

```
In [1]: # Multicollinearity Analysis
#Libraries

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
import yfinance as yf
import statsmodels.formula.api as smf
from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn.linear_model import RidgeCV, LassoCV, Ridge, Lasso
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from sklearn.model_selection import TimeSeriesSplit, cross_val_score
from sklearn.metrics import mean_squared_error
import os

plt.rcParams["figure.figsize"] = (12, 9)
sns.set(style="whitegrid")
```

```
In [26]: #Dataset
## Tickers: S&P 500 ETF plus ten large components

tickers = ["^GSPC", "AAPL", "MSFT", "GOOG", "META", "AMZN", "NVDA", "TSLA", "BRK-
start, end = "2010-01-01", "2025-09-01"

for t in tickers:
    df = yf.download(t, start=start, end=end, interval="3mo", progress=False)
    print("\nTicker:", t)
    print("Columns:", df.columns.tolist())
    print("Head:\n", df.head(2))
```

Ticker: ^GSPC

Columns: [('Close', '^GSPC'), ('High', '^GSPC'), ('Low', '^GSPC'), ('Open', '^GSPC'), ('Volume', '^GSPC')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	^GSPC	^GSPC	^GSPC	^GSPC	^GSPC
Date					
2010-01-01	1169.430054	1180.689941	1044.500000	1116.560059	279192470000
2010-04-01	1030.709961	1219.800049	1028.329956	1171.229980	354511440000

Ticker: AAPL

Columns: [('Close', 'AAPL'), ('High', 'AAPL'), ('Low', 'AAPL'), ('Open', 'AAPL'), ('Volume', 'AAPL')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	AAPL	AAPL	AAPL	AAPL	AAPL
Date					
2010-01-01	7.054727	7.129178	5.711327	6.407194	38099247200
2010-04-01	7.550961	8.375914	5.981509	7.127077	47101037200

Ticker: MSFT

Columns: [('Close', 'MSFT'), ('High', 'MSFT'), ('Low', 'MSFT'), ('Open', 'MSFT'), ('Volume', 'MSFT')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	MSFT	MSFT	MSFT	MSFT	MSFT
Date					
2010-01-01	21.930445	23.390477	20.642620	22.926262	3544531400
2010-04-01	17.308956	23.755621	17.263822	22.078134	4710971300

Ticker: GOOG

Columns: [('Close', 'GOOG'), ('High', 'GOOG'), ('Low', 'GOOG'), ('Open', 'GOOG'), ('Volume', 'GOOG')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	GOOG	GOOG	GOOG	GOOG	GOOG
Date					
2010-01-01	14.029052	15.572415	12.863427	15.509087	9030100641
2010-04-01	11.006886	14.788982	11.001198	14.133689	8611573755

Ticker: META

Columns: [('Close', 'META'), ('High', 'META'), ('Low', 'META'), ('Open', 'META'), ('Volume', 'META')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	META	META	META	META	META
Date					
2012-05-01	21.576982	33.245053	21.477597	28.712991	1188100200
2012-08-01	20.980661	24.101422	17.442472	21.368271	3311526900

Ticker: AMZN

Columns: [('Close', 'AMZN'), ('High', 'AMZN'), ('Low', 'AMZN'), ('Open', 'AMZN'), ('Volume', 'AMZN')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	AMZN	AMZN	AMZN	AMZN	AMZN
Date					
2010-01-01	6.7885	6.9095	5.6910	6.8125	11980988000
2010-04-01	5.4630	7.5545	5.3005	6.7900	8946270000

Ticker: NVDA

Columns: [('Close', 'NVDA'), ('High', 'NVDA'), ('Low', 'NVDA'), ('Open', 'NVDA'), ('Volume', 'NVDA')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	NVDA	NVDA	NVDA	NVDA	NVDA
Date					
2010-01-01	0.398845	0.434604	0.347270	0.424289	37507632000
2010-04-01	0.234035	0.415578	0.234035	0.400679	51575292000

Ticker: TSLA

Columns: [('Close', 'TSLA'), ('High', 'TSLA'), ('Low', 'TSLA'), ('Open', 'TSLA'), ('Volume', 'TSLA')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	TSLA	TSLA	TSLA	TSLA	TSLA
Date					
2010-06-01	1.298667	1.728	0.998667	1.666667	1194210000
2010-09-01	2.355333	2.400	1.300000	1.308000	793632000

Ticker: BRK-B

Columns: [('Close', 'BRK-B'), ('High', 'BRK-B'), ('Low', 'BRK-B'), ('Open', 'BRK-B'), ('Volume', 'BRK-B')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	BRK-B	BRK-B	BRK-B	BRK-B	BRK-B
Date					
2010-01-01	81.269997	83.570000	64.720001	66.000000	841106900
2010-04-01	79.690002	81.949997	68.480003	81.599998	497986800

Ticker: JPM

Columns: [('Close', 'JPM'), ('High', 'JPM'), ('Low', 'JPM'), ('Open', 'JPM'), ('Volume', 'JPM')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	JPM	JPM	JPM	JPM	JPM
Date					
2010-01-01	29.927181	30.796574	24.764324	27.947641	2660845800
2010-04-01	24.512856	32.273139	24.445897	30.150610	3060674400

Ticker: JNJ

Columns: [('Close', 'JNJ'), ('High', 'JNJ'), ('Low', 'JNJ'), ('Open', 'JNJ'), ('Volume', 'JNJ')]

Head:

Price	Close	High	Low	Open	Volume
Ticker	JNJ	JNJ	JNJ	JNJ	JNJ
Date					
2010-01-01	41.002075	41.473725	38.920530	40.693932	708414300
2010-04-01	37.425018	41.949476	36.468163	41.423527	950979900

```
In [49]: #Add SPY and XLK quarterly returns to our DataFrame
for etf in ["SPY", "XLK"]:
    df = yf.download(etf, start="2010-01-01", end="2025-09-01",
                      interval="3mo", progress=False)
    print("\nTicker:", t)
    print("Columns:", df.columns.tolist())
    print("Head:\n", df.head(2))
```

Ticker: JNJ

Columns: [('Close', 'SPY'), ('High', 'SPY'), ('Low', 'SPY'), ('Open', 'SPY'), ('Volume', 'SPY')]

Head:

	Price	Close	High	Low	Open	Volume
Ticker		SPY	SPY	SPY	SPY	SPY
Date						
2010-01-01	88.040848	88.921255	78.694975	84.556840	12058443200	
2010-04-01	77.991447	92.271998	77.734546	89.007873	17007028100	

Ticker: JNJ

Columns: [('Close', 'XLK'), ('High', 'XLK'), ('Low', 'XLK'), ('Open', 'XLK'), ('Volume', 'XLK')]

Head:

