L4
Relational Algebra

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Supplemental Materials

Helpful References

https://en.wikipedia.org/wiki/Relational_algebra

Overview

Last time, learned about

- pre-relational models
- an informal introduction to relational model
- an introduction to the SQL query language.

Learn about formal relational query languages

- Relational Algebra (algebra: perform operations)
- Relational Calculus (logic: are statements true?)

Keys to understanding SQL and query processing

Relational algebra is the basis for the most popular **query language** in the universe*

*known universe

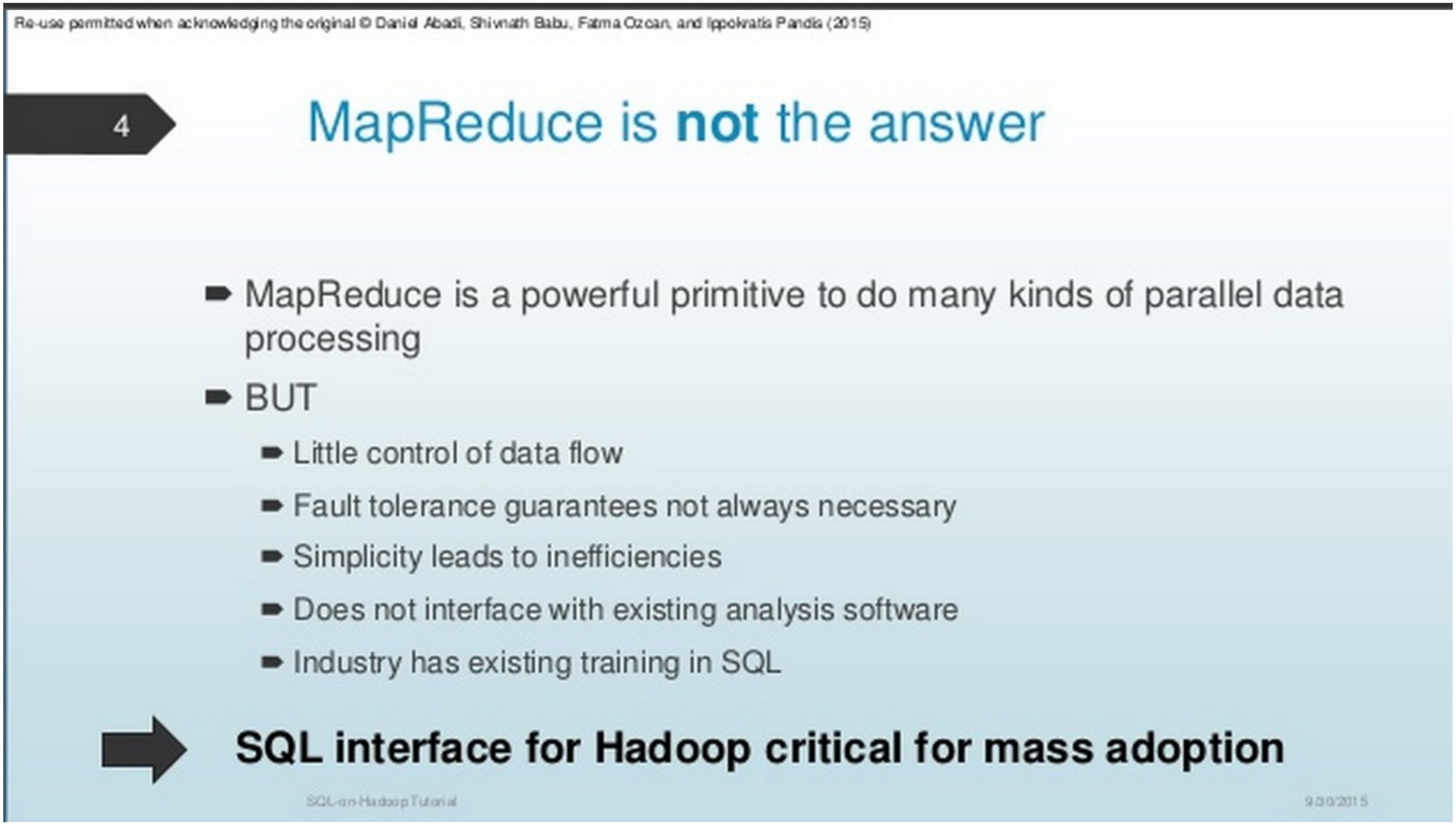
What’s a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language
- Doesn’t need to be turing complete
- *Not* designed for computation
- Supports easy, efficient access to (very) large databases

Recent years
- Scaling to large datasets is a reality
- Powerful way to think about…
  - data algorithms that scale
  - asynchronous/parallel programming

What's a Query Language?



