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Math for Machine Learning

Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

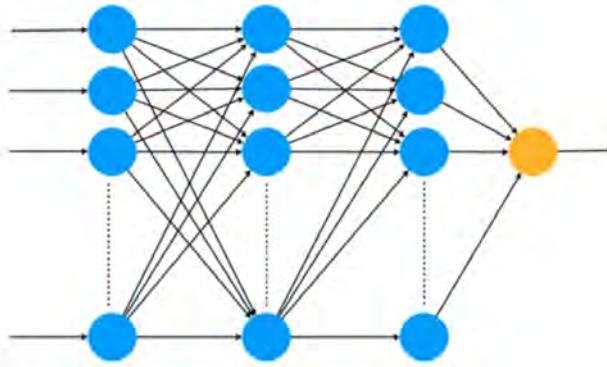


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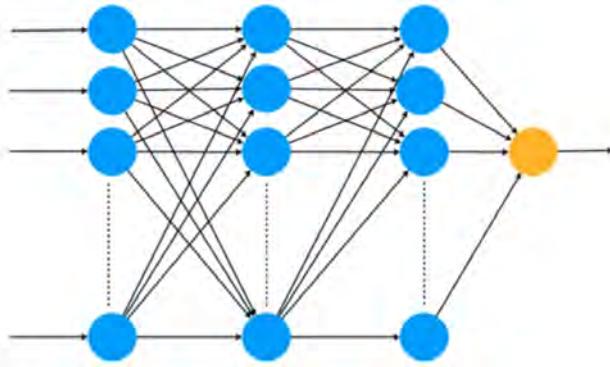
Solving System of Linear Equations

Machine learning motivation

Neural networks - Matrix operations

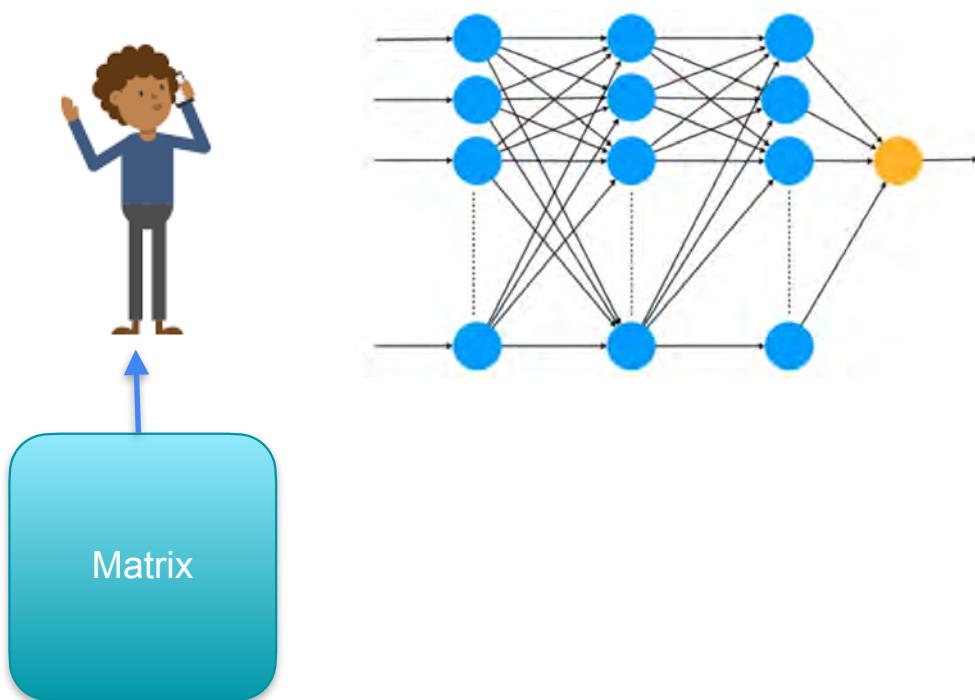


Neural networks - Matrix operations



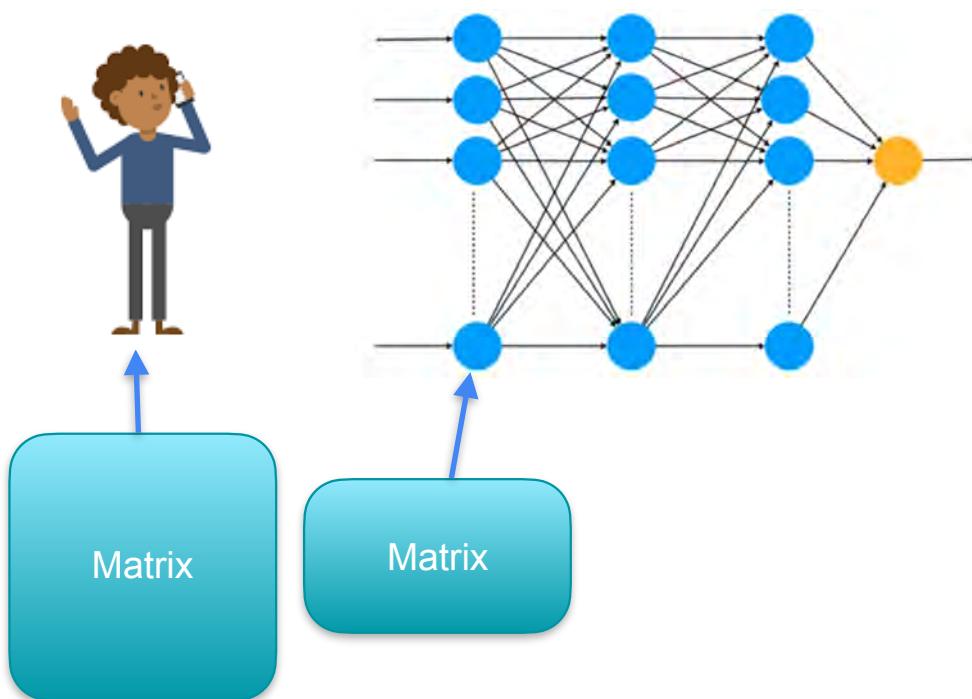
"Hello! Welcome to
Math for Machine
Learning!"

Neural networks - Matrix operations



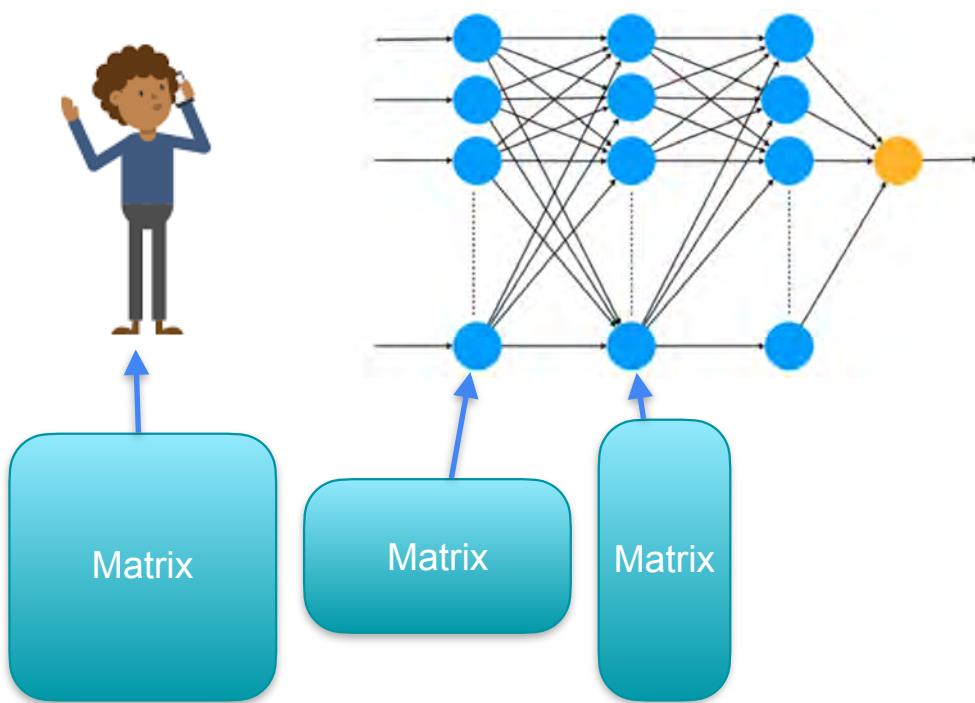
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Neural networks - Matrix operations



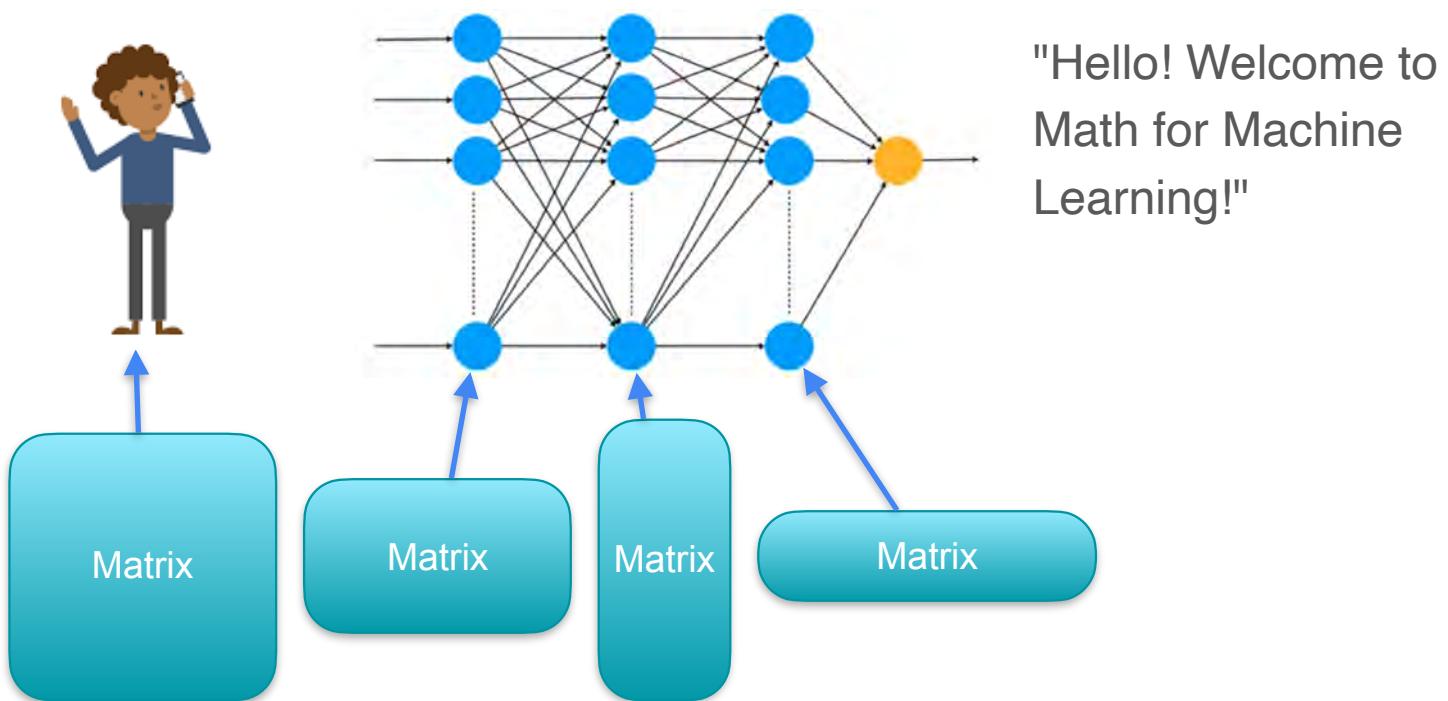
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Neural networks - Matrix operations

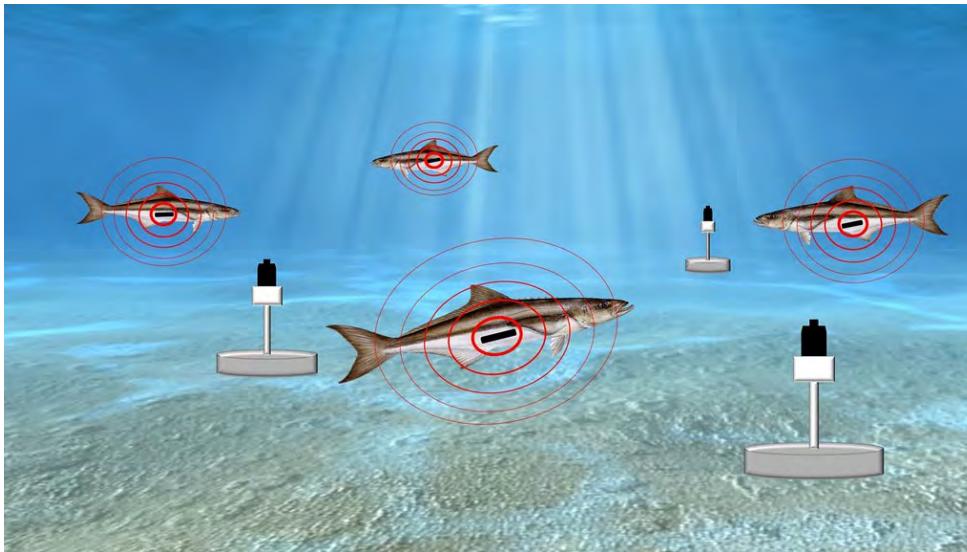


"Hello! Welcome to
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Neural networks - Matrix operations