	Price	Close	High	Low	Open	Volume
Ticker		XLK	XLK	XLK	XLK	XLK
Date						
2010-01-01	18.671961	18.898287	16.740099	18.704292	718489100	
2010-04-01	16.542023	19.590945	16.493370	18.780061	822849300	

In [50]: *#Quarterly Prices*

```
tickers = [ "^GSPC", "AAPL", "MSFT", "GOOG", "META", "AMZN", "NVDA", "TSLA", "BRK
start, end = "2010-01-01", "2025-09-01"

frames = []
for t in tickers:
    df = yf.download(t, start=start, end=end, interval="3mo", progress=False)
    col = next((c for c in ["Adj Close", "Close"] if c in df.columns), None)
    s = df[col].copy()
    s.name = t                    # set the Series name
    frames.append(s)

prices_q = pd.concat(frames, axis=1)
print("Quarterly prices:\n", prices_q.head())
```

Quarterly prices:

Ticker	^GSPC	AAPL	MSFT	GOOG	META	AMZN	\
Date							
2010-01-01	1169.430054	7.054727	21.930445	14.029052	NaN	6.7885	
2010-04-01	1030.709961	7.550961	17.308956	11.006886	NaN	5.4630	
2010-06-01	NaN	NaN	NaN	NaN	NaN	NaN	
2010-07-01	1141.199951	8.518209	18.505398	13.006658	NaN	7.8530	
2010-09-01	NaN	NaN	NaN	NaN	NaN	NaN	

Ticker	NVDA	TSLA	BRK-B	JPM	JNJ	SPY	\
Date							
2010-01-01	0.398845	NaN	81.269997	29.927181	41.002075	88.040848	
2010-04-01	0.234035	NaN	79.690002	24.512856	37.425018	77.991447	
2010-06-01	NaN	1.298667	NaN	NaN	NaN	NaN	
2010-07-01	0.267731	NaN	82.680000	25.512232	39.621323	86.645172	
2010-09-01	NaN	2.355333	NaN	NaN	NaN	NaN	

Ticker	XLK
Date	
2010-01-01	18.671961
2010-04-01	16.542023
2010-06-01	NaN
2010-07-01	18.732578
2010-09-01	NaN

```
In [83]: # Quarterly simple returns
returns_q = prices_q.pct_change().dropna()

print("Quarterly returns:\n", returns_q.head())
#Describe the prepared quarterly return
print("\nSummary statistics for all quarterly returns:\n")
print(returns_q.describe().T) # .T gives a nicer vertical layout
```

Quarterly returns:

Ticker	^GSPC	AAPL	MSFT	GOOG	META	AMZN	\
Date							
2012-06-01	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
2012-07-01	0.057636	0.142295	-0.020750	0.300705	0.000000	0.113729	
2012-08-01	0.000000	0.000000	0.000000	0.000000	-0.027637	0.000000	
2012-09-01	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
2012-10-01	-0.010051	-0.198839	-0.096541	-0.062452	0.000000	-0.013566	

Ticker	NVDA	TSLA	BRK-B	JPM	JNJ	SPY	\
Date							
2012-06-01	0.000000	-0.033221	0.000000	0.000000	0.000000	0.000000	
2012-07-01	-0.034732	0.000000	0.058442	0.140357	0.029911	0.063306	
2012-08-01	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
2012-09-01	0.000000	0.185835	0.000000	0.000000	0.000000	0.000000	
2012-10-01	-0.080960	0.000000	0.017007	0.095272	0.026513	-0.005555	

Ticker	XLK
Date	
2012-06-01	0.000000
2012-07-01	0.077261
2012-08-01	0.000000
2012-09-01	0.000000
2012-10-01	-0.060358

Summary statistics for all quarterly returns:

	count	mean	std	min	25%	50%	75%	max
Ticker								
^GSPC	159.0	0.010969	0.043736	-0.200011	0.0	0.0	0.005259	0.199529
AAPL	159.0	0.021080	0.093785	-0.298781	0.0	0.0	0.000000	0.437987
MSFT	159.0	0.021604	0.069957	-0.165252	0.0	0.0	0.001102	0.327705
GOOG	159.0	0.020868	0.078316	-0.216807	0.0	0.0	0.000000	0.395102
META	159.0	0.029146	0.121936	-0.414456	0.0	0.0	0.000000	0.613211
AMZN	159.0	0.022877	0.092519	-0.348395	0.0	0.0	0.000000	0.414983
NVDA	159.0	0.052690	0.168944	-0.524685	0.0	0.0	0.022405	0.901198
TSLA	159.0	0.054406	0.270173	-0.421158	0.0	0.0	0.000000	1.983952
BRK-B	159.0	0.012678	0.052507	-0.226375	0.0	0.0	0.000000	0.192874
JPM	159.0	0.018729	0.073316	-0.348987	0.0	0.0	0.002469	0.332796
JNJ	159.0	0.009526	0.044940	-0.116895	0.0	0.0	0.000000	0.186584
SPY	159.0	0.012490	0.044648	-0.195249	0.0	0.0	0.010669	0.203388
XLK	159.0	0.017255	0.060369	-0.198673	0.0	0.0	0.010243	0.306901