Tutorial at VLDB 2015 Abadi et al.
http://www.slideshare.net/abadid/sqlonhadoop-tutorial

Formal Relational Query Languages

Relational Algebra

Operational, used to represent execution plans

$\pi_{name}(\sigma_{age<30}(Sailors))$ sailor names younger than 30

Relational Calculus

Logical, describes what data users want (declarative)

`{ s: name | s ∈ Sailors ^ s.age < 30 }`

`(this is shorthand)`

Journey of a Query

SQL	SELECT … FROM …
Relational Algebra	$\pi_b(P \bowtie Q \bowtie \ldots)$
Query Rewriting	$\pi_b(P \bowtie S) \bowtie Q \ldots)$

Query execution

https://cseweb.ucsd.edu/classes/fa19/cse132A-a/slides/relational-calculus.pdf

Prelims

Query is a function over relation instances

$$Q(R_1,\ldots,Rn) = R_{result}$$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation

- Position easier for formal defs
  - one-indexed (not 0-indexed!!!)
- Named is more readable
- Both used in SQL

Prelims

Relation (for this lecture)

- Instance: **set** of tuples (important!)
- Schema: list of field names and types (domains)
- Students(<u>sid</u> int, name text, major text, gpa int)

How are relations different than generic sets ($\mathbb{R}$)?

- Can assume item structure due to schema
- Some algebra operations (x) need to be modified
- Will use this later

Relational Algebra Overview

Core 5 operations

- PROJECT (π)
- SELECT (σ)
- UNION (U)
- SET DIFFERENCE (-)
- CROSSPRODUCT (x)

Additional operations

- RENAME (ρ)
- INTERSECT ($\cap$)
- JOIN ($\bowtie$)
- DIVIDE (/)

Instances Used Today: Library

Students, Reservations

Use positional or named field notation

Fields in query results are inherited from input relations (unless specified)

R1

sid	rid	day
1	101	10/10
2	102	11/11

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

Project

$$\pi_{<attr1,\ldots>}(A) = R_{result}$$

Pick out desired attributes (subset of columns)

Schema is subset of input schema in the projection list

- $\pi_{<a,b,c>}(A)$ has output schema (a,b,c) w/ types carried over

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

$\pi_{name,age}(S2) =$

name	age
aziz	21
barb	21
tanya	88
rusty	21

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

$$\pi_{name,name,age}(S2) =$$

name	name	age
aziz	aziz	21
barb	barb	21
tanya	tanya	88
rusty	rusty	21

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

$\pi_{age}(S2) =$

age
21
88

Where did all the rows go?
Real systems typically don't remove duplicates. Why?

Select

$$\sigma_{<p>}(A) = R_{result}$$

Select subset of rows that satisfy condition *p*

Won't have duplicates in result. Why?

Result schema same as input

Select

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

$$\sigma_{age<30}(S1) =$$

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21

$$\pi_{name}(\sigma_{age<30}(S1)) =$$

name
eugene
barb

Commutative Operations

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

Associative Operations

$$A + (B + C) = (A + B) + C$$

$$A + (B * C) = (A + B) * C$$

Commutative Operations

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

Associative Operations

$$A + (B + C) = (A + B) + C$$

$$\cancel{A + (B * C) = (A + B) * C}$$

Commutatively

$\pi_{age}(\sigma_{age<30}(S1))$

$\sigma_{age<30}$ (

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

)

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21

Commutatively

$\pi_{age}(\sigma_{age<30}\ (S1))$

$\sigma_{age<30}$ (

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

)

π_{age} (

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21

) =

age
20
21

Commutatively

$\sigma_{age<30}(\pi_{age}(S1))$

π_{age} (

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

)

age
20
21
88

Commutatively

$$\sigma_{age<30}(\pi_{age}(S1))$$

π_{age} (

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

)

$\sigma_{age<30}$ (

age
20
21
88

) =

age
20
21

Commutatively

Does Project and Select **always** commute?

$$\boldsymbol{\pi}_{age}(\sigma_{age<30}\,(S1)) = \sigma_{age<30}(\boldsymbol{\pi}_{age}(S1))$$

What about

$$\boldsymbol{\pi}_{name}(\sigma_{age<30}\,(S1))?$$

Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name}(S1))$$

Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name,\ age}(S1))$$

Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) = \pi_{name}(\sigma_{age<30}(\pi_{name, age}(S1)))$$

OK!

Union, Set-Difference

$$A \text{ op } B = R_{result}$$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema borrowed from first arg (A)

- A(id int, imgid int) U B(blah int, gloop int) = ?