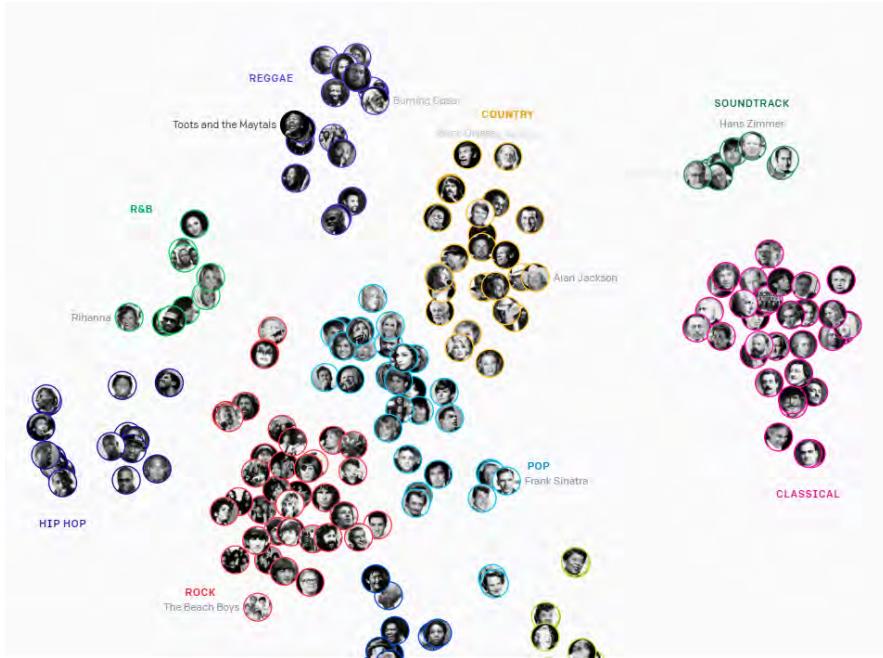
Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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Solving System of Linear Equations

**Solving non-singular system
of linear equations**

Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 

Solving systems of equations

System

$$\cdot a + b = 10$$


$$\cdot a + 2b = 12$$



$$= \$10$$


$$= \$12$$

Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 

 +  = \$10

 +  +  = \$12

Solving systems of equations

System

$$\cdot a + b = 10$$


$$\cdot a + 2b = 12$$



$$= \$10$$


$$= \$12$$

Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 

 +  = \$10

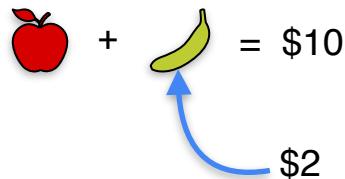
 +  +  = \$12
 
 \$2

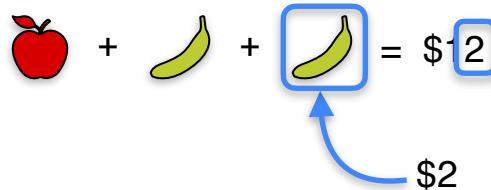
Solving systems of equations

System

- $a + b = 10$

- $a + 2b = 12$



$$\text{apple} + \text{banana} = \$10$$


$$\text{apple} + \text{banana} + \text{banana} = \$12$$

Solving systems of equations

System

- $a + b = 10$

- $a + 2b = 12$


$$\begin{array}{c} \text{apple} \\ + \\ \text{banana} \end{array} = \$10$$

\$8  

$$\begin{array}{c} \text{apple} \\ + \\ \text{banana} \\ + \\ \boxed{\text{banana}} \end{array} = \$12$$



Solving systems of equations

Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 

Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 



Solved system

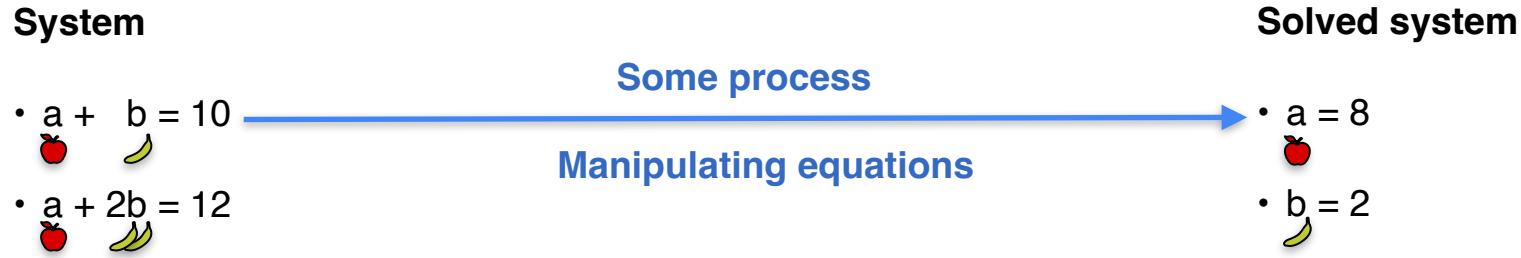
- $a = 8$

- $b = 2$

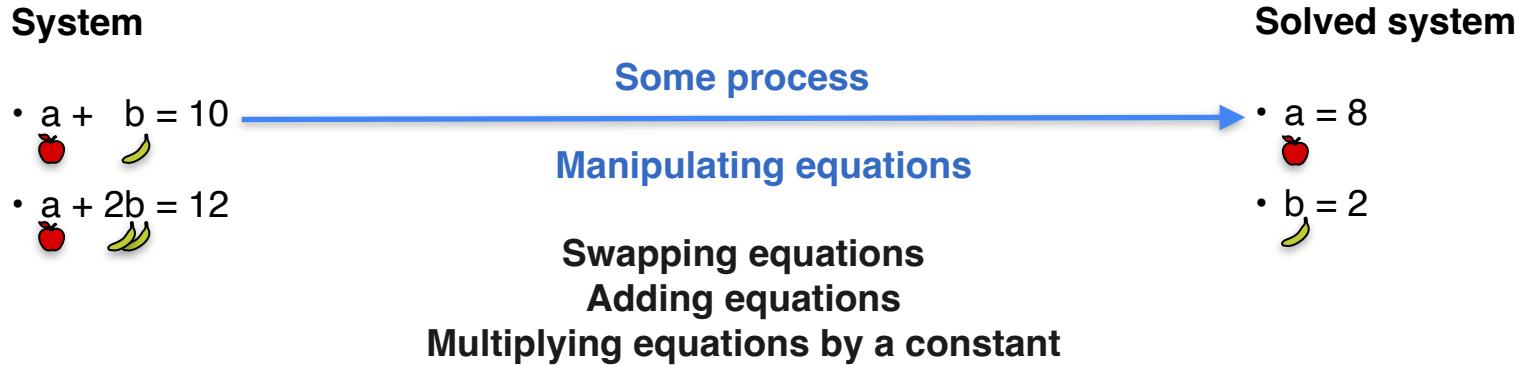

Solving systems of equations



Solving systems of equations



Solving systems of equations



Solving systems of equations

System

$$\begin{aligned} \bullet \quad & a + b = 10 \\ & \begin{array}{c} \text{apple} \\ \text{banana} \end{array} \end{aligned}$$
$$\begin{aligned} \bullet \quad & a + 2b = 12 \\ & \begin{array}{c} \text{apple} \\ \text{banana} \end{array} \end{aligned}$$

Solved system

$$\begin{aligned} \bullet \quad & a = 8 \\ & \begin{array}{c} \text{apple} \\ \text{banana} \end{array} \end{aligned}$$
$$\bullet \quad b = 2$$

Solving systems of equations

System

$$\bullet \ a + \text{apple} \text{banana} = 10$$

$$\bullet \ a + 2\text{banana} = 12$$

Solved system

$$\bullet \ a = 8 \text{apple}$$

$$\bullet \ b = 2 \text{banana}$$

Eliminate 'a' from this equation

Manipulating equations

Manipulating equations

Multiplying by a constant

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \\ \times \quad \quad \quad 7 \\ \hline \end{array}$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \qquad \qquad \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \quad \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \quad \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \quad \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \quad \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \quad 2a + 3b = 26 \\ \hline \end{array}$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \quad \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \quad 2a + 3b = 26 \\ \hline 3a + 4b = 36 \end{array}$$

Let's do a harder example

Systems of equations

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$



Eliminate 'a'
from this equation

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

$$\bullet 5a + b = 17$$

$$\bullet 4a - 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet a - 0.75b = 1.5$$

Solved system

$$\bullet a = ?$$

$$\bullet b = ?$$



Eliminate 'a'
from this equation

Systems of equations

System

$$\bullet 5a + b = 17$$

$$\bullet 4a - 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet a - 0.75b = 1.5$$

Solved system

$$\bullet a = ?$$

$$\bullet b = ?$$

Subtract equation 1 from equation 2

Eliminate 'a'
from this equation



Systems of equations

System

$$\bullet 5a + b = 17$$

$$\bullet 4a - 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet a - 0.75b = 1.5$$

Eliminate 'a'
from this equation

Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$

Solved system

$$\bullet a = ?$$

$$\bullet b = ?$$

Systems of equations

System

$$\bullet 5a + b = 17$$

$$\bullet 4a - 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet a - 0.75b = 1.5$$

