Introduction to Robotics

Lecture 6

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Contents

1 Forward kinematics

From Wikipedia: Forward kinematics refers to the use of the kinematic equations of a robot to compute the position of the end-effector from specified values for the joint parameters (configuration vectors)

$$Q^{a} \ni q \to x^{b} = k(q) \quad \text{where} \quad x \in X^{c} \tag{1}$$

[a]Configuration space
[b]Position an orientation of end-effector
[c]Task space

2 Different coordinates systems

2.1 Cartesian

Unique – point $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ describes one point exactly.

2.2 Cylindrical

Non-unique – the same point can be described by $r = 0$ and any angle $\gamma \in [0, 2\pi]$. Sensitivity to error – for

2.3 Spherical

3 Inverse kinematics

Having given point (generalized position) find the configuration that realizes it.
Given the task space and forward kinematics find such configuration(s) that solve the equation.
Given: $x_p \in X, k = k(q)$ find: $q^* : \ x_f = k(q^*)$.

$$x^{(t)} = k(q^{(t)}) \quad \Big/ \frac{\partial}{\partial t} \tag{2}$$

$$\dot{x} = \frac{\partial k(q)}{\partial q} \cdot \dot{q} = J(q)\dot{q} \tag{3}$$

3.1 Double pendulum

Photo

3.2 Custom example

Photo

Function approximation using Taylor series

How to get q_{i+1} from the complicated equation:

$$\xi \left(x_f - k(q_i)\right) = J(q_i) \left(q_{i+1} - q_i\right) \tag{4}$$

$$q_{i+1} = q_i + \xi J^{-1}(q_i)(x_f - k(q_i)) \tag{5}$$

Newton algorithm of inverse kinematics for non-redundant manipulator.

Only for inversible Jacobi matrix (aka. entering the singular configuration)

Stop condition – when the difference between q_i and q_f is small enough.

regular configuration q: $rank J(q) = m$ (full rank)

singular configuration q: $rank J(q) < m$

Checking the rank: 2x3 -> at least one determinant of square submatrix must be non-zero

What about redundant manipulators, with non-square matrices:

pseudo-inverse of Jacobi matrix: (still not working for singular configuration)

$$J^{\#} = J^T(JJ^T)^{-1} \tag{6}$$

Manipulabity matrix

$$J_{(q)}J_{(q)}^T = M(q) \tag{7}$$

3rd method – robust (against singularity) inverse:

$$J^{\#} = J^T(JJ^T + \lambda I_m) \tag{8}$$

When to switch to robust? When the determinant of manipulabity matrix is below given threshold