Union, Set-Difference

$A \text{ op } B = R_{result}$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema borrowed from first arg (A)

- A(id int, imgid int) U B(blah int, gloop int) = R_{result}(id int, imgid int)

Union, Intersect, Set-Difference

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

S1 U S2 =

sid	name	gpa	age
1	eugene	4	20
4	aziz	3.2	21
5	rusty	3.5	21
3	tanya	2	88
2	barb	3	21

Union, Intersect, Set-Difference

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

S1 - S2 =

sid	name	gpa	age
1	eugene	4	20

Note on Set Difference & Performance

Notice that most operators are monotonic

increasing size of inputs → outputs grow

if $A \supseteq B \rightarrow Q(A,T) \supseteq Q(B,T)$

can compute *incrementally*

Set Difference is *not monotonic*

if $A \supseteq B \rightarrow T - A \subseteq T - B$

e.g., $5 > 1 \rightarrow 9 - 5 < 9 - 1$

Set difference is *blocking*:

For T – S, must wait for all S tuples before any results

Cross-Product

$$A(a_1,\ldots,a_n) \times B(a_{n+1},\ldots,a_m) = R_{result}(a_1,\ldots,a_m)$$

Each row of A paired with each row of B

- Result schema concats A and B's fields, inherit if possible
- Conflict: students and reservations have *sid* field

Different than mathematical "X" by flattening results:

- math $A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$
- e.g., $\{1, 2\} \times \{3, 4\} = \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$
- what is $\{1, 2\} \times \{3, 4\} \times \{5, 6\}$?

Cross-Product

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

S1 x R1 =

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
2	barb	3	21	1	101	10/10
3	tanya	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barb	3	21	2	102	11/11
3	tanya	2	88	2	102	11/11

Rename

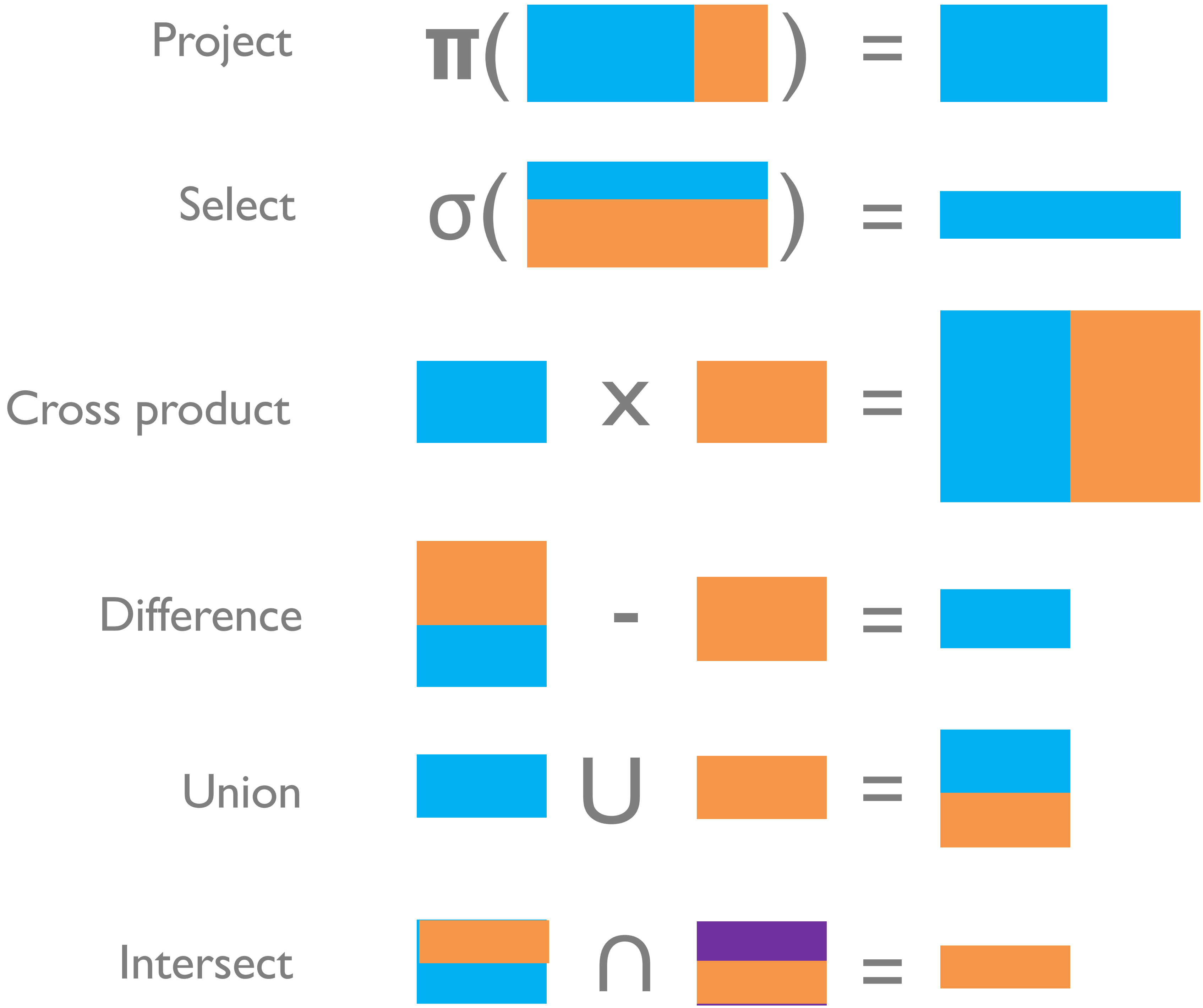
p(<newRelationName>(<mappings>), Q)