Eliminate 'a'
from this equation

Solved system

$$\bullet a = ?$$

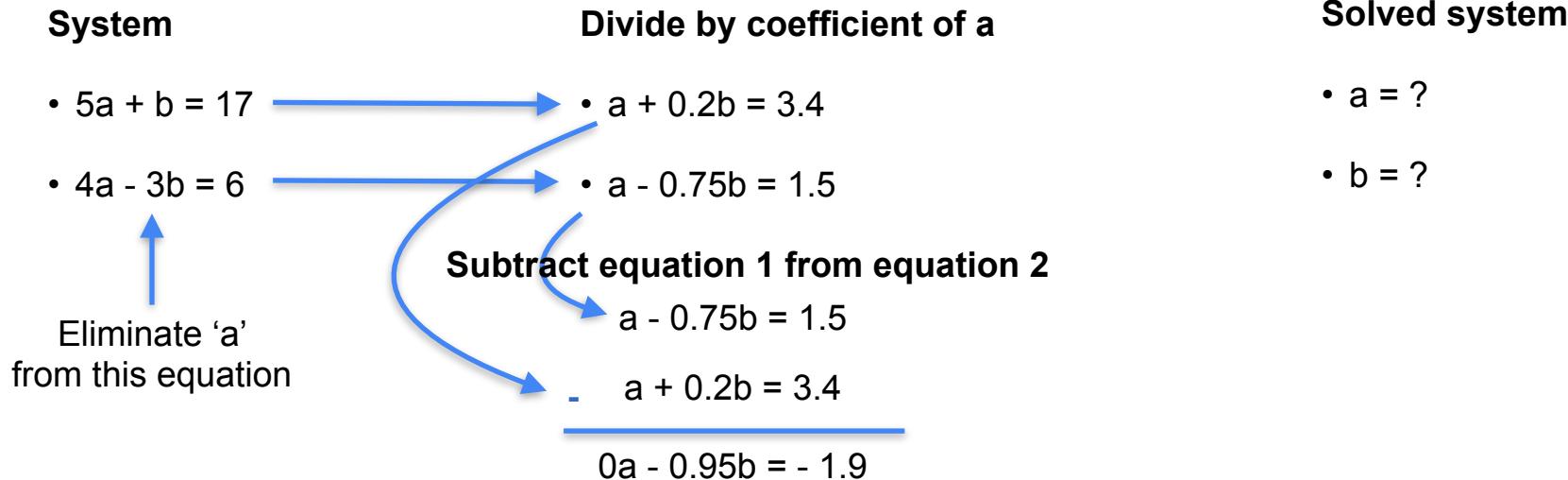
$$\bullet b = ?$$

Subtract equation 1 from equation 2

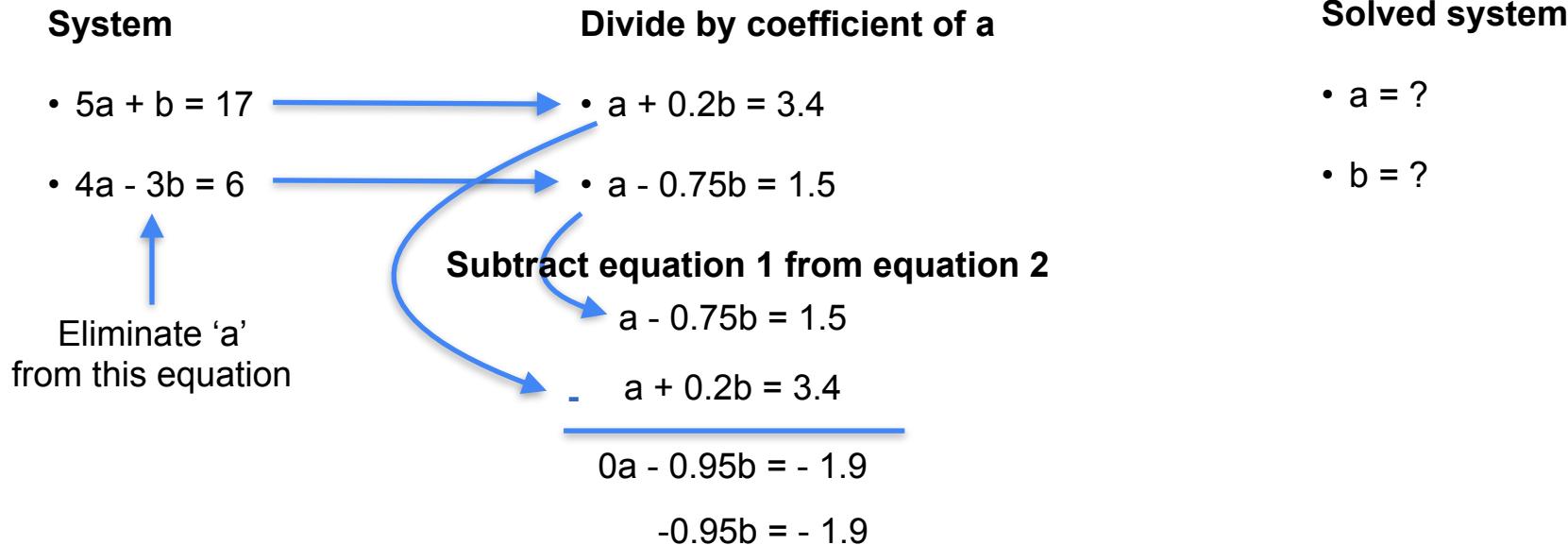
$$a - 0.75b = 1.5$$

$$- a + 0.2b = 3.4$$

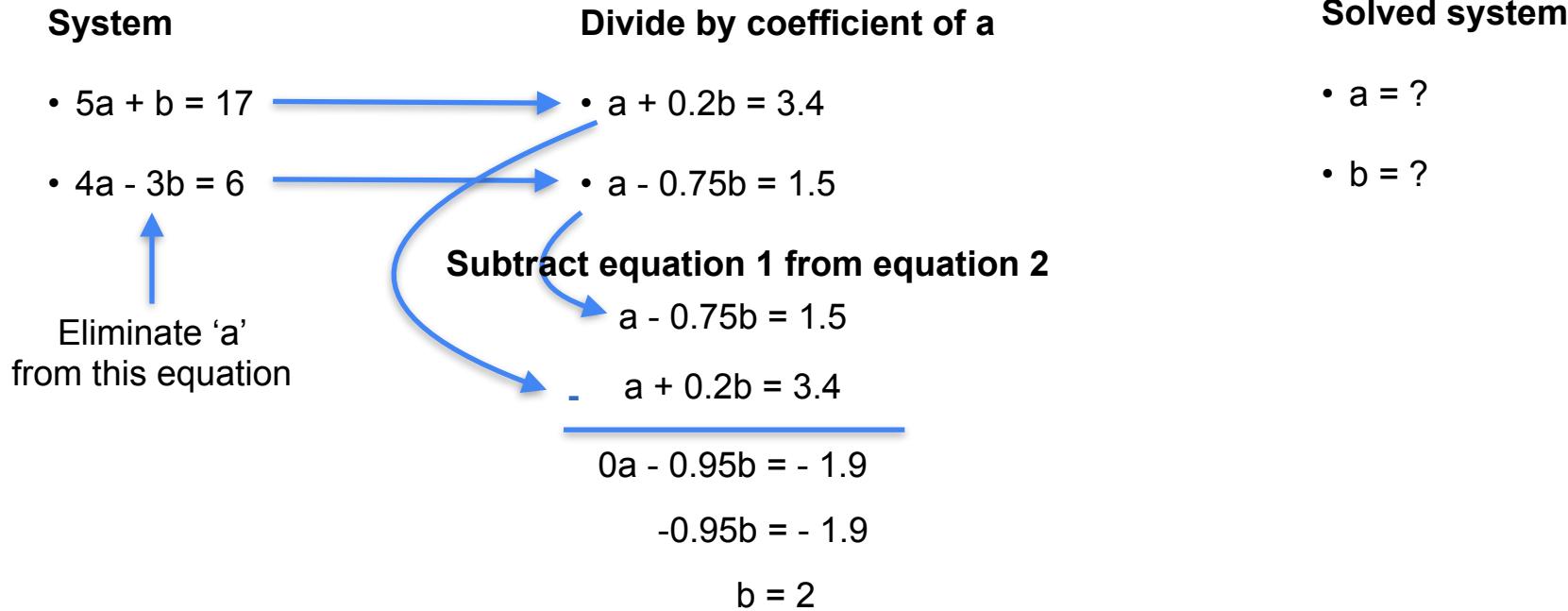
Systems of equations



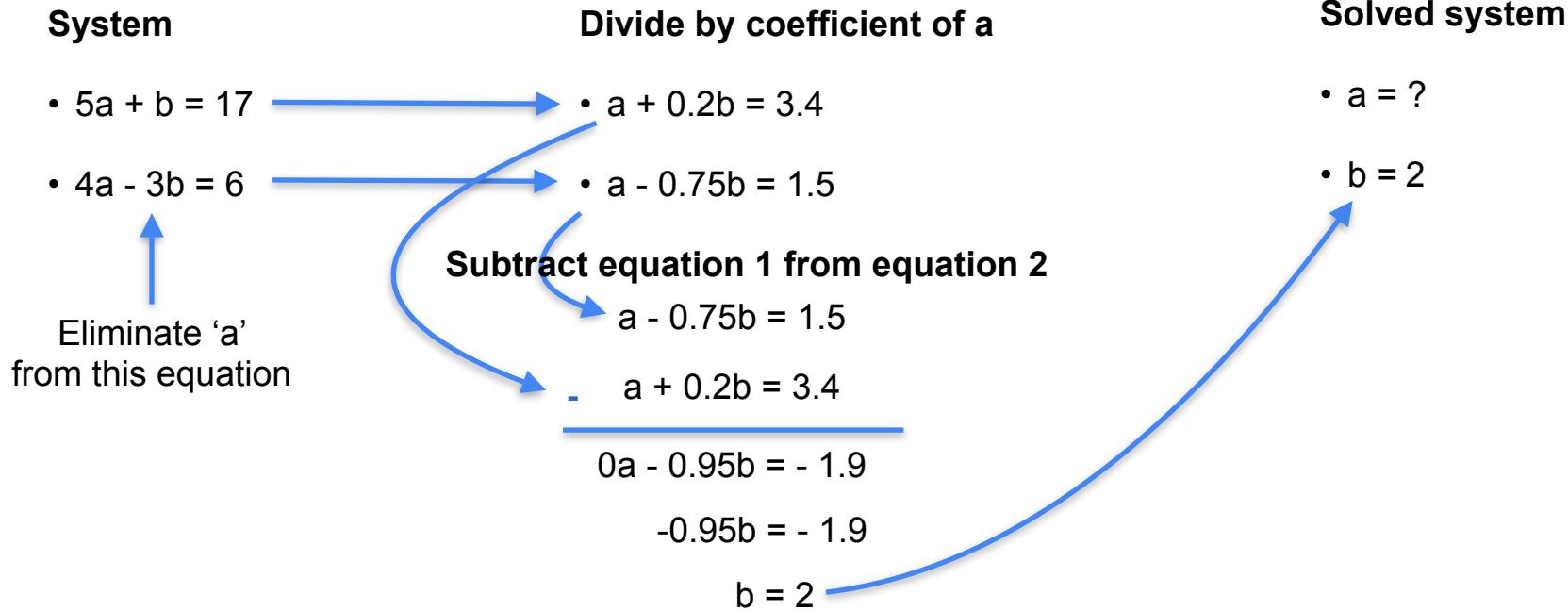
Systems of equations



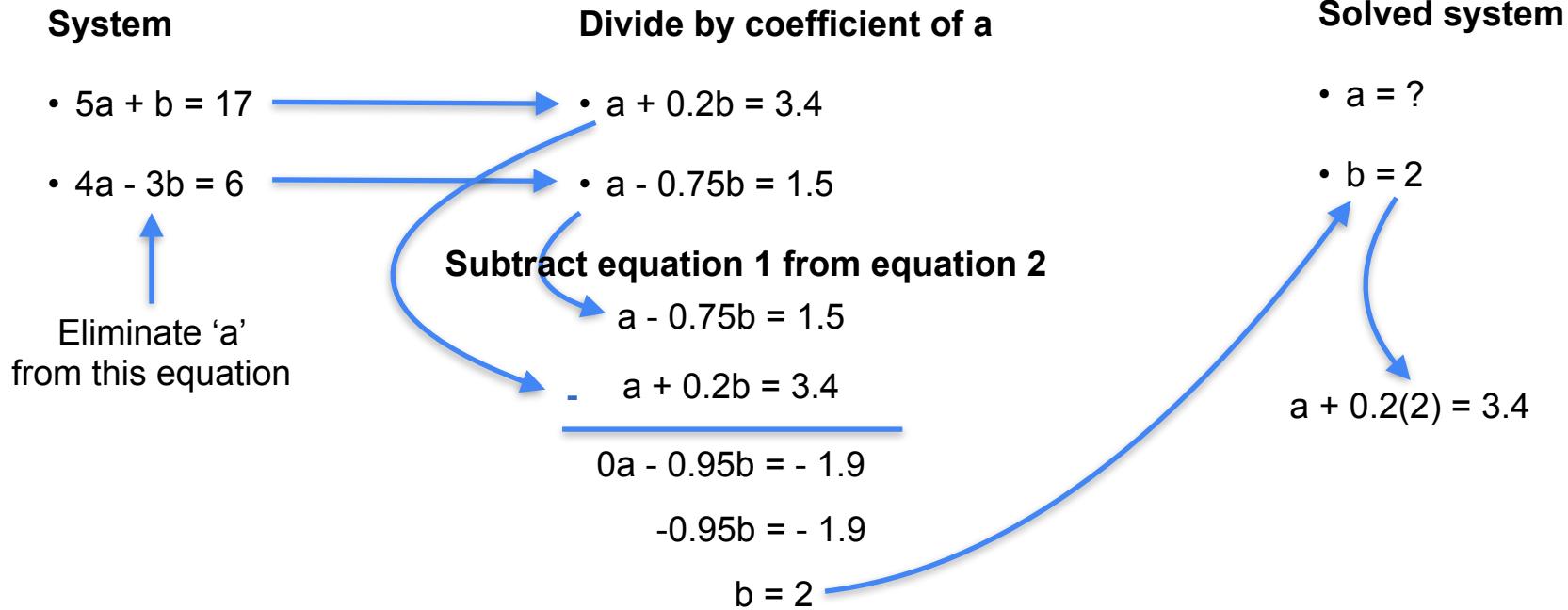
Systems of equations



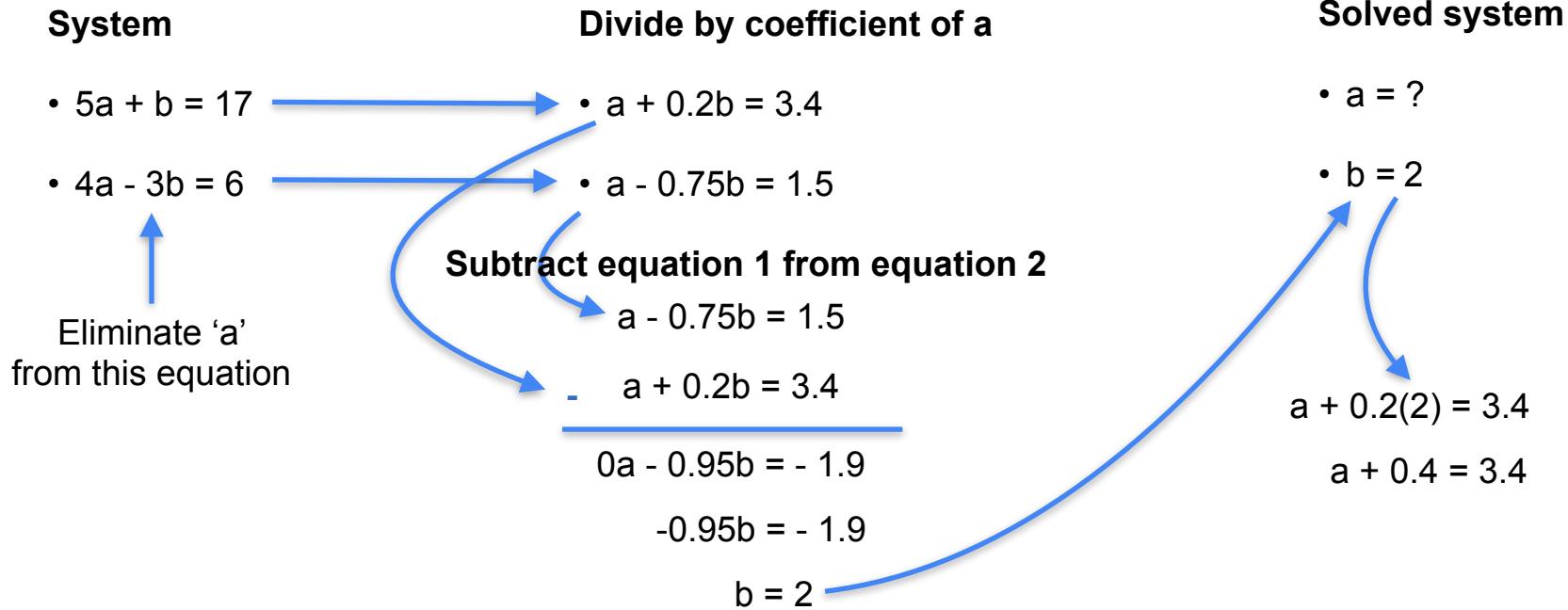
Systems of equations



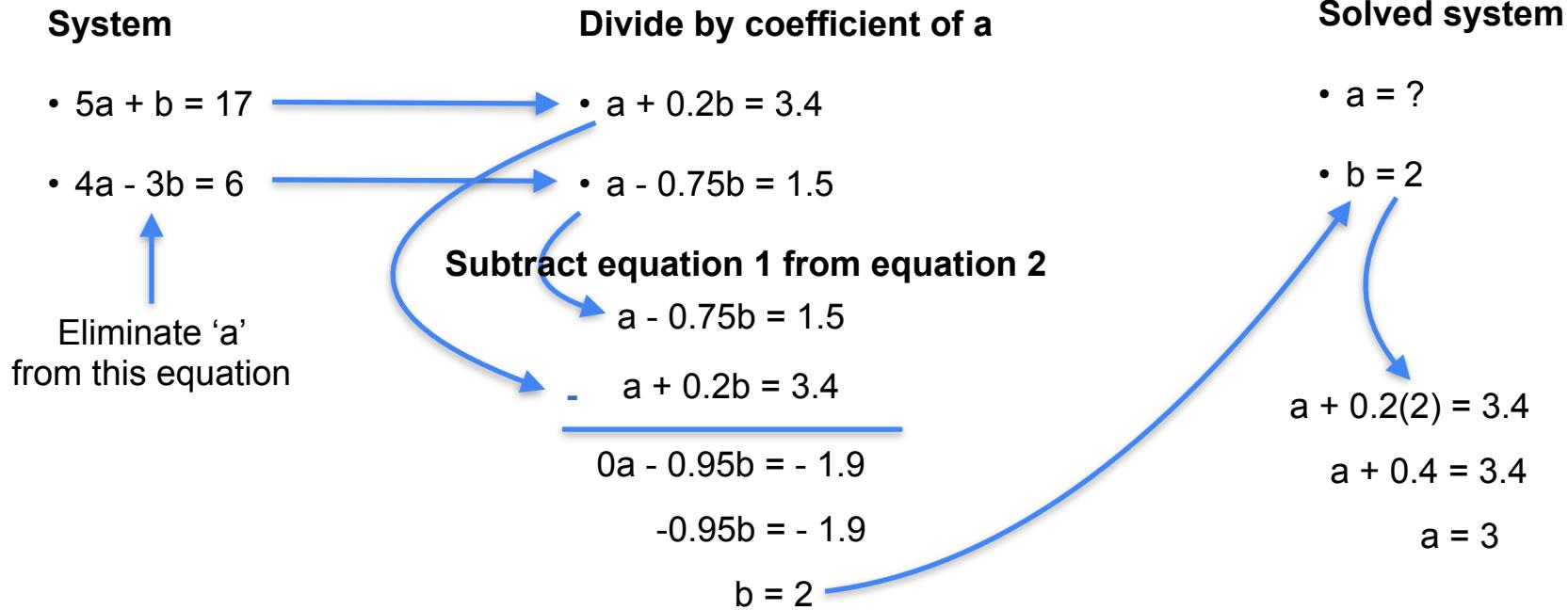
Systems of equations



Systems of equations



Systems of equations



Systems of equations

System

$$\bullet 5a + b = 17$$

$$\bullet 4a - 3b = 6$$

Eliminate 'a'
from this equation

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet a - 0.75b = 1.5$$

Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$

$$a + 0.2b = 3.4$$

$$\underline{0a - 0.95b = -1.9}$$

$$-0.95b = -1.9$$

$$b = 2$$

Solved system

$$\bullet a = 3$$

$$\bullet b = 2$$

$$a + 0.2(2) = 3.4$$

$$a + 0.4 = 3.4$$

$$a = 3$$

What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$



