Introduction to Robotics

Lecture 2

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Date: Tuesday 11:15, 7.03.2023

Contents

$$R \in SO(3) \tag{1}$$

1 Elementary rotational matrices

$$rot(z, \gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$rot(y, \beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{3}$$

$$rot(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{4}$$

2 Representation of $R \in SO(3)$

Roll-pitch-yaw angles:

$$RPY(\alpha, \beta, \gamma) = rot(z, \alpha) \cdot rot(y, \beta) \cdot rot(x, \gamma) \tag{5}$$

Euler angles:

$$Euler(\alpha, \beta, \gamma) = rot(z, \alpha) \cdot rot(y, \beta) \cdot rot(z, \gamma) \tag{6}$$

3 Operations on rotational matrices

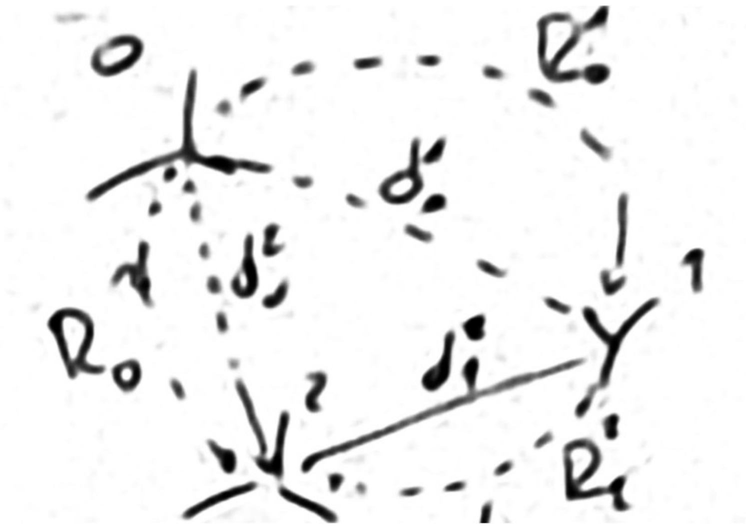
$$rot(x, \alpha) \cdot rot(z, \beta) \stackrel{?}{=} rot(z, \beta) \cdot rot(x, \alpha) \tag{7}$$

Those two operations are **not** interchangeable, because matrix multiplication is not commutative.

$$R_0^n = R_0^1 \cdot R_1^2 \cdots R_{n-2}^{n-1} \cdot R_{n-1}^n \tag{8}$$

p_n - position in n-th frame

$$p_0 = R_0^1 \cdot p_n$$
$$p_n = R_n^0 \cdot p_0$$
$$p_0 = R_1^0 \cdot p_0$$
$$R_1^0 = (R_0^1)^{-1} = (R_0^1)^T$$



$$p_0 = p_1 + d_0^1 \text{(pure translation)}$$
$$p_0 = R_0^1 \cdot p_1 \text{(pure rotation)}$$
$$p_0 = R_0^1 p_1 + d_0^1$$
$$p_0 = R_0^2 p_2 + d_0^2$$
$$p_1 = R_1^2 p_2 + d_1^2$$

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