Introduction to Robotics

Lecture 8

Kacper Jastrzębski
260607@student.pwr.edu.pl

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Contents

Until now we mainly considered manipulator, the robots we can encounter eg on factory floors. Today we'll talk about a different kind: mobile robots. As for manipulator, our goal is to find a model, mathematical description.

1 Mobile manipulators and how to model them

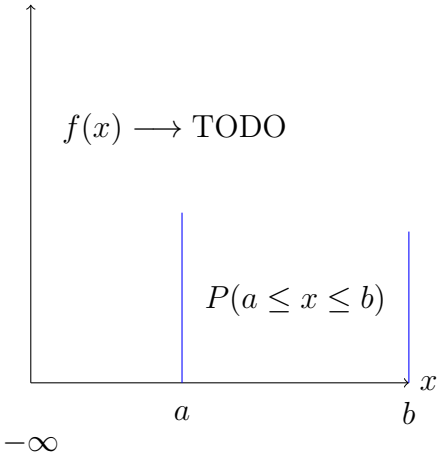

Figure 1: Model od unicycle (top view)

Let's consider the simplest mobile robot – a unicycle. What is its confiquration (set of data required to uniquely describe the manipulator)

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

With assumption that there is no lateral slippage:

$$\frac{\partial y}{\partial x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \tag{2}$$

$$\sin\theta \cdot \partial x - \cos\theta \cdot \partial y = 0 \quad \Big/ \cdot \frac{\partial}{\partial t} \tag{3}$$

$$\sin\theta \cdot \dot{x} - \cos\theta \cdot \dot{y} = 0 \tag{4}$$

$$\begin{bmatrix} \sin\theta & -\cos\theta & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \tag{5}$$

In robotics those constrians are called constraints in **Pfaff form**:

$$A(q)\dot{q} = 0 \tag{6}$$

$$rank\ A(q) = r \tag{7}$$

no longitudinal slippage, odometry, motion of point
no lateral slppage and no longitudinal slippage coupled together

constriants (2) x

$$\begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 0 & -R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} \tag{8}$$

Lets complicate the system:

2 Configuration vs controllability

n = 3 <- dimension of configuration space
r = 1 <- number of constrians
m = n - r = 2 <- difference (controls?)

2.1 Vector field vs vectors

Lets find a perpendicular vector field

$$a_1 \sin\theta - a_2 \cos\theta + a_3 \cdot 0 = 0 \; \forall \; q \tag{9}$$

2.2 How to determine if system is controllable?