C:\Users\OkechPC\AppData\Local\Temp\ipykernel_38808\368859145.py:2: FutureWarning: The default fill_method='pad' in DataFrame.pct_change is deprecated and will be removed in a future version. Either fill in any non-leading NA values prior to calling pct_change or specify 'fill_method=None' to not fill NA values.

```
returns_q = prices_q.pct_change().dropna()
```

```
In [53]: #S&P 500's quarterly return as the dependent variable and the others as predictors
#Rename the S&P 500 column
```

```

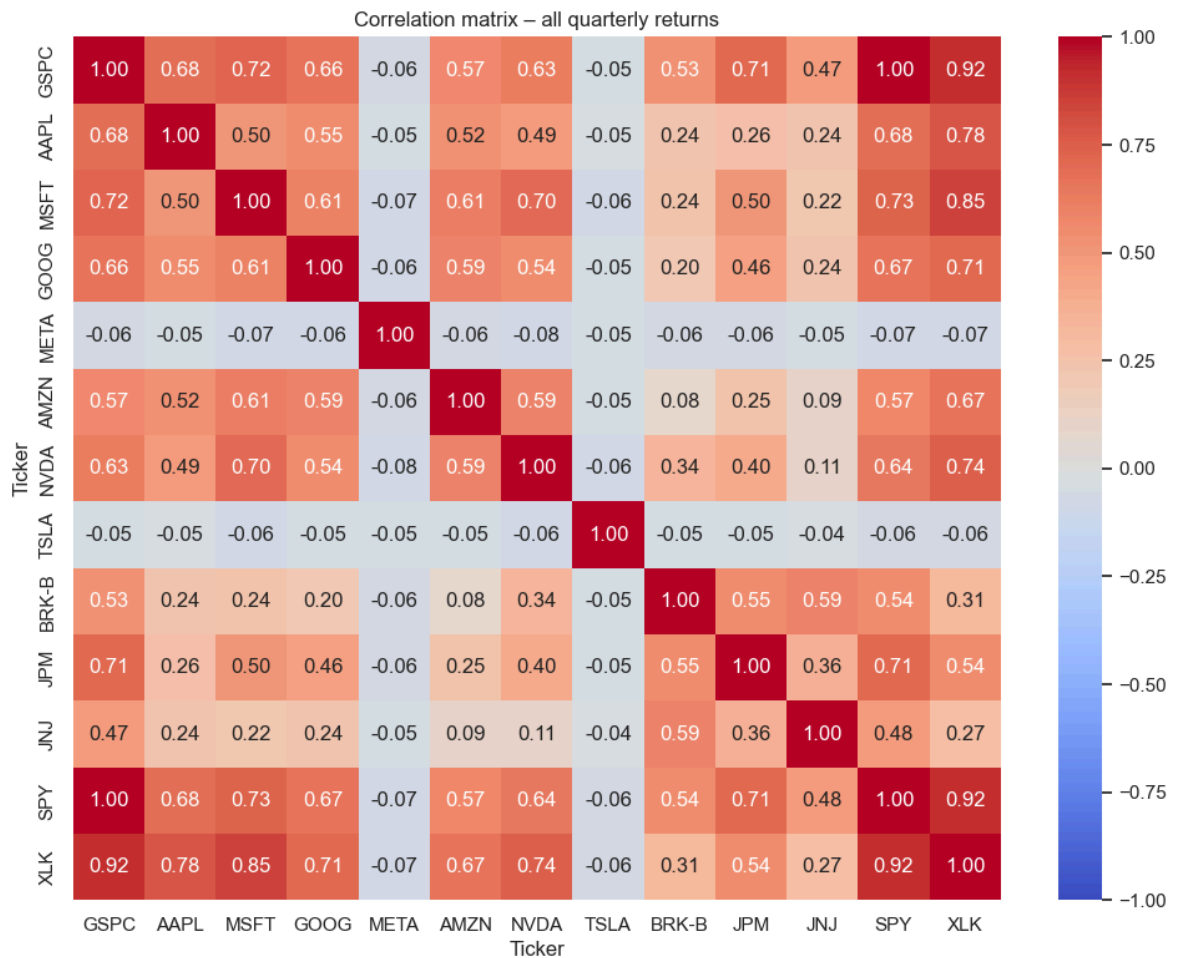
returns_q = returns_q.rename(columns={"^GSPC": "GSPC"})
y = returns_q["GSPC"]
X = returns_q[["AAPL", "MSFT", "GOOG", "META", "AMZN", "NVDA", "TSLA", "BRK-B", "

#Correlation plot of all variables

corr_all = returns_q.corr()

plt.figure(figsize=(12,9))
sns.heatmap(corr_all, annot=True, fmt=".2f", cmap="coolwarm",
            vmin=-1, vmax=1)
plt.title("Correlation matrix - all quarterly returns")
plt.show()

