Master DSC/MLDM - First year
Introduction to Artificial Intelligence
First exam - November 2023

Time allocated: 3h - No documents allowed.
TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Truth table ($\backsimeq$ 2 points)

1. Using the truth table method, prove that the premises: $p \Rightarrow q$ and $r \Rightarrow p \vee q$, logically entail $r \Rightarrow q$

2. Using the truth table method, prove that $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ is valid. For that question, you should present your table in the same way as the example below showing the truthtable of $(p \vee q) \wedge \neg p$:

p	q	(p	$\vee$	q)	$\wedge$	$\neg$	p
T	T	T	T	T	**F**	F	T
T	F	T	T	F	**F**	F	T
F	T	F	T	T	**T**	T	F
F	F	F	F	F	**F**	T	F

2 Problem solving using a truth table ($\backsimeq$ 3 points)

John, Paul, and Mike find themselves trapped in a dark and cold dungeon. After a quick search the boys find three doors, the first one red, the second one blue, and the third one green. Behind **one** of the doors is a path to freedom. Behind **the other two** doors, however, is an evil fire-breathing dragon. Opening a door to the dragon means almost certain death. On each door there is an inscription:

1. On the red door is written: "Freedom is behind this door"
2. On the blue door is written: "Freedom is not behind this door"
3. On the green door is written: "Freedom is not behind the blue door"

Given the fact that **at least one** of the three statements on the three doors is true and **at least one** of them is false, which door would lead the boys to safety?

3 Problem modeling and solving in PL ($\backsimeq$ 5 points)

Here are some information about a simple world:

When the days are getting shorter, the leaves on the trees turn yellow and it gets cold. When the days are getting shorter, John gets depressed and doesn't talk to anyone. When the leaves on the trees turn yellow, they fall to the ground and the trees are leafless. When the birds are no longer safe, they move to another country or hide in the sand. When the trees are leafless, the monkeys climb them. When John is depressed, he needs to go for a walk in the forest and eat chocolate. If John needs to go for a walk in the forest and it gets cold, he asks Mary to go with him. If the monkeys climb trees, they throw stones at John and Mary. When John doesn't talk to anyone, Mary is sad and cries. When the monkeys throw stones at Mary, she gets hurt and screams. If Mary or John scream, the birds are no longer safe. The days are getting shorter. The birds do not hide in the sand.

You are asked to:

1. Choose proposition constants wisely (list all your constants on your examn sheet).
2. Convert these English sentences into propositional logic sentences (list all those sentences on your examn sheet).
3. Convert these proposition logic sentences into CNF (list all your clauses on your examn sheet).
4. Use resolution reasoning in proposition logic to prove that, in this world: *The birds move to another country and Mary cries* (write all the steps of the reasoning on your exam sheet).

4 Validity, unsatisfiability, contingency ($\backsimeq$ 3 points)

Using resolution reasonning and the method studied during the course, say whether the following sentences are unsatisfiable, valid or contingent

1. $(p \vee (\neg q \wedge r)) \Rightarrow ((q \vee \neg r) \Rightarrow p)$
2. $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
3. $\forall X.\forall Y.((p(X) \wedge p(Y)) \Rightarrow q(X, Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X, Y))$

Note: you must write down all the details of the conversion of each formula to CNF.

5 Resolution principle ($\backsimeq$ 2 points)

For each pair of clauses below, say whether the resolution principle can be applied. If yes, explain the steps you follow and give the resolvent(s), if no, explain why.

1. $\{p(A, B, C), q(C, A), q(a, b), r(B)\}$ and $\{s(D), t(Z), \neg p(D, e, E)\}$
2. $\{p(a, f(X, g(Y), Z), X, b, T), t(T), w(X, Y, Z)\}$ and $\{q(A, B), \neg p(A, f(a, B, c), b, C, g(B)), r(C)\}$
3. $\{p(X), \neg q(b)\}$ and $\{\neg p(a), q(X)\}$
4. $\{p(A), q(A, B), r(B), q(a, C), s(C), q(A, b)\}$ and $\{\neg q(A, b), \neg q(a, B), t(A), t(B)\}$

6 Problem modeling and solving in FOL ($\backsimeq$ 5 points)

You can find below some information about a simple world.
1) For any person, if that person is young and plays the guitar then that person is happy.
2) For any person, if that person is old and plays the violin then that person is happy.
3) For any person, if that person plays the drums then that person is happy.
4) Mary is young, owns a guitar, has learned the guitar and loves Paul.
5) John is old and he knows how to dance.
6) For any person and musical instrument, if that person owns that musical instrument and has learned to play it then that person plays that instrument.
7) For any person and musical instrument, if that person is a genius and that musical instrument is a string instrument, then that person plays that instrument.
8) For any person and musical instrument, if that person person has built that musical instrument then that person plays that instrument.
9) Lindsey has built a violin.
10) The guitar and violin are string instruments.
11) George is a genius.
12) For any person p1 and p2,
    a) If p1 is happy and loves p2, then p1 is a happy lover of p2.
    b) If p1 knows how to dance and p2 is happy then p1 is a happy dancer with p2.
    c) If p1 is a happy dancer with p2 then p1 dances with p2.
    d) If p1 is a happy lover of p2 then p1 dances with p2.
    e) If p1 has built and instrument, then p1 dances with this instrument.

You are asked to:

1. Convert these English sentences into first-order logic sentences.
2. Convert these first-order logic sentences into CNF (you can directly write the clauses without showing the steps of conversion).
3. Use resolution reasoning in first-order logic, to get the answers to the question: *"Who dances with whom/what?"* (write all the steps of the reasoning on your exam sheet)