Generative Adversarial Nets

Ian J. Goodfellow, Jean Pouget-Abadie∗ , Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair† , Aaron Courville, Yoshua Bengio‡

Presented by: Kithdara Hansamal

Generative Adversarial Nets

Generative model (G)

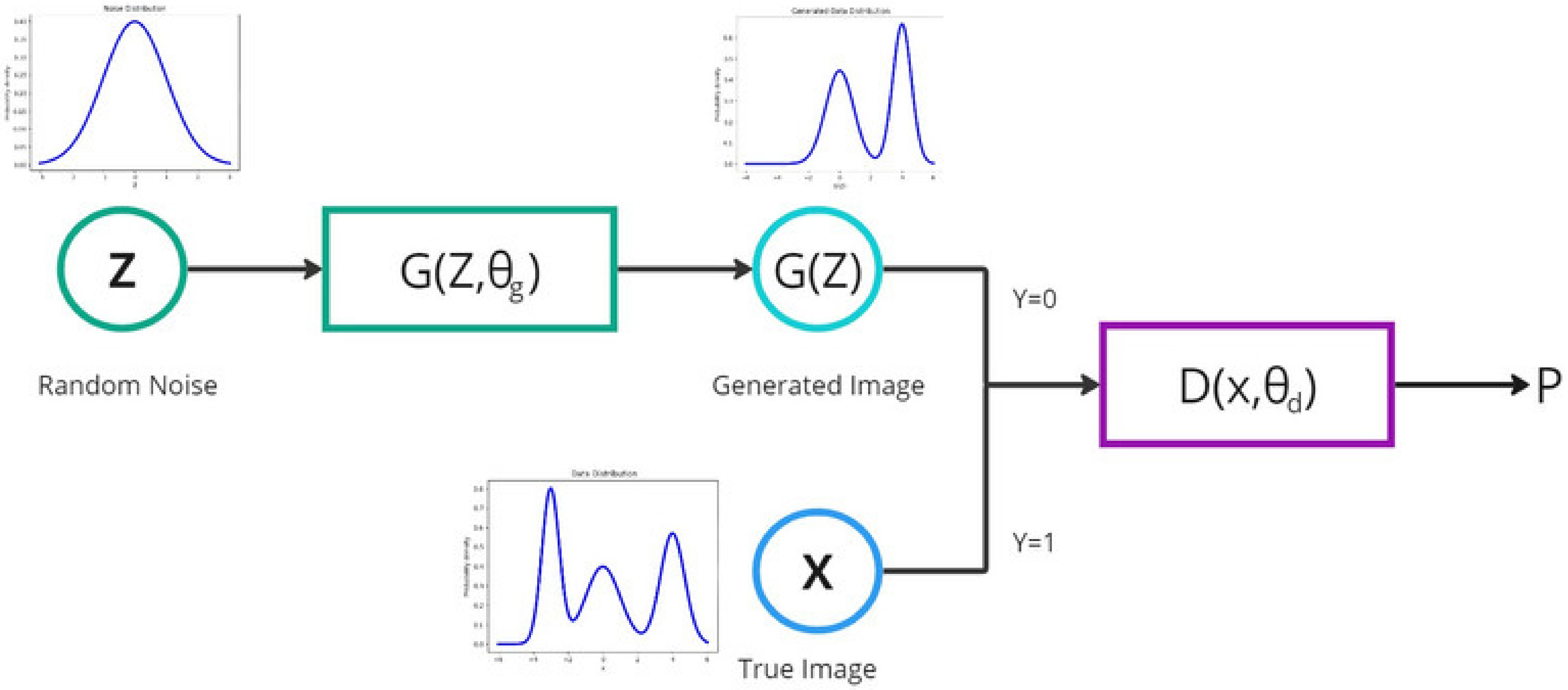
Discriminative model (D)

