Introduction to Robotics

Lecture 3

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1 Block Matrix of translation and rotation

To create single object consisting of rotational and translation matrices we can group them in so called *block matrix (A)*. For easier calculations, we can make it square by filling it with constant values that does not interfere with operations performed on it (zeros and ones in our case). The final object will look something like this:

$$A_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ \mathbf{0}^a & \mathbf{1}^b \end{bmatrix} \tag{1}$$

[a]this is actually a 3×1 matrix, so it should be denoted $[\,0\;0\;0\,]$, but for the simplification we'll just be using $\mathbf{0}$
[b]Similarly, in this case $\mathbf{1}$ represents a 1×1 unit matrix.

Let's check if this object obeys the *chain rule*:

$$A_0^2 = \begin{bmatrix} R_0^2 & d_0^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \left[\begin{array}{c:c} R_0^1 R_1^2 & R_0^1 d_1^2 + d_0^1 \\ \hdashline 0 & 1 \end{array}\right] \tag{2}$$

As we can see, the chain multiplication can be performed without issues. But what about an operation like $p_0 = A_0^1 p_1$? We cannot perform it, because sizes of vector p and object A are different. Let's add one missing dimension to the vector then:

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = A_0^1 \cdot \begin{bmatrix} p_1 \\ 1 \end{bmatrix} \tag{3}$$

We can shortly denote is

2 Building blocks for SE(3) group

2.1 Rotational functions

2.2 Translation functions

2.3 Order of operations

2.4 How to check if matrix belongs to SE(3)?

3 Euler angles

3.1 Regular case

3.2 Simple case