Artificial Neural Networks for Pattern Recognition: applications to Face Detection and Recognition

Sébastien Marcel

marcel@idiap.ch

IDIAP Research Institute
Martigny, Switzerland

http://www.idiap.ch

Outline

- Introduction to Statistical Machine Learning
- Artificial Neural Networks
- Application Examples

Outline

- Introduction to Statistical Machine Learning
  - Learning and Learning
  - Capacity and Generalization
  - Regression, Classification and Density Estimation
  - Applications
- Artificial Neural Networks
- Application Examples

Learning and Learning

- Learning by heart:

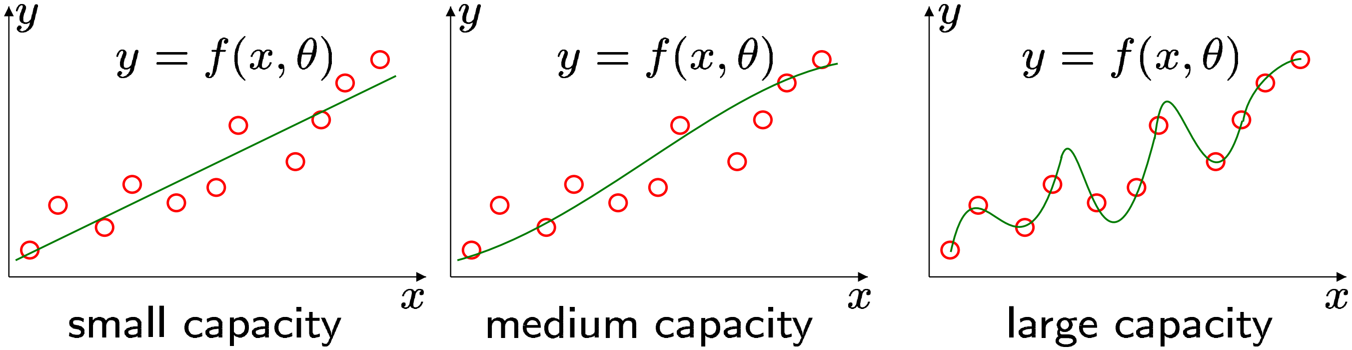
$$\begin{array}{cc} 1+0=1 & 1\times 0=0 \\ 1+1=2 & 1\times 1=1 \\ 1+2=3 & 1\times 2=2 \\ \ldots & \ldots \end{array}$$

any computer can do that !

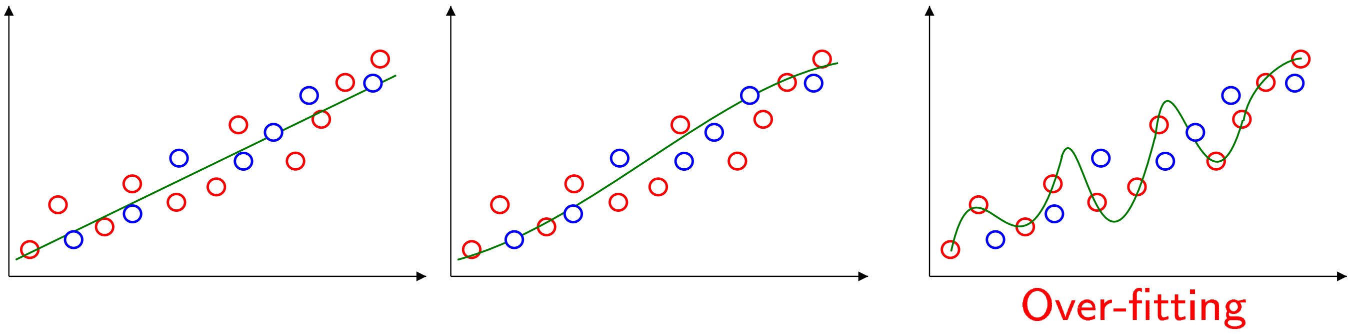
- Learning by heart is not learning:
  - learning is "to gain knowledge or understanding of or skill in by study, instruction, or experience" (Merriam-Webster),
  - the difficulty of learning is to be able to **generalize**.

Capacity and Generalization

- Capacity: # of parameters (θ) required to fit the data with a function

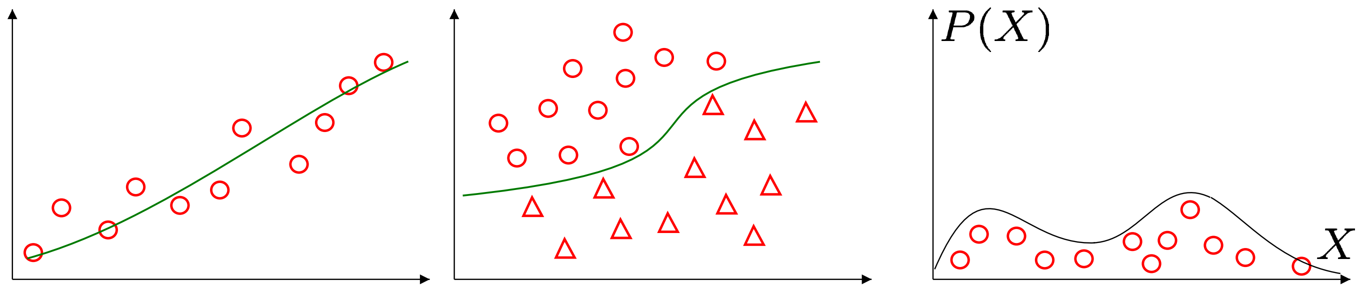


- Generalization: the performance of the above function on unseen data



Regression, Classification and Density Estimation

- there are 3 kinds of problems:



regression, classification and density estimation

- Machine Learning Algorithms address the above problems using various tools:
  - Artificial Neural Networks,
  - Support Vector Machines,
  - Gaussian Mixture Models,
  - Hidden Markov Models,
  - and many others ...

Applications

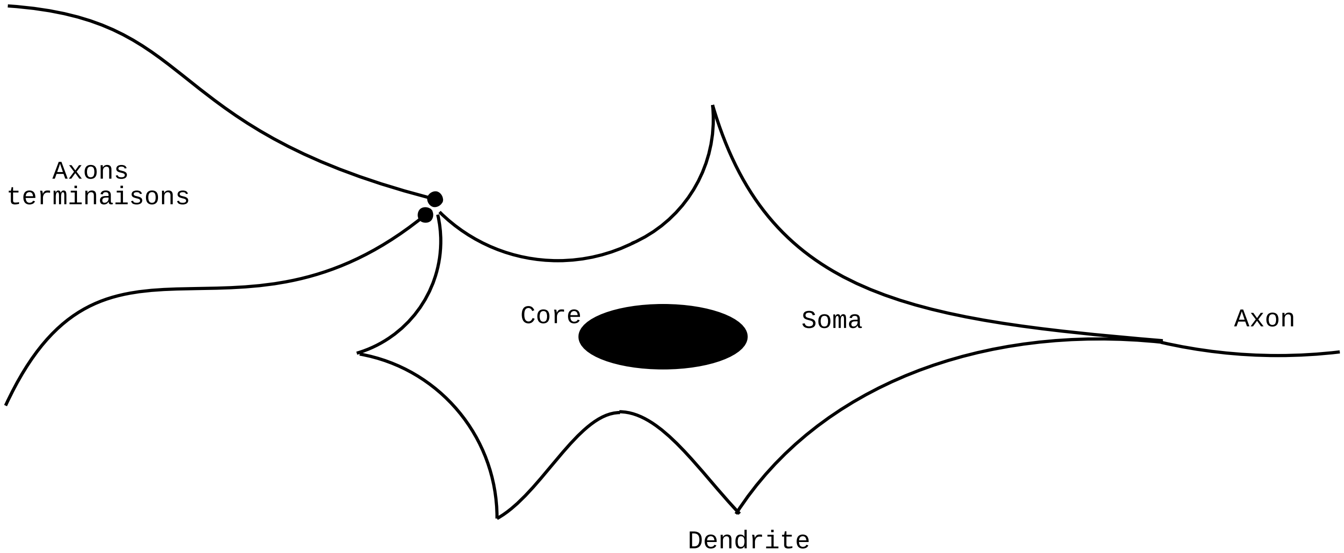
- in Computer Vision:
  - Face detection, face recognition, face orientation estimation,
  - Gesture recognition,
  - Optical character recognition,
  - Handwritten recognition.
- in Speech Processing:
  - Speech recognition,
  - Speaker recognition.
- but also in Finance, Telecoms, Games, Robotic and more ...

Outline

- Introduction to Statistical Machine Learning
- Artificial Neural Networks
  - Biological Bases
  - History
  - The Formal Neuron
  - The Perceptron
  - Linearly Separable
  - The problem of XOR
  - The Multi Layer Perceptron
  - Cost function and Criterion
  - Gradient Descent
  - More about classification and MLP tricks
- Application Examples

ANN: Biological Bases

- the brain:
  - 10^{12} neurons massively connected,
  - 1 neuron is connected with 10^3 others in average,



- the neuron:
  - core: where DNA belongs (approx. 30000 genes)
  - dendrite: receives a signal from other neurons (via their axon),
  - axon: propagates the signal to other neurons.