Generative Adversarial Nets

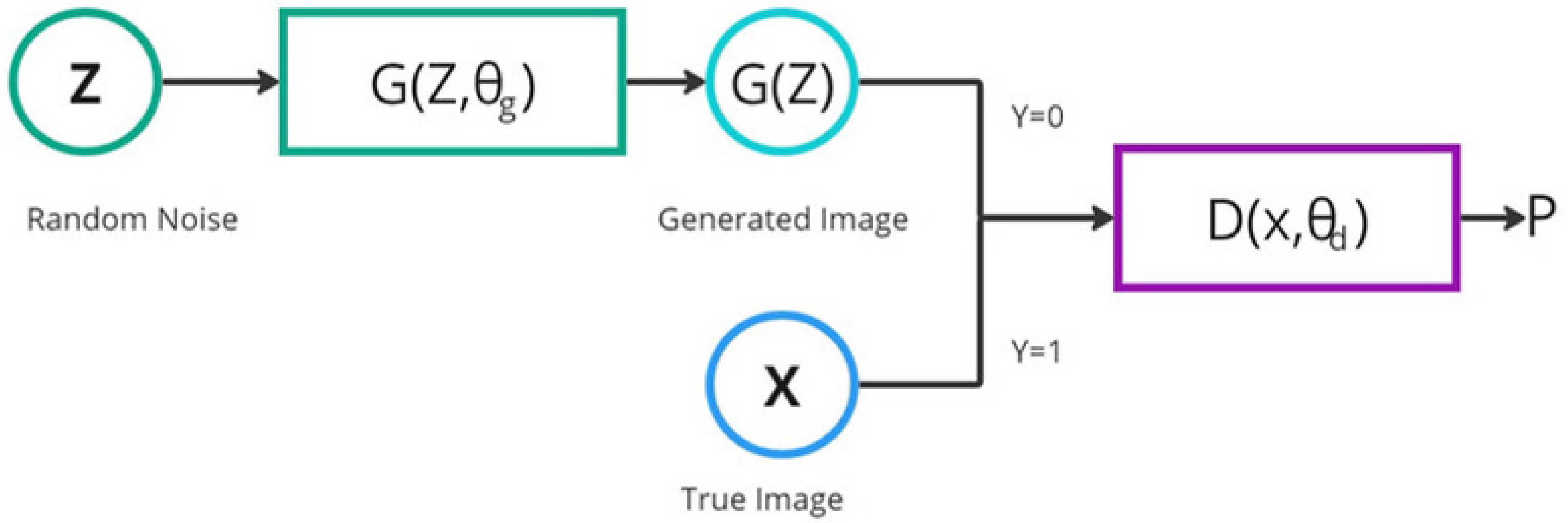
Generative model (G)

Discriminative model (D)

