Introduction to Robotics

Lecture 4

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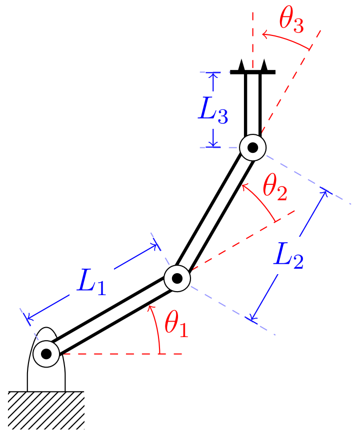
Contents

1 Forward kinematics

Let's recapitulate the naming conventions for a exemplary manipulator:



2 Denavit-Hartenberg (1955) solution

Jacques Denavit and Richard Hartenberg introduced this convention in 1955 in order to standardize the coordinate frames for spatial linkages[1]. They came up with an universal algorithm for describing the motion (or in other words: attaching a reference frames to the links) of a manipulator.

2.1 Preliminary assumptions

1. motion allowed only along z-axis
2. rigid body assumed

2.2 Algorithm:

1. Step: assign axes of rotation $z_0 \dots z_{n-1}$
2. Step: describe base frame $O_0 x_0 y_0 z_0$[2]
3. Step: create a loop $i = 1, \dots, n-1$ (repeat steps 4-6)
4. Step: determine O_i (the origin of next frame), consider 3 cases:
   (a) case: $O_i = z_{i-1} \bigcap z_i$
   (b) case parallel: a point where normal line passing through O_{i0-1} crosses Z_i
   (c) case: a point where normal line to both Z_{i-1} and Z_i crosses Z_i
5. Step: determine x_i axis, for each case:
   (a) $x_i = z_{i-1} \times Z_i$
   (b) b and c: x_i along normal line selected previously
6. Step: calculate missing axis y_i such the $x_i y_i z_i$ is a right-handed frame
7. Step: end-effector frame:
   (a) origin O_n – between fingers of a grabbing, two fingered effector
   (b) $z_n \,||\, z_{n-1}$ – inherited from the last joint
   (c) y_n – finger motion direction

[1]Description borrowed from Wikipedia: Denavit–Hartenberg parameters.
[2]Axis should be chosen wisely, in respect to the surroundings, context, and the use case.

(d) $x_n \rightarrow x_n y_n z_n \rightarrow$ right-handed

8. Step: determine D-H parameters described in table below:

	θ_i	d_i	a_i	α_i
1	θ_1	d_1	a_1	α_1
2	θ_2	d_2	a_2	α_2
$\vdots$				
n	θ_n	d_n	a_n	α_n

This is the procedure that is using th D-H parameters

$$A_{i-1}^{i}(q_i) = Rot(z, \theta_i)\ Tran(z, d_i)\ Tran(x, a_i)\ Rot(x, \alpha_i) \tag{1}$$

9. Describe full kinematic:

$$A_0^n(q) = A_0^1(q1) \cdot A_1^2(q2) \cdots A_{n-1}^n(q_n) \tag{2}$$

note-1 with pictures

3 Planar double pendulum

Simple but not trivial example of a system:
notes-2

	θ_i	d_i	a_i	α_i
1	q_1	0	a_1	0
2	q_2	0	a_2	0

$$q = \begin{bmatrix} q1 \\ q2 \end{bmatrix} \tag{3}$$

$$A_0^2(q) = A_0^1(q_1) \cdot A_1^2(q_2) \tag{4}$$

$$Rot(z,q_1) \cdot Tran(x, a_1) = \left[\begin{array}{ccc:c} c_1 & -s_1 & 0 & \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & \\ \hdashline & 0 & & 1 \end{array}\right] = \tag{5}$$