherent mathematical operations

C. multiple input multiple output

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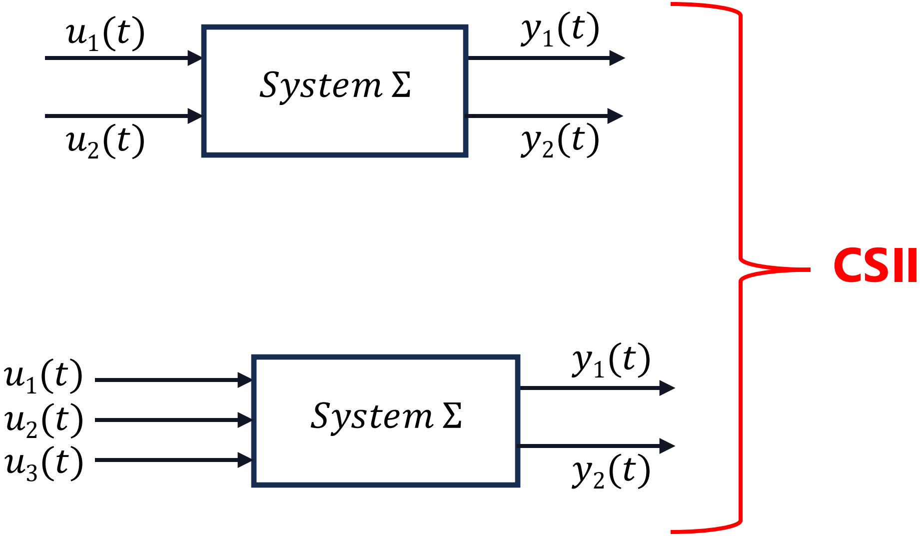

Quiz 3

What are the basic three objectives of a controller?

A. Linearity, Predictability, Affordability

B. Performance, Robustness, Stability

C. Data Analysis, Efficiency, Tracking

D. Stability, Performance, Linearity

Quiz 3: Solution

What are the basic three objectives of a controller?

A. Linearity, Predictability, Affordability

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C. Data Analysis, Efficiency, Tracking

D. Stability, Performance, Linearity

Quiz 4

What is the meaning of the state vector $x(t)$?

A. How the system changes internally over time.

B. Represents the memory of a system, i.e. the summary of the effects of all past inputs.

C. The effects of the outside world on the system.

Quiz 4

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C. The effects of the outside world on the system.

Course Schedule

	Subject	Week
	Introduction, Control Architectures, Motivation	1
Modeling	Modeling, Model examples	2
	System properties, Linearization	3
	Analysis: Time response, Stability	4
	Transfer functions 1: Definition and properties	5
	Transfer functions 2: Poles and Zeros	6
Analysis	Proportional feedback control, Root Locus	7
	Time-Domain specifications, PID control, Computer implementation	8
	Frequency response, Bode plots	9
	The Nyquist condition, Time delays	10
	Frequency-domain Specifications, Dynamic Compensation, Loop Shaping	11
Synthesis	Time delays, Successive loop closure, Nonlinearities	12
	Describing functions	13
	Intro to Uncertainty and Robustness	14

Today

1. Modeling
2. Interconnection Basics / Block diagrams

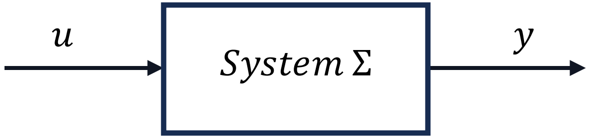
1. Modeling

disclaimer: there were entire courses only talking about system modeling. What we'll cover here is just a short introduction. In the exam you don't have to actually model a system, but you'll need to match the right set of equations or definitions to a given system.

Modeling

IMPORTANT:
All models are wrong,
but some are useful.

We want to learn how to mathematically represent dynamic systems. Specifically we want to write down equations that express the output as a function of the input and some internal parameters.