Generative Adversarial Nets



Minmax Game

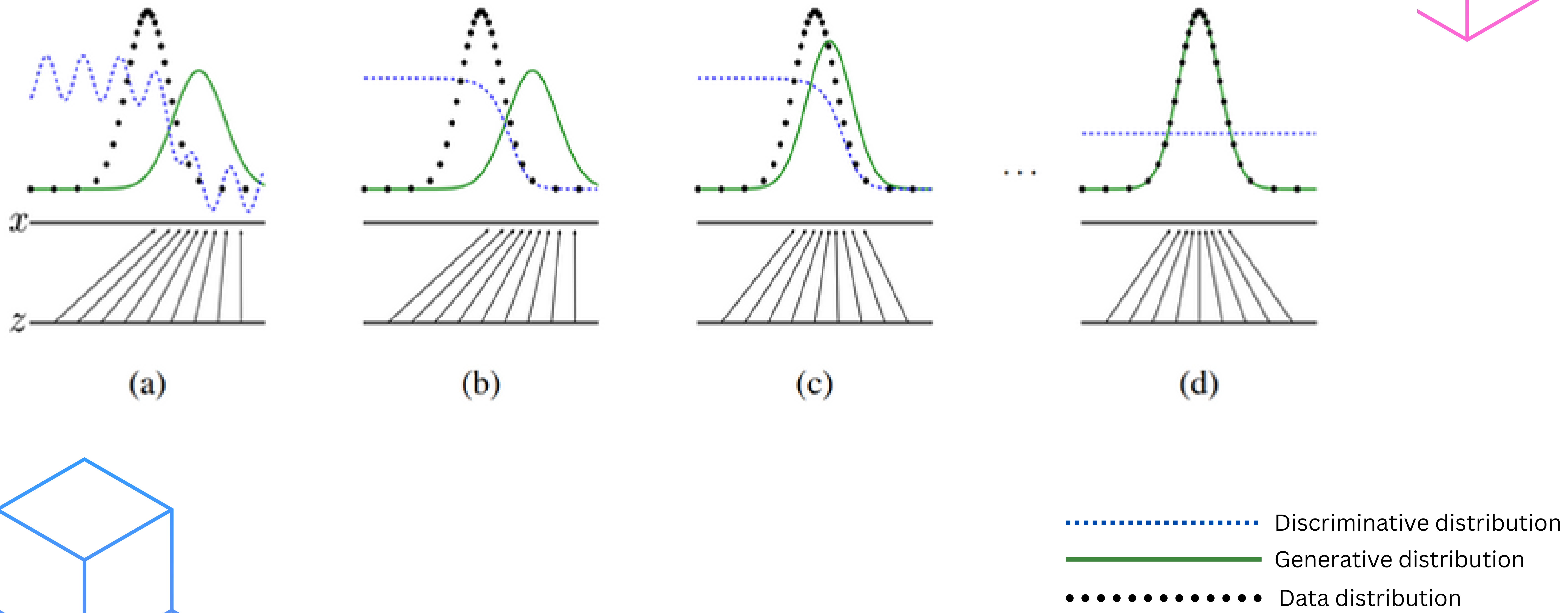


Z
Random Noise
G(Z,θg)
G(Z)
Generated Image
Y=0
D(x,θd)
P
Y=1
X
True Image

Value Function

$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))].$$

Theoretical Results



(a) (b) (c) (d)

Algorithm

for number of training iterations **do**

for k steps **do**

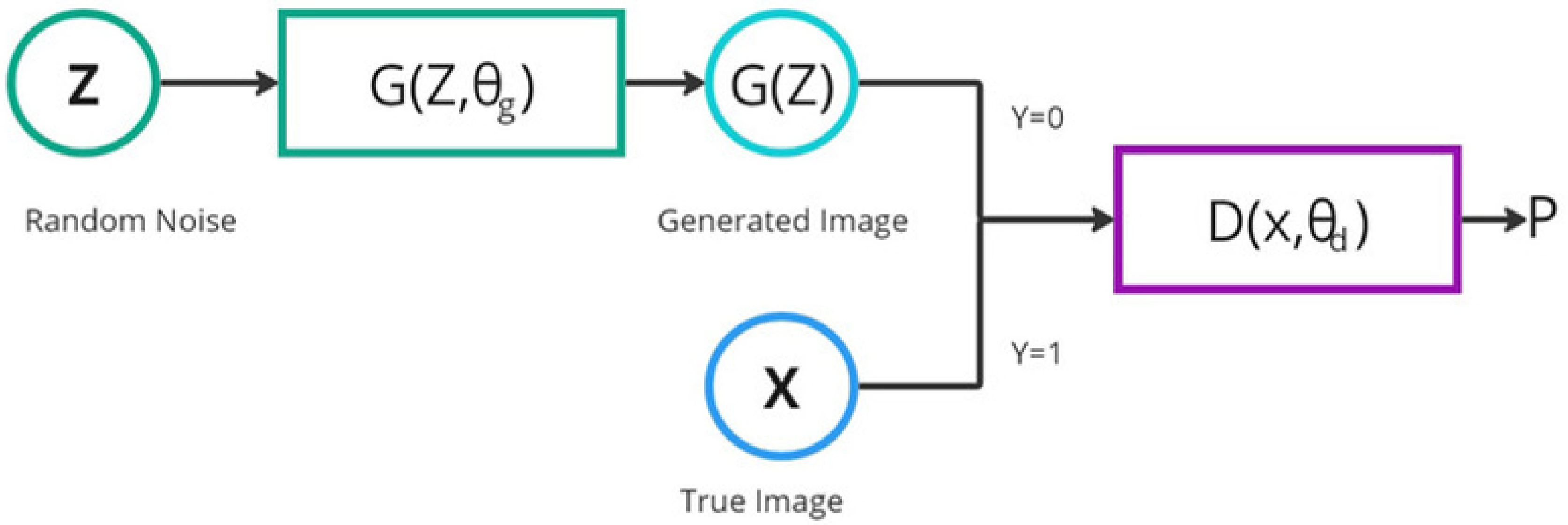
- Sample minibatch of m noise samples {z (1) , . . . , z (m)} from noise prior $p_g(z)$.
- Sample minibatch of m examples {x (1) , . . . , x (m)} from data generating distribution $p_{data}(x)$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right].$$