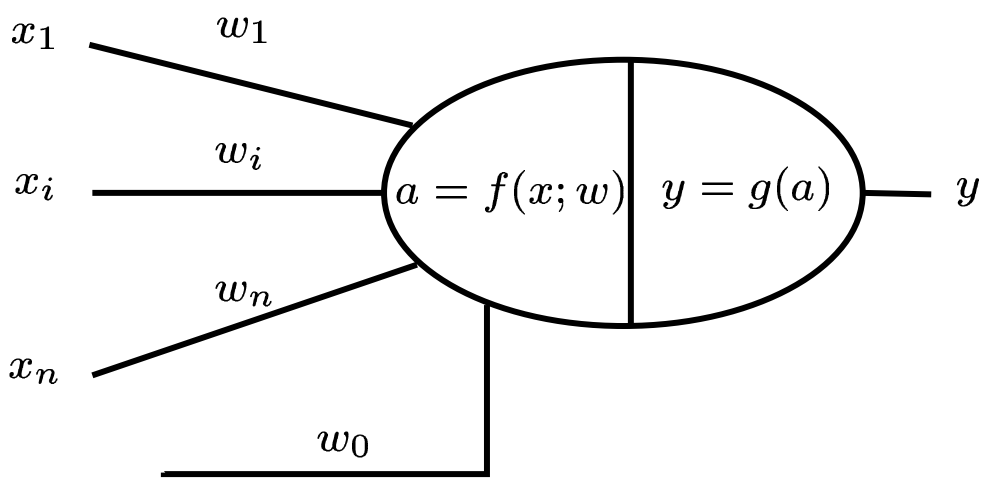
Now, let's forget about biology and let's come back to Mathematics !

History of ANN

- Mc Culloch and Pitts (1943): formal neuron inspired from biology
- Hebb (1949): the first training rule
- Rosenblatt (1962): the Perceptron
- Minsky and Papert (1969): limitations of Perceptron
- then, nothing during 16 years, research goes for symbolic AI
- Hopfield (1982): auto-associative memory
- Werbos (1974), Rumelhart (1986), Parker (1985), Le Cun (1985): Multi Layer Perceptron and Back-propagation algorithm
- Kohonen (1995): auto-organizing maps

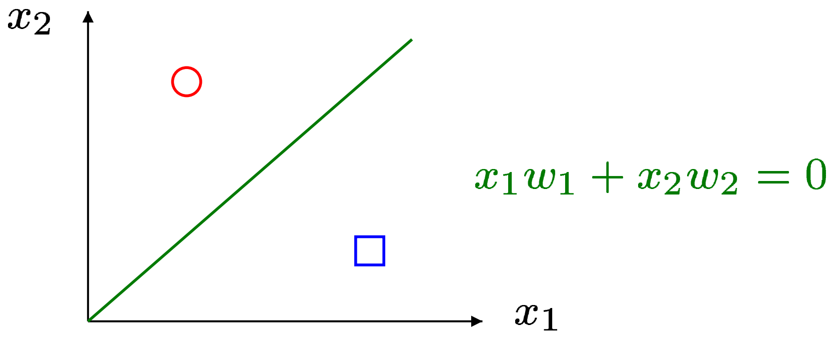
The Formal Neuron

- Mc Culloch and Pitts (1943):
  - x is the input $\in \mathbb{R}^n$,
  - $w_1 ... w_n$ are the weights,
  - w_0 is the bias,
  - a is the result of the integration function $f(x;w) = \sum_{i=1}^{n} w_i x_i + w_0$,
  - y is the output of the transfer function $g(a) = tanh(a)$.



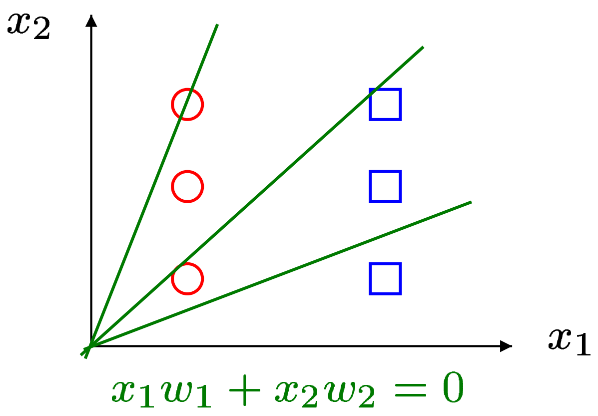
The Role of the Bias

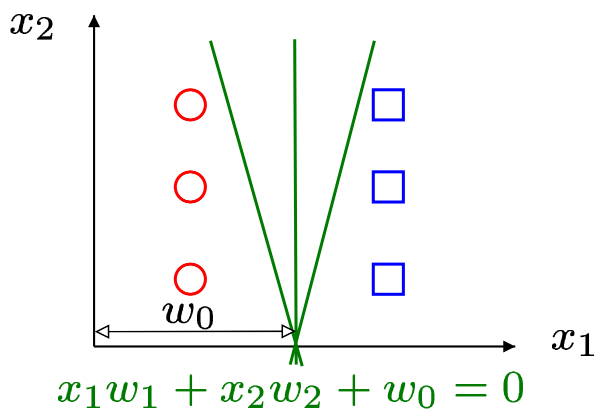
- the formal neuron is a linear separator:



$$x_1w_1 + x_2w_2 = 0 \quad \Leftrightarrow \quad x_2 = -\frac{w_1}{w_2}x_1$$

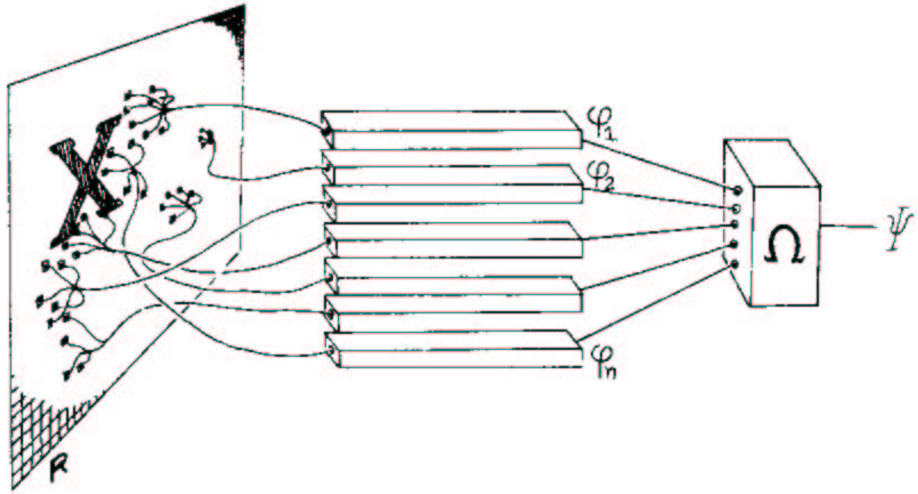
- without the bias the linear separation is not always possible:





The Perceptron

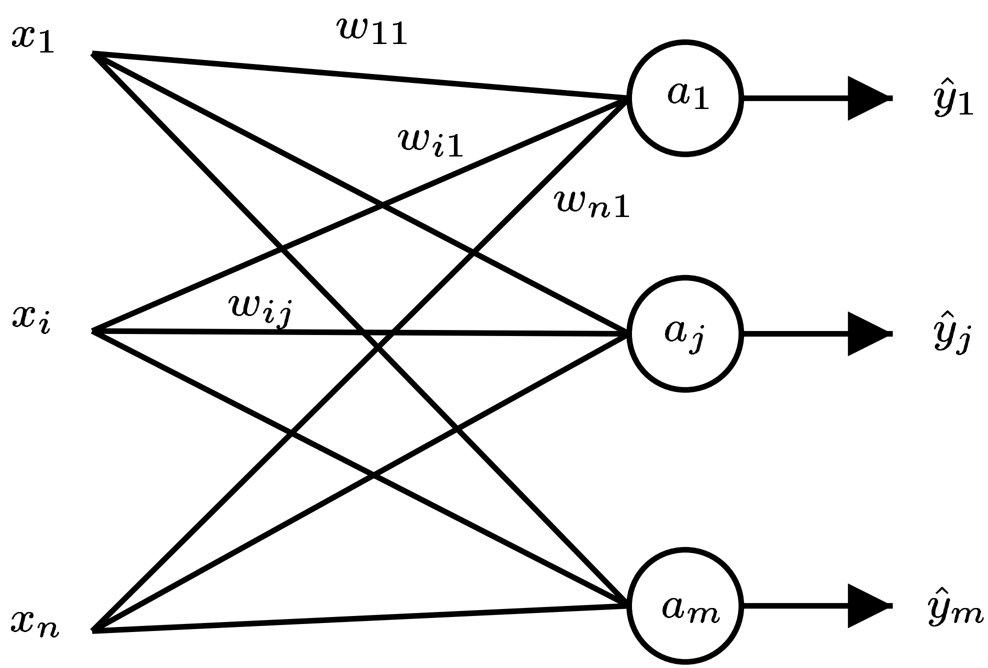
- Rosenblatt (1962):



  - a retina: binary input of the perceptron,
  - association cells: “pre-processing”,
  - decision cells: linear units.