Inputs can be:

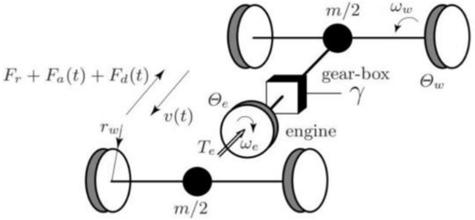
- Endogenous: can be manipulated by the designer, e.g. control inputs
- Exogenous: generated by the environment and can't be controlled, e.g. *disturbances*

Outputs (what we observe over time) can be classified as:

- Measured outputs: what we can measure (sensors), e.g. speed of a car
- Performance outputs: not directly measurable, but we want to control, e.g. avg fuel consumption

Basic Idea

abstraction



Physical first principles

$Plant\ P$

We are doing *model-based* control

Ways of finding a model:

1. White box: Apply physical first principles (Dynamics, Thermo, etc.)
2. Black box: Observe input and output behaviour of a system with unknown dynamics
3. Grey box: mix between white and black box. Physical first princples with unknown parameters

Modeling

We want to describe the system with the **state-space form:**

$$\boxed{\begin{aligned} \dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{f}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t})) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{f}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t})) \end{aligned}}$$

$\boldsymbol{x}(\boldsymbol{t})$: state → a physical observable that evolves with e.g speed, position
→ desribes system's evolution
→ you don't/can't measure a state

procedure:

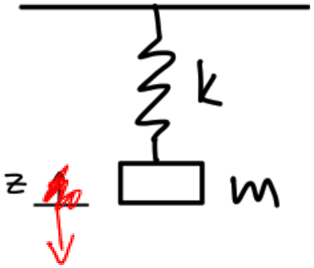
1. derive the ODE describing your system
   - LMB: $m\ddot{x}(t) = \sum F$
   - AMB: $J\ddot{\theta}(t) = \sum T$
   - Reservoirmethod: $\frac{d}{dt}[\text{relevant quantity}] = \sum \text{flows in} - \sum \text{flows out}$
2. identify input and output
3. write down state space form

note: the degree of your ODE tells you about the number of states

Hint: Use $x(t) = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$

Example

Modeling a simple spring oscillator



LMB: $\sum F = ma$

$mg - kz = m\ddot{z}$

$\ddot{z} = g - \frac{k}{m} z$

in terms of states $x(t)$

$x(t) = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

state-space:

$\frac{d}{dt} x(t) = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{k}{m} x_1 \end{bmatrix}$

Exam Question 2017WA6

u: input "what we can influence"
y: output "what we measure"
r: reference "our goal"
x: state "changes with time"

Description: Consider an electric motor which you would like to operate at a constant rotational speed ω_0. Applying a voltage $U(t)$ results in a change in the circuit current $I(t)$, which is governed by the differential equation

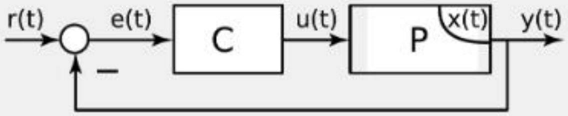
$$L \cdot \frac{d}{dt} I(t) = -R \cdot I(t) - \kappa \cdot \omega(t) + U(t) \ , \qquad (1)$$

whereby L is the circuit inductance, R its resistance and κ a constant relating the motor speed $\omega(t)$ to an electro motor-force (EMF). The dynamics of the motor speed are given by

$$\Theta \cdot \frac{d}{dt} \omega(t) = -d \cdot \omega(t) + T(t) \ , \qquad (2)$$

where Θ represents its mechanical inertia, d a friction constant and $T(t) = \kappa \cdot I(t)$ the current-dependent motor torque.

Box 2: Question 6



Question 6 *Choose the correct answer. (1 Point)*

Relate the variables in the block diagram above to the correct signals.

A $u(t) = U(t), x(t) = \begin{bmatrix} \omega(t) \\ I(t) \end{bmatrix}, y(t) = \omega(t), r(t) = \omega_0$

B $u(t) = U(t), x(t) = \begin{bmatrix} U(t) \\ T(t) \end{bmatrix}, y(t) = T(t), r(t) = \omega_0$

C $u(t) = U(t), x(t) = \begin{bmatrix} \omega(t) \\ I(t) \end{bmatrix}, y(t) = I(t), r(t) = \omega_0$

D $\cancel{u(t) = I(t)}, x(t) = \begin{bmatrix} \omega(t) \\ U(t) \end{bmatrix}, y(t) = I(t), r(t) = \omega_0$

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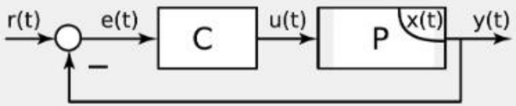