end for

- Sample minibatch of m noise samples {z (1) , . . . , z (m)} from noise prior pg(z).
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$



end for

Global Optimality

Proposition

For G fixed, the optimal discriminator D is

$$D_G^*(\boldsymbol{x}) = \frac{p_{data}(\boldsymbol{x})}{p_{data}(\boldsymbol{x}) + p_g(\boldsymbol{x})}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$
$y \to a \log(y) + b \log(1 - y)$
maximum in $[0, 1]$ at $\frac{a}{a+b}$

$$\begin{aligned} V(G, D) &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_{z} p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz \\ &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx \end{aligned}$$

Global Optimality

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}}[\log(1 - D_G^*(G(\boldsymbol{z})))] \\
&= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))] \\
&= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}\left[\log \frac{p_{\text{data}}(\boldsymbol{x})}{P_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] + \mathbb{E}_{\boldsymbol{x} \sim p_g}\left[\log \frac{p_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right]
\end{aligned}
$$

Global Optimality

Theorem

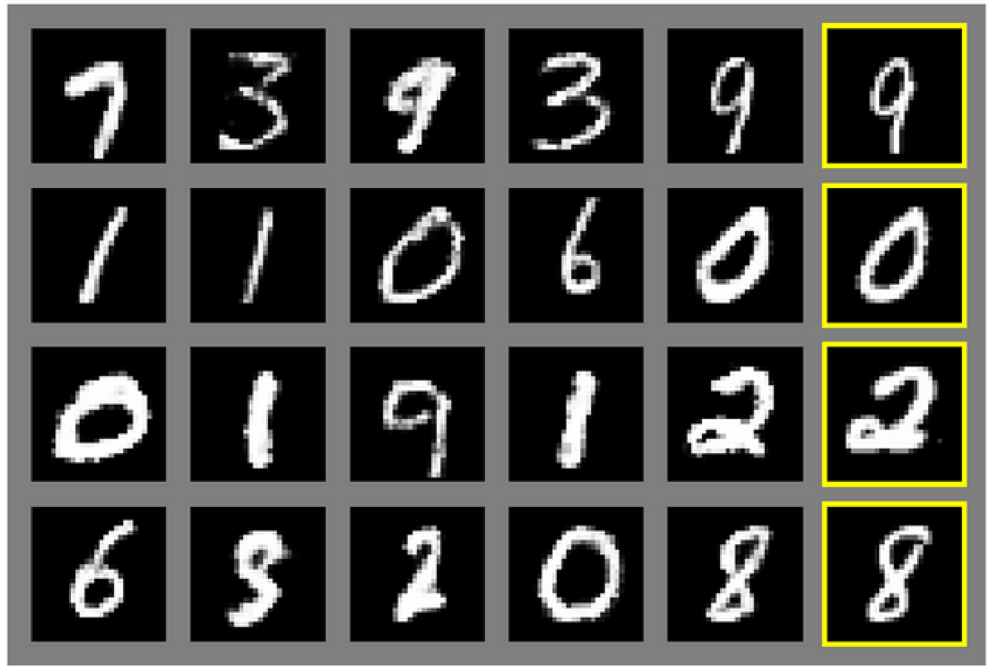
The global minimum of the virtual training criterion C(G) is achieved if and only if pg = pdata.
At that point, C(G) achieves the value − log 4.

