(*Include this page in your submission of the Reading Portfolio*)

Petroleum Engineering 648 — Pressure Transient Testing
Final Project [East Texas Gas Well – SPE 114947]
Thursday 30 April 2020 [by 5:00 p.m. (*i.e.*, 16:59:59 US CDT)]

Assignment Coversheet

[This sheet (or the sheet provided for a given assignment) must be included with EACH work submission]

Required Academic Integrity Statement: (Texas A&M University Policy Statement)

Academic Integrity Statement

All syllabi shall contain a section that states the Aggie Honor Code and refers the student to the Honor Council Rules and Procedures on the web.

Aggie Honor Code
"An Aggie does not lie, cheat, or steal or tolerate those who do."

Upon accepting admission to Texas A&M University, a student immediately assumes a commitment to uphold the Honor Code, to accept responsibility for learning and to follow the philosophy and rules of the Honor System. Students will be required to state their commitment on examinations, research papers, and other academic work. Ignorance of the rules does not exclude any member of the Texas A&M University community from the requirements or the processes of the Honor System. For additional information please visit: www.tamu.edu/aggiehonor/

On all course work, assignments, and examinations at Texas A&M University, the following Honor Pledge shall be preprinted and signed by the student:

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Aggie Code of Honor:

An Aggie does not lie, cheat, or steal or tolerate those who do.

Required Academic Integrity Statement:

"*On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.*"

Kittipong Limchuchua (Print your name)

Kittipong Limchuchua (Your signature)

Kittipong Limchuchua

Pressure Transient Analysis (PTA)

SPE 114947 - East Texas Gas Well

Problem Description/Data/Reference: Final Project (PTA) - SPE 114947 (Ilk)

Problem Description:

Elements:

- This is the high-frequency pressure buildup test (PBU) with 13,898 data point for a gas well with a finite conductivity fracture. The bilinear flow regime dominates PBU.
- The gas well requires pseudo-pressure and pseudo-time calculations, which I decided to compute it from scratch.
- Need to account for phase redistribution in early time
- The equivalent producing time is used instead of super-position time.

Challenges:

- Need to compute pseudo-pressure and pseudo-time.
- Need to use 'De-superposition technique to incorporate finite conductivity element from Lee-Brockenbrough's trilinear flow model into Ozkan's fractured well solution.

Results from publication/reference:

- Shut-in pressure (P_{ws}) is provided as a time series
- The preliminary analysis is provided by Dr. Blasingame.
- The use of well-test derivative helps identify flow regime. (Exhibit a quarter slope (1/4) which indicates bilinear flow)

Data Description:

- There are 13,898 data points $(\Delta t, P_{ws},)$ with excellent quality.
- The maximum shut-in pseudo time (Δt_a (max) =136.26 hr) is shorter than the (pseudo) producing time (2339 hr), so the superposition could be neglected. For this work, I decided to use Agarwal's effective pseudo time $(\Delta t_{sa},)$ to minimize producing time effect.

Reference:

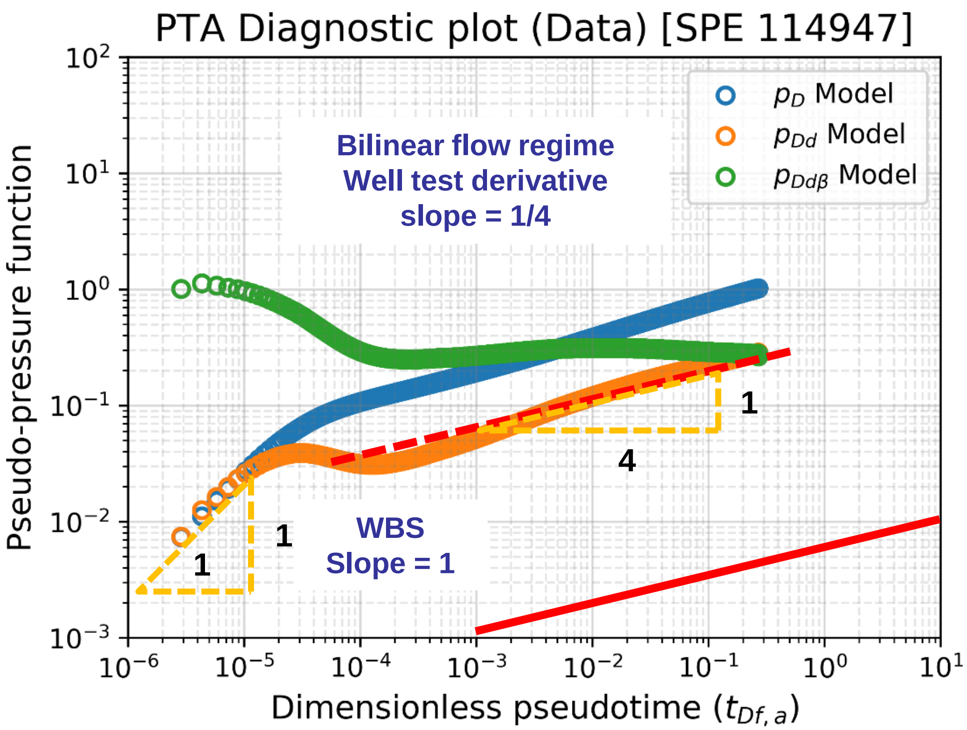
Ilk, D., Perego, A. D., Rushing, J. A., and Blasingame, T. A. (2008) Integrating Multiple Production Analysis Techniques to Assess Tight Gas Sand Reserves: Defining a New Paradigm for Industry Best Practices. Society of Petroleum Engineers. doi:10.2118/114947-MS

Table of Properties

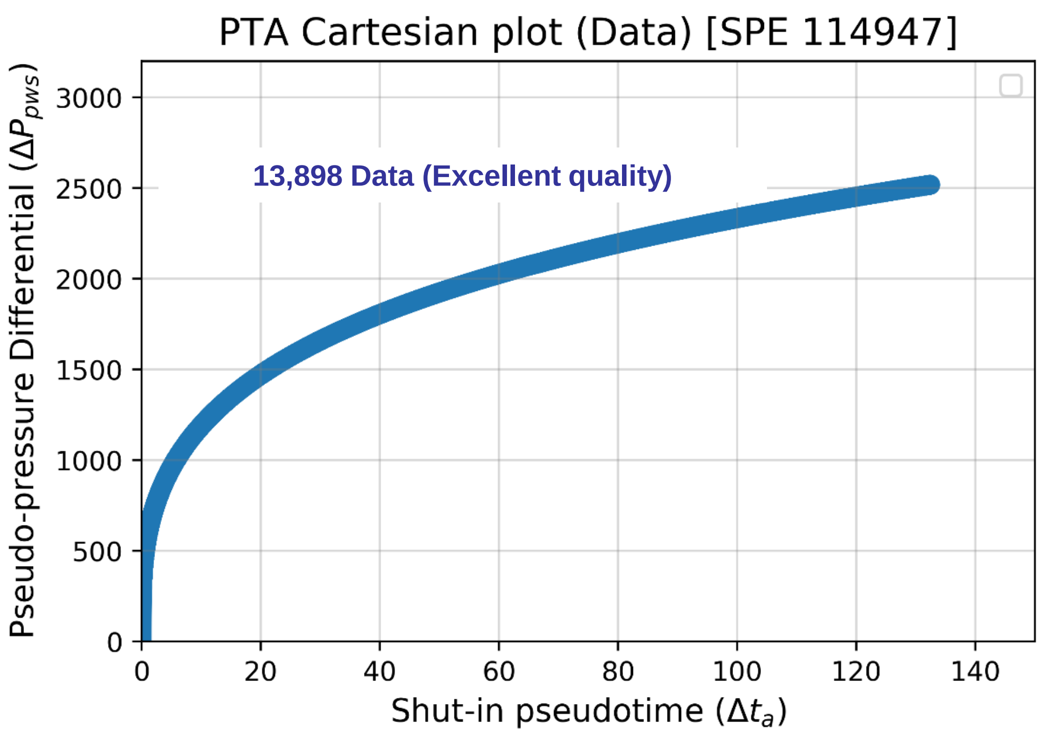
```
Fluid Properties:
 REF. GAS FVF, MSCF/STB                    = 0.5483
 REF. GAS VISCOSITY, cp                    = 0.03605
 REF. TOTAL COMPRESSIBILITY, 1/psia        = 5.0975E-5
 TEMPERATURE, deg F                        = 300

Reservoir Properties:
 RESERVOIR THICKNESS, ft                   = 170
 WELLBORE RADIUS, ft                       = 0.333
 POROSITY, fraction                        = 0.088
 EQUIVALENT POROSITY, fraction             = 0.07647 (*)

 (*) Adjusted to the irreducible water saturation
     Swi = 0.131

Production Properties:
 INITIAL PRESSURE (Pi), psia               = 9330
 FBHP AT SHUT-IN (Pwf [tp]), psia          = 1521
 PSEUDO INITIAL PRESSURE (Ppi),psia        = 7540.13
 PSEUDO FBHP AT SHUT-IN (Ppwf),psia        = 389.649

Properties to be Solved: (Final Estimates)

 (from Python Program)
 (Bilinear flow model)

 PERMEABILITY (k), md                      = 0.016
 FRACTURE HALF-LENGTH (xf), ft             = 119
 EQUIVALENT SKIN (S), Dim-Less             = -5.19 (*)
 WBS COEF. (C),Dim-less                    = 4.73E-3
 DIM-LESS WBS COEF. (CDf),Dim-less         = 4.5E-4 (**)
 FRACTURE COND.(FcD),Dim-less              = 4

 (Extra work on phase redistribution)

 P PHASE-REDISTRIBUTION (Cφ),psi           = 320
 T PHASE-REDISTRIBUTION (α),hr             = 0.035

 (*) Calculated from S = -ln(xf/2rw)
 (**) Normalized to fracture half-length square
```