The Perceptron

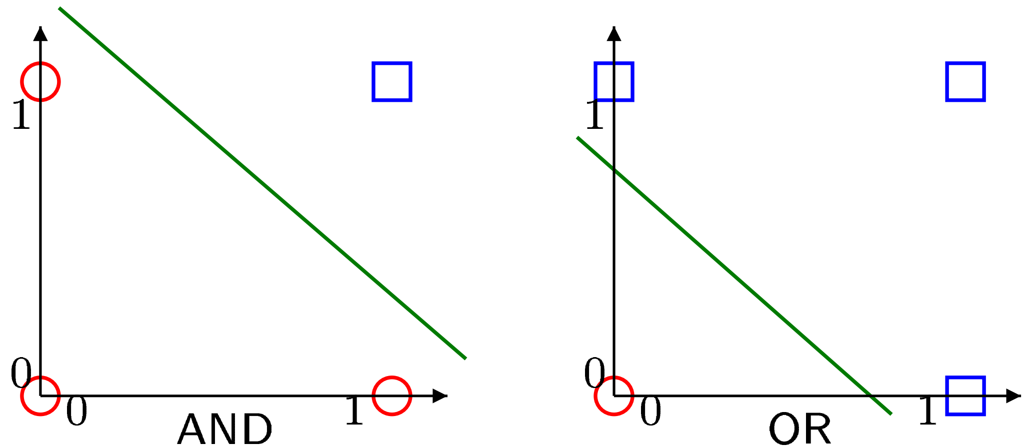
- Rosenblatt (1962):



- Training rules:
  - $w_{ij}^{t+1} = w_{ij}^{t} + \alpha(\hat{y}_j - y_j)x_i$ (Rosenblatt)
  - $w_{ij}^{t+1} = w_{ij}^{t} + \alpha(\hat{y}_j - a_j)y_i$ (Widrow-Hoff)
- Limitations of Perceptron: restricted to linearly separable problems

Linearly Separable

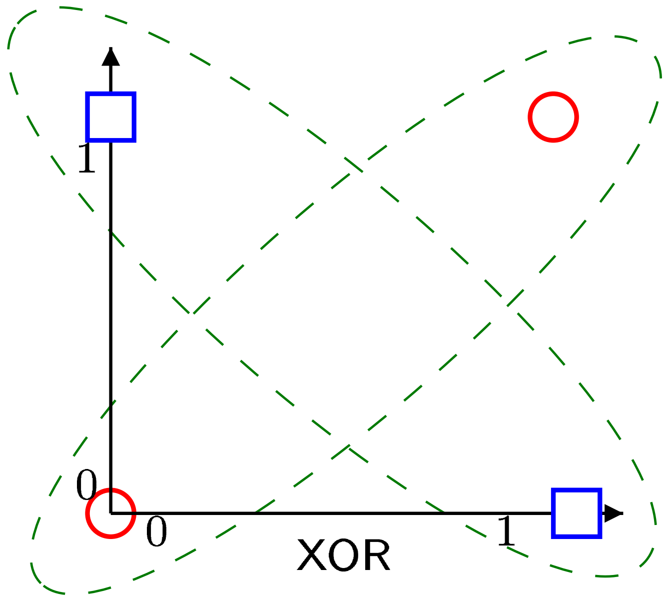
- OR and AND: are linearly separable:



- one solution to AND: $w_1 = 1$, $w_2 = 1$ and $w_0 = 1.5$
- one solution to OR: $w_1 = 1$, $w_2 = 1$ and $w_0 = -0.5$

The problem of XOR

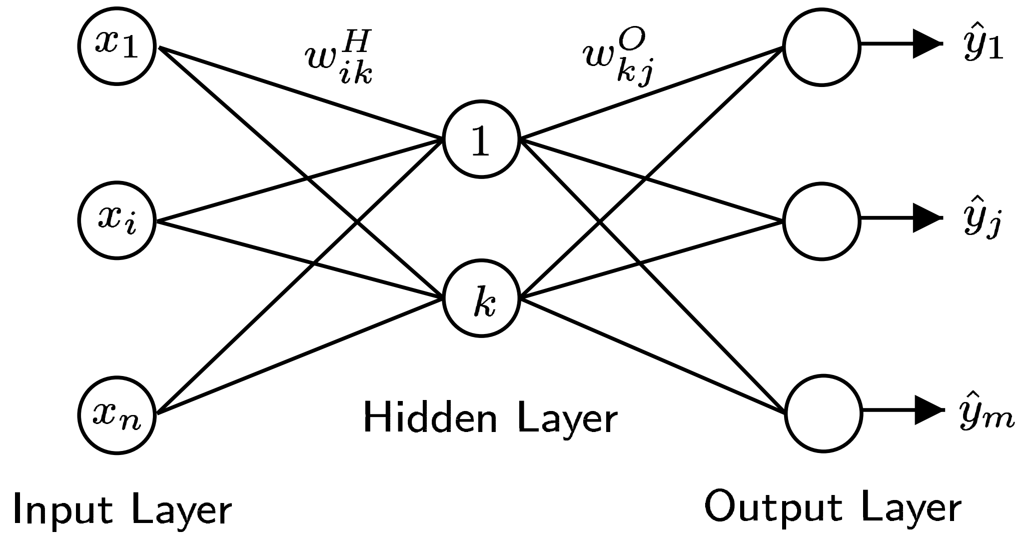
- XOR is not linearly separable:



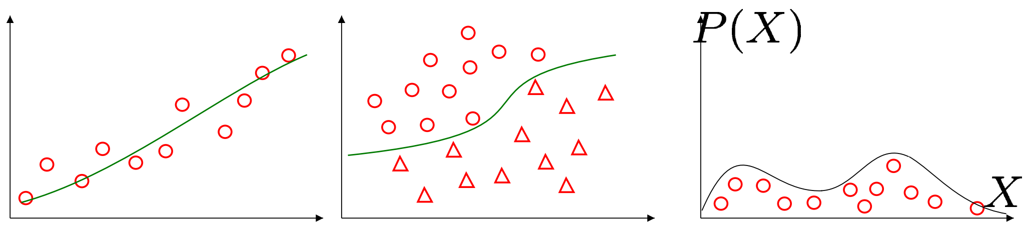
- impossible to solve $x_1 w_1 + x_2 w_2 + w_0 = 0$, but what about multiple equations $x_1 w_{11} + x_2 w_{21} + w_{01} = 0$, $x_1 w_{12} + x_2 w_{22} + w_{02} = 0$,...
- Solution: the Multi Layer Perceptron

The Multi Layer Perceptron

- It contains 1 input layer, 1 or several hidden layer and 1 output layer:



- It can approximate any continuous functions,



regression, classification and density estimation

- Problem: how to modify the weights ?

Outline

- Introduction to Statistical Machine Learning
- Artificial Neural Networks
  - Biological Bases
  - History
  - The Formal Neuron
  - The Perceptron
  - Linearly Separable
  - The problem of XOR
  - The Multi Layer Perceptron
  - Cost function and Criterion
  - Gradient Descent
  - More about classification and MLP tricks
- Application Examples

The Multi Layer Perceptron

- A Multi Layer Perceptron (MLP) is a function: $\hat{y} = MLP(x; W)$,
- W is the set of parameters $\{w_{ij}^l, w_{i0}^l\} \, \forall i, j, l$
- For each unit i on layer l of the MLP:
  - integration: $a_i^l = \sum_j^{H_l} y_j^{l-1} w_{ij}^l + w_{i0}^l$,
  - transfer: $y_i^l = f(a_i^l)$ where $f(x) = tanh(x)$ or $\frac{1}{1+exp(-x)}$ or x
- Input/Output limit cases:
  - on the input layer $(l = 0)$ $y_i^l = x_i \, \forall i = 1..n$,
  - on the output layer $(l = L)$ $\hat{y}_i = y_i^L \, \forall i = 1..m$.
- the data $D_P = \{z_1, z_2, ..., z_P\} \in \mathcal{Z}$ is independently and identically distributed and is drawn from an unknown distribution $p(Z)$,
- 3 forms of data for 3 types for problems:
  - classification: $Z = (X, Y) \in \mathbb{R}^n \times \{-1, 1\}$
  - regression: $Z = (X, Y) \in \mathbb{R}^n \times \mathbb{R}^m$
  - density estimation: $Z \in \mathbb{R}^n$

Cost function and Criterion

- The goal is to minimize a cost function C over the set of data D_P:

$$C(D_P, W) = \sum_{p=1}^{P} L(y(p), \hat{y}(p))$$

- $x(p)$ is the input vector for example p,
- $y(p)$ is the output target vector for example p,
- $\hat{y}$ is the output of the MLP ($\hat{y} = MLP(x; W)$),

(from now let's omit p index)

- L is a criterion to optimize such as the mean squared error (MSE):