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \,||\, p_g\right)$$

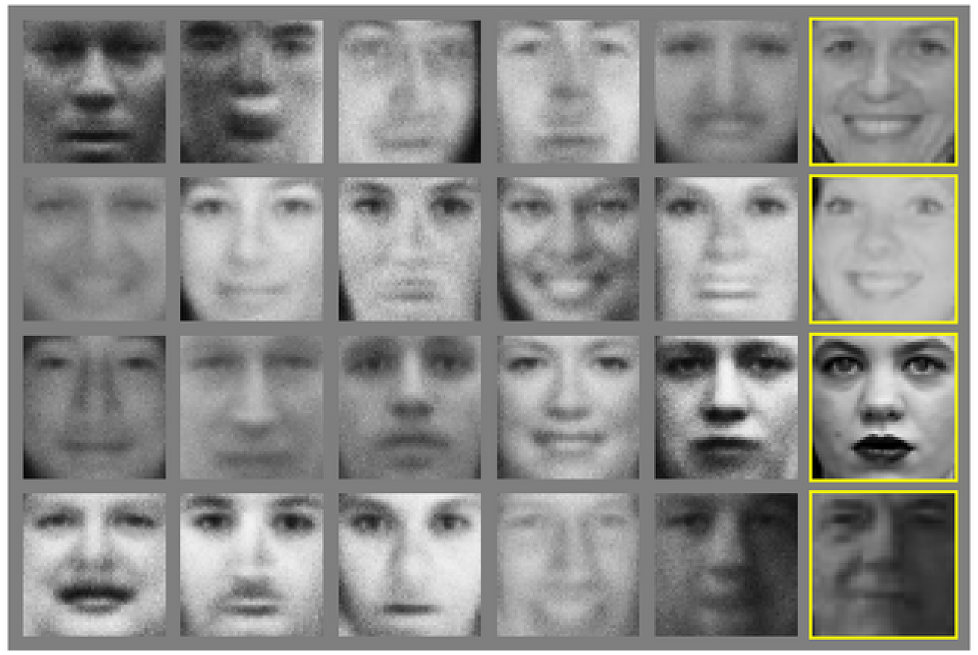
Jensen–Shannon divergence between two distributions is always non-negative, and zero iff they are equal

Advantages	Disadvantages
• Markov chains are never needed • Only backprop is used to obtain gradients • No inference is needed during the learning • A wide variety of functions can be incorporated into the model	• No explicit representation of pg(x) • D must be synchronized well with G during training