Explicitly defines/changes field names of schema

p(C(1 → sid1, 5 → sid2), S1 x R1)

C =

sid1	name	gpa	age	**sid2**	rid	day
1	eugene	4	20	1	101	10/10
2	barb	3	21	1	101	10/10
3	tanya	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barb	3	21	2	102	11/11
3	tanya	2	88	2	102	11/11


Project
π( ) =
Select
σ( ) =
Cross product
x =
Difference
- =
Union
∪ =
Intersect
∩ =

Compound/Convenience Operators

INTERSECT ($\cap$)

JOIN ($\bowtie$)

DIVIDE ($/$)

Intersect

$$A \cap B = R_{result}$$

A, B must be *union-compatible*

Intersect

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barb	3	21
3	tanya	2	88
5	rusty	3.5	21

$S1 \cap S2 =$

sid	name	gpa	age
2	barb	3	21
3	tanya	2	88

Intersect

$$A \cap B = R_{result}$$

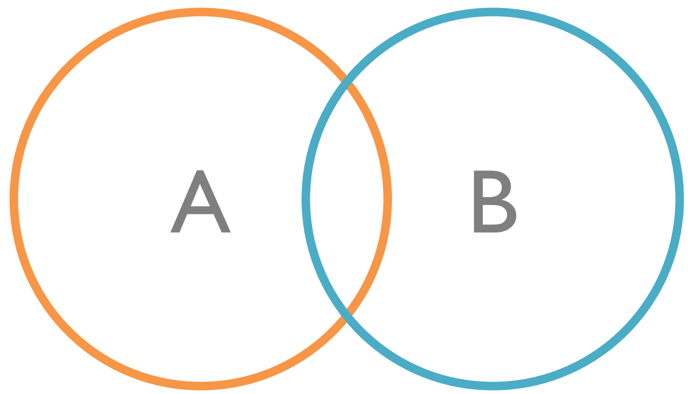
A, B must be *union-compatible*

Can we express using core operators?

$A \cap B = ?$

Intersect

$$A \cap B = R_{result}$$

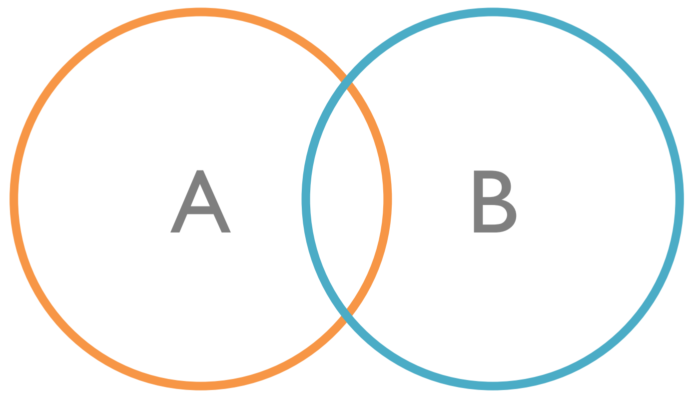


Can we express using core operators?

$A \cap B = A - ?$ (think venn diagram)