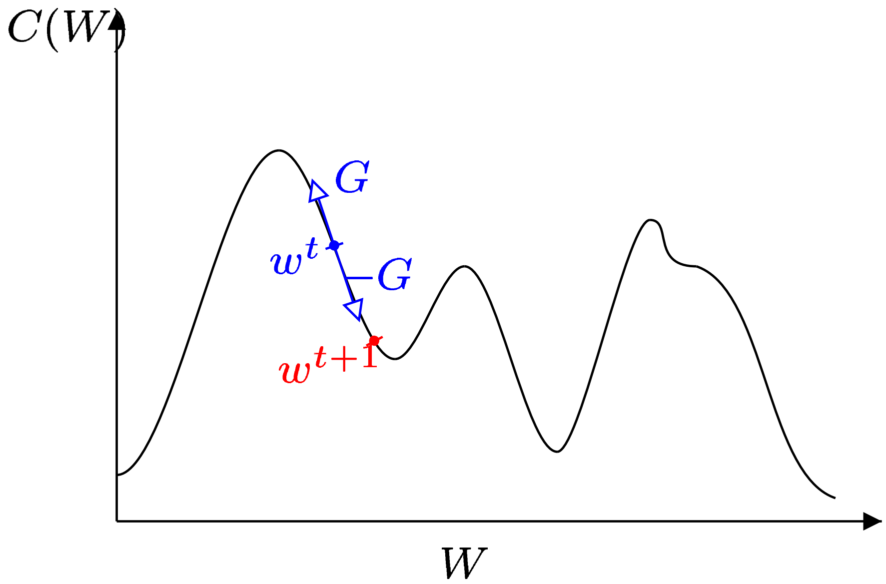
$$MSE(y, \hat{y}) = \frac{1}{2} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$

Gradient Descent

- the gradient descent is an iterative procedure to modify the weights:

$$W^{t+1} = W^t - \eta \frac{\partial C(D, W^t)}{\partial W^t}$$

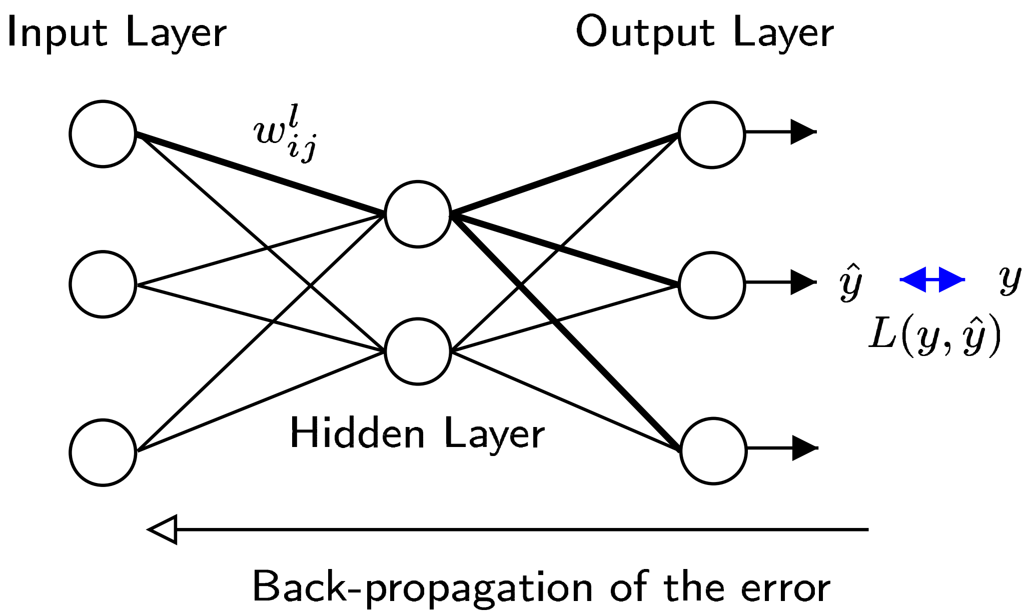
  where η is the learning rate (neither too small or too big)



- the goal is to "move" w^t in the opposite direction of the gradient to reach the global minimum.

Gradient Descent

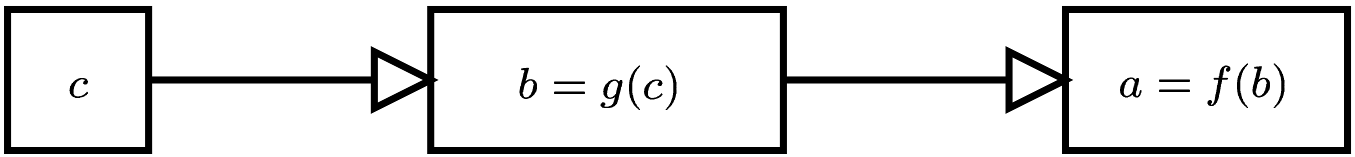
- Computing the gradient and updating the weights is performed from the output neurons to the input neurons, in the inverse order of the propagation (Gradient Back-Propagation).



Gradient Descent

- the chain rule:
  - let us denote $a = f(b)$ and $b = g(c)$
  - then

$$\frac{\partial a}{\partial c} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial c} = f'(b) \cdot g'(c) \tag{1}$$

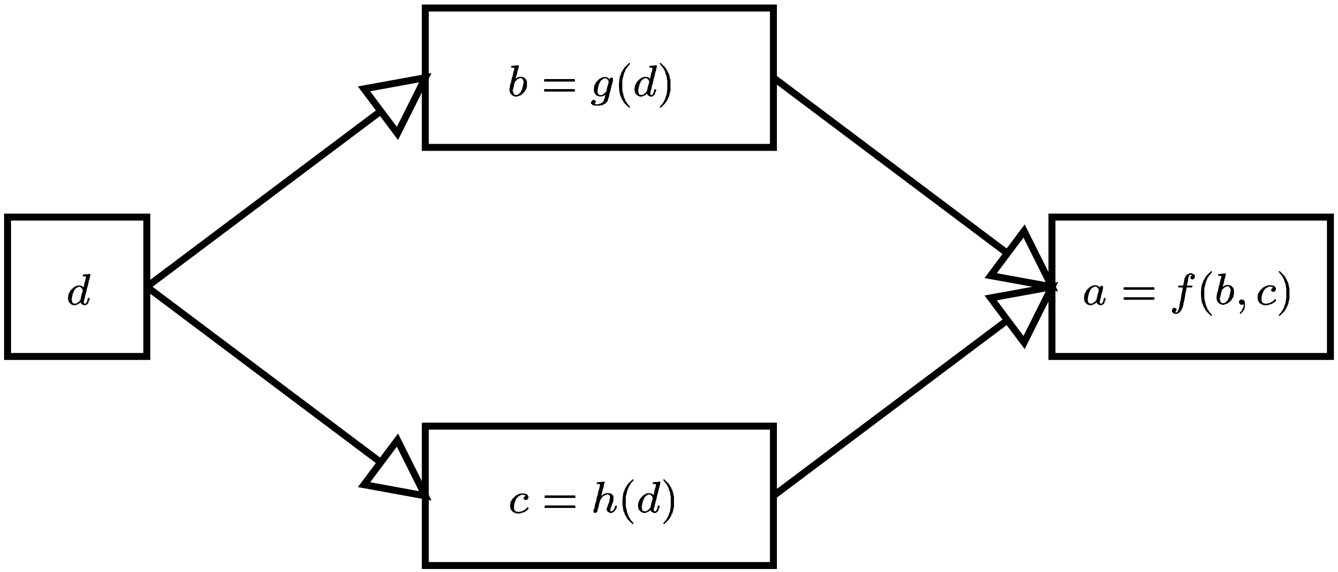


Gradient Descent

- the sum rule:
  - let us denote $a = f(b, c)$, $b = g(d)$ and $c = h(d)$,
  - then

$$\frac{\partial a}{\partial d} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial d} + \frac{\partial a}{\partial c} \cdot \frac{\partial c}{\partial d} \tag{2}$$

$$= \frac{\partial f(b, c)}{\partial b} \cdot g'(d) + \frac{\partial f(b, c)}{\partial c} \cdot h'(d) \tag{3}$$



Gradient Descent

- cost function derivative $\Leftrightarrow$ criterion derivative:

$$\frac{\partial C(D_P, W)}{\partial W} \quad \Leftrightarrow \quad \frac{\partial C_p(W)}{\partial W}$$

- remember that:

$$\begin{aligned} C(D_P, W) &= \sum_{p=1}^{P} L(y(p), \hat{y}(p)) \\ C_p(W) &= \frac{1}{2} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2 = \frac{1}{2} \sum_{i=1}^{m} (y_i - y_i^L)^2 \end{aligned}$$

Gradient Descent

- computes the derivative of the criterion with respect to weights w_{ij}^l.

$$\begin{aligned}
\frac{\partial C_p(W)}{\partial w_{ij}^l} &= \frac{\partial C_p(W)}{\partial a_j^l} \cdot \frac{\partial a_j^l}{\partial w_{ij}^l} \\
&= \frac{\partial C_p(W)}{\partial a_j^l} \cdot y_i^{l-1} \\
&= \frac{\partial C_p(W)}{\partial y_j^l} \cdot \frac{\partial y_j^l}{\partial a_j^l} \cdot y_i^{l-1} \\
&= \Phi_j^l \cdot f'(a_j^l) \cdot y_i^{l-1} \qquad (4)
\end{aligned}$$

- now let's compute Φ_j^l

Gradient Descent

- for $l = L$ (output layer):

$$\begin{aligned} \Phi_j^L &= \frac{\partial C_p(W)}{\partial y_j^L} \\ &= \frac{\partial \frac{1}{2}\sum_{i=1}^{m}(y_i - y_i^L)^2}{\partial y_j^L} \\ &= (y_j^L - y_j) \end{aligned} \quad (5)$$

Thus, we compute for each output neuron j, the difference between the output y_j^L and the target y_j (for example p).

