

1) Calcular as derivadas das expressões abaixo, usando as fórmulas de derivação:

a) $y = x^2 + 4x$

R: $\frac{dy}{dx} = 2x + 4$

b) $f(x) = \frac{2}{x^2}$

R: $f'(x) = -\frac{4}{x^3}$

c) $y = \frac{x^3}{2} + \frac{3x}{2}$

R: $\frac{dy}{dx} = \frac{3}{2}(x^2 + 1)$

d) $y = \sqrt[3]{x}$

R: $\frac{dy}{dx} = \frac{1}{3\sqrt[3]{x^2}}$

e) $f(x) = \left(3x + \frac{1}{x}\right) \cdot (6x - 1)$

R: $\frac{df(x)}{dx} = 36x + \frac{1}{x^2} - 3$

f) $y = \frac{x^5}{a+b} - \frac{x^2}{a-b} - x$

R: $\frac{dy}{dx} = \frac{5x^4}{a+b} - \frac{2x}{a-b} - 1$

g) $y = \frac{(x+1)^3}{x^{3/2}}$

R: $\frac{dy}{dx} = \frac{3(x+1)^2(2x-1)}{2x^{5/2}}$

h) $y = x(2x-1)(3x+2)$

R: $\frac{dy}{dx} = 2(9x^2 + x - 1)$

2) Calcule as derivadas abaixo **através da definição** $\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$.

a) $f(x) = 3x + 2$

c) $f(x) = 1 - 4x^2$

b) $f(x) = \frac{1}{x+2}$

d) $f(x) = 2x^2 - x - 1$

Respostas:

a) 3 b) -8x c) $\frac{-1}{(x+2)^2}$ d) 4x - 1

3) Para cada função $f(x)$, determine a derivada $f'(x)$ no ponto x_0 indicado:

a) $f(x) = x^2$ para $x_0 = 4$

b) $f(x) = 2x + 3$ para $x_0 = 3$

c) $f(x) = -3x$ para $x_0 = 1$

d) $f(x) = x^2 - 3x$ para $x_0 = 2$

e) $f(x) = x^2 - 4$ para $x_0 = 0$

f) $f(x) = 5x^4 + x^3 - 6x^2 + 9x - 4$ para $x_0 = 0$

g) $f(x) = \frac{1}{x}$ para $x_0 = 2$

h) $f(x) = \frac{5x^2 + 3x - 9}{x^2 + 5}$ para $x_0 = 5$

i) $f(x) = x^2 - 3x + 4$ para $x_0 = 6$

Respostas: a) 8 b) 2 c) -3 d) 1 e) 0 f) 9 g) -1/4 h) 14/45 i) 9

1.

$$\begin{aligned} \text{a) } y &= x^2 + 4x \\ 2x^2 - 1 + 4(1) \\ 2x + 4 \end{aligned}$$

$$\begin{aligned} \text{b) } f(x) &= 2/x^2 \\ 2 \cdot x^{-2} \\ 2 \cdot (-2x^{-2-1}) \\ -4x^{-3} \\ -4/x^3 \end{aligned}$$

$$\begin{aligned} \text{c) } y &= x^3/2 + 3x/2 \\ 3x^2/2 + 3/2 \\ 3/2(x^2 + 1) \end{aligned}$$

$$\begin{aligned} \text{d) } y &= \sqrt[3]{x} \\ x^{1/3} \\ y' &= \frac{1}{3} \cdot x^{1/3 - 1} \\ y' &= \frac{1}{3} x^{-2/3} \end{aligned}$$

$$\begin{aligned} \text{e) } f(x) &= (3x + 1/x) \cdot (6x - 1) \\ 18x^2 - 3x + 6 - 1/x \\ 18x^2 - 3x + 6 - x^{-1} \\ 18 \cdot 2x^{2-1} - 3 \cdot 1 \cdot x^{1-1} + 0 - x^{-1-1} \\ 36x^1 - 3x^0 - (-1)x^{-2} \\ 36x - 3 + 1/x^2 \end{aligned}$$

$$\begin{aligned} \text{f) } y &= (x^5/a+b) - (x^2/a-b) - x \\ (5x^4/a+b) - (2x/a-b) - 1 \end{aligned}$$

$$\begin{aligned} \text{g) } y &= (x + 1)^3 / x^{3/2} \\ 3 \cdot x + 1^3 x^{-1} / x^{3/2} \\ 3x^1 x^{-1} / 3x^1 x^{-1} / 2 \\ (x+1)^3 / (x x^3) \cdot 2 \\ -(x+1)^2 (x+4) / 2x^5 \end{aligned}$$

$$\begin{aligned} \text{h) } y &= x(2x-1)(3x+2) \\ x(2x-1)(3x+2) \\ x(6x^2+4x-3x-2) \\ (6x^3+4x^2-3x^2-2x) \\ 18x^2+2x-2 \end{aligned}$$

2.

a)

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} (f(x+h) - f(x)) / h \\f'(x) &= \lim_{h \rightarrow 0} (3(x+h)-2-(3x-2)) / h \\f'(x) &= \lim_{h \rightarrow 0} (3x+3h-2-3x+2) / h \\f'(x) &= \lim_{h \rightarrow 0} 3h / h \\f'(x) &= 3\end{aligned}$$

c)

$$\begin{aligned}f(x) &= 1-4x^2 \\f'(x) &= \lim_{x \rightarrow h} f(x+h)-f(x)/h \Rightarrow \\&= \lim_{x \rightarrow h} 1-4(x+h)^2/h \Rightarrow \\&= \lim_{x \rightarrow h} 1-4(x^2+2xh+h^2)/h \\&= 1-4x^2-8xh-4h^2/h \\&= \lim_{x \rightarrow h} f(x) \Rightarrow \\&= \lim_{x \rightarrow h} 1-4(x^2) \\&= \lim_{x \rightarrow h} 1-4x^2-8xh-4h^2-1+4x^2/h \Rightarrow \\&= (1,-1)(-4x^2,+4x^2) \\&= \lim_{x \rightarrow h} -8xh-4h^2/h \Rightarrow \\&= \lim_{x \rightarrow h} h(-8x-4h)/h \Rightarrow \\&= \lim_{x \rightarrow h} -8x\end{aligned}$$

d)

$$\begin{aligned}&2x^2 - x + 1 \\&\lim_{h \rightarrow 0} [2(x+h)^2 - 2(x+h) - 1 - (2x^2 - 2x - 1)] / h \\&\lim_{h \rightarrow 0} [2(x^2 + 2xh + h^2) - 2x - 2h - 1 - 2x^2 + 2x + 1] / h \\&\lim_{h \rightarrow 0} [2x^2 + 4xh + 2h^2 - 2x - 2h - 1 - 2x^2 + 2x + 1] / h \\&\lim_{h \rightarrow 0} [4xh + 2h^2 - 2h] / h \\&\lim_{h \rightarrow 0} [h(4x + 2h - 2)] / h \\&\lim_{h \rightarrow 0} (4x + 2h - 2) \\&\lim_{h \rightarrow 0} 4x - 2 \\&4x - 2\end{aligned}$$

3

a) $f(x) = x^2$ para $x_0 = 4$

$$f'(x) = 2x$$

$$f'(x) = 2 \cdot 4 = 8$$

b) $f(x) = 2x+3$ para $x_0 = 3$

$$f'(x) = 2$$

c) $f(x) = -3x$ para $x_0 = 1$

$$f'(x) = -3$$

d) $f(x) = x^2-3x$ para $x_0 = 2$

$$f(x)' = 2x-3$$

$$f(x)' = 2 \cdot 2 - 3 = 1$$

$$\text{e) } f(x) = x^2 - 4 \text{ para } x_0 = 0$$

$$f(x)' = 2x$$

$$f(x) = 2 \cdot 0 = 0$$

$$\text{f) } f(x) = 5x^4 + x^3 - 6x^2 + 9x - 4 \text{ para } x_0 = 0$$

$$f(x)' = 20x^3 + 3x^2 - 6x + 9$$

$$f(x) = 20 \cdot 0^3 + 3 \cdot 0^2 - 6 \cdot 0 + 9 = 9$$