Eliminate 'a'
from this equation

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

$$\bullet 5a + b = 17$$

$$\bullet 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet ???$$

Solved system

$$\bullet a = ?$$

$$\bullet b = ?$$



Eliminate 'a'
from this equation

What if one of the coefficients of a is zero?

System

$$\bullet 5a + b = 17$$

$$\bullet 3b = 6$$

Divide by coefficient of a

$$\bullet a + 0.2b = 3.4$$

$$\bullet ???$$

Solved system

$$\bullet a = ?$$

$$\bullet b = ?$$

Eliminate 'a'
from this equation

$$b = 2$$

What if one of the coefficients of a is zero?

System

$$\begin{array}{l} \bullet \ 5a + b = 17 \\ \bullet \ 3b = 6 \end{array} \xrightarrow{\quad} \begin{array}{l} \bullet \ a + 0.2b = 3.4 \\ \bullet \ ??? \end{array}$$

Divide by coefficient of a

Solved system

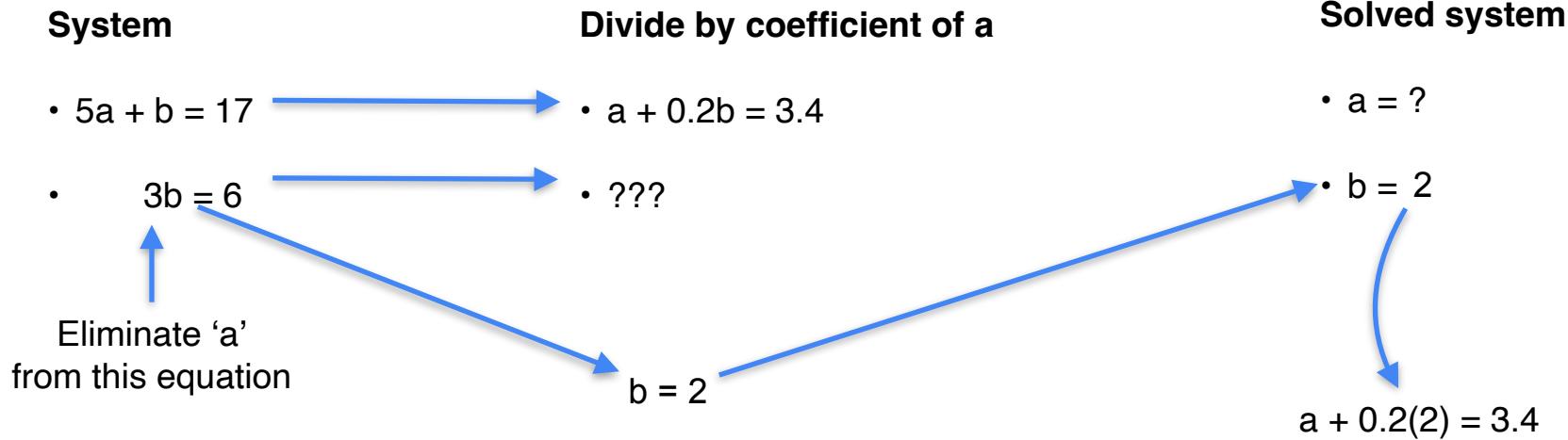
$$\bullet \ a = ?$$

$$\bullet \ b = 2$$

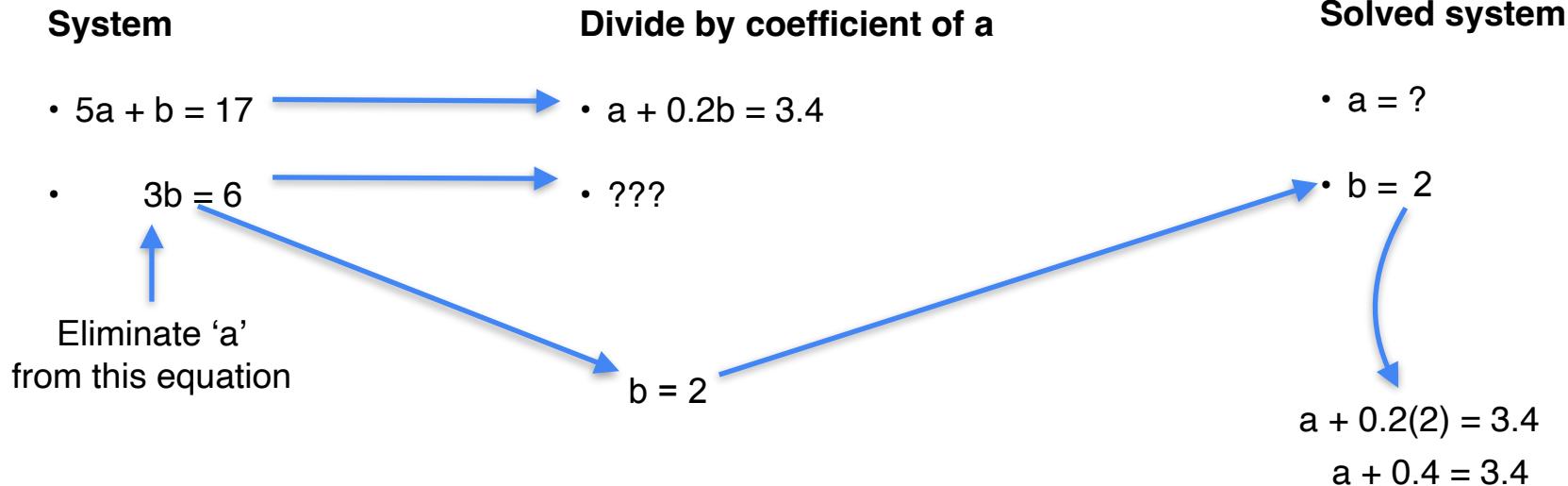
Eliminate 'a'
from this equation

$$b = 2$$

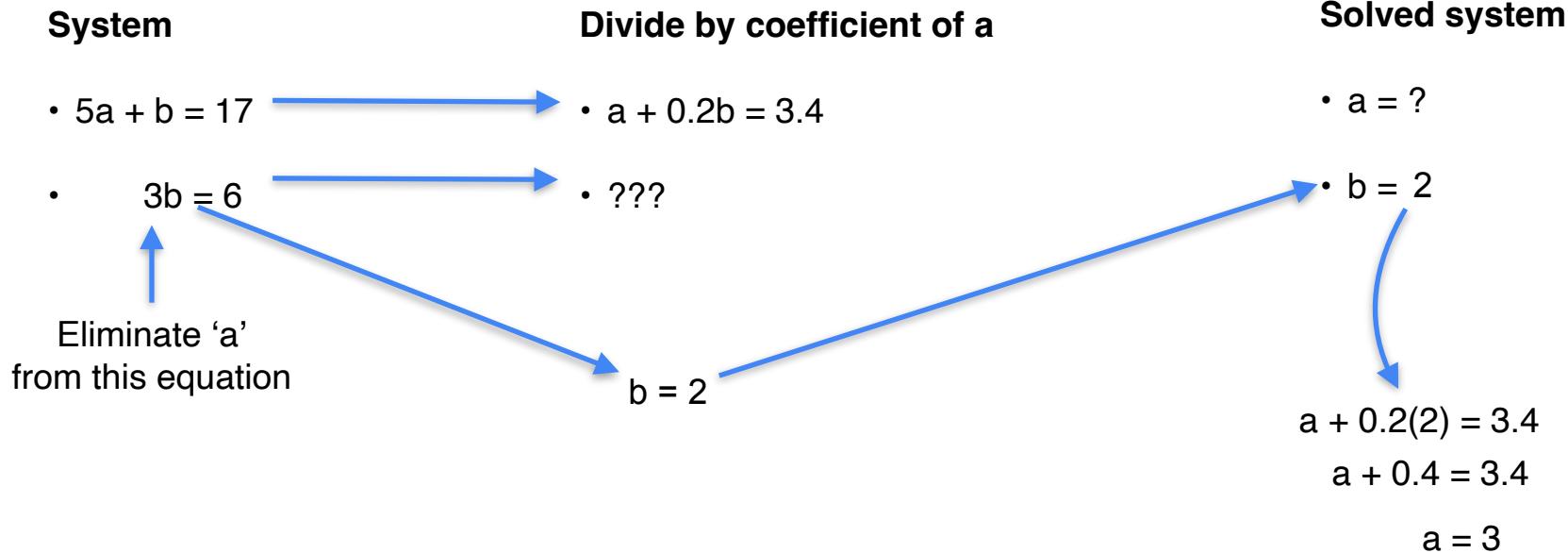
What if one of the coefficients of a is zero?



