Introduction to Robotics

Lecture 5

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Contents

1 More complex manipulator setups

Table of D-H parameters (for frames of motion):

	θ_i	d_i	a_i	α_i
1	q_1	d_1	a_1	$\frac{\pi}{2}$
2	q_2	0	0	$\frac{\pi}{2}$
3	0	q_3	0	0

$$Rot(z,\theta_i)\ Tran(z,d_i)\ Tran(x,\theta_i)\ Rot(x,\alpha_i) \tag{1}$$

$$A_0^1(q_1) = \left[\begin{array}{ccc:c} c_1 & -s_1 & 0 & \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & \\ \hdashline & 0 & & 1 \end{array}\right] \left[\begin{array}{c:c} & a_1 \\ I_3 & 0 \\ & d_1 \\ \hdashline 0 & 1 \end{array}\right] \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \\ \hdashline & 0 & & 1 \end{array}\right] = \left[\begin{array}{ccc:c} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ \hdashline & 0 & & 1 \end{array}\right] \tag{2}$$

$$A_1^2(q_2) = \left[\begin{array}{ccc:c} -c_2 & s_2 & 0 & \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 1 & \\ \hdashline & 0 & & 1 \end{array}\right] \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \\ \hdashline & 0 & & 1 \end{array}\right] = \left[\begin{array}{ccc:c} -c_2 & 0 & -s_2 & \\ -s_2 & 0 & c_2 & 0 \\ 0 & 1 & 0 & \\ \hdashline & 0 & & 1 \end{array}\right] \tag{3}$$

$$A_2^3(q_3) = \left[\begin{array}{c:c} & 0 \\ I_3 & 0 \\ & q_3+d_3 \\ \hdashline 0 & 1 \end{array}\right] \tag{4}$$

$$A_0^3(q) = A_0^1(q_1)\cdot A_1^2(q_2)\cdot A_2^3(q_3) \qquad \text{where} \qquad q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{5}$$

2 Configuration space vs task space

Configuration space – denoted Q – is a space that contain all configuration vectors, $q \in Q$. Its usually assumed to be rectangular or parallelepiped. But in the real live it may resemble different figures, because of the sizes of joints, of the volume taken by the motors and of the mass of transmission elements (e.g. gears). Configuration space can be decided after the creation of manipulator.

Task space (described by $X \ni x$), on the other hand, is defined by the user. Since it's a space, it has a dimensionality, usually denoted m where $m \leq n$ (less or equal to the dimensionality of configuration space). By the relation between m and n we can distinguish two possibilities:

- $m = n$ – non-redundant manipulator,
- $m < n$ – redundant manipulator,

And the value $n - m$ is called the **degree of redundancy**.

$$A_0^n(q) = K(q) = \left[\begin{array}{c:c} R_0^n(q) & d_0^n(q) \\ \hdashline O & 1 \end{array}\right] \xrightarrow{\text{Kinematics in coordinates}} \begin{bmatrix} x & y & \xi \end{bmatrix} \tag{6}$$

$$X = K(Q) \tag{7}$$

Image of configuration space via kinematics in coordinates.

2.1 Exercise with double pendulum

Task space – all the points that we can reach, provided in coordinates (x, y).
Two dimensional planar double pendulum.
1^st example – q_1 $q_2 \rightarrow$ unconstrained
2^nd example – $q_1 \rightarrow$ unconstrained and $q_2 \in [0, \frac{\pi}{2}]$