Introduction to Robotics

Lecture 7

Kacper Jastrzębski
260607@student.pwr.edu.pl

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Contents

1 Rewind — Inverse kinematics

Inverse kinematics – we have given position and orientation and we have to find the configuration that realizes this problem. One of the methods, jacobi based, for solving that kind of task.

Given: $x = k(1), x_f \in X$

$$q_{i+1} = q_i + \xi_i J^{\#}(q_i)(x_f - k(q_i)) \tag{1}$$

$$J^{\#} = J^T(JJ^T)^{-1} \tag{2}$$

This is an iterative algorithm, so $q_0 \to q_1 \to q$. That kind of thask comes in two flavours:

- regular configuration: $rank\ J(q) = m$
- singular configuration: $rank\ J(q) < m$

1.1 Singular case

How to determine singular configuratoin:

1. $rank\ J(q)$
   $J\ as\ m \times n\ rectangle$ select all $m \times m$ – sub-matrices of J (there are $\binom{n}{k} = \dfrac{n!}{m!(n-m)!}$) singular cont: if all determinants of all submatrices are equal to 0
2. $det\left(J(q)J^T(q)\right)$
   Example calculation *(3 DOF planar pendulum)*

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k_1(q) \\ k_2(q) \end{bmatrix} = \begin{bmatrix} a_1c_1 + a_2c_{12} + a_3c_{123} \\ a_1s_1 + a_2s_{12} + a_3s_{123} \end{bmatrix} \tag{3}$$

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{4}$$

$$J = \frac{\partial k}{\partial q} \tag{5}$$

$$m = dim\ X \tag{6}$$

$$n = dim\ Q \tag{7}$$

$$det\ J_{23} = a_1a_2s_3 = 0 \tag{8}$$

$$det\ J_{12} = a_1a_2s_{@} + a_1a_3s_{23} = 0 \tag{9}$$

$$det\ J_{13} = a_1a_3s_{23} + a_2a_3s_3 = 0 \tag{10}$$

therefore:

$$s_3 = 0 \to q_3 = \{0, \pi\} \tag{11}$$

$$s_{23} = 0 \to q_2 = \{0, \pi\} \tag{12}$$

$$q = \begin{bmatrix} * \\ \{0, \pi\} \\ \{0, \pi\} \end{bmatrix} \tag{13}$$

At regular configuration we can move in every direction infinitesimally (like arm in L shape). in singular it is not possible (like straight extended arm).

2 Other methods

2.1 Geometrical method

Works only or special kind of robots, lets excercise with planar double pendulum:

photo

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 S_{12} \end{bmatrix} \tag{14}$$

Having given x_f and y_f we can easily obtain $r = \sqrt{x_f^2 + y_f^2}$. Using cosinus theorem:

$$r^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos\beta \to \beta \tag{15}$$

$$a_2^2 = a_1^2 + r^2 - 2a_1 r \cos\alpha \to \alpha \tag{16}$$

therefore

$$q_2 = \beta - \pi \tag{17}$$

$$q_1 = \gamma + \alpha \tag{18}$$

$$\tan\gamma = \frac{x_f}{y_f} \to \gamma \tag{19}$$

There is also another solution, mirrored, called elbow-up or elbow-down. For singular case (extended arm)

2.2 Kinematics decoupling

Applicable only when three last axes cross at a single point. Let's have a manipulator with $n = 6$

$$K(q) = \left[\begin{array}{c:c} R_0^6(q) & d_0^6(q) \\ \hdashline 0 & 1 \end{array}\right] \tag{20}$$

$$r(q_1, q_2, q_3) = d_0^6 - d_6 R_0^6 e_3 \tag{21}$$

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{22}$$

We used to calculate in forward kinematics this general approach:

$$A_0^n(q) = \dots \tag{23}$$

Now we have case that:

$$R_0^6 = R_0^3(q_1, q_2, q_3) R_0^6(q_4, q_5, q_6) \tag{24}$$

So at the end we decoupled this 6-dimensional problem in two 3-dimensional problems. This is always a good idea, because of the complexity