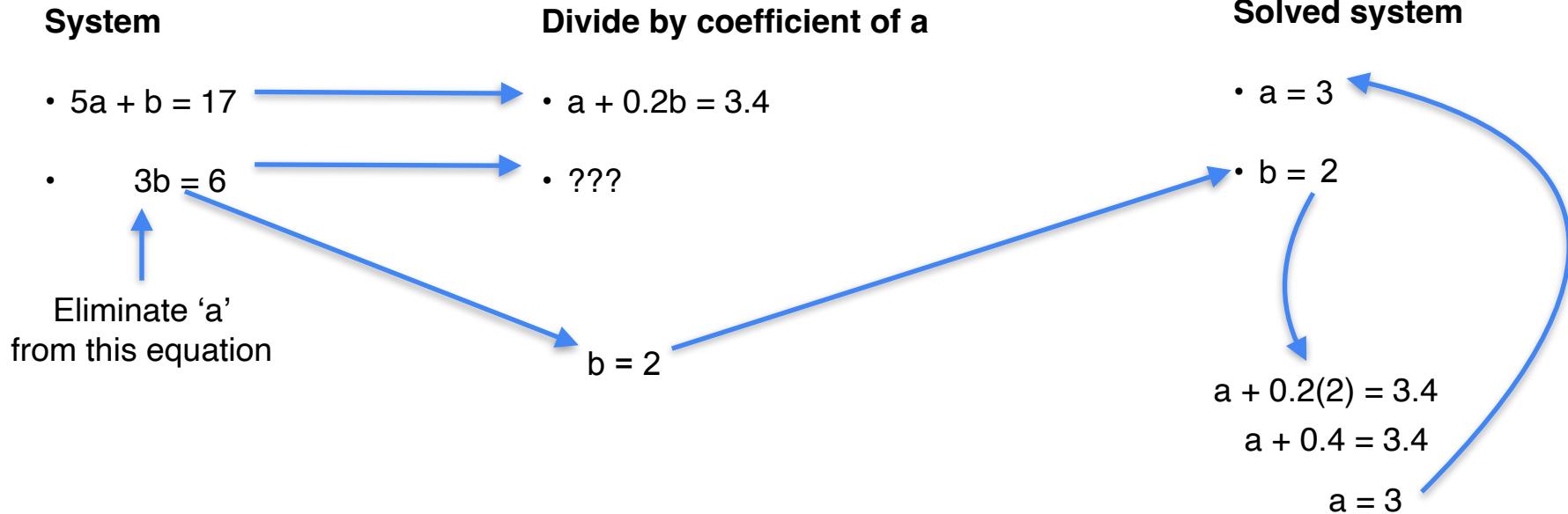
What if one of the coefficients of a is zero?



What if one of the coefficients of a is zero?



What if one of the coefficients of a is zero?



Quiz

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- $a = 3$
- $b = 8$



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Solving System of Linear Equations

**Solving singular system of
linear equations**

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (redundant)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Divide by coefficient of a



$$\cdot a + b = 10$$



$$\cdot a + b = 10$$

Solved system

$$\cdot a = ?$$

$$\cdot b = ?$$



Eliminate 'a'
from this equation

What if the system is singular (redundant)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Divide by coefficient of a

$$\cdot a + b = 10$$

$$\cdot a + b = 10$$

Solved system

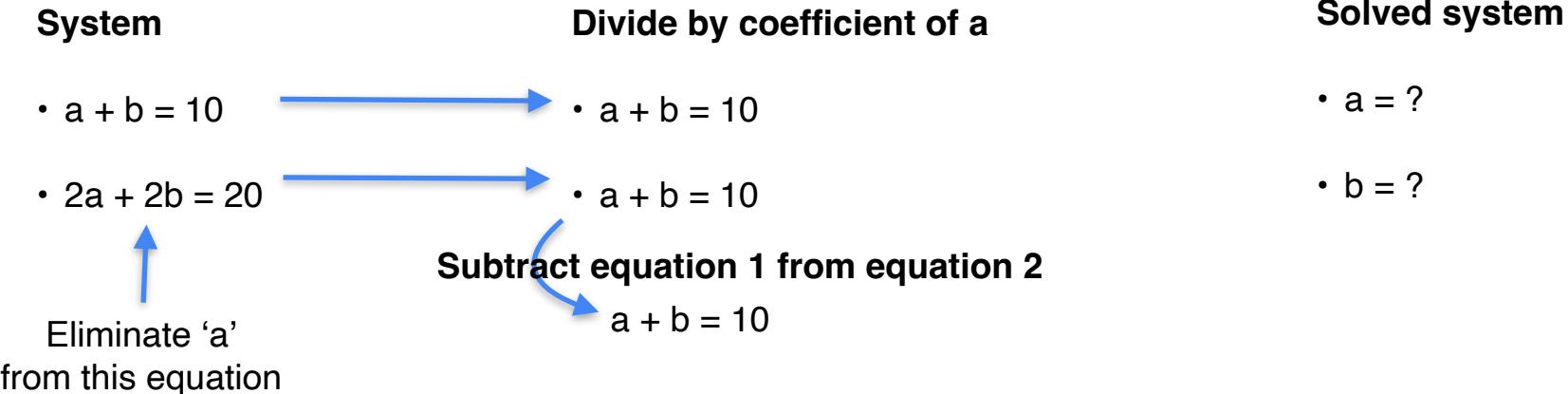
$$\cdot a = ?$$

$$\cdot b = ?$$

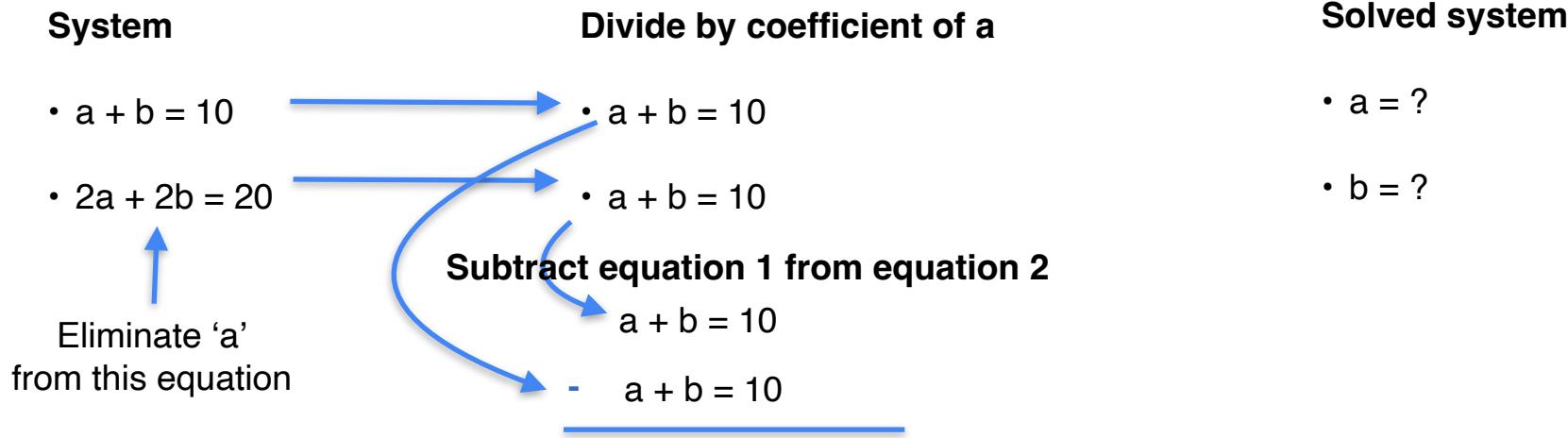
Subtract equation 1 from equation 2

Eliminate 'a'
from this equation

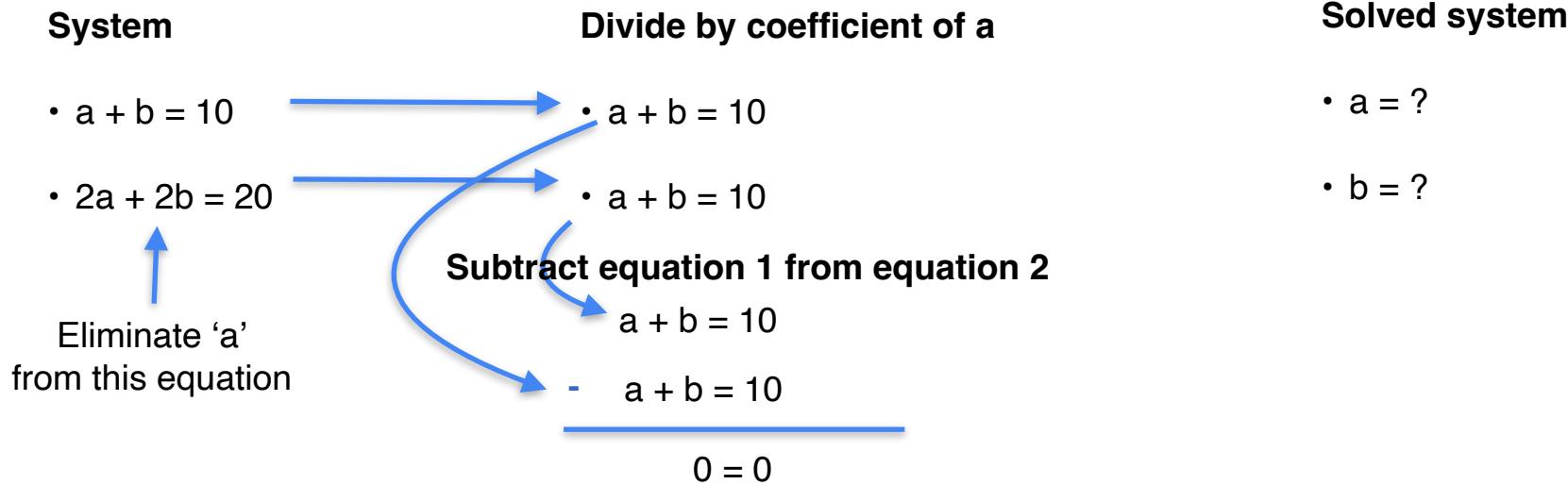
What if the system is singular (redundant)?



