Problems in Allufi's Algebra

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1 Groups, first encounter

Group Objects in Categories

Problem 10.2: Show that groups are "group objects in the category of sets"

Proof. Take the morphism $m : G \times G \to G$ to be the morphism that takes the Cartesian product of a group G and maps it to itself via its multiplication table. Let $e : 1 \to G$ be the morphism that takes the set with a unique element and maps its element to the identity of G and $i : G \to G$ be the morphism which maps every element in G to its inverse. The diagrams shown in the figure on page 116 commute with these morphisms as we can easily check. □

Problem 10.3: Let $(G, \cdot)$ be a group, and suppose $\circ : G \times G \to G$ is a group homomorphism (w.r.t *cdot*) such that $(G, \circ)$ is also a group. Prove that $\circ$ and $\cdot$ coincide.