Intersect

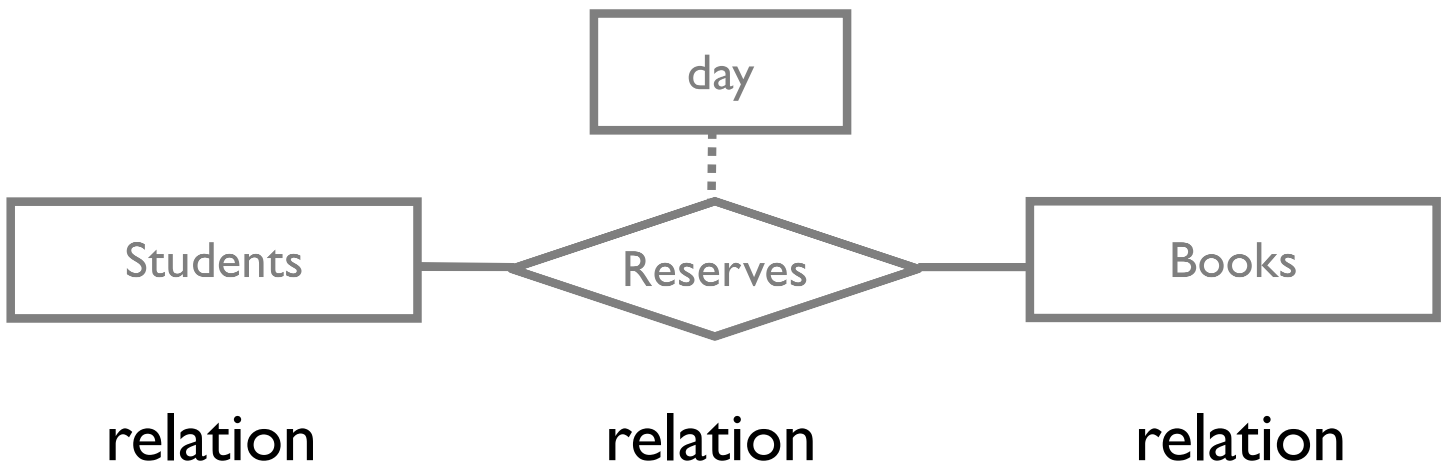
$$A \cap B = R_{result}$$



Can we express using core operators?

$A \cap B = A - (A - B)$

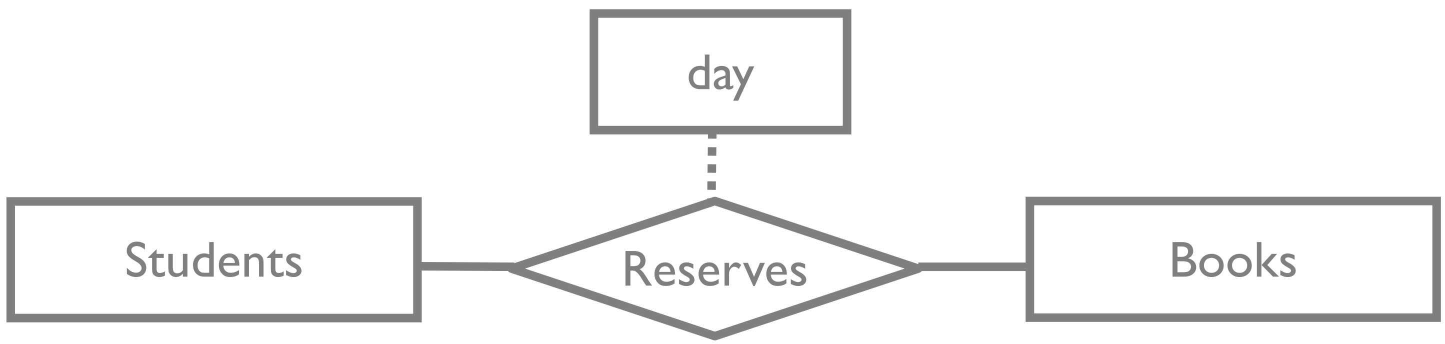
Joins (high level)



What if you want to query across all three tables?
e.g., all names of students that reserved "The Purple Crayon"

Need to combine these tables
Cross product? But that ignores foreign key references

Joins (high level)



relation relation relation

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

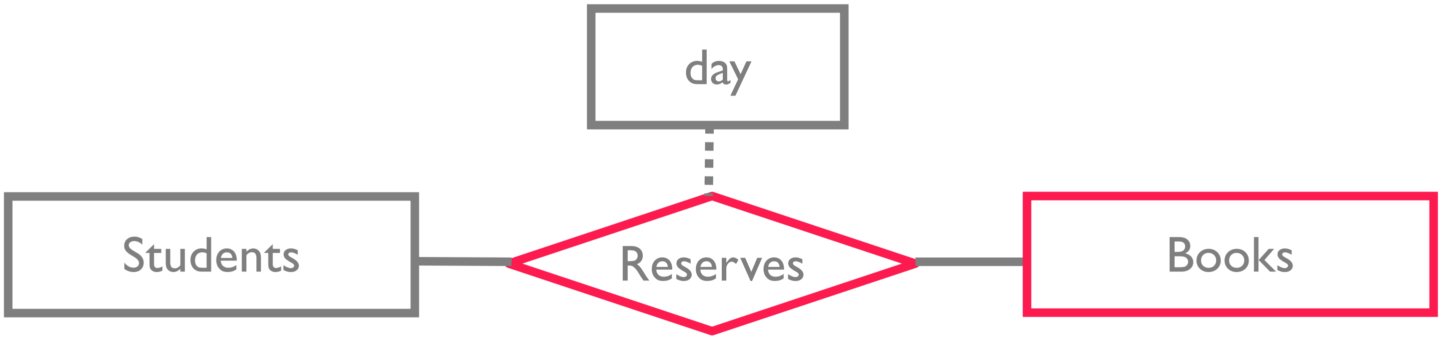
R1

sid	rid	day
1	101	10/10
2	102	11/11

B1

rid	name
101	The Purple Crayon
102	1984

Joins (high level)