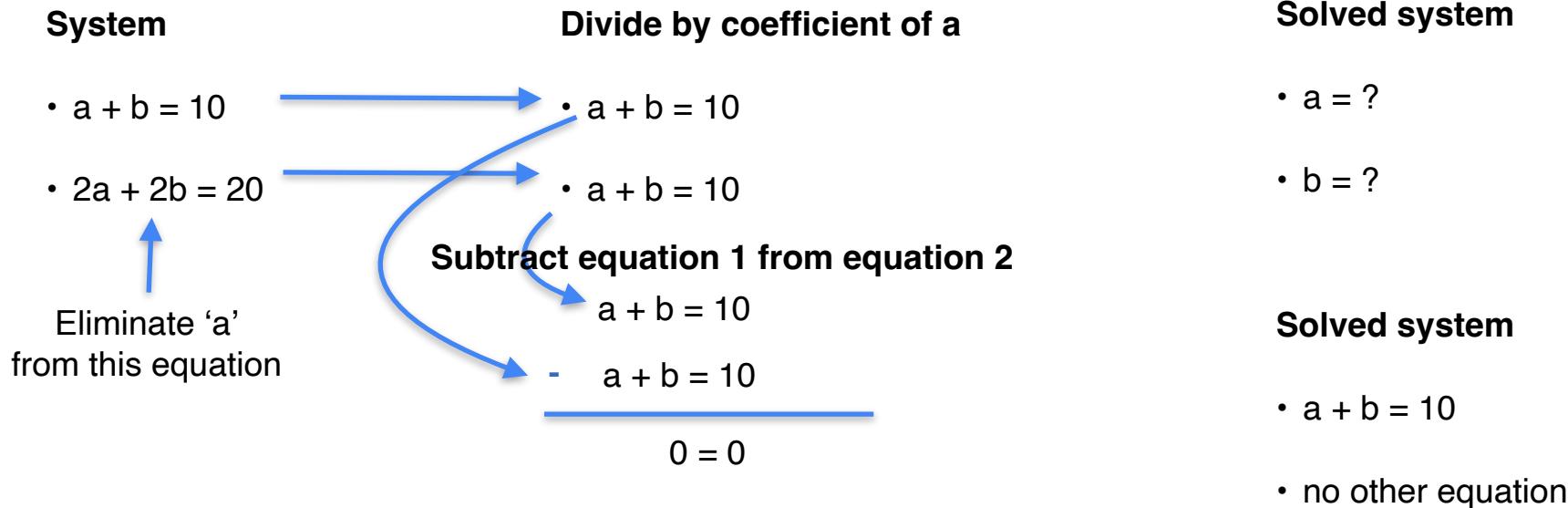
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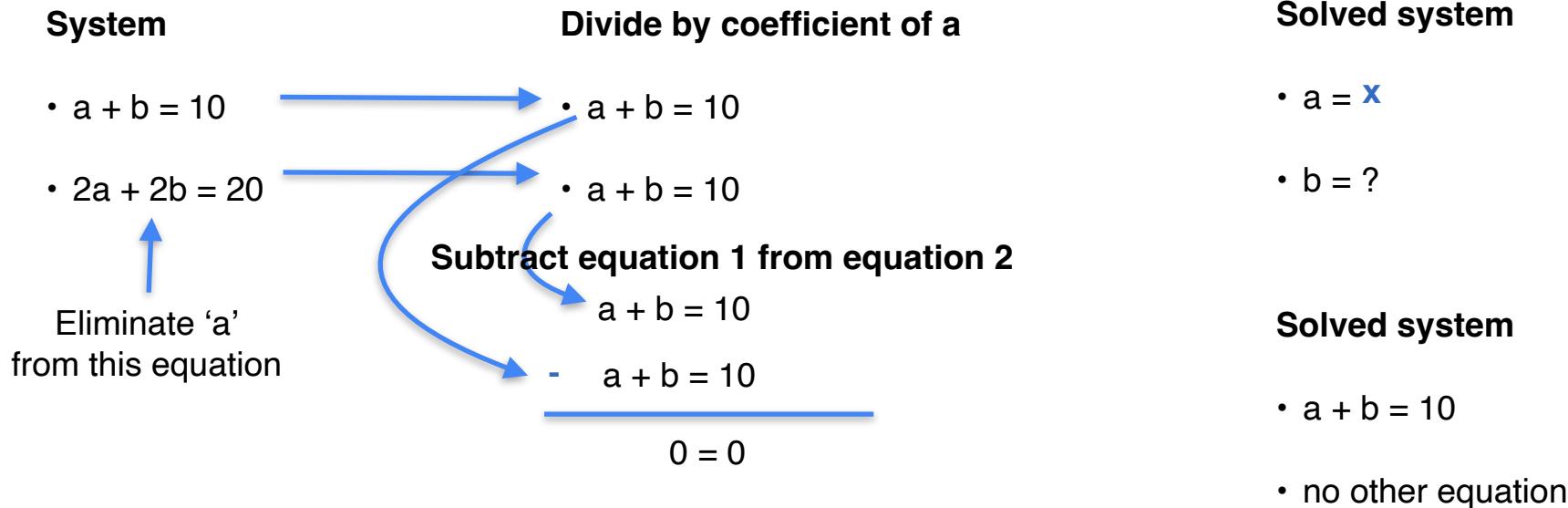
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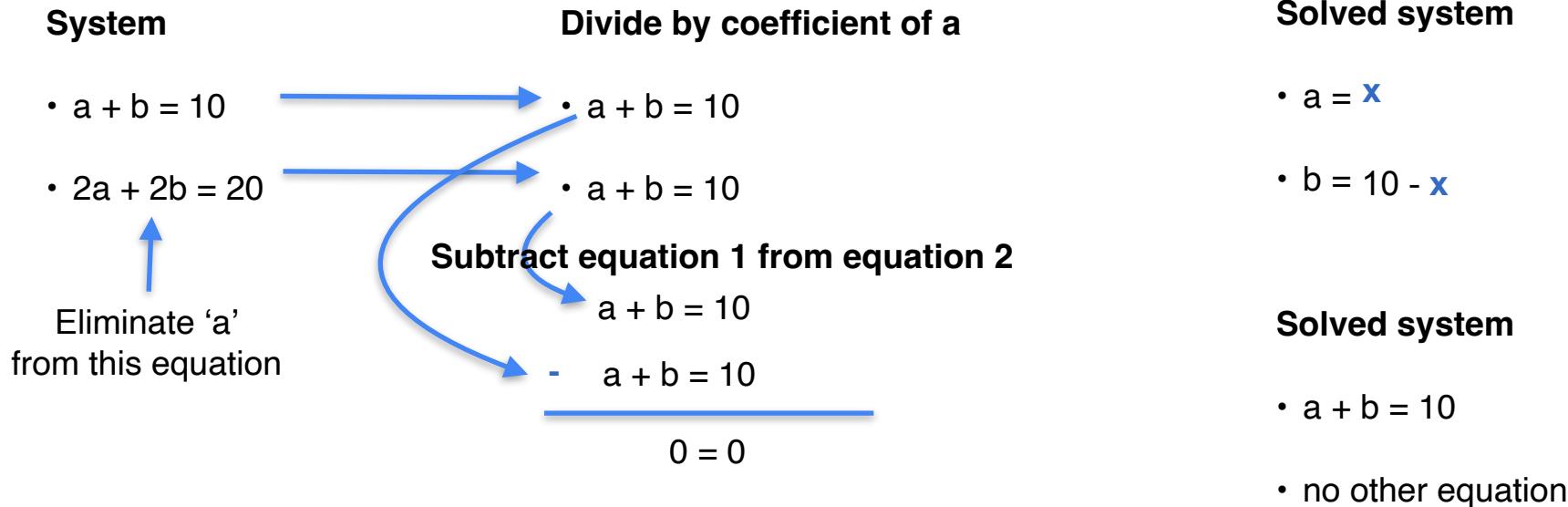
What if the system is singular (redundant)?



What if the system is singular (redundant)?



What if the system is singular (redundant)?



What if the system is singular (redundant)?

System

$$\bullet a + b = 10$$

$$\bullet 2a + 2b = 20$$

Eliminate 'a'
from this equation

Divide by coefficient of a

$$\bullet a + b = 10$$

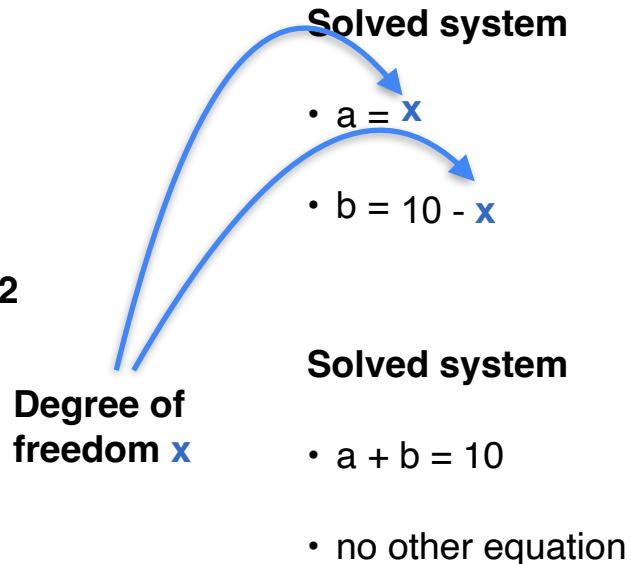
$$\bullet a + b = 10$$

Subtract equation 1 from equation 2

$$a + b = 10$$

$$- a + b = 10$$

$$\hline 0 = 0$$



What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$



Eliminate 'a'
from this equation

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 24$$

Divide by coefficient of a



$$\cdot a + b = 10$$



$$\cdot a + b = 12$$

Solved system

$$\cdot a = ?$$

$$\cdot b = ?$$



Eliminate 'a'
from this equation

What if the system is singular (contradictory)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 24$$

Divide by coefficient of a

$$\cdot a + b = 10$$

$$\cdot a + b = 12$$

Solved system

$$\cdot a = ?$$

$$\cdot b = ?$$

Subtract equation 1 from equation 2

Eliminate 'a'
from this equation

What if the system is singular (contradictory)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 24$$