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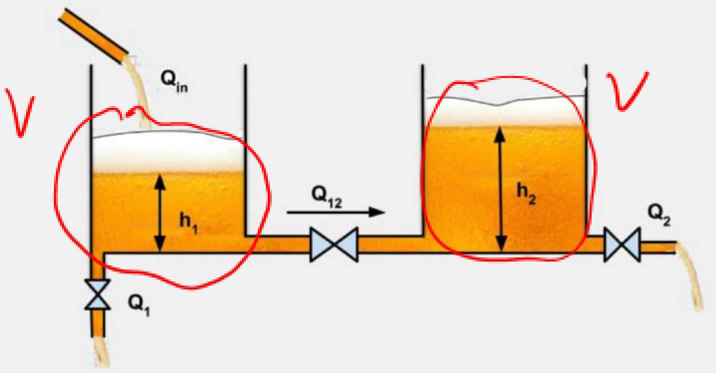
→ A)

Reservoirmethod: $\frac{d}{dt}$[relevant quantity] $= \sum$flows in $- \sum$flows out

Exam Question 2018WA7



You have decided to brew some beer and you want to go big. So you bought a tank system (shown in the Figure) that you will use for processing and storage. However, you need to model it first. Input into your system is the flow of beer into the first tank Q_{in}, h_1 and h_2 are heights of the beer in tank 1 and 2, respectively. Surface area of tank 1 is A_1 and the surface area of tank 2 is A_2. Each valve produces a flow through it that is dependent on the relative difference of heights of the fluid on both of its sides. The flow through the valve i is then described as $Q_i = K_i\sqrt{2g\Delta h}$ (in m^3/s) where K_i is a constant that is associated to valve i (assumed to be given in the correct unit), $g = 9.81\,m/s^2$ refers to the gravitational acceleration and Δh is the height difference on both sides of the valve.

A

$$A_1\frac{dh_1}{dt} = Q_{in} - Q_1 - Q_{12}$$
$$A_2\frac{dh_2}{dt} = Q_{12} - Q_2$$
$$Q_1 = K_1\sqrt{2gh_1}$$
$$Q_2 = K_2\sqrt{2gh_2}$$
$$\boxed{Q_{12} = K_{12}\sqrt{2g|h_1 - h_2|}\,sign(h_1 - h_2)}$$

B

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$$\boxed{Q_{12} = K_{12}\sqrt{2g(h_1 - h_2)}}$$

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$$\frac{dh_2}{dt} = Q_{12} - Q_2$$
$$Q_1 = K_1\sqrt{2gh_1}$$
$$Q_2 = K_2\sqrt{2gh_2}$$
$$Q_{12} = K_{12}\sqrt{2g(h_1 - h_2)}$$

$$\frac{dV_1}{dt} = A_1\frac{dh_1}{dt} = Q_{in} - Q_{12} - Q_1$$
$$A_2\frac{dh_2}{dt} = Q_{12} - Q_2$$

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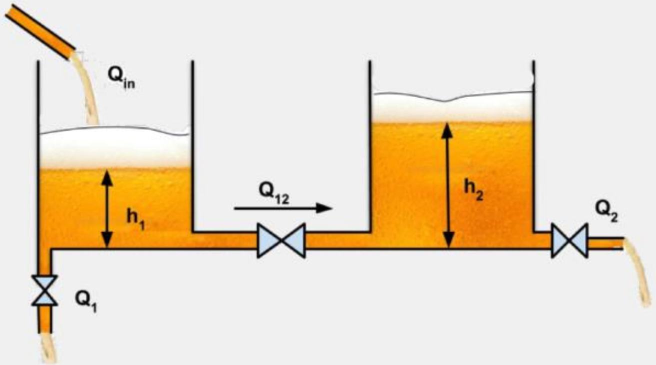
Exam Question: Solution



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$$

D

$$
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\end{aligned}
$$

→ **A)**

2. Interconnection Basics / Block diagrams

Interconnection Basics

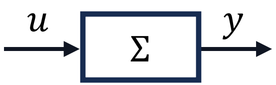
Σ : sigma (🗿) stands for a controller /plant/system

in MIMO systems they are matrices ($AB \neq BA$) (CSII)

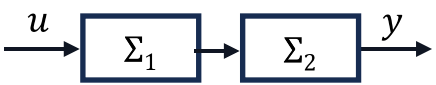
in SISO systems they are scalars ($ab = ba$)

→ it doesn't matter if you write $\Sigma_2\Sigma_1$ or $\Sigma_1\Sigma_2$!

