

Additional Useo of $\epsilon - \delta$ technique

August 16, 2018

Question: Find

$$\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$$

Solution:

$$\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) = \lim_{t \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \right)$$

Perform a change of variables using $a = \sqrt{1+t}$. Then

$$a^2 - 1 = t$$

Thus,

$$\lim_{t \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \right) = \lim_{a \rightarrow 1} \left(\frac{1 - a}{a(a^2 - 1)} \right) = \lim_{a \rightarrow 1} - \left(\frac{a - 1}{a(a^2 - 1)} \right) = \lim_{a \rightarrow 1} - \left(\frac{1}{a(a + 1)} \right) = -\frac{1}{2}$$

as desired.