```



In [36]: #S&P500's quarterly return as the dependent variable and the others as predictor

```

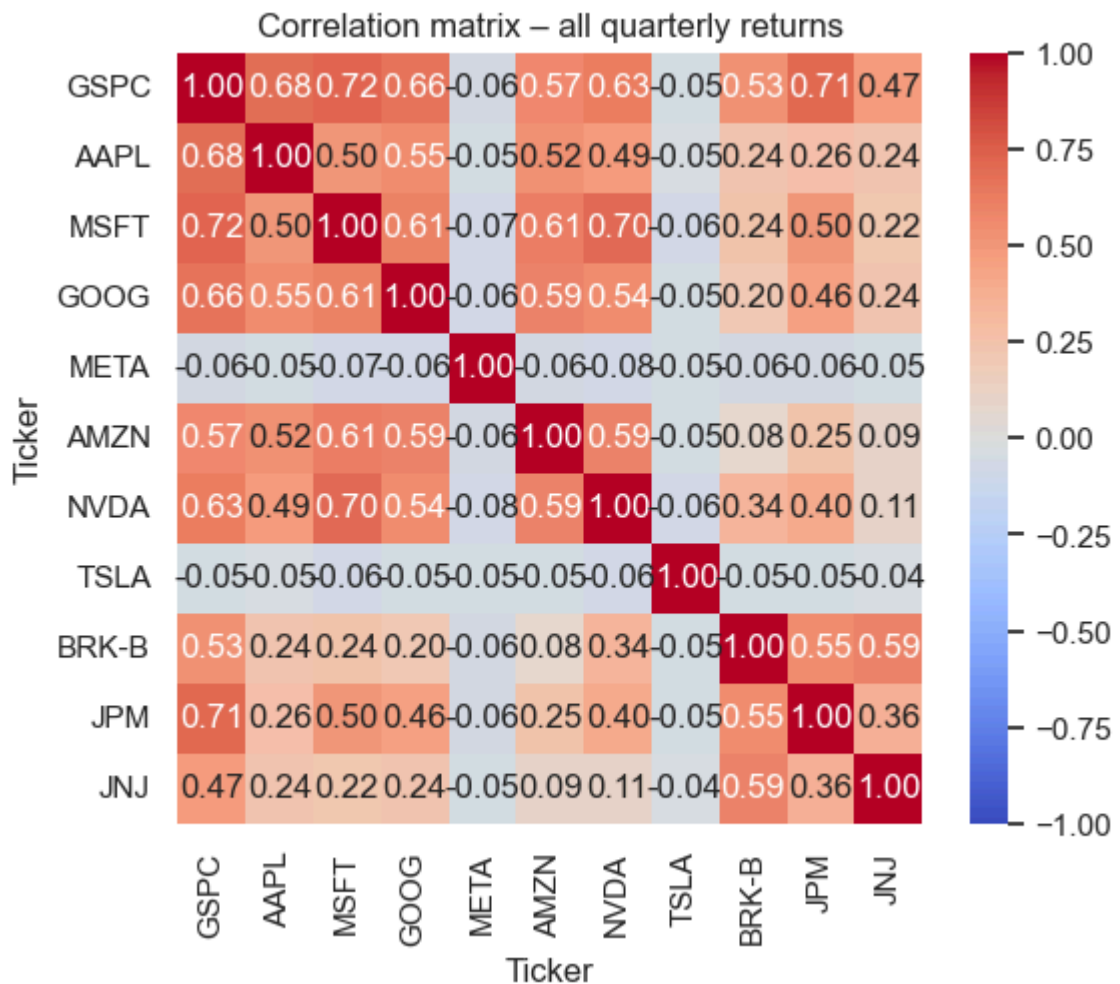
y = returns_q["GSPC"]
X = returns_q[["AAPL", "MSFT", "GOOG", "META", "AMZN", "NVDA", "TSLA", "BRK-B", "

#Correlation plot of all variables

corr_all = returns_q.corr()

plt.figure(figsize=(6,5))
sns.heatmap(corr_all, annot=True, fmt=".2f", cmap="coolwarm",
            vmin=-1, vmax=1)
plt.title("Correlation matrix - all quarterly returns")
plt.show()

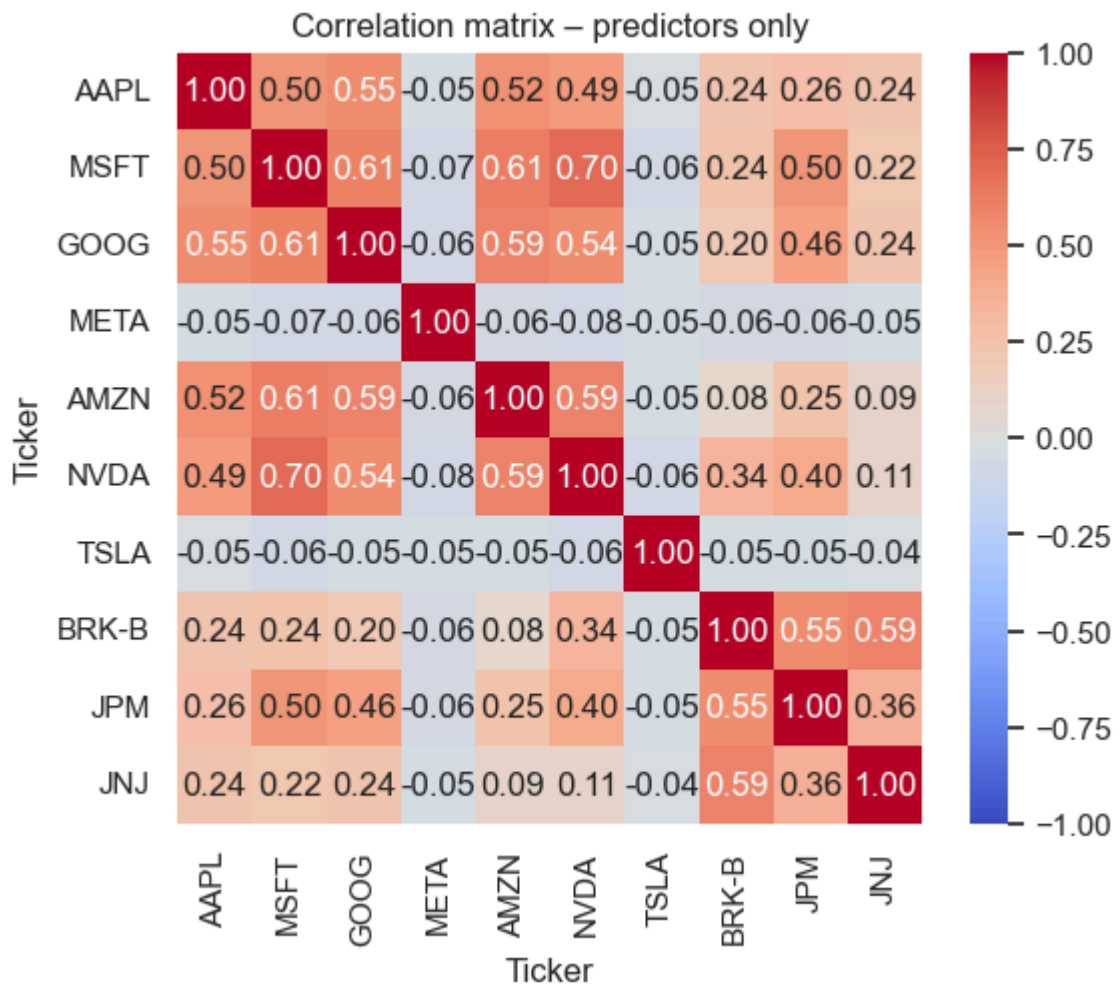
```



```
In [37]: # Correlation plot of all independent variables

corr_predictors = X.corr()

plt.figure(figsize=(6,5))
sns.heatmap(corr_predictors, annot=True, fmt=".2f", cmap="coolwarm",
            vmin=-1, vmax=1)
plt.title("Correlation matrix - predictors only")
plt.show()
```



```
In [54]: #S&P500 Excess Return Regression Model with Independent Variables

returns_q = returns_q.rename(columns={"BRK-B": "BRK_B"})

model_1 = smf.ols(
    formula="GSPC ~ AAPL + MSFT + GOOG + META + AMZN + NVDA + TSLA + BRK_B + JPM
    data=returns_q
).fit()

print(model_1.summary())
```


OLS Regression Results

=====						
Dep. Variable:	GSPC		R-squared:	0.998		
Model:	OLS		Adj. R-squared:	0.998		
Method:	Least Squares		F-statistic:	7313.		
Date:	Mon, 29 Sep 2025		Prob (F-statistic):	2.55e-196		
Time:	13:52:27		Log-Likelihood:	781.32		
No. Observations:	159		AIC:	-1537.		
Df Residuals:	146		BIC:	-1497.		
Df Model:	12					
Covariance Type:	nonrobust					
=====						
	coef	std err	t	P> t	[0.025	0.975]

Intercept	-0.0011	0.000	-6.370	0.000	-0.001	-0.001
AAPL	-0.0039	0.003	-1.226	0.222	-0.010	0.002
MSFT	-0.0046	0.005	-0.884	0.378	-0.015	0.006
GOOG	-0.0047	0.003	-1.680	0.095	-0.010	0.001
META	0.0020	0.001	1.648	0.101	-0.000	0.004
AMZN	-0.0045	0.002	-1.886	0.061	-0.009	0.000
NVDA	-0.0026	0.002	-1.693	0.093	-0.006	0.000
TSLA	0.0008	0.001	1.413	0.160	-0.000	0.002
BRK_B	-0.0091	0.005	-1.894	0.060	-0.019	0.000
JPM	-0.0069	0.004	-1.930	0.056	-0.014	0.000
JNJ	-0.0119	0.005	-2.469	0.015	-0.021	-0.002
SPY	1.0286	0.017	59.784	0.000	0.995	1.063
XLK	-0.0016	0.016	-0.100	0.921	-0.033	0.030
=====						
Omnibus:	44.495	Durbin-Watson:	2.558			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	71.301			
Skew:	-1.505	Prob(JB):	3.29e-16			
Kurtosis:	4.306	Cond. No.	157.			
=====						