Diagnostics / Hand analysis plots

Diagnostic Plot [log-log]:



Cartesian Plot:



Comment:

1) This is not a perfect 1/4 slope line, but it is close enough.
2) This is obviously not linear flow (slope = 1/2). Bilinear flow is more pronounced in PTA Data
3) The wellbore storage is changing in the beginning, as there is some hump in well test derivative during the early time due to phase redistribution. Fair's exponential phase redistribution is used.
4) Use a power law trick integral for the first panel when calculating gas property integration.

Pseudo-pressure/ Pseudo-time calculations:

1) Note that pseudo-pressure (P_{pws}) and effective shut-in pseudo-time (Δt_{as}) must be used to calculate these dimensionless variables. Use well shut-in pressure when calculating pseudo-time.

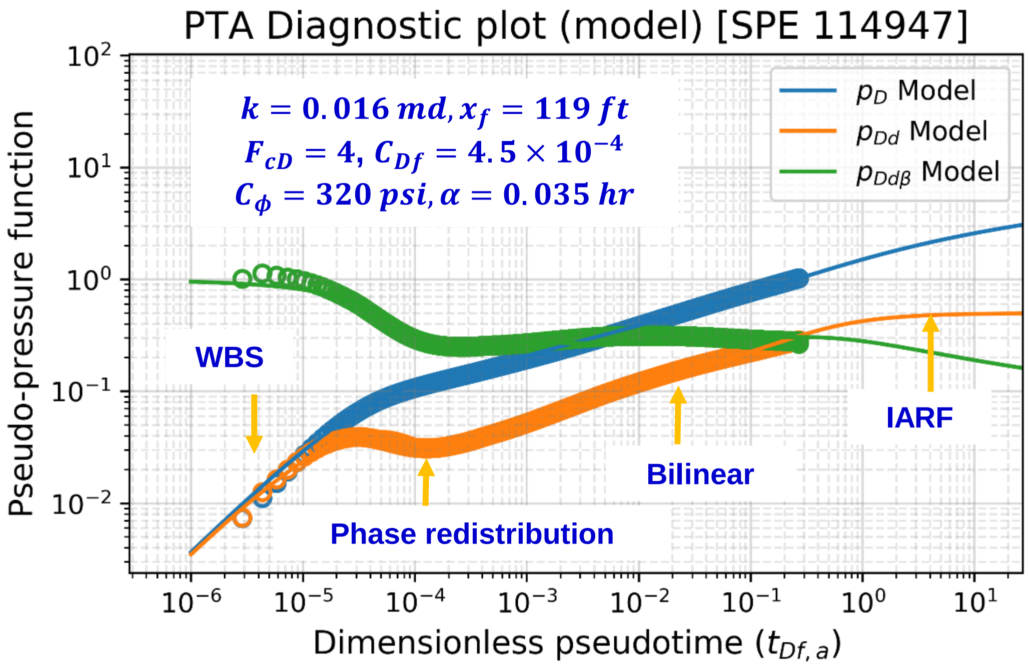
$$\Delta t_a = \mu_{gi} c_{ti} \int_0^{\Delta t} \frac{1}{\mu_g(P_{ws}) c_t(P_{ws})} d(\Delta\tau)$$