Divide by coefficient of a

$$\cdot a + b = 10$$

$$\cdot a + b = 12$$

Solved system

$$\cdot a = ?$$

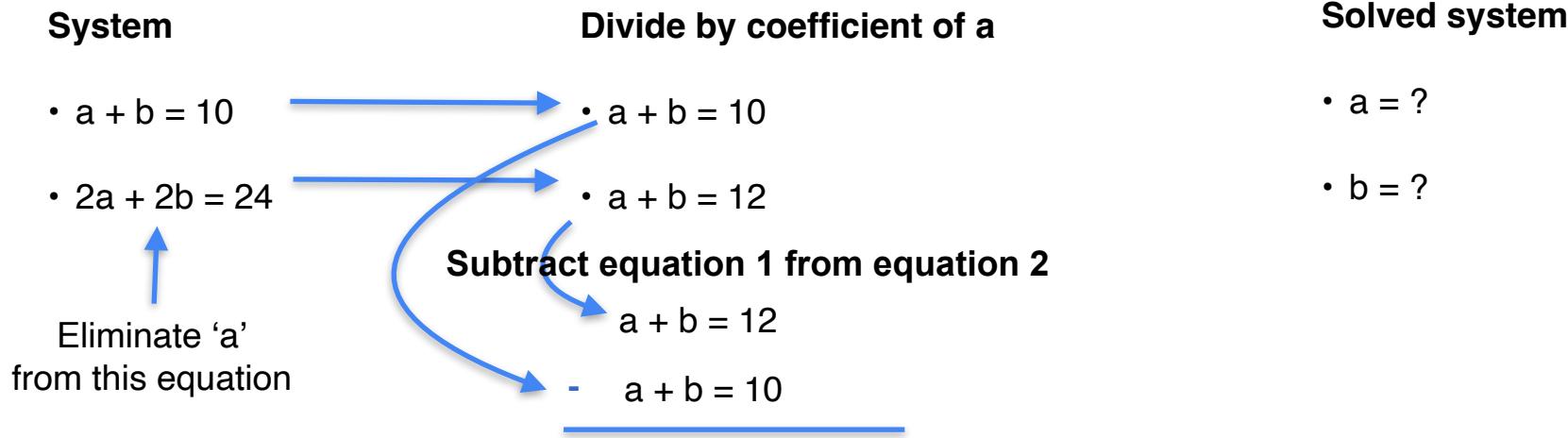
$$\cdot b = ?$$

Subtract equation 1 from equation 2

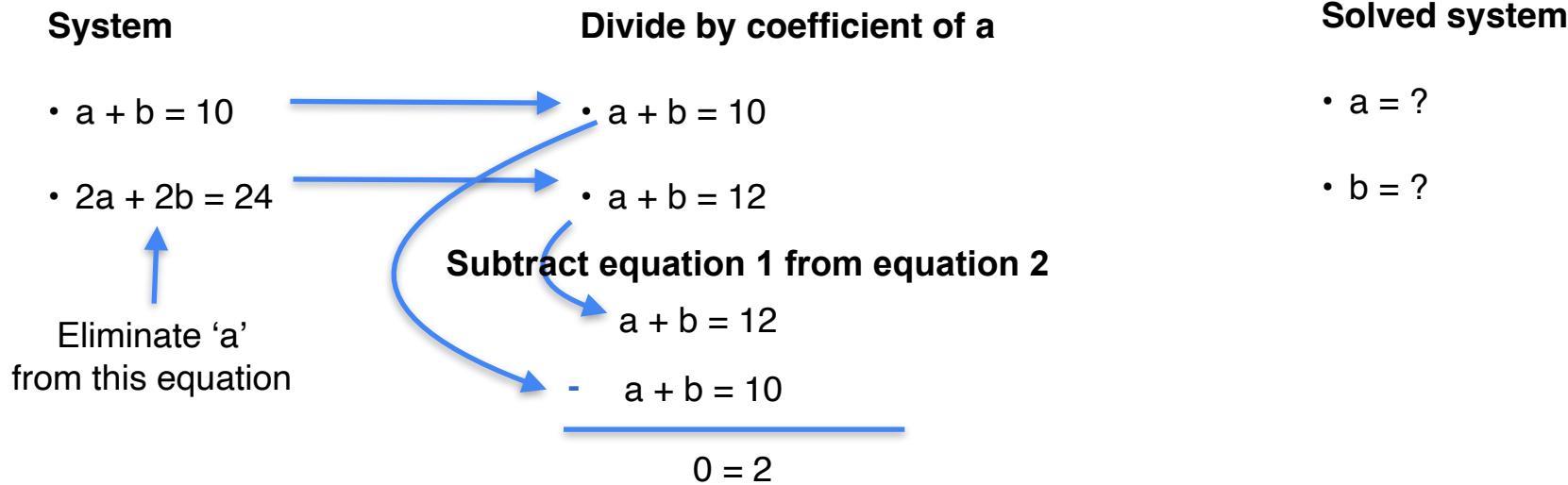
$$a + b = 12$$

Eliminate 'a'
from this equation

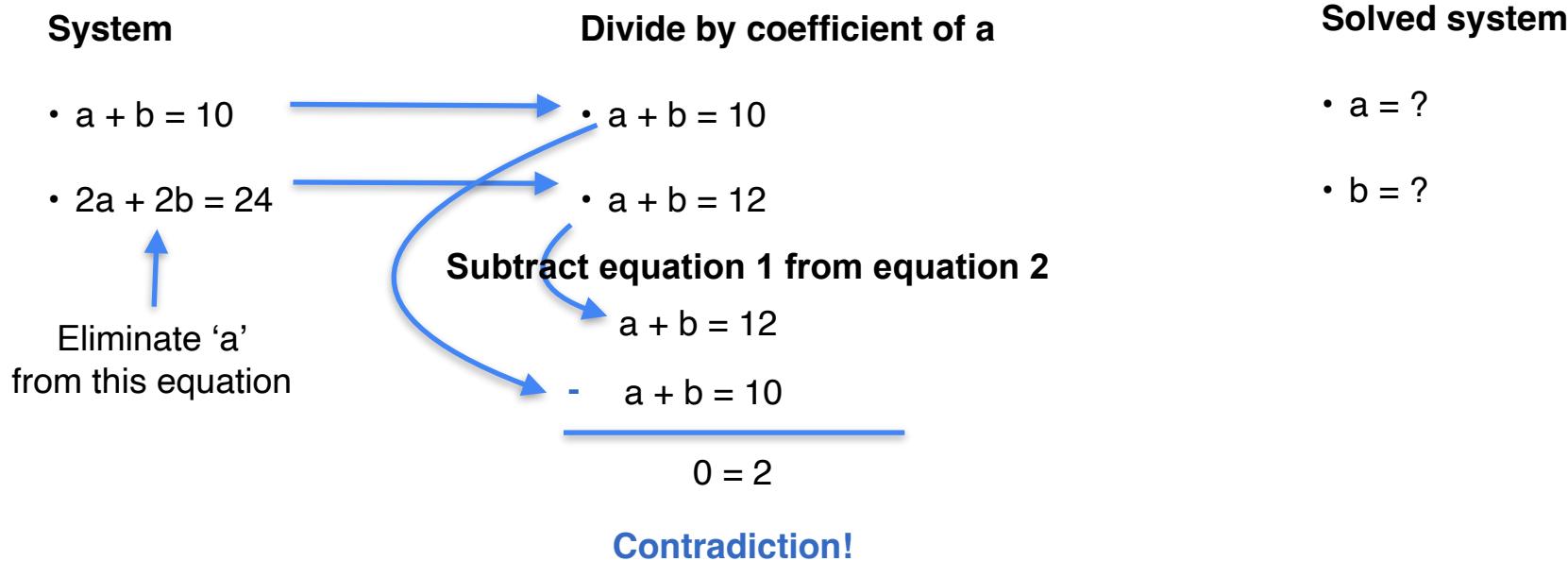
What if the system is singular (contradictory)?



What if the system is singular (contradictory)?



What if the system is singular (contradictory)?



Quiz

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution: If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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Solving System of Linear Equations

**Solving system of equations
with more variables**

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method

System

$$\bullet a + b + 2c = 12$$

$$\bullet 3a - 3b - c = 3$$

$$\bullet 2a - b + 6c = 24$$

Leave 'a' by
itself

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each
row by the
coefficient of 'a'

Elimination method

System

$$\bullet a + b + 2c = 12$$

$$\bullet 3a - 3b - c = 3$$

$$\bullet 2a - b + 6c = 24$$

$$\bullet a + b + 2c = 12$$

$$\bullet a - b - 1/3 c = 1$$

$$\bullet a - b/2 + 3c = 12$$

Divide each row by the coefficient of 'a'

Elimination method

System

$$\bullet a + b + 2c = 12$$

$$\bullet 3a - 3b - c = 3$$

$$\bullet 2a - b + 6c = 24$$

$$\bullet a + b + 2c = 12$$

$$\bullet a - b - 1/3 c = 1$$

$$\bullet a - b/2 + 3c = 12$$

Divide each row by the coefficient of 'a'

Use the first equation to remove 'a' from the others

Elimination method

System

$$\bullet a + b + 2c = 12$$

$$\bullet 3a - 3b - c = 3$$

$$\bullet 2a - b + 6c = 24$$

Divide each row by the coefficient of 'a'

$$\bullet a + b + 2c = 12$$

$$\bullet a - b - 1/3 c = 1$$

$$\bullet a - b/2 + 3c = 12$$

Use the first equation to remove 'a' from the others

$$\bullet a + b + 2c = 12$$

$$\bullet -2b - 7/3 c = -11$$

$$\bullet -3/2 b + c = 0$$

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first equation to remove 'a' from the others

- $a - b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Isolated 'a'

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first equation to remove 'a' from the others

- $a - b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Isolated 'a'

Solve this new system of 2 equations

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two
rows by the
coefficient of b

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$
- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Divide last two
rows by the
coefficient of b

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $c = 3$

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$  $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Elimination method

System

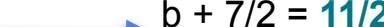
- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$ 
 $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

Elimination method

System

- $a + b + 2c = 12$  $a + 2 + 6 = 12$
 $a = 4$
- $b + 7/6 c = 11/2$  $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

Elimination method

System

- $a + b + 2c = 12$  $a + 2 + 6 = 12$
 $a = 4$
- $b + 7/6 c = 11/2$  $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

The solution is
 $a = 4$
 $b = 2$
 $c = 3$



DeepLearning.AI

Solving System of Linear Equations

Matrix row reduction

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$



Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$



Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$



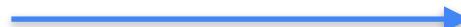
Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$



Original matrix

1	1
2	2

Systems of equations to matrices

Original system

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Intermediate System

$$\cdot a + b = 10$$

$$\cdot 0a + 0b = 0$$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Systems of equations to matrices

Original system

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Intermediate System

$$\cdot a + b = 10$$

$$\cdot 0a + 0b = 0$$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



Original matrix

5	1
10	2

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



Original matrix

0	0
0	0

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form



DeepLearning.AI

Solving System of Linear Equations

**Row operations that
preserve singularity**

Switching rows

5	1
4	3

Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

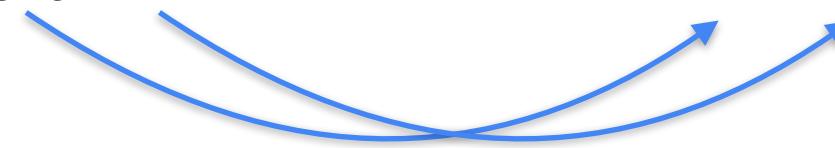
Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

Multiplying a row by a (non-zero) scalar

5	1
4	3

Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 =$$

4	3
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

4	3
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\text{Determinant} = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Determinant} &= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) \\ &= 10 \cdot 11 \end{aligned}$$

Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

5	1
4	3

5	1
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

5	1
4	3

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

5	1
4	3

$$\begin{array}{cc} 5 & 1 \\ + & \hline 4 & 3 \end{array}$$

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

$$= 11$$

Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & 3 \\ \hline \end{array} \\ + \end{array} \begin{array}{r} \begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & 3 \\ \hline \end{array} \\ \hline \end{array} \begin{array}{|c|c|} \hline 9 & 4 \\ \hline \end{array}$$

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & 3 \\ \hline \end{array} \\ + \end{array} \begin{array}{r} \begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & 3 \\ \hline \end{array} \\ \hline \end{array} \begin{array}{r} \begin{array}{|c|c|} \hline 9 & 4 \\ \hline \end{array} \end{array}$$

9	4
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{cc} 5 & 1 \\ 4 & 3 \end{array} \\ + \\ \hline \begin{array}{cc} 9 & 4 \end{array} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{cc} 5 & 1 \\ 4 & 3 \end{array} \\ + \\ \hline \begin{array}{cc} 9 & 4 \end{array} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\text{Determinant} = 9 \cdot 3 - 4 \cdot 4$$

Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{cc} 5 & 1 \\ 4 & 3 \end{array} \\ + \\ \hline \begin{array}{cc} 9 & 4 \end{array} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Determinant} &= 9 \cdot 3 - 4 \cdot 4 \\ &= 11 \end{aligned}$$



DeepLearning.AI

Solving System of Linear Equations

Rank of a matrix

Compressing Images - Reducing rank

Compressing Images - Reducing rank



Compressing Images - Reducing rank

Original (Rank 200)

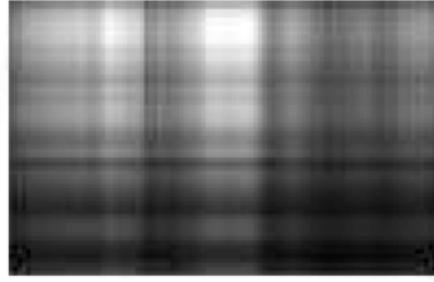


Compressing Images - Reducing rank

Original (Rank 200)



Rank 1

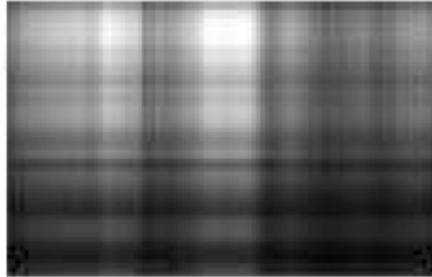


Compressing Images - Reducing rank

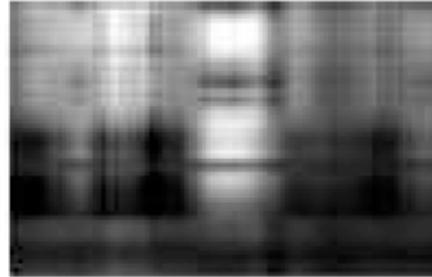
Original (Rank 200)



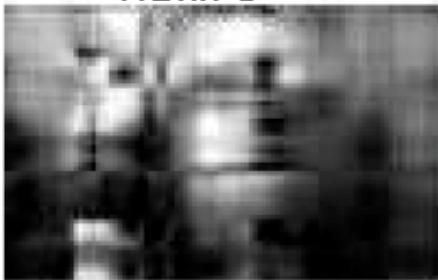
Rank 1



Rank 2



Rank 5

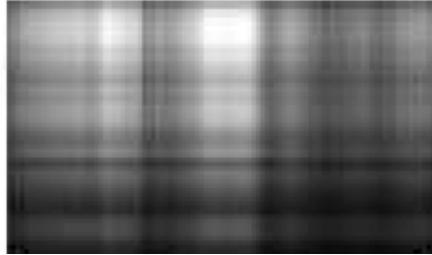


Compressing Images - Reducing rank

Original (Rank 200)



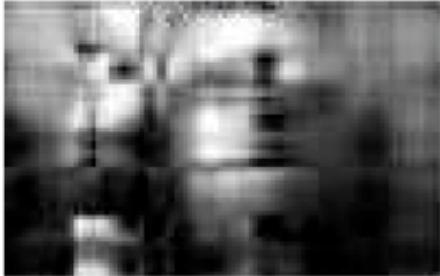
Rank 1



Rank 2



Rank 5



Rank 15

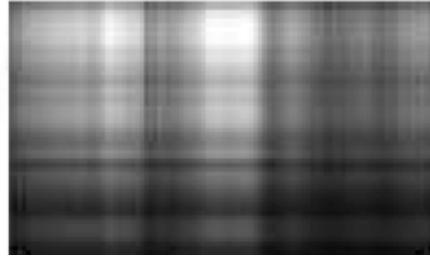


Compressing Images - Reducing rank

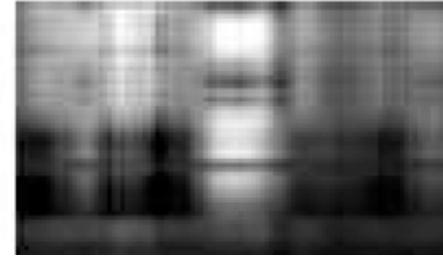
Original (Rank 200)



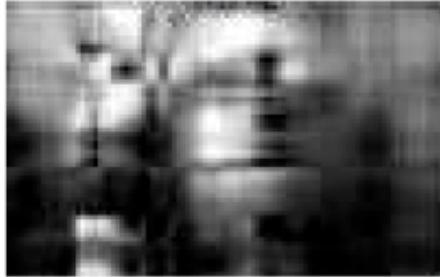
Rank 1



Rank 2



Rank 5



Rank 15



Rank 50



Systems of information

Systems of information

System 1

-  The dog is **black**
-  The cat is **orange**

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

 The dog
 The dog

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

 The dog
 The dog

Two sentences

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

 The dog
 The dog

Two sentences

Two pieces of information

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

System 2

 The dog is **black**
 The dog is **black**

Two sentences

System 3

 The dog
 The dog

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

System 3

 The dog
The dog

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

System 3

 The dog
The dog

Two sentences

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

System 3

 The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

System 3

 The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

 The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

 The dog
 The dog

Two sentences

Zero pieces of information

Rank = 0

Systems of equations

Systems of equations

System 1

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} + \text{banana} = 0 \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} + 2\text{banana} = 0 \end{array}$$