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [41]: # Parameters with high precision
         model_1.summary2().tables[1]
```

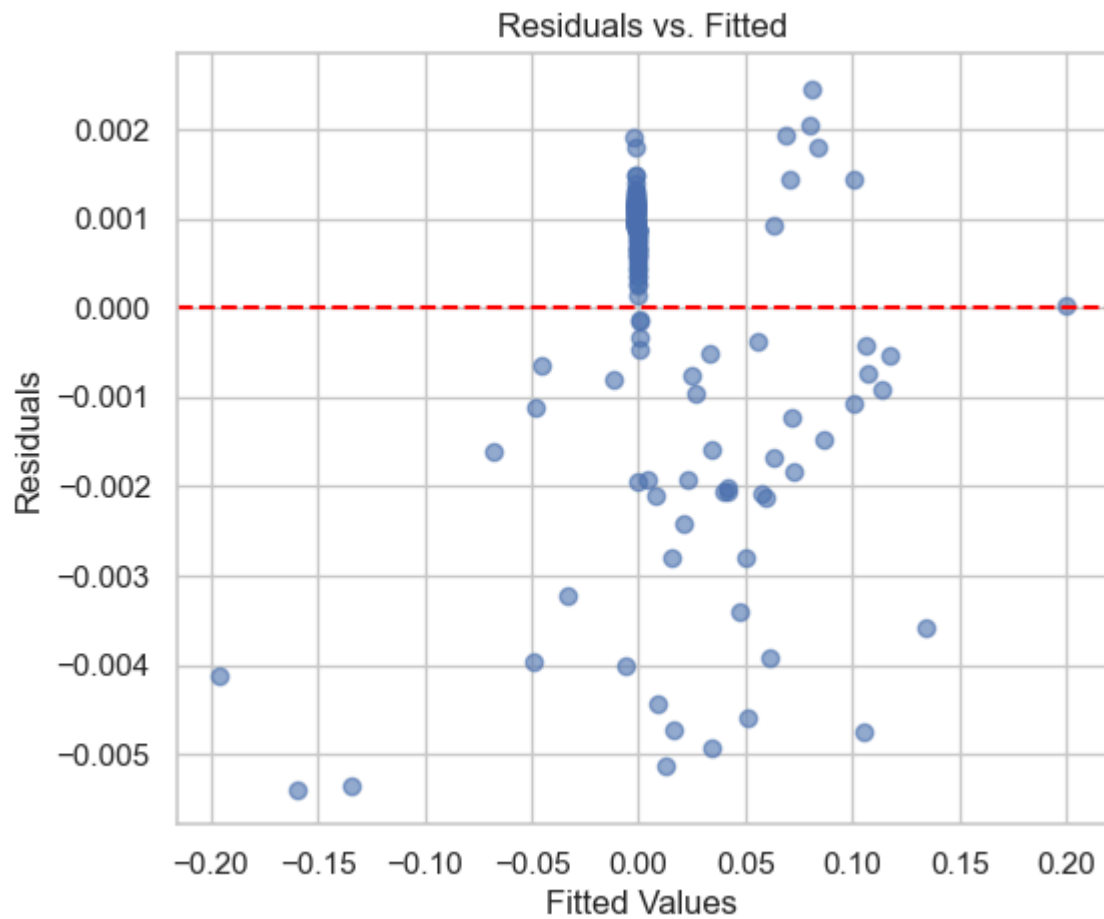
Out[41]:

	Coef.	Std.Err.	t	P> t	[0.025	0.975]
Intercept	-0.004045	0.001525	-2.652182	8.869392e-03	-0.007059	-0.001031
AAPL	0.148737	0.018724	7.943856	4.552458e-13	0.111737	0.185737
MSFT	0.126245	0.031951	3.951200	1.200054e-04	0.063106	0.189384
GOOG	0.038868	0.025373	1.531826	1.277001e-01	-0.011273	0.089008
META	0.007546	0.011127	0.678140	4.987419e-01	-0.014442	0.029533
AMZN	0.048658	0.021131	2.302652	2.269211e-02	0.006900	0.090415
NVDA	0.012014	0.012845	0.935314	3.511501e-01	-0.013370	0.037398
TSLA	0.002915	0.005016	0.581196	5.619926e-01	-0.006996	0.012826
BRK_B	0.083064	0.038901	2.135291	3.438333e-02	0.006192	0.159937
JPM	0.206532	0.025678	8.043155	2.593367e-13	0.155789	0.257274
JNJ	0.135825	0.039593	3.430568	7.807945e-04	0.057585	0.214065

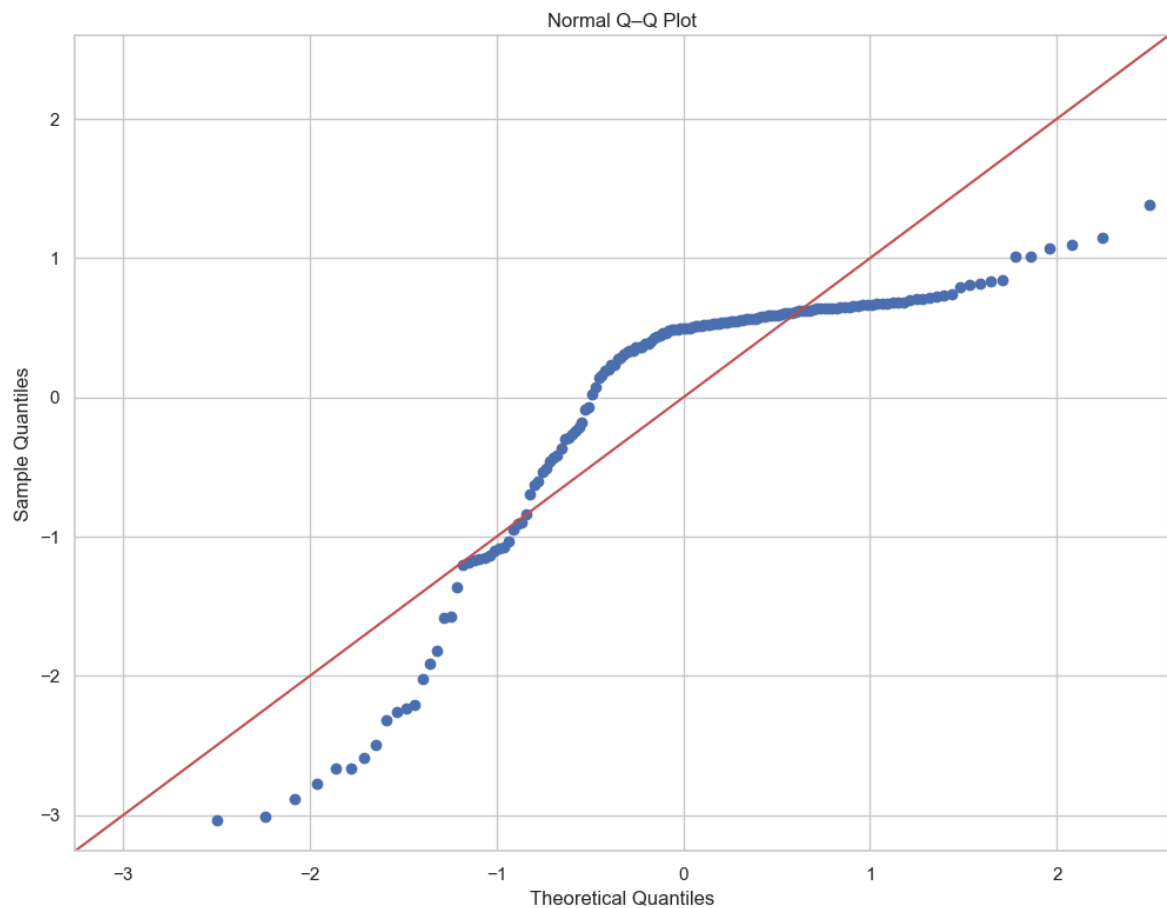
In the above results, we can see Johnson & Johnson and SPY estimates are significant (p-values less than 0.05).