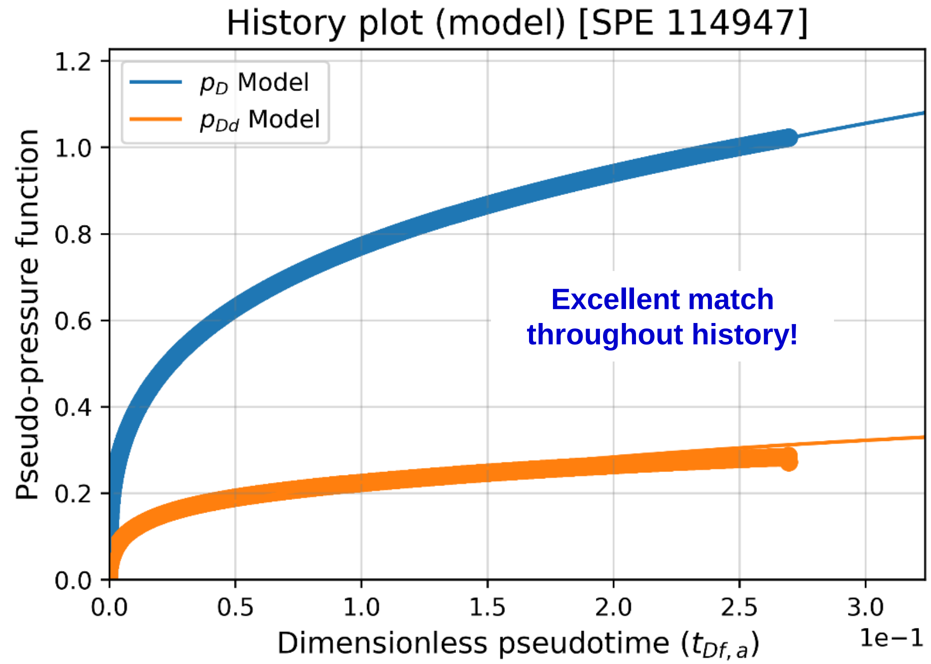
2) Producing pseudo time is approximately producing time $\Delta t_{pa} \approx t_p = 2339\ hr$
3) Effective shut-in pseudo time

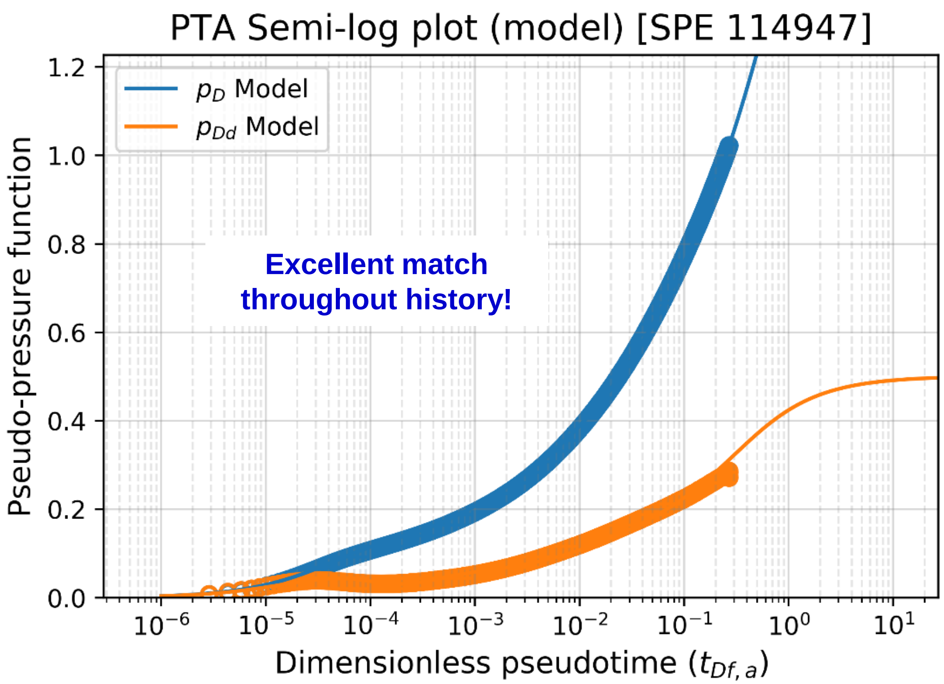
$$\Delta t_{sa} = \frac{t_p \Delta t_a}{t_p + \Delta t_a}$$