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

System 2

$$\begin{array}{r} a + b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$a + b = 0$$


$$a + 2b = 0$$


System 2

$$a + b = 0$$


$$2a + 2b = 0$$


System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Two pieces of information

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Two pieces of information

Rank = 2

Systems of equations

System 1

$$a + b = 0$$


$$a + 2b = 0$$


Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Systems of equations

System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

	
1	1
1	2

System 2

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



1	1
2	2

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



1	1
2	2

Rank = 1

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



1	1
2	2

Rank = 1

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$



0	0
0	0

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



1	1
2	2

Rank = 1

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$



0	0
0	0

Rank = 0

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

Rank and solutions to the system



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

Rank and solutions to the system

	
1	1
1	2

Rank = 2

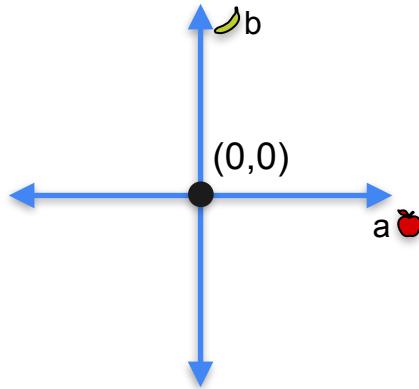
	
1	1
2	2

Rank = 1

	
0	0
0	0

Rank = 0

Dimension of solution space = 0



Rank and solutions to the system

	
1	1
1	2

Rank = 2

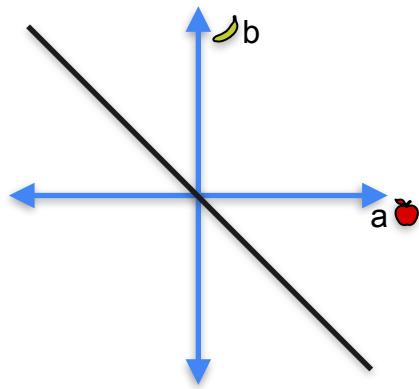
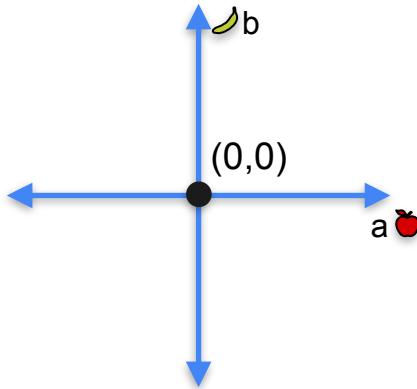
	
1	1
2	2

Rank = 1

	
0	0
0	0

Rank = 0

Dimension of solution space = 0 Dimension of solution space = 1



Rank and solutions to the system

	
1	1
1	2

Rank = 2

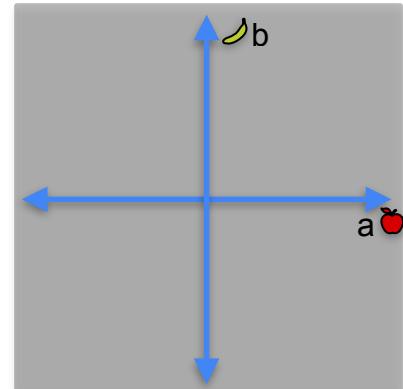
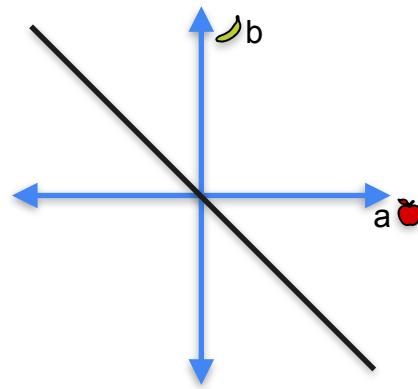
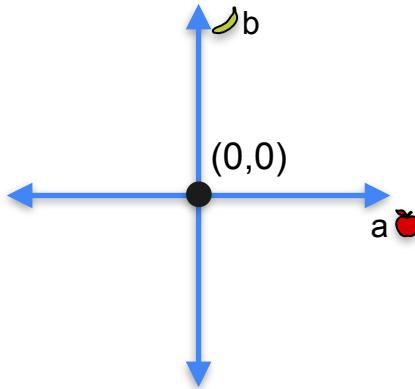
	
1	1
2	2

Rank = 1

	
0	0
0	0

Rank = 0

Dimension of solution space = 0 Dimension of solution space = 1 Dimension of solution space = 2



Rank of a matrix

	
1	1
1	2

Rank = 2

	
1	1
2	2

Rank = 1

	
0	0
0	0

Rank = 0

Dimension of solution space = 0 Dimension of solution space = 1 Dimension of solution space = 2

Rank = 2 - (Dimension of solution space)

Rank and singularity



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

Rank and singularity



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

Non-singular

Rank and singularity



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

Non-singular

Singular

Rank and singularity



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

Non-singular

Singular

Singular

Rank and singularity

	
1	1
1	2

Rank = 2

Non-singular

	
1	1
2	2

Rank = 1

Singular

	
0	0
0	0

Rank = 0

Singular

Quiz: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1: Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

Matrix 2: Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



DeepLearning.AI

Solving System of Linear Equations

**Rank of a matrix:
General case**

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$



System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \\ 0a + 0b + 0c &= 0 & \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank 0

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank 0

0	0	0
0	0	0
0	0	0

Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



DeepLearning.AI

Solving System of Linear Equations

Row echelon form

Row echelon form of a matrix

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

0	0
0	0

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

0	0
0	0

0	0
0	0

Row echelon form

Original matrix

5	1
4	-3

Row echelon form

Original matrix

5	1
4	-3

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

5	1
4	-3



Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \quad \xrightarrow{\hspace{100pt}} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{100pt}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array}$$

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline & \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline & \\ \hline \end{array} \quad - \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline & \\ \hline \end{array} \quad \underline{\quad}$$

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array}$$
$$\begin{array}{r} \\ - \\ \hline \end{array}$$

Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & -0.95 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array}$$
$$\begin{array}{r} \\ - \\ \hline \end{array}$$

Row echelon form

Original matrix

5	1		
4	-3		
		1	0.2
		1	-0.75

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \\ 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

5	1	1	0.2	1	0.2	1	0.2
4	-3	1	-0.75	0	-0.95		

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

5	1	1	0.2	1	0.2	1	0.2
4	-3	1	-0.75	0	-0.95	0	1

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
0	1

Divide each row by the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Row echelon form

Divide the second row by the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2

Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \quad \xrightarrow{\hspace{100pt}} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{10em}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline \end{array}$$

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline - & 1 & 0.2 \\ \hline \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array}$$
$$\begin{array}{r} \\ - \\ \hline \end{array}$$

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline - & 1 & 0.2 \\ \hline \hline 0 & 0 \\ \hline \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & \begin{array}{r} 1 & 0.2 \\ \hline 0 & 0 \end{array} \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1	1	0.2	1	0.2	1	0.2
10	2	1	0.2	0	0	1	0.2

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline 0 & 0 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1	1	0.2	1	0.2	?	?
10	2	1	0.2	0	0	?	?

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline 0 & 0 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

1	0.2
?	?

Divide each row by
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline 0 & 0 \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

0	0
0	0

Row echelon form for singular matrices

Original matrix

0	0
0	0

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

0	0
0	0



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \quad \xrightarrow{\hspace{100pt}} \quad \begin{array}{|c|c|} \hline ? & ? \\ \hline \end{array}$$

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



0	0	?	?
0	0	?	?

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Row echelon form

Original matrix



0	0	?	?
0	0	?	?

Divide each row by
the leftmost coefficient

Row echelon form, singularity, and rank

5	1
4	-3

5	1
10	2

0	0
0	0

Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{10em}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

2 ones in the diagonal

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

0	0
0	0



0	0
0	0

Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

Rank 2

2 ones in the diagonal

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

1 one in the diagonal

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0



0	0
0	0

Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0



0	0
0	0

0 ones in the diagonal

Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0



0	0
0	0

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0



0	0
0	0

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0



0	0
0	0

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

Rank 2

2 ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

Rank 1

1 one in the diagonal

Singular matrix

0	0
0	0



0	0
0	0

Rank 0

0 ones in the diagonal



DeepLearning.AI

Solving System of Linear Equations

**Row echelon form:
General case**

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Row echelon form

System

$$\bullet a + b + 2c = 12$$

$$\bullet 3a - 3b - c = 3$$

$$\bullet 2a - b + 6c = 24$$



System

$$\bullet a + b + 2c = 12$$

$$\bullet -6b - 7c = -33$$

$$\bullet 6c = 18$$

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$



Matrix

1	1	2
3	-3	-1
2	-1	6

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Matrix

1	1	2
3	-3	-1
2	-1	6



Row echelon form matrix

1	1	2
0	-6	7
0	0	6

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

Rank 5

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

Another example

Matrix

1	1	1
1	2	1
1	1	2

Another example

Matrix

1	1	1
1	2	1
1	1	2

Subtract the first row
from the second and
the third ones

Another example

Matrix			Row echelon form		
1	1	1			
1	2	1	→	0	1
1	1	2		0	0

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix



1	1	1
1	1	2
1	1	3

1	1	1
0	0	1
0	0	2

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3



1	1	1
0	0	1
0	0	2

Subtract the first row
from the second and
the third ones

Subtract twice the
second row from the
third one

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3



Row echelon form

1	1	1
0	0	1
0	0	2



1	1	1
0	0	1
0	0	0

Subtract the first row
from the second and
the third ones

Subtract twice the
second row from the
third one

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

Subtract twice the
first row from the
second row

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3



1	1	1
0	0	0
3	3	3

Subtract twice the
first row from the
second row

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3



1	1	1
0	0	0
3	3	3

Subtract twice the first row from the second row

Subtract three times the first row from the third row

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3



Row echelon form

1	1	1
0	0	0
3	3	3



1	1	1
0	0	0
0	0	0

Subtract twice the first row from the second row

Subtract three times the first row from the third row

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Number of pivots = 3

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0



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Solving System of Linear Equations

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$



Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$



Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Reduced row echelon form

Row echelon form

1	0.2
0	1

Reduced row echelon form

Row echelon form

1	0.2
0	1

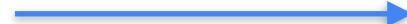


0	1
---	---

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

0.2

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

$$\begin{matrix} x & 0.2 \end{matrix}$$

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

x

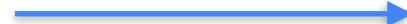
0.2

0	0.2
---	-----

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

1	0.2
---	-----

x

0.2

0	0.2
---	-----

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

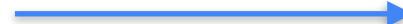
$$\begin{array}{r} 0 \quad 1 \\ \hline x \quad 0.2 \\ \hline 0 \quad 0.2 \end{array}$$

$$\begin{array}{r} 1 \quad 0.2 \\ - \quad 0 \quad 0.2 \\ \hline 0 \quad 0.2 \end{array}$$

Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

$$\begin{array}{r} 0 \quad 1 \\ \hline x \quad 0.2 \\ \hline 0 \quad 0.2 \end{array}$$

$$\begin{array}{r} 1 \quad 0.2 \\ - \quad 0 \quad 0.2 \\ \hline 1 \quad 0 \end{array}$$

Reduced row echelon form

Row echelon form

1	0.2
0	1



1	0
0	1

0	1
---	---

x

0.2

$$\begin{array}{r} \\ \hline 0 & 0.2 \end{array}$$

$$\begin{array}{r} \\ - \\ \hline 1 & 0.2 \\ 0 & 0.2 \end{array}$$

1	0
---	---

Reduced row echelon form

Row echelon form

1	0.2
0	1

Reduced row echelon form

1	0
0	1

0	1
---	---

$$x \quad 0.2$$

0	0.2
---	-----

1	0.2
0	0.2

1	0
---	---

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

Reduced row echelon form

Reduced row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by
the value of the pivot

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the value of the pivot

Reduced row echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a pivot to 0

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the
second row from the
first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1

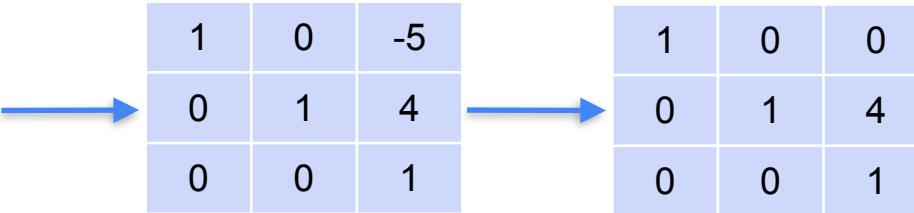
Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Subtract 4 times the third row from the second one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1



1	0	0
0	1	0
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Subtract 4 times the third row from the second one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1



Reduced row echelon form

1	0	0
0	1	0
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Subtract 4 times the third row from the second one



DeepLearning.AI

Solving System of Linear Equations

Conclusion