```
In [86]: # Basic residuals & fitted values
resid = model_1.resid
fitted = model_1.fittedvalues

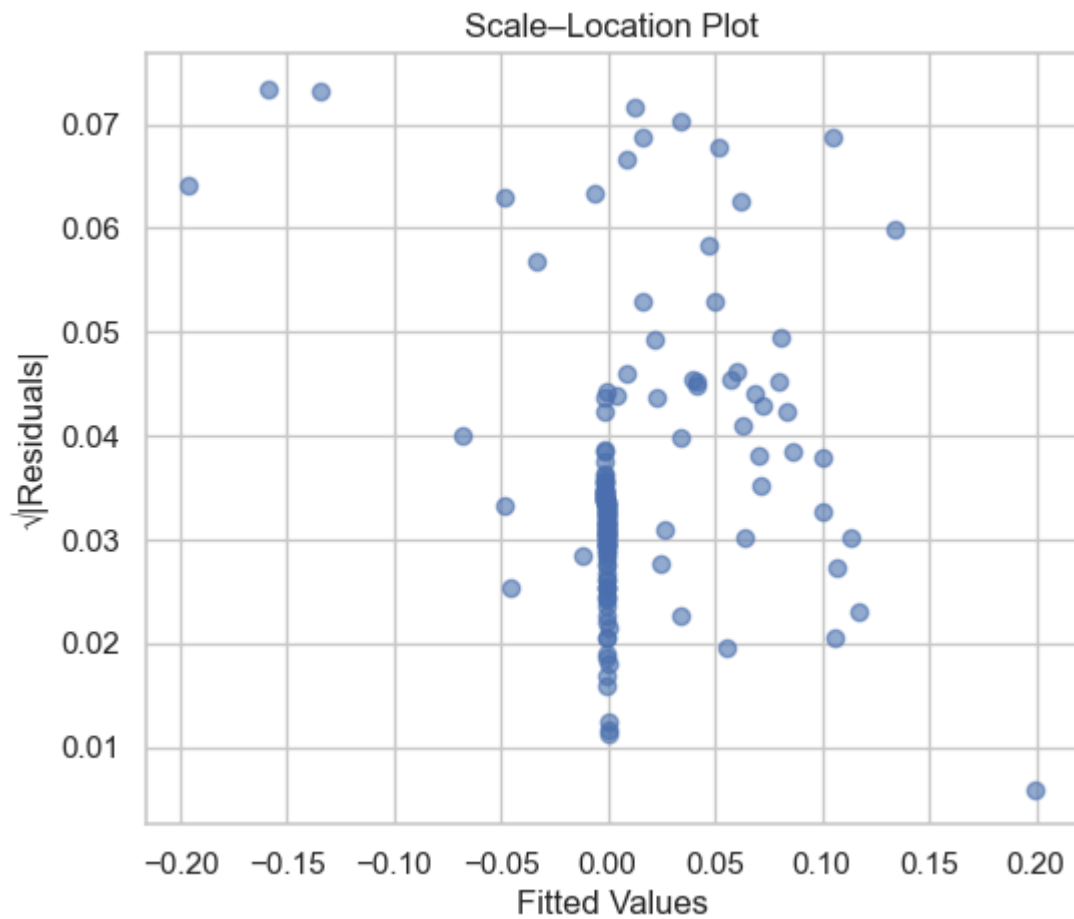
#Residuals vs Fitted
fig, ax = plt.subplots(figsize=(6,5))
ax.scatter(fitted, resid, alpha=0.6)
ax.axhline(0, color='red', linestyle='--')
ax.set_xlabel("Fitted Values")
ax.set_ylabel("Residuals")
ax.set_title("Residuals vs. Fitted")
plt.show()
```



```
In [89]: # Normal Q-Q Plot  
sm.qqplot(resid, line="45", fit=True)  
plt.title("Normal Q-Q Plot")  
plt.show()
```

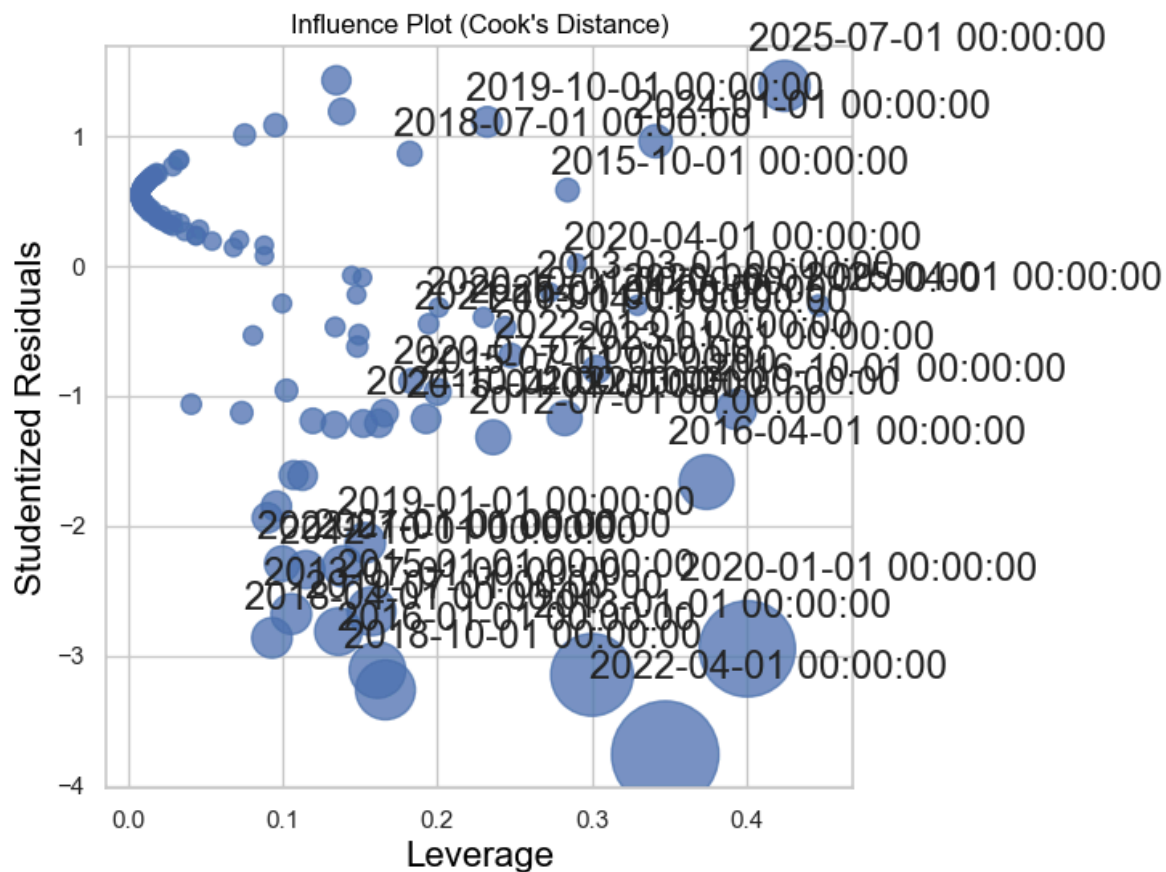


