Introduction to Robotics

Lecture 5

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Date: Tuesday 11:15, 28-03-2023

Contents

1 More complex manipulator setups

Table of D-H parameters (for frames of motion):

	θ_i	d_i	a_i	α_i
1	q_1	d_1	a_1	$\frac{\pi}{2}$
2	q_2	0	0	$\frac{\pi}{2}$
3	0	q_3	0	0

$$Rot(z, \theta_i)\ Tran(z, d_i)\ Tran(x, \theta_i)\ Rot(x, \alpha_i) \tag{1}$$

$$A_0^1(q_1) = \left[\begin{array}{ccc:c} c_1 & -s_1 & 0 & \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & \\ \hdashline & 0 & & 1 \end{array}\right] \left[\begin{array}{c:c} & a_1 \\ I_3 & 0 \\ & d_1 \\ \hdashline 0 & 1 \end{array}\right] \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \\ \hdashline & 0 & & 1 \end{array}\right] = \left[\begin{array}{ccc:c} c_1 & 0 & s_1 & a_1 c_1 \\ s_1 & 0 & -c_1 & a_1 s_1 \\ 0 & 1 & 0 & d_1 \\ \hdashline & 0 & & 1 \end{array}\right] \tag{2}$$

$$A_1^2(q_2) = \left[\begin{array}{c:c} 1 & 1 \\ \hdashline 1 & 1 \end{array}\right] \tag{3}$$

$$A_2^3(q_3) = \tag{4}$$

$$A_0^3(q) = A_0^1(q_1) \cdot A_1^2(q_2) \cdot A_2^3(q_3) \quad \text{where} \quad q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{5}$$

2 Configuration space vs task space

Configuration space – denoted Q – is a space that contain all configuration vectors, $q \in Q$. Its usually assumed to be rectangular or parallelepiped. But in the real live, because the joints of the "motor" volume (size) and mass or transmission of motion (gears). Configuration space can be decided after the creation of manipulator.

Task space (described by $X \ni x$), on the other hand will be defined by the user. Since it's a space, it has a dimensionality, usually denoted m where $m \leq n$ (less or equal to the dimensionality of configuration space). If $m = n$ we obtain non-redundant manipulator, if $m < n$ – redundant, and the value $n - m$ is called degree of redundancy.

$$A_0^n(q) = K(q) = \left[\begin{array}{c:c} R_0^n(q) & d_0^n(q) \\ \hdashline O & 1 \end{array}\right] \xrightarrow{\text{Kinematics in coordinates}} \begin{bmatrix} x & y & \xi \end{bmatrix} \tag{6}$$

2.1 Exercise with double pendulum

Task space – all the points that we can reach (x, y).

$$X = K(Q) \tag{7}$$

Image of configuration space via kinematics in coordinates.
1st example – $q_1\ q_2 \to$ unconstrained
2nd example – $q_1 \to$ unconstrained and $q_2 \in [0, \frac{\pi}{2}]$