

# Introduction to Robotics

## Lecture 4

Kacper Jastrzębski  
260607@student.pwr.edu.pl

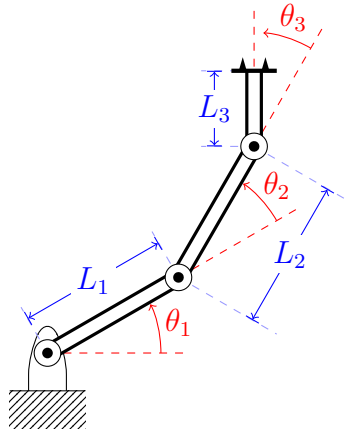
Date: Tuesday 11:15, 21-03-2023

### Contents

<b>1</b>	<b>Forward kinematics</b>	<b>2</b>
<b>2</b>	<b>Denavit-Hartenberg (1955) solution</b>	<b>2</b>
2.1	Preliminary assumptions . . . . .	2
2.2	Algorithm: . . . . .	2
<b>3</b>	<b>Planar double pendulum</b>	<b>3</b>

# 1 Forward kinematics

Let's recapitulate the naming conventions for a exemplary manipulator:



## 2 Denavit-Hartenberg (1955) solution

Jacques Denavit and Richard Hartenberg introduced this convention in 1955 in order to standardize the coordinate frames for spatial linkages<sup>1</sup>. They came up with an universal algorithm for describing the motion (or in other words: attaching a reference frames to the links) of a manipulator.

### 2.1 Preliminary assumptions

1. motion allowed only along z-axis
2. rigid body assumed

### 2.2 Algorithm:

1. Step: assign axes of rotation  $z_0 \dots z_{n-1}$
2. Step: describe base frame  $O_0x_0y_0z_0$ <sup>2</sup>
3. Step: create a loop  $i = 1, \dots, n - 1$  (repeat steps 4-6)
4. Step: determine  $O_i$  (the origin of next frame), consider 3 cases:
  - (a) case:  $O_i = z_{i-1} \cap z_i$
  - (b) case parallel: a point where normal line passing through  $O_{i0-1}$  crosses  $Z_i$
  - (c) case: a point where normal line to both  $Z_{i-1}$  and  $Z_i$  crosses  $Z_i$
5. Step: determine  $x_i$  axis, for each case:
  - (a)  $x_i = z_{i-1} \times Z_i$
  - (b) b and c:  $x_i$  along normal line selected previously
6. Step: calculate missing axis  $y_i$  such the  $x_iy_iz_i$  is a right-handed frame
7. Step: end-effector frame:
  - (a) origin  $O_n$  – between fingers of a grabbing, two fingered effector
  - (b)  $z_n \parallel z_{n-1}$  – inherited from the last joint
  - (c)  $y_n$  – finger motion direction

<sup>1</sup>Description borrowed from Wikipedia: [Denavit–Hartenberg parameters](#).

<sup>2</sup>Axis should be chosen wisely, in respect to the surroundings, context, and the use case.

(d)  $x_n \rightarrow x_n y_n z_n \rightarrow$  right-handed

8. Step: determine D-H parameters described in table below:

	$\theta_i$	$d_i$	$a_i$	$\alpha_i$
1	$\theta_1$	$d_1$	$a_1$	$\alpha_1$
2	$\theta_2$	$d_2$	$a_2$	$\alpha_2$
$\vdots$				
n	$\theta_n$	$d_n$	$a_n$	$\alpha_n$

This is the procedure that is using the D-H parameters

$$A_{i-1}^i(q_i) = Rot(z, \theta_i) Tran(z, d_i) Tran(x, a_i) Rot(x, \alpha_i) \quad (1)$$

9. Describe full kinematic:

$$A_0^n(q) = A_0^1(q_1) \cdot A_1^2(q_2) \cdots A_{n-1}^n(q_n) \quad (2)$$

note-1 with pictures

### 3 Planar double pendulum

Simple but not trivial example of a system:

notes-2

	$\theta_i$	$d_i$	$a_i$	$\alpha_i$
1	$q_1$	0	$a_1$	0
2	$q_2$	0	$a_2$	0

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad (3)$$

$$A_0^2(q) = A_0^1(q_1) \cdot A_1^2(q_2) \quad (4)$$

$$Rot(z, q_1) \cdot Tran(x, a_1) = \left[ \begin{array}{ccc|c} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline & 0 & & 1 \end{array} \right] = \quad (5)$$