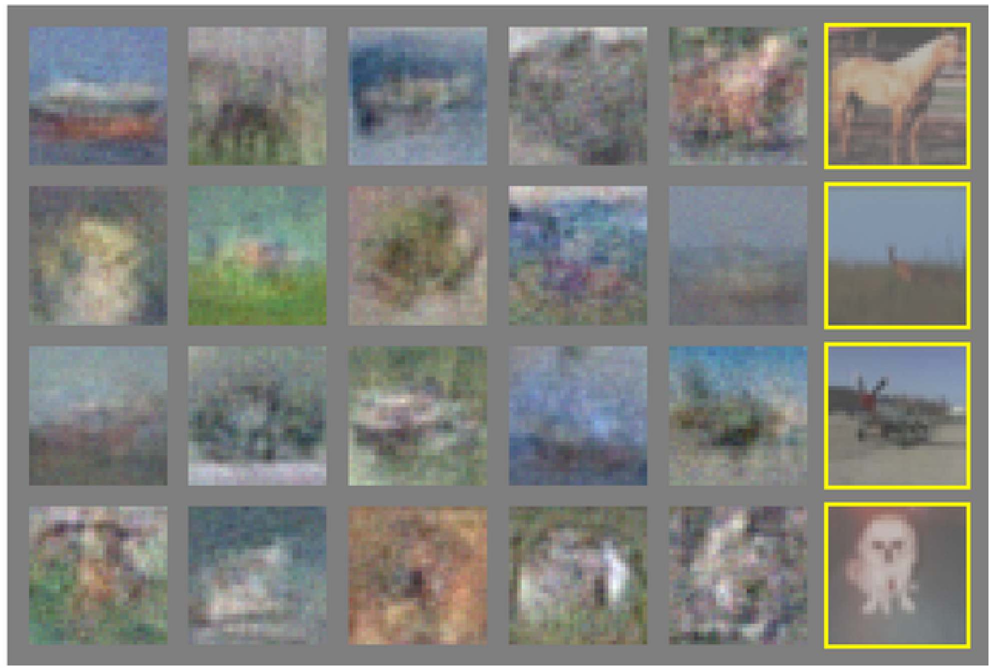
Conclusions



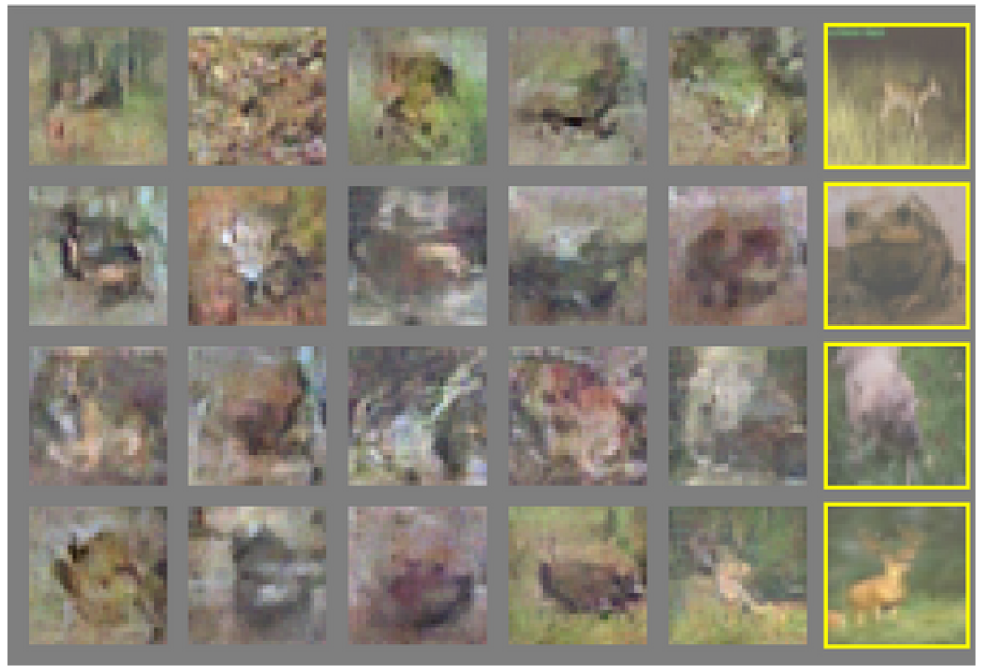
MNIST



TFD



CIFAR-10
(Fully connected model)



CIFAR-10
(convolutional discriminator and "deconvolutional" generator)

Thank You