Gradient Descent

- for $l \neq L$ (hidden layers):

$$
\begin{aligned}
\Phi_j^l &= \frac{\partial C_p(W)}{\partial y_j^l} = \sum_{k=1}^{H_{l+1}} \frac{\partial C_p(W)}{\partial a_k^{l+1}} \cdot \frac{\partial a_k^{l+1}}{\partial y_j^l} \\
&= \sum_{k=1}^{H_{l+1}} \frac{\partial C_p(W)}{\partial a_k^{l+1}} \cdot \frac{\partial \sum_{i=1}^{H_l} w_{ik}^{l+1} y_i^l}{\partial y_j^l} \\
&= \sum_{k=1}^{H_{l+1}} \frac{\partial C_p(W)}{\partial a_k^{l+1}} \cdot w_{jk}^{l+1} = \sum_{k=1}^{H_{l+1}} \frac{\partial C_p(W)}{\partial y_k^{l+1}} \cdot \frac{\partial y_k^{l+1}}{\partial a_k^{l+1}} \cdot w_{jk}^{l+1} \\
&= \sum_{k=1}^{H_{l+1}} \Phi_k^{l+1} \cdot f'(a_k^{l+1}) \cdot w_{jk}^{l+1} \qquad (6)
\end{aligned}
$$

Thus, Φ_j^l can be computed using layer $l+1$.

Gradient Descent

- For each weight, the update is done using the following rule:

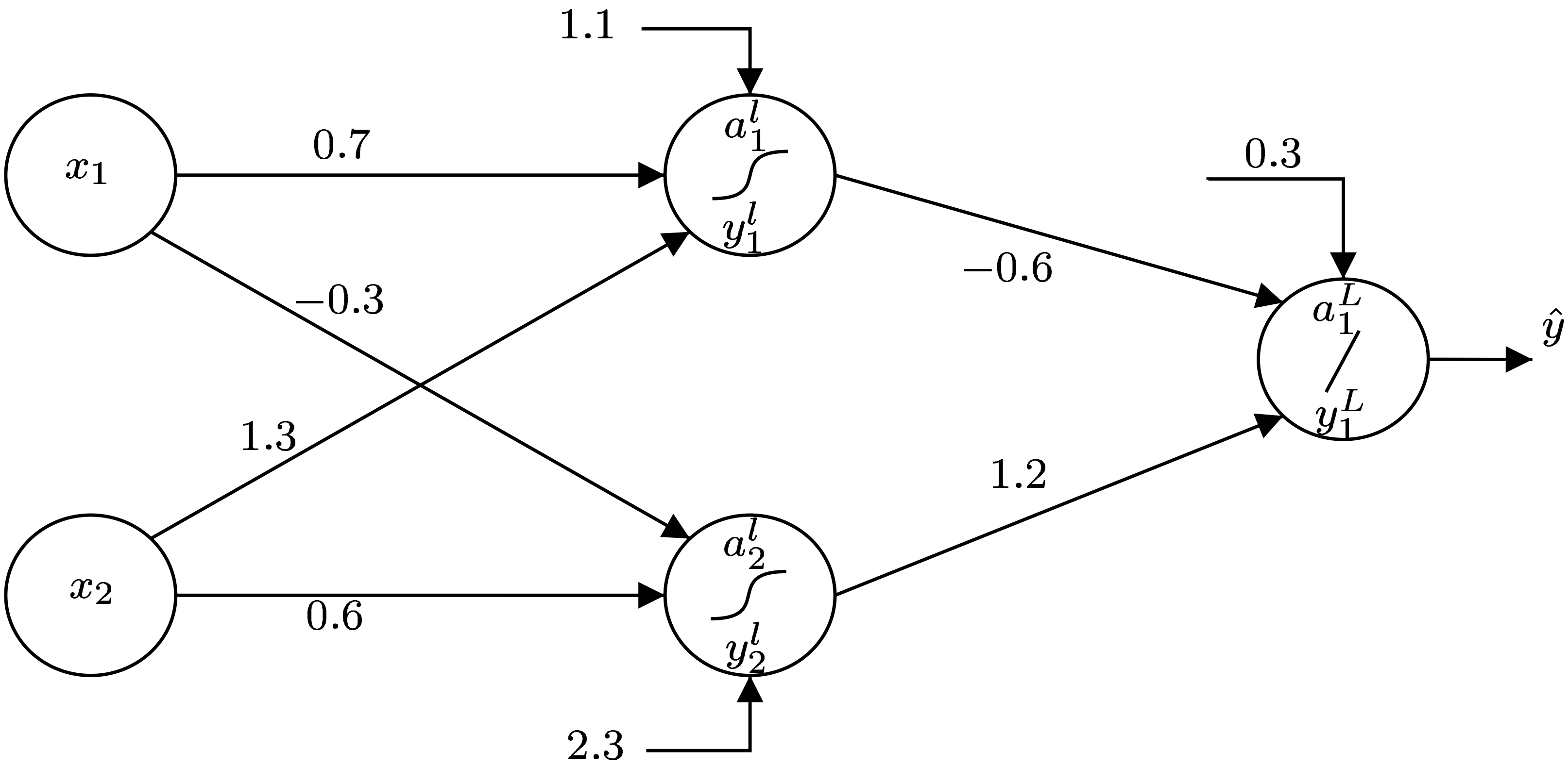
$$w_{ij,t+1}^{l} = w_{ij,t}^{l} - \eta \cdot \frac{\partial C_p}{\partial w_{ij,t}^{l}} \tag{7}$$

where η is the learning rate, and $\frac{\partial C_p}{\partial w_{ij,t}^{l}}$ is defined by:

$$\frac{\partial C_p}{\partial w_{ij,t}^{l}} = \begin{cases} l = L & : \quad f'(a_j^l) \cdot y_i^{l-1} \cdot (y_j^L - y_j) \\ l \neq L & : \quad f'(a_j^l) \cdot y_i^{l-1} \cdot \left[\sum_{k=1}^{H_{l+1}} \Phi_k^{l+1} \cdot f'(a_k^{l+1}) \cdot w_{jk}^{l+1} \right] \end{cases}$$

Gradient Descent: Example

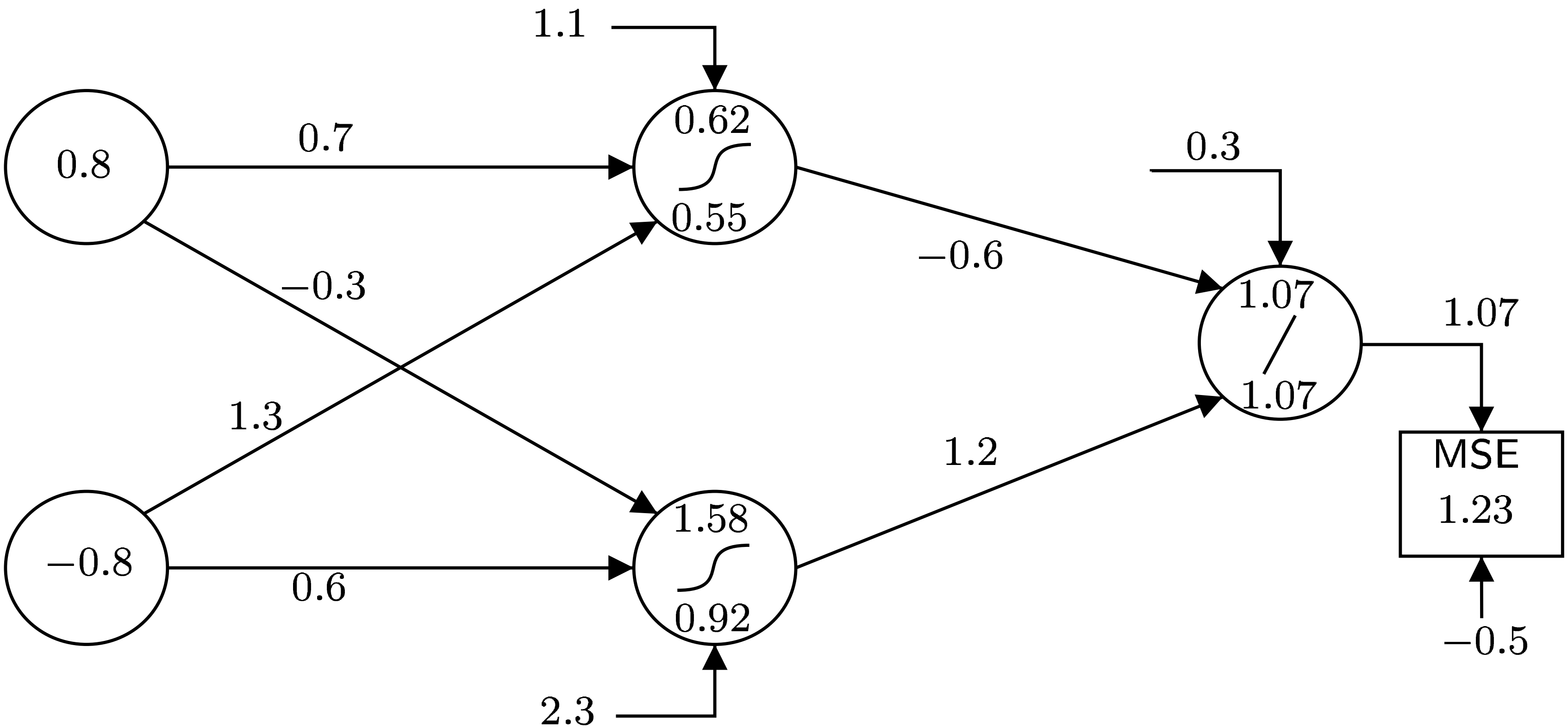
- Initial MLP:



- Note that: $y_1^L = a_1^L$ and $y_j^l = tanh(a_j^l)$

Gradient Descent: Example

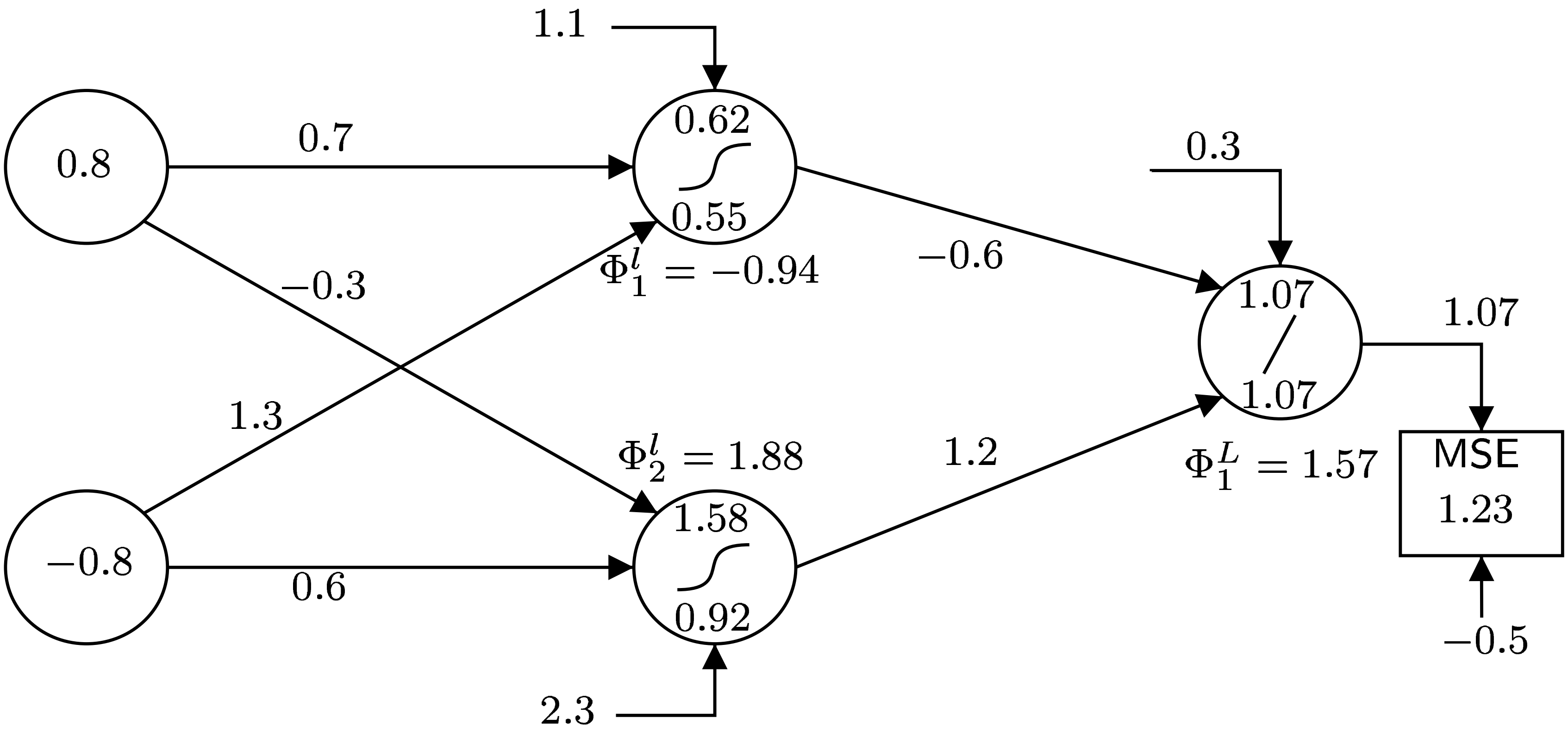
- Forward:



- Note that: $MSE = \frac{1}{2}\sum_j (y_j - y_j^L)^2$

Gradient Descent: Example

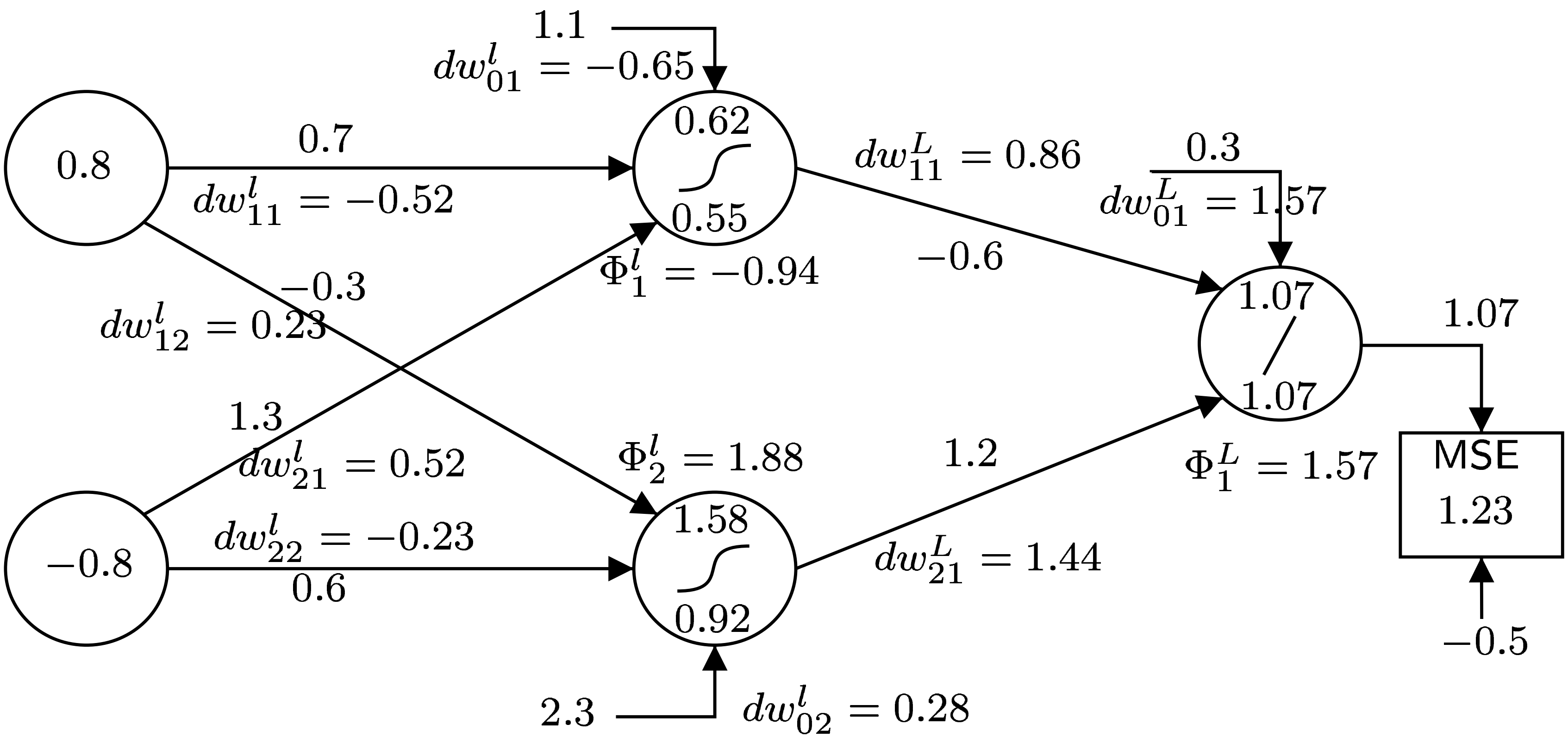
- Backward:



- Note that: $\Phi_j^L = (y_j^L - y_j)$,
- and that: $\Phi_j^l = \Phi_1^L \cdot f'(a_1^L) \cdot w_{j1}^L$.

Gradient Descent: Example

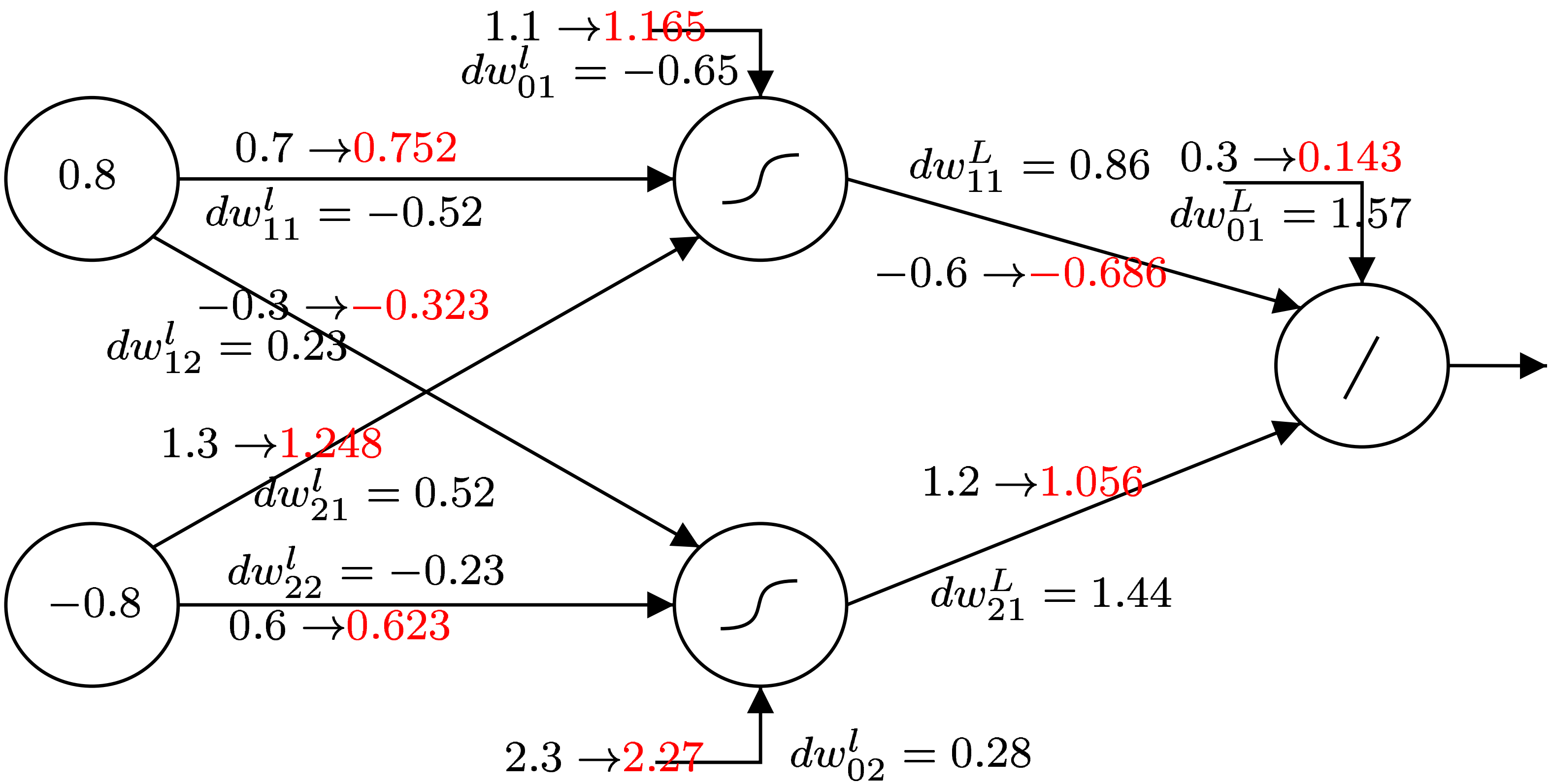
- Backward (cont):



- Note that: $\frac{\partial C}{\partial w^l_{ij}} = dw^l_{ij} = \Phi^l_j \cdot f'(a^l_j) \cdot y^{l-1}_i$,
- and that: $y^l_{0j} = a^l_{0j}$, $tanh'(a) = 1 - tanh(a)^2 = 1 - y^2$.

Gradient Descent: Example

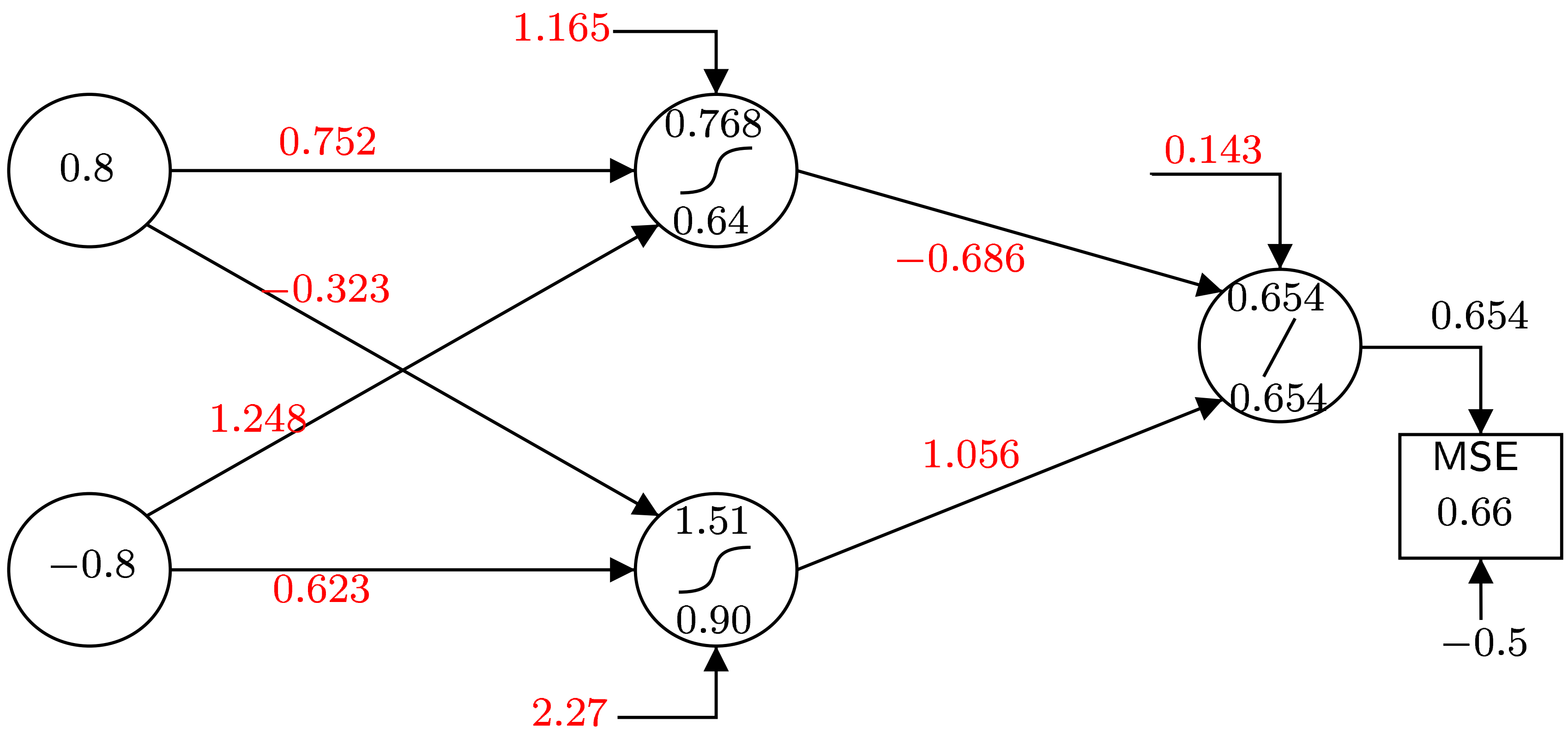
- Update:



- Note that: $w^l_{ij,t+1} = w^l_{ij,t} - \eta \cdot dw^l_{ij}$ with $\eta = 0.1$ for instance

Gradient Descent: Example

- Re-Forward:



Gradient Descent: Summary

- For each iteration t
  - Initialize the gradients $\frac{\partial C_p}{\partial w^l_{ij,t}}$ to 0
  - For each example p $(x(p), y(p))$:
    * Compute $\hat{y}(p) = MLP(x(p); W)$
    * Compute $f'(a^L_j)$
    * Compute Φ^L_j using Equation (5)
    * Compute gradient $\frac{\partial C_p}{\partial w^L_{ij,t}}$ using Equation (4)
    * Accumulate the above gradient
    * For each layer l from $L-1$ to 1:
      · Compute $f'(a^l_j)$
      · Compute Φ^l_j using Equation (6)
      · Compute gradient $\frac{\partial C_p}{\partial w^l_{ij,t}}$ using Equation (4)
      · Accumulate the above gradient
  - Update weights w^l_{ij} using Equation (7)

More about Classification

- 2-class problem:
  - use 1 output,
  - encode the target as $\{+1, -1\}$ or $\{0, 1\}$ depending on the transfer function (linear, tanh, sigmoid),
- multi-class problem:
  - use 1 output per class
  - encode the target as $(0, ..., 1, ..., 0)$

MLP Tricks

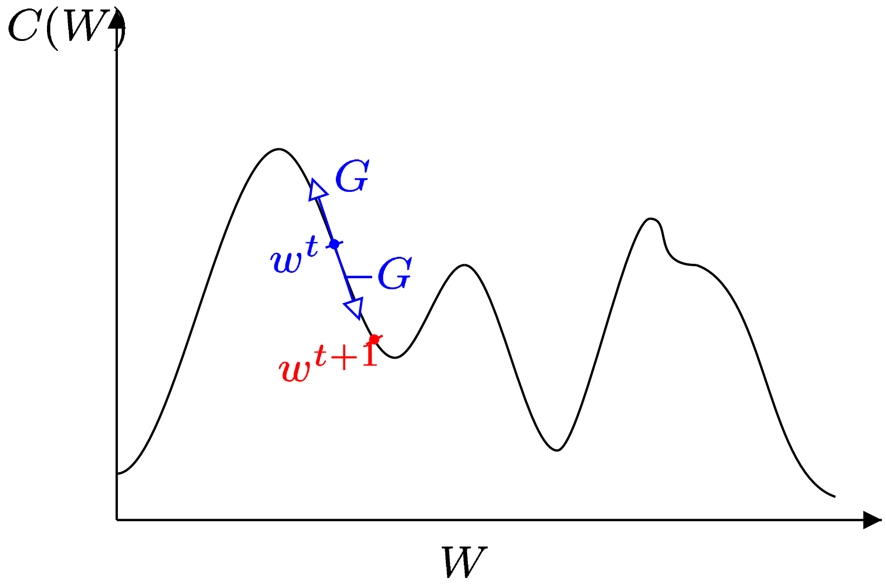
- Stochastic gradient:
  - use stochastic gradient instead of global (batch) gradient,
  - adjust the weights at each example,
- Initialization: to avoid the saturation of the transfer function (gradient tends toward 0)
- Learning rate:
  - if too big the optimization diverges,
  - if too small the optimization is very slow or is stuck into a local minima
- more in the book: Orr, G. B. and Muler, K. “Neural Networks: Tricks of the Trade”, Springer, 1998

MLP Tricks: initialization

- input data: normalized with zero mean and unit variance,
- targets:
  - for regression: normalized with zero mean and unit variance,
  - for classification, if output transfer function is:
    * $tanh(.)$ targets should be 0.6 and -0.6,
    * $sigmoid(.)$ targets should be 0.8 and 0.2,
    * $linear(.)$ targets should be 0.6 and -0.6.
- weights w_{ij}: uniformly distributed in $\left[\frac{-1}{\sqrt{\textsf{fan in}_j}}, \frac{1}{\sqrt{\textsf{fan in}_j}}\right]$ where fan in$_j$ is the number of units preceding unit j.

MLP Tricks: inertia momentum

- to avoid to be stucked in a local minima:



$$w^l_{ij,t+1} = w^l_{ij,t} - \eta \cdot dw^l_{ij} + \beta \cdot (w^l_{ij,t} - w^l_{ij,t-1})$$

where β is the inertia momentum rate

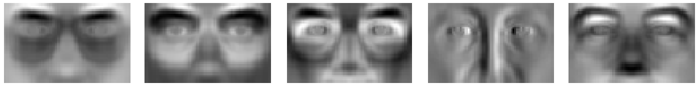
Outline

- Introduction to Statistical Machine Learning
- Artificial Neural Networks
- Application Examples
  - Face detection
  - Face recognition

Face Processing using MLP

- the input x of the MLP is a particular representation of the face image
- face representations:
  - Raw pixels:

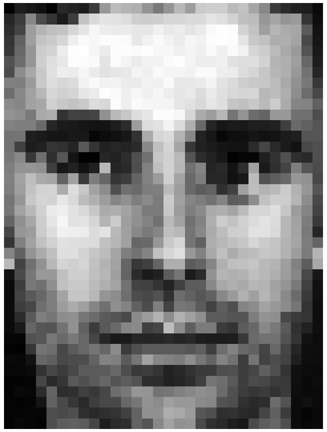
  - Principal Componant subspace obtained by PCA:



- target coding:
  - Face detection: face $(+1)$ vs non-face (-1),
  - Face authentication: client $(+1)$ vs impostor (-1)

Face Processing using MLP

- Raw pixels:
  - let us denote the image I of size $n = w \times h$,



  - then the input of the MLP is $x \in \mathbb{R}^n$,
  - for an image 30×40, a MLP with 90 hidden units has:

$$(1200 \text{ inputs } + 1 \text{ bias }) \times 90 + 90 + 1 \text{ bias } = 109291$$

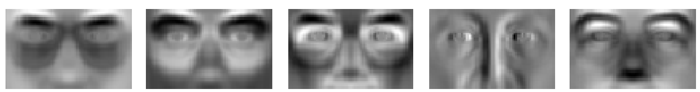
  - Warning !!: a large number of parameters $\Rightarrow$ a large number of examples which is not always possible
  - Solution: reduce the dimensionality $m \ll n$ using Principal Component Analysis (PCA)

Face Processing using MLP

- Principal Componant subspace obtained by PCA:
  - $\mathbf{u} = \mathbf{W}\mathbf{x}$ where $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{W}$ is a $m \times n$ matrix,

$$\begin{aligned} \boldsymbol{\mu} &= \frac{1}{P}\sum_{i=1}^{P}\mathbf{x}_i \\ \boldsymbol{\Sigma} &= \frac{1}{P}\sum_{i=1}^{P}(\mathbf{x}_i - \boldsymbol{\mu})(\mathbf{x}_i - \boldsymbol{\mu})^T \end{aligned}$$

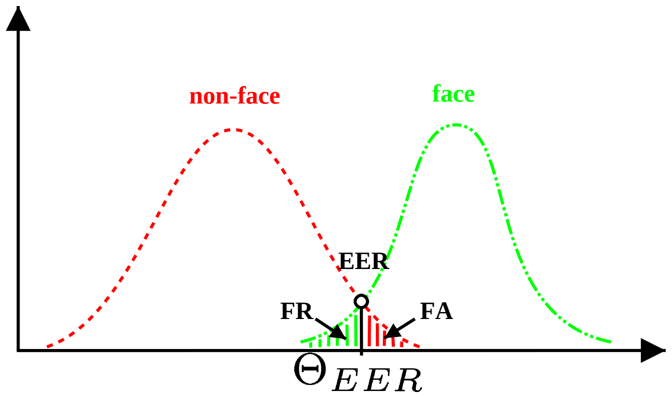
  - compute the m eigenvectors $\mathbf{e}_1 ... \mathbf{e}_m$ corresponding to the m largest non-zero eigenvalues $(\boldsymbol{\Sigma} - \alpha_i \mathbf{I})\mathbf{e}_i = 0$, $i = 1..m$,
  - $\mathbf{W} = [\mathbf{e}_1 ... \mathbf{e}_m]^T$,



  - the input of the MLP for a given face $\mathbf{x}$ becomes $\mathbf{u} = \mathbf{W}\mathbf{x}$

Face Processing using MLP

- Select a threshold to take the final decision:



  - False Rejection (FRR): when the system rejects a face,
  - False Acceptance Rate (FAR): when the system accepts a non-face,
  - the decision threshold Θ chosen on a evaluation data set.

Future Lectures

- Artificial Neural Networks:
  - Hopfield auto-associative memory
  - Kohonen auto-organizing maps
- Gaussian Mixture Models
- Hidden Markov Models
- Support Vector Machines and links with MLP

References

- This lecture is at `http://www.idiap.ch/~marcel`
- Machine learning Library: *http://www.torch.ch*
- Books:
  - Bishop, C. "Neural Networks for Pattern Recognition", 1995
  - Vapnik, V. "The Nature of Statistical Learning Theory", 1995
  - Orr, G. B. and Muler, K. "Neural Networks: Tricks of the Trade", Springer, 1998
- Extended Lectures on Machine Learning Algorithms:
  - Bengio, Y. `http://www.iro.umontreal.ca/~pift6266/A03`
  - Bengio, S. `http://www.idiap.ch/~bengio/lectures/index.html`
  - Jordan, M. `http://www.cs.berkeley.edu/~jordan/courses.html`