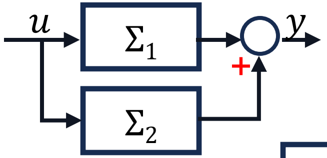
- Basic connection $y = \Sigma u$



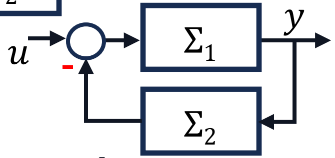
- Serial connection $y = \Sigma_2\Sigma_1 u$



- Parallel connection $y = (\Sigma_1 + \Sigma_2)u$



- Negative Feedback $y = \frac{\Sigma_1}{(1+ \Sigma_2\Sigma_1)} u$



→ our goal is to find a function that relates u to y → $y = \Sigma_{gen} u$

Why do we need this?

Modeling

- Provides precise, quantitative representation of the system.
- Equations describe dynamics, inputs, states, outputs.

Block Diagrams

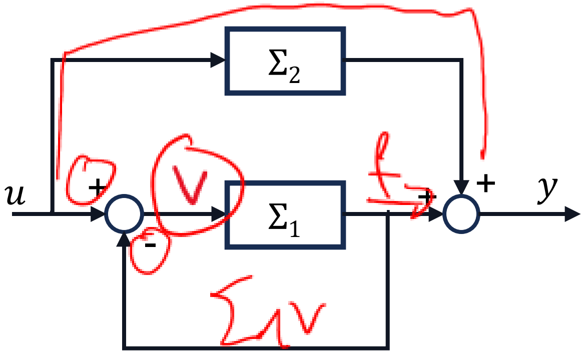
- Offer a visual and intuitive understanding of the system.
- Show how subsystems interact and how signals flow.

Together

- diagrams explain *how the system is organized*, while equations predict *how the system behaves*.