```
In [90]: # Scale-Location (Spread-Level) Plot
fig, ax = plt.subplots(figsize=(6,5))
ax.scatter(fitted, np.sqrt(np.abs(resid)), alpha=0.6)
ax.set_xlabel("Fitted Values")
ax.set_ylabel("|√|Residuals|")
ax.set_title("Scale-Location Plot")
plt.show()
```



In [93]: *#Influence Plot (Cook's Distance) ---*

```
fig, ax = plt.subplots(figsize=(6,6))
sm.graphics.influence_plot(model_1, ax=ax, criterion="cooks")
plt.title("Influence Plot (Cook's Distance)")
plt.show()
```



```
In [55]: # Regression model to check multicollinearity among independent variables

model_AAPL = smf.ols(
    formula="AAPL ~ MSFT + GOOG + META + AMZN + NVDA + TSLA + BRK_B + JPM + JNJ
    data=returns_q
).fit()

print(model_AAPL.summary())
```

OLS Regression Results

=====						
Dep. Variable:	AAPL		R-squared:	0.755		
Model:	OLS		Adj. R-squared:	0.737		
Method:	Least Squares		F-statistic:	41.15		
Date:	Mon, 29 Sep 2025		Prob (F-statistic):	2.16e-39		
Time:	13:52:39		Log-Likelihood:	262.97		
No. Observations:	159		AIC:	-501.9		
Df Residuals:	147		BIC:	-465.1		
Df Model:	11					
Covariance Type:	nonrobust					
=====						
	coef	std err	t	P> t	[0.025	0.975]

Intercept	0.0036	0.004	0.814	0.417	-0.005	0.012
MSFT	-0.8288	0.117	-7.107	0.000	-1.059	-0.598
GOOG	0.0426	0.073	0.584	0.560	-0.102	0.187
META	-0.0066	0.032	-0.210	0.834	-0.069	0.056
AMZN	0.0679	0.061	1.108	0.270	-0.053	0.189
NVDA	-0.1036	0.039	-2.678	0.008	-0.180	-0.027
TSLA	-0.0026	0.014	-0.180	0.857	-0.031	0.026
BRK_B	0.3762	0.121	3.107	0.002	0.137	0.616
JPM	-0.1221	0.092	-1.329	0.186	-0.304	0.060
JNJ	0.2165	0.123	1.753	0.082	-0.028	0.460
SPY	-1.5939	0.427	-3.733	0.000	-2.438	-0.750
XLK	3.1376	0.320	9.801	0.000	2.505	3.770
=====						
Omnibus:	47.182	Durbin-Watson:	2.012			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	448.223			
Skew:	-0.695	Prob(JB):	4.67e-98			
Kurtosis:	11.107	Cond. No.	139.			
=====						

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [56]: # Parameters with 6 significant digits
model_AAPL.summary2().tables[1]
```

Out[56]:

	Coef.	Std.Err.	t	P> t	[0.025	0.975]
Intercept	0.003561	0.004377	0.813749	4.171047e-01	-0.005088	0.012211
MSFT	-0.828794	0.116618	-7.106916	4.762701e-11	-1.059258	-0.598329
GOOG	0.042588	0.072978	0.583572	5.604027e-01	-0.101634	0.186810
META	-0.006644	0.031616	-0.210134	8.338538e-01	-0.069125	0.055837
AMZN	0.067852	0.061252	1.107742	2.697827e-01	-0.053197	0.188900
NVDA	-0.103580	0.038677	-2.678061	8.247812e-03	-0.180016	-0.027145
TSLA	-0.002567	0.014249	-0.180128	8.573003e-01	-0.030725	0.025592
BRK_B	0.376202	0.121089	3.106822	2.270256e-03	0.136902	0.615502
JPM	-0.122129	0.091922	-1.328614	1.860342e-01	-0.303790	0.059531
JNJ	0.216453	0.123489	1.752808	8.171981e-02	-0.027591	0.460497
SPY	-1.593886	0.426923	-3.733428	2.694290e-04	-2.437586	-0.750187
XLK	3.137576	0.320116	9.801387	9.155974e-18	2.504953	3.770200

In [57]: *# Add a constant column for the intercept*

```
import statsmodels.api as sm
X_with_const = sm.add_constant(X)

vif = pd.DataFrame()
vif["Variable"] = X_with_const.columns
vif["VIF"] = [variance_inflation_factor(X_with_const.values, i)
               for i in range(X_with_const.shape[1])]

print(vif)
```

	Variable	VIF
0	const	1.320004
1	AAPL	4.079076
2	MSFT	6.096314
3	GOOG	2.232055
4	META	1.013497
5	AMZN	2.207639
6	NVDA	3.052805
7	TSLA	1.010518
8	BRK-B	2.936833
9	JPM	3.133554
10	JNJ	2.143507
11	SPY	27.118045
12	XLK	42.098410

We can see Microsoft, SPY and XLK Indexes have VIF values larger than 5 indicating severe multicollinearity. Since these variables have severe multicollinearity issues, we need to dig further into this variable and maybe drop them from the model. We can also use Principle Component Analysis.

