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- 1) Determine a integral indefinida  $\int (\sin(x) + \cos(x)) dx$  e faça a verificação derivando o resultado.

$$\int \sin(x) dx + \int \cos(x) dx = \int \sin(x) dx = -\cos(x) \quad \int \cos(x) dx = \sin(x) = -\cos(x) + \sin(x) + C$$

$$-\frac{d}{dx}(\cos(x)) + \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(C) = \frac{d}{dx}(C) = 0 \quad \sin(x) + \cos(x)$$

- 2) Calcule as integrais indefinidas utilizando o método da substituição.

a)  $\int \frac{1}{4+3x} dx$

$$= \int \frac{1}{3u} du = \frac{1}{3} \cdot \int \frac{1}{u} du = \frac{1}{3} \ln|u| = \frac{1}{3} \ln|4+3x| = \frac{1}{3} \ln|4+3x| + C$$

b)  $\int (3x^2 + 1)^3 dx$

$$\begin{aligned}
&= \int \frac{(u^2+1)^3}{\sqrt{3}} du = \frac{1}{\sqrt{3}} \cdot \int (u^2+1)^3 du = \frac{1}{\sqrt{3}} \cdot \int u^6 + 3u^4 + 3u^2 + 1 du \\
&= \frac{1}{\sqrt{3}} \left( \int u^6 du + \int 3u^4 du + \int 3u^2 du + \int 1 du \right) = \frac{1}{\sqrt{3}} \left( \frac{u^7}{7} + \frac{3u^5}{5} + u^3 + u \right) \\
&= \frac{1}{\sqrt{3}} \left( \frac{(\sqrt{3}x)^7}{7} + \frac{3(\sqrt{3}x)^5}{5} + (\sqrt{3}x)^3 + \sqrt{3}x \right) \\
&= \frac{1}{35} x (135x^6 + 189x^4 + 105x^2 + 35) \\
&= \frac{1}{35} x (135x^6 + 189x^4 + 105x^2 + 35) + C
\end{aligned}$$


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3) Calcule as integrais utilizando o método da integração por partes:

a)  $\int x e^{-5x} dx$

$$\begin{aligned}
\int \frac{e^u u}{25} du &= \frac{1}{25} \cdot \int e^u u du = \frac{1}{25} (e^u u - \int e^u du) = \frac{1}{25} (e^u u - e^u) \\
\frac{1}{25} (e^{-5x} (-5x) - e^{-5x}) &= \frac{1}{25} (-5e^{-5x} x - e^{-5x}) = \frac{1}{25} (-5e^{-5x} x - e^{-5x}) + C
\end{aligned}$$

b)  $\int (2x+1) \sin(x) dx$

$$\begin{aligned}
-\cos(x)(2x+1) - \int -2\cos(x) dx &= -\cos(x)(2x+1) - (-2\sin(x)) \\
-\cos(x)(2x+1) + 2\sin(x) + C
\end{aligned}$$


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4) Calcule a integral  $\int \frac{2x+1}{(x+1)(x-1)^2} dx$

$$= \frac{2x}{(x+1)(x-1)^2} + \frac{1}{(x+1)(x-1)^2}$$

$$= \int \frac{2x}{(x+1)(x-1)^2} dx + \int \frac{1}{(x+1)(x-1)^2} dx = -\frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| - \frac{1}{x-1}$$

$$= \frac{1}{4} \ln|x+1| - \frac{1}{4} \ln|x-1| - \frac{1}{2(x-1)} =$$

$$= -\frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| - \frac{1}{x-1} + \frac{1}{4} \ln|x+1| - \frac{1}{4} \ln|x-1| - \frac{1}{2(x-1)} \quad \text{div}$$

$$- \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| - \frac{1}{x-1} - \frac{1}{2(x-1)}$$

$$= -\frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| - \frac{1}{x-1} - \frac{1}{2(x-1)}$$

$$= -\frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| - \frac{1}{x-1} - \frac{1}{2(x-1)} + C$$


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5) Calcule a integral definida  $\int_1^3 (x^2 + 3x + 2) dx$

$$\int_1^3 x^2 dx + \int_1^3 3x dx + \int_1^3 2 dx = \int_1^3 x^2 dx = \frac{26}{3} \quad \int_1^3 2 dx = 4 = \frac{26}{3} + 12 + 4 = \frac{74}{3}$$

6) Calcule o valor da integral  $\int_0^1 \frac{x^2}{(x^3+1)^5} dx =$

$$\int_1^2 \frac{1}{3u^5} du = \frac{1}{3} \cdot \int_1^2 \frac{1}{u^5} du = \frac{1}{3} \cdot \int_1^2 u^{-5} du = \frac{1}{3} \left[ \frac{u^{-5+1}}{-5+1} \right]_1^2 = \frac{1}{3} \left[ -\frac{1}{4u^4} \right]_1^2 = \frac{1}{3} \cdot \frac{15}{64} = \frac{5}{64}$$