relation relation relation

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

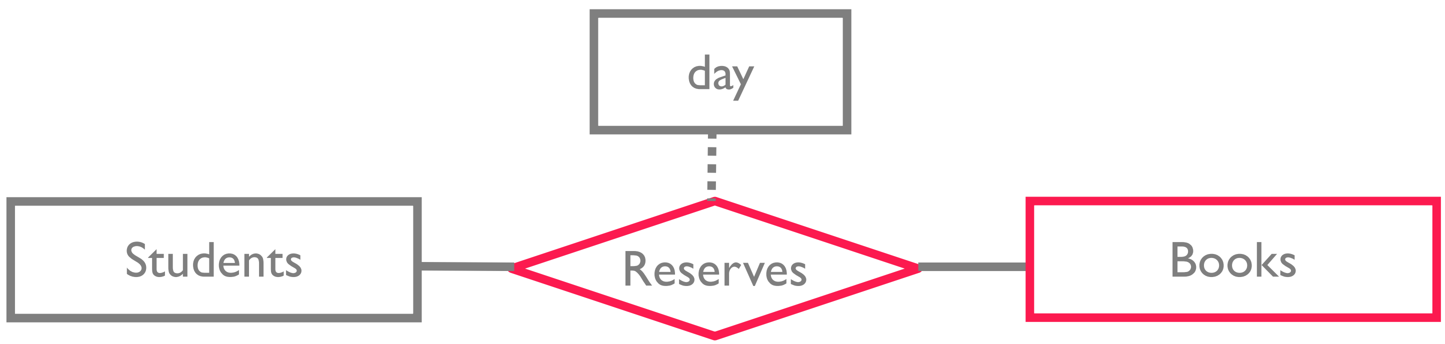
R1

sid	rid	day
1	101	10/10
2	102	11/11

B1

rid	name
101	The Purple Crayon
102	1984

Joins (high level)



relation relation relation

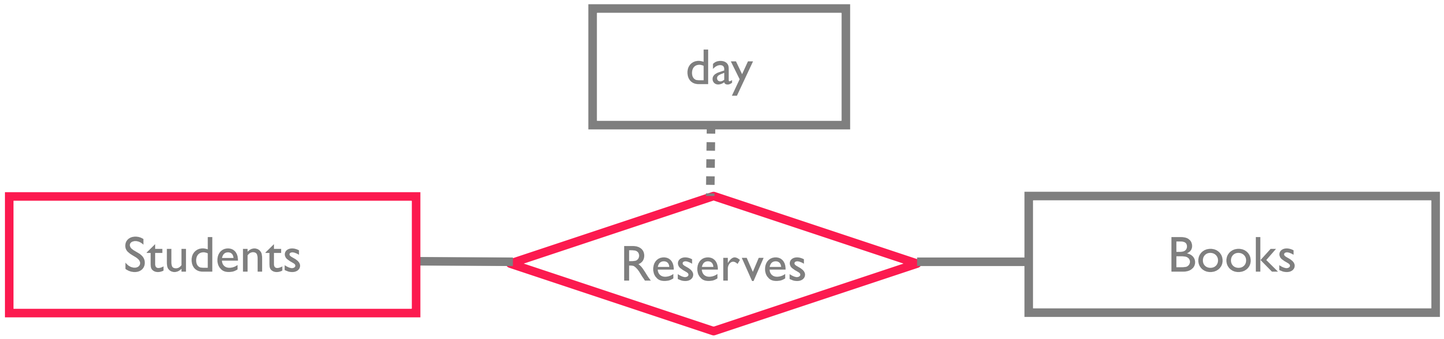
S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