Example

find the transfer function from u → y



$$y = \Sigma_2 u + f$$

$$f = \Sigma_1 v$$

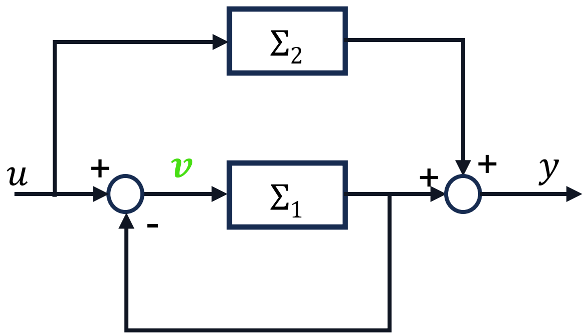
$$\boxed{v = u - \Sigma_1 v}$$

$$(1+\Sigma_1)v = u \rightarrow \underline{v} = \frac{u}{1+\Sigma_1}$$

$$y = \Sigma_2 u + \Sigma_1 v = \Sigma_2 u + \frac{\Sigma_1}{1+\Sigma_1} u$$

Example: Solution

find the transfer function from u → y



$$v = u - \Sigma_1 v$$

$$u = v + \Sigma_1 v = (1 + \Sigma_1) v$$

$$v = \frac{1}{1 + \Sigma_1} u$$

$$y = \Sigma_2 u + \Sigma_1 v$$

$$y = \Sigma_2 u + \frac{\Sigma_1}{1 + \Sigma_1} u$$

$$\Rightarrow \; \Sigma_{u \to y} = \frac{\Sigma_1}{1 + \Sigma_1} + \Sigma_2$$

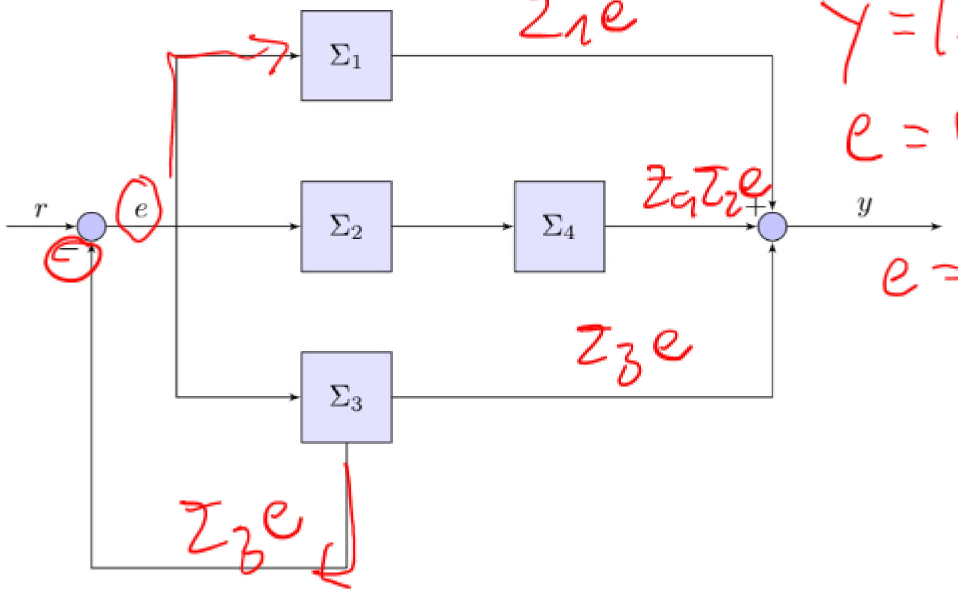
Exam Question 2018WA5

1. $y = \dots$
2. $e = \dots$

$y = (\Sigma_1 + \Sigma_4 \Sigma_2 + \Sigma_3) e$

$e = r - \Sigma_3 e$

$e = \dfrac{r}{1+\Sigma_3}$



You are given the above system diagram. What is the associated transfer function from $r \to y$.

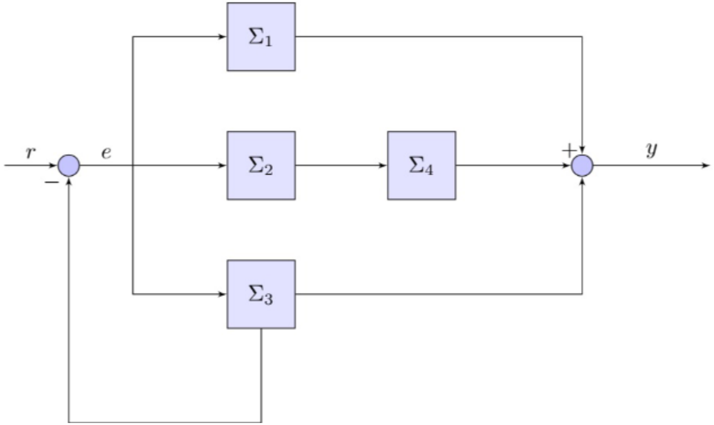
A $\Sigma_{r\to y} = (\Sigma_1 + \Sigma_2\Sigma_4 + \Sigma_3)$

B $\Sigma_{r\to y} = \frac{(\Sigma_1+\Sigma_2\Sigma_4+\Sigma_3)}{1+\Sigma_2\Sigma_4}$

C $\Sigma_{r\to y} = \frac{(\Sigma_1+\Sigma_2\Sigma_4+\Sigma_3)}{1+\Sigma_1+\Sigma_2\Sigma_4+\Sigma_3}$

D $\Sigma_{r\to y} = \frac{(\Sigma_1+\Sigma_2\Sigma_4+\Sigma_3)}{1+\Sigma_3}$

Exam Question: Solution



$$e = r - \Sigma_3 e$$

$$r = e + \Sigma_3 e = (1 + \Sigma_3)e$$

$$e = \frac{1}{1 + \Sigma_3} r$$

$$y = (\Sigma_1 + \Sigma_4\Sigma_2 + \Sigma_3)e$$

$$y = \underbrace{\frac{\Sigma_1 + \Sigma_4\Sigma_2 + \Sigma_3}{1 + \Sigma_3}}_{\Rightarrow \boldsymbol{\Sigma_{r\to y}}} r$$

→ D)

Problem Set

PS01

- Problem 1 (Water Tank):
- Problem 2 (Longitudinal Dynamics of a Car)
- Problem 3 (Inverted Pendulum)
- Problem 4 (Thermal Model of two Houses)
- Problem 5 (Block Diagram Algebra)

Feedback?

Too fast? Too slow? Less theory, more exercises?

I would appreciate your feedback. Please let me know.

https://docs.google.com/forms/d/e/1FAIpQLSdHl0kjWo63aNzDkAV0cnmQadCAj5L0D7v7aSh0BK7BBdEgpA/viewform?usp=header