In [70]: *#PCA(Principal Component Analysis)**#Libraries*

```
from scipy.stats import boxcox
from sklearn import preprocessing
```



```
from sklearn.preprocessing import scale
from sklearn.decomposition import PCA
```

```
In [80]: #Select Twelve Variables & Standardize

columns_to_use = ["AAPL", "MSFT", "GOOG", "META", "AMZN",
                  "NVDA", "TSLA", "BRK_B", "JPM", "JNJ", "SPY", "XLK"]

pc = returns_q[columns_to_use]
# Standardize
pca_data = scale(pc)

# PCA
pca = PCA(n_components=12)
pca.fit(pca_data)
```

```
Out[80]: PCA
PCA(n_components=12)
```

```
In [78]: # Get proportions of variance and cumulative proportion of variance
pr_var = pca.explained_variance_ratio_
cum_pr = np.cumsum(pca.explained_variance_ratio_)
ind = ["Proportion of variance", "Cumulative proportion of variance"]
cols = ["PC1", "PC2", "PC3", "PC4", "PC5", "PC6", "PC7", "PC8", "PC9", "PC10", "PC11", "PC12"]
pd.DataFrame(np.vstack((pr_var, cum_pr)), ind, columns=cols)
```

```
Out[78]:
```

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12
Proportion of variance	0.467415	0.132597	0.087362	0.078195	0.060061	0.045638	0.040971	0.032415	0.025638	0.020971	0.015638	0.010971
Cumulative proportion of variance	0.467415	0.600012	0.687374	0.765569	0.825630	0.871268	0.912240	0.944655	0.970283	0.991254	1.000000	1.000000

Looking at the proportion of variance and cumulative proportion in the above chart we can see that the first principal component accounts for about 46.7% of the total variance in the data. The first seven principal components explain about 91.2% of the total variance.

```
In [79]: # Coefficients (Loadings) of 12 Principal Components
pc_res = pd.DataFrame(pca.components_.T, index=list(pc.columns), columns=cols)
pc_res
```

Out[79]:

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	
AAPL	0.309535	-0.148488	-0.002510	0.043996	0.565727	0.089767	0.581412	-0.05
MSFT	0.353194	-0.154890	0.000112	-0.005376	-0.231410	0.039169	-0.198365	0.60
GOOG	0.326337	-0.148612	-0.000909	0.014233	0.058669	-0.529424	-0.179600	-0.46
META	-0.042809	-0.028931	-0.699319	0.710901	-0.025403	0.026300	-0.033561	-0.00
AMZN	0.296498	-0.345422	0.001024	-0.014688	0.115723	0.067595	-0.501865	-0.31
NVDA	0.326891	-0.167354	0.001577	-0.030023	-0.295965	0.576736	-0.092894	-0.11
TSLA	-0.035934	-0.023634	0.714777	0.696447	-0.022517	0.022907	-0.029133	-0.00
BRK_B	0.210070	0.578836	-0.001791	0.015041	-0.072145	0.430482	0.053073	-0.39
JPM	0.280945	0.327083	-0.002472	0.028709	-0.531885	-0.415462	0.224015	-0.04
JNJ	0.178814	0.554386	-0.002234	0.027275	0.479700	-0.072423	-0.466694	0.27
SPY	0.398953	0.119375	-0.003956	0.056018	0.020096	-0.092397	0.154982	0.09
XLK	0.401069	-0.139737	-0.002466	0.036934	0.046890	0.001573	0.181827	0.24



For $PC1$, the XLK Index excess return has the highest absolute value of the coefficient. This means that the XLK Index excess return has the largest impact on $PC1$. Therefore, we can use the XLK Index excess return as a proxy for $PC1$. Following the same logic, we can use BRK_B excess return as a proxy for $PC2$. We also know that the first seven principal components cover around 91.2% of the data variation. In this case, we can just choose XLK Index excess return, BRK_B excess return, S&P500's revenue growth, TSLA excess return, JNJ excess return and NVDA excess return to run a regression model.

```
In [76]: # OLS for revised model
model_3 = smf.ols(
    "GSPC ~ XLK + BRK_B + TSLA + JNJ + NVDA",
    data=returns_q,
).fit()
model_3.summary()
```

Out[76]:

OLS Regression Results

Dep. Variable:	GSPC	R-squared:	0.925
Model:	OLS	Adj. R-squared:	0.922
Method:	Least Squares	F-statistic:	377.1
Date:	Mon, 29 Sep 2025	Prob (F-statistic):	4.26e-84
Time:	16:31:07	Log-Likelihood:	478.36
No. Observations:	159	AIC:	-944.7
Df Residuals:	153	BIC:	-926.3
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-0.0021	0.001	-1.975	0.050	-0.004	1.02e-06
XLK	0.6658	0.025	26.771	0.000	0.617	0.715
BRK_B	0.1974	0.025	8.048	0.000	0.149	0.246
TSLA	0.0015	0.004	0.416	0.678	-0.006	0.009
JNJ	0.0946	0.028	3.360	0.001	0.039	0.150
NVDA	-0.0366	0.009	-4.068	0.000	-0.054	-0.019

Omnibus:	77.446	Durbin-Watson:	2.043
Prob(Omnibus):	0.000	Jarque-Bera (JB):	442.966
Skew:	-1.686	Prob(JB):	6.47e-97
Kurtosis:	10.449	Cond. No.	36.1

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

From the result, we can see that the XLK Index has become significant, and the adjusted R^2 has improved to 0.925. By using the information provided by PCA, we can reduce the impact of multicollinearity among independent variables.

```
In [82]: #Full PCA regression
k = 6 #just enough PCs for ~85% variance
pc_scores = pca.transform(pca_data[:, :k])
pc_df = pd.DataFrame(pc_scores, columns=[f"PC{i}" for i in range(1, k+1)],
                    index=returns_q.index)

pc_df["GSPC"] = returns_q["GSPC"]
model_pc = smf.ols("GSPC ~ PC1 + PC2 + PC3 + PC4 + PC5 + PC6", data=pc_df).fit()
print(model_pc.summary())
```

OLS Regression Results

=====						
Dep. Variable:	GSPC		R-squared:	0.911		
Model:	OLS		Adj. R-squared:	0.907		
Method:	Least Squares		F-statistic:	258.2		
Date:	Mon, 29 Sep 2025		Prob (F-statistic):	4.76e-77		
Time:	23:03:11		Log-Likelihood:	464.51		
No. Observations:	159		AIC:	-915.0		
Df Residuals:	152		BIC:	-893.5		
Df Model:	6					
Covariance Type:	nonrobust					
=====						
	coef	std err	t	P> t	[0.025	0.975]

Intercept	0.0110	0.001	10.377	0.000	0.009	0.013
PC1	0.0173	0.000	38.676	0.000	0.016	0.018
PC2	0.0051	0.001	6.092	0.000	0.003	0.007
PC3	-0.0002	0.001	-0.193	0.848	-0.002	0.002
PC4	0.0028	0.001	2.606	0.010	0.001	0.005
PC5	0.0009	0.001	0.706	0.481	-0.002	0.003
PC6	-0.0043	0.001	-3.015	0.003	-0.007	-0.001
=====						
Omnibus:	70.616	Durbin-Watson:	2.157			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	254.408			
Skew:	-1.713	Prob(JB):	5.70e-56			
Kurtosis:	8.164	Cond. No.	3.20			
=====						

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In []: