1)

a) $\mathcal{L} = \{ w \in \{a, b\}^*, b \in w \}$

$G = (\{S\}, \{a, b\}, P, S)$

$P = \{ S \to aS \mid bS \mid Sb \mid b \}$

b) $\mathcal{L} = \{ \cancel{w \in \{a, b\}^*} \, axb, x \in \{a, b\}^*$

$G = (\{S, A\}, \{a, b\}, P, S\})$

$P = \{ S \to aA, A \to aA \mid bA \mid Ab \mid b \}$

c) $\mathcal{L} = \{ xbb; x \in \{a, b\}^*$

$G = \{ \{S, A, B, C\}, \{a, b\}, P, S \}$

$P = \{ \cancel{\{S, A, B, C\}, \{a, b\}, P}$

$S \to AB, A \to aA \mid bA \mid \varepsilon, B \to bC, C \to b\}$

d) $\mathcal{L} = \{ w \in \{a, b\}^*, aab \in w \}$

$G = \{ \{S, X, Y, A, B, C\}, \{a, b\}, P, S \}$

$P = \{ S \to XAY, X \to \cancel{a/X, b/X} \, aX \mid bX \mid \varepsilon, Y \to aY \mid bY \mid \varepsilon$

$A \to aB, B \to aC, C \to b$

e) $\mathcal{L} = \{ w \in \{a, b\}^* \mid |w|_a \equiv_3 0 \}$

$G = \{ \{S, A, B\}, \{a, b\}, P, S \}$

$P = \{ S \to A \mid B, A \to aaaA \mid B, B \to bB \mid \varepsilon \}$

2)

a) $S \to bS \mid aT$

$T \to \varepsilon \mid aS \mid bU$

$U \to aS \mid bU$

Hệ phương trình:

$$\begin{cases} X_S = \emptyset + bX_S + aX_T + \emptyset X_U & (1) \\ X_T = \varepsilon + aX_S + \emptyset X_T + bX_U & (2) \\ X_U = \emptyset + aX_S + \emptyset X_T + bX_U & (3) \end{cases}$$

ĐL 3.2 $\Rightarrow X_U = aX_S + bX_U \to X_U = b^*aX_S$ (

Thay X_U vào (2):

$X_T = \varepsilon + aX_S + bb^*aX_S = \varepsilon + (a + bb^*a)X_S$

$\to X_T = (a + bb^*a)^*$

Thay X_T vào (1):

$X_S = bX_S + a(a + bb^*a)^*$

$\Leftrightarrow X_S = b^*a(a + bb^*a)^*$

b) $S \to abS \mid abT$

$T \to aT \mid U$

$U \to b \mid \varepsilon$.

HPT:

$$\begin{cases} X_S \to \emptyset + abX_S + abX_T & (1) \\ X_T \to \emptyset + \quad + aX_T + X_U & (2) \\ X_U \to \varepsilon + b & (3) \end{cases}$$

* Thay x_U vào (2)

$x_T = ax_T + \varepsilon + b \Rightarrow x_T = a^*(\varepsilon + b)$

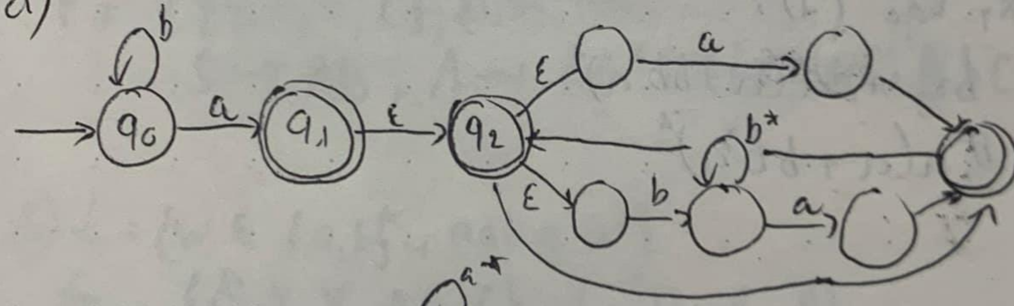
Thay x_T vào (1):

$x_S = abx_S + aba^*(\varepsilon + b)$

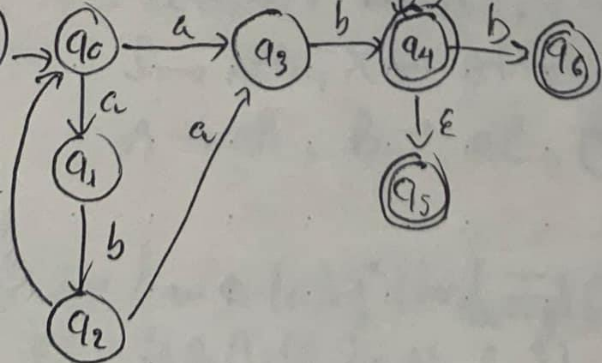
$x_S = (ab)^*aba^*(\varepsilon + b)$

3

a)

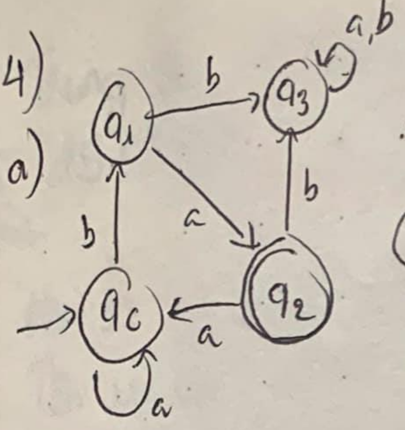


b)



4)

a)



: Biểu thức chính qui:

$(a + baa)^* ba$

$$\begin{cases} x_0 = ax_0 + bx_1 = ax_0 + ba + baax_0 \\ x_1 = ax_2 + bx_3 = a + aax_0 \\ x_2 = \varepsilon + ax_0 + bx_3 = \varepsilon + ax_0 \\ x_3 = (a+b)x_3 \to x_3 = \emptyset \end{cases}$$

$S \to BC$

$A \to a \mid baa$

$B \to AB \mid \varepsilon$

$C \to \cancel{BA}\ ba$

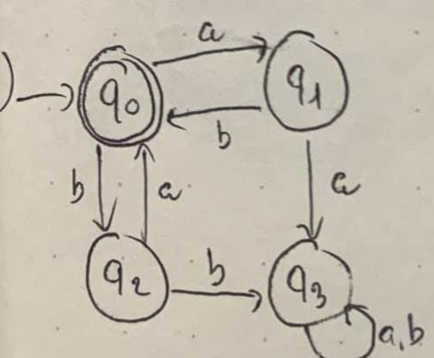
$V_0 \to aV_0 \mid bV_1$

$V_1 \to aV_2 \mid bV_3 \mid \varepsilon$

$V_2 \to aV_0 \mid bV_3$

$V_3 \to aV_3 \mid bV_3$

b)



Biểu thức chính qui

$$\begin{cases} x_0 = \varepsilon + ax_1 + bx_2 \\ x_1 = bx_0 + ax_3 \Rightarrow x_1 = bx_0 \\ x_2 = ax_0 + bx_3 \Rightarrow x_2 = ax_0 \end{cases}$$

$x_3 = (a+b) \Rightarrow x_3 = \emptyset$

$\Rightarrow x_0 = \varepsilon + abx_0 + bax_0$

$= \varepsilon + (ab + ba)x_0$

$\Rightarrow (ab + ba)^*$

$S \to AS \mid \varepsilon$

$A \to ab \mid ba$

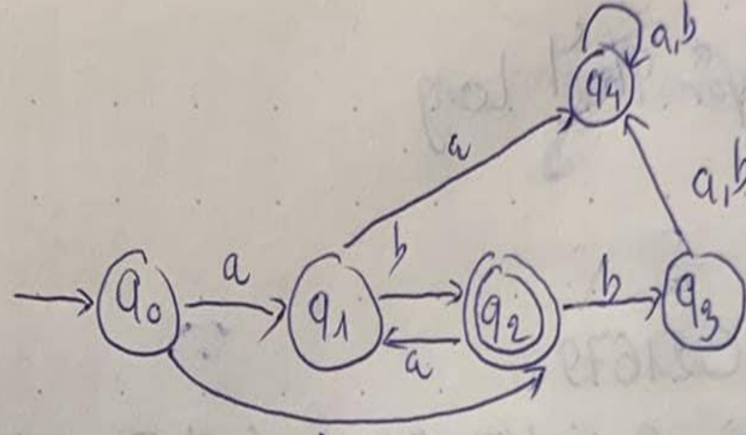
$V_0 \to aV_1 \mid bV_2 \mid \varepsilon$

$V_1 \to aV_3 \mid bV_0$

$V_2 \to aV_0 \mid bV_3$

$V_3 \to aV_3 \mid bV_3$

c)



BT chính qui hpt,

$$b(\varepsilon + ax_1) = b + (ba)x_1$$

$$\begin{cases} x_0 = ax_1 + bx_2 \\ x_1 = bx_2 + ax_4 & \Rightarrow x_1 = \cancel{bax_1} \Rightarrow \cancel{bax_1 = (ba)^*} \Rightarrow (ba)^* b \\ x_2 = \varepsilon + ax_1 + bx_3 & \Rightarrow x_2 = \varepsilon + ax_1\ (*) \rightarrow \text{Thay vào } (x_1) \\ x_3 = (a+b)x_4 & \Rightarrow x_3 = \emptyset \\ x_4 = (a+b)x_4 & x_4 = \emptyset \end{cases}$$

$\Rightarrow \cancel{x_0 = a(b + b}\ a(ba)^* b + b(\varepsilon + ax_1)$. $x_1 = (ba)^* b$

$$x_0 = a(ba)^* b + b(\varepsilon + ax_1) = a(ba)^* b + b + bax_1 =$$

$$= a(ba)^* b + b + ba(ba)^* b$$

$V_0 \rightarrow aV_1 \mid bV_2$

$V_1 \rightarrow aV_4 \mid bV_2$

$V_2 \rightarrow aV_1 \mid bV_3 \mid \varepsilon$

$V_3 \rightarrow aV_4 \mid bV_4$

$V_4 \rightarrow aV_4 \mid bV_4$