RB1

sid	(rid)	day	(rid)	name
1	101	10/10	101	The Purple Crayon
2	102	11/11	102	1984

Joins (high level)



relation relation relation

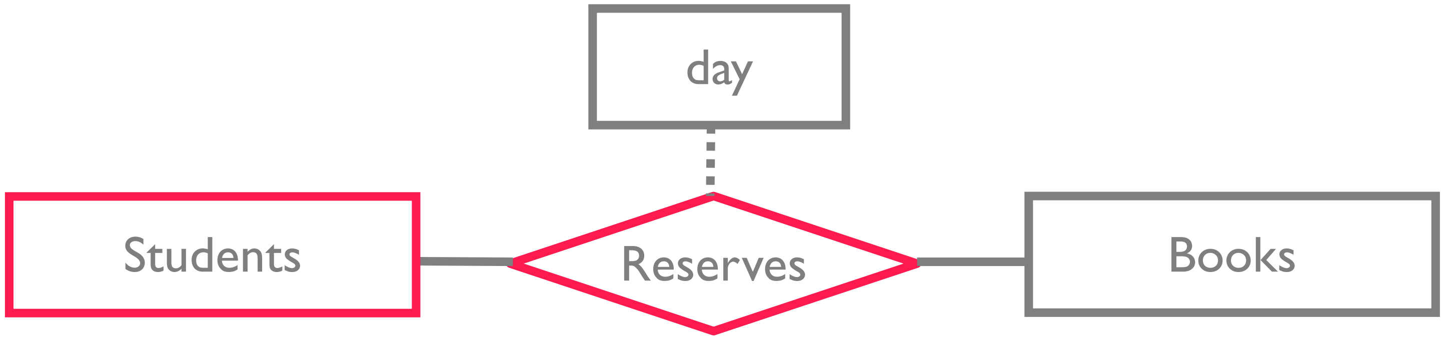
S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

RB1

sid	(rid)	day	(rid)	name
1	101	10/10	101	The Purple Crayon
2	102	11/11	102	1984

Joins (high level)



relation relation relation

SRB1

(sid)	(name)	gpa	age	(sid)	(rid)	day	(rid)	(name)
1	eugene	4	20	1	101	10/10	101	The Purple Crayon
2	barb	3	21	2	102	11/11	102	1984

theta (θ) Join

$$A \bowtie_c B = \sigma_c(A \times B)$$

Most general form

Result schema same as cross product

Often *far* more efficient to compute than cross product

Commutative

$$(A \bowtie_c B) \bowtie_c C = A \bowtie_c (B \bowtie_c C)$$

theta (θ) Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

$\sigma_{S1.sid \leq R1.sid}(S1 \times R1) =$

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
2	barb	3	21	1	101	10/10
3	tanya	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barb	3	21	2	102	11/11
3	tanya	2	88	2	102	11/11

theta (θ) Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

$\sigma_{S1.sid \leq R1.sid}(S1 \times R1) =$

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
2	barb	3	21	1	101	10/10
3	tanya	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barb	3	21	2	102	11/11
3	tanya	2	88	2	102	11/11

theta (θ) Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{S1.sid \leq \mathbf{R1.sid}} R1 =$

$\sigma_{S1.sid \leq \mathbf{R1.sid}}(S1 \times R1) =$

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barb	3	21	2	102	11/11

Equi-Join

$$A \bowtie_{\text{attr}} B = \boldsymbol{\pi}_{\text{all attrs } \textit{except} \text{ B.attr}}(A \bowtie_{\text{A.attr = B.attr}} B)$$

Special case where the condition is attribute equality

Result schema only keeps *one copy* of equality fields

Natural Join ($A \bowtie B$):

Equijoin on *all* shared fields (fields w/ same name)

Equi-Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barb	3	21
3	tanya	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{sid} R1 =$

sid	name	gpa	age	rid	day
1	eugene	4	20	101	10/10
2	barb	3	21	102	11/11

Equi-Join

S1

sid	name	day	gpa	age
1	eugene	10/10	4	20
2	barb	12/12	3	21
3	tanya	3/3	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{sid,name} R1$ = INVALID! name not in R1

$S1 \bowtie_{sid,day} R1 = S1 \bowtie_{s1.sid=R1.sid \wedge s1.day = R1.day} R1$

sid	name	day	gpa	age	rid
1	eugene	10/10	4	20	101

$S1 \bowtie R1 = S1 \bowtie_{sid,day} R1$

Division (skipped during lecture)

Let us have relations A(x, y), B(y)

$$A/B = \{ \langle x \rangle \mid \forall y \in B \ \langle x,y \rangle \in A \}$$

Find all students that have reserved all books

A/B = all x (students) s.t. for every y (reservation), $\langle x,y \rangle \in A$

Good to ponder, not supported in most systems (why?)

Generalization

y can be a list of fields in B

x U y is fields in A

Examples

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

R1

rid
2

R2

rid
2
4

R3

rid
1
2
4

A/R1

A/R2

A/R3

Which sid values have relationship with all vals in Rx?

Examples

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

R1

rid
2

R2

rid
2
4

R3

rid
1
2
4

sid
1
2
3
4

A/R1

A/R2

A/R3

Which sid values have relationship with all vals in Rx?

Examples

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

R1

rid
2

sid
1
2
3
4

A/R1

R2

rid
2
4

sid
1
4

A/R2

R3

rid
1
2
4

A/R3

Which sid values have relationship with all vals in Rx?

Examples

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

R1

rid
2

sid
1
2
3
4

A/R1

R2

rid
2
4

sid
1
4

A/R2

R3

rid
1
2
4

sid
1

A/R3

Which sid values have relationship with all vals in Rx?

Is A/B a Fundamental Operation?

No. Shorthand like Joins

joins natively supported: ubiquitous, can be optimized

Hint: Find all *x*s not 'disqualified' by some *y* in *B*.

A(x, y), B(y)

x value is *disqualified* if

1. by attaching *y* value from *B* (e.g., create <x, y>)
2. we obtain an <*x*,*y*> that is not in *A*.

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

Disqualified =

A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_{sid}(A)$

sid
1
2
3
4

Disqualified =

A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_{sid}(A)$

sid
1
2
3
4

$\pi_{sid}(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

Disqualified = $(\pi_{sid}(A) \times B)$

A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_{sid}(A)$

sid
1
2
3
4

$\pi_{sid}(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

$(\pi_{sid}(A) \times B) - A$

sid	rid
2	4
3	4

Disqualified = $((\pi_{sid}(A) \times B) - A)$

A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_{sid}(A)$

sid
1
2
3
4

$\pi_{sid}(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

$(\pi_{sid}(A) \times B) - A$

sid	rid
2	4
3	4

sid
1
4

A/B

$$\text{Disqualified} = \pi_{sid}((\pi_{sid}(A) \times B) - A)$$

$$A/B = \pi_{sid}(A) - \text{Disqualified}$$

Different Plans, Same Results

Semantic equivalence:

results are *always* the same

Note that it is independent

of the database instance!

Names of students that reserved book 2

$$\pi_{name}(\sigma_{rid=2}\ (R1) \bowtie S1)$$

Equivalent Queries

$$p(tmp1,\ \sigma_{rid=2}\ (R1))$$
$$p(tmp2,\ tmp1 \bowtie S1)$$
$$\pi_{name}(tmp2)$$

$$\pi_{name}(\sigma_{rid=2}(R1 \bowtie S1))$$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}$ (Book)

Names of students that reserved db books

Book(rid, type)   Reserve(sid, rid)   Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}(Book) \bowtie Reserve$

Names of students that reserved db books

Book(rid, type)   Reserve(sid, rid)   Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student$

Names of students that reserved db books

Book(rid, type)   Reserve(sid, rid)   Student(sid, name)

Need to join DB books with reserve and students

$\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid, name)

Need to join DB books with reserve and students

$$\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$$

More efficient query

$$\pi_{name}(\pi_{sid}((\pi_{rid}\, \sigma_{type='db'}(Book)) \bowtie Reserve) \bowtie Student)$$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type)   Reserve(sid, rid)   Student(sid, name)

Need to join DB books with reserve and students

$$\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$$

More efficient query

$$\pi_{name}(\pi_{sid}((\pi_{rid}\ \sigma_{type='db'}(Book)) \bowtie Reserve) \bowtie Student)$$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type)   Reserve(sid, rid)   Student(sid, name)

Need to join DB books with reserve and students

$\pi_{name}(\sigma_{type=\text{'db'}}(\text{Book}) \bowtie \text{Reserve} \bowtie \text{Student})$

More efficient query

$\pi_{name}(\pi_{sid}((\pi_{rid}\,\sigma_{type=\text{'db'}}(\text{Book})) \bowtie \text{Reserve}) \bowtie \text{Student})$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid, name)

Need to join DB books with reserve and students

$\pi_{name}(\sigma_{type=\text{'db'}}(\text{Book}) \bowtie \text{Reserve} \bowtie \text{Student})$

More efficient query

$\pi_{name}(\pi_{sid}((\pi_{rid}\, \sigma_{type=\text{'db'}}(\text{Book})) \bowtie \text{Reserve}) \bowtie \text{Student})$

Query optimizer can find the more efficient query!

Students that reserved DB or HCI book

1. Find all DB or HCI books
2. Find students that reserved one of those books

$$\rho(tmp, (\sigma_{type='DB' \vee type='HCI'} (Book))$$

$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

"v" means logical OR

Alternatives

define tmp using UNION (how?)

Using UNION

$p(tmp1, (\sigma_{type='DB'}\ (Book))$

$p(dbnames, \pi_{name}(tmp1 \bowtie Reserve \bowtie Student))$

$p(tmp2, (\sigma_{type='HCI'}\ (Book))$

$p(hcinames, \pi_{name}(tmp2 \bowtie Reserve \bowtie Student))$

dbnames UNION hcinames