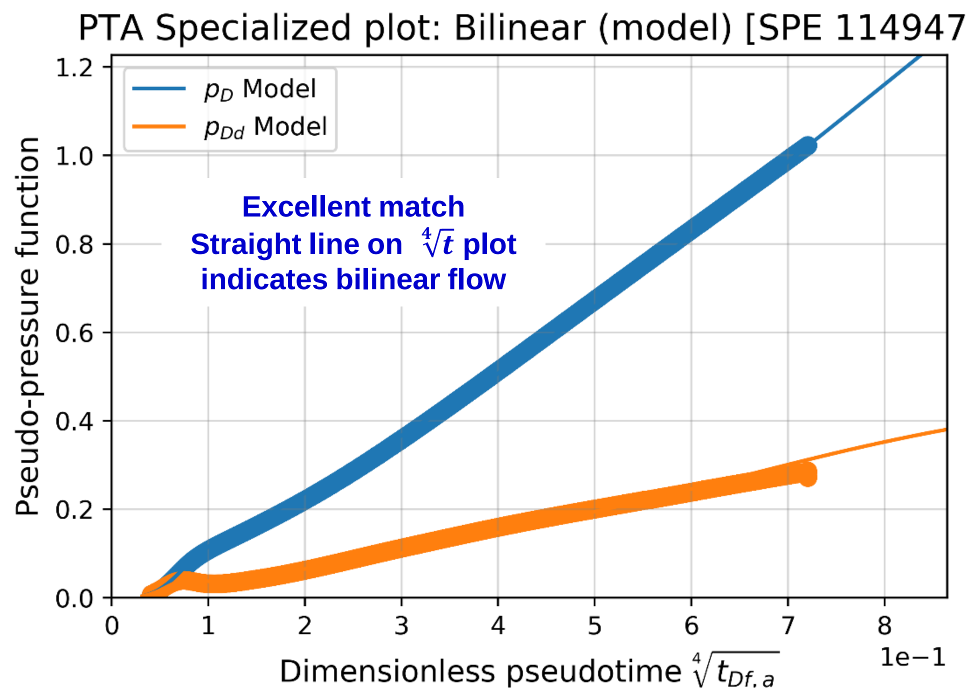
Analysis Summary Plots (Model)

"Model" matching: **(by Python)**









Method of Work / Discussion of Results

Method of Work:

Starting Point:
- The buildup test requires the adjustment of the time and pressure plotting functions. In diagnostic plot, Agarwal effective shut-in pseudo time is used.
- Type curve matching requires Stehfest's algorithm.

Governing Equations:

Blasingame-Poe Desuperposition "Trilinear Pseudo radial"	$\bar{p}_{TPR,inf}(C_{fD},s_f) = \bar{p}_{LBD,inf}(C_{fD},s_f) - \bar{p}_{LBD,inf}(C_{fD}=\infty,s_f=0) + \bar{p}_{ORD,inf}(\lvert x_D(C_{fD})\rvert \le 1, y_D=0)$
Lee-Brockenbrough Trilinear flow equation	$\bar{p}_{LBD,inf}(C_{fD},s_f) = \frac{\pi}{\mu C_{fD}} \frac{1}{\psi \tanh(\psi)}$, $\psi = \sqrt{\frac{2}{C_{fD}}\frac{\alpha}{1+\alpha s_f}+\frac{\mu}{\eta_{fD}}}, \alpha = \sqrt{\mu+\sqrt{\mu}}$
Infinite Cond. Fracture soln. (Okzan/Laplace)	$\bar{p}_{sD}(x_D=0.732,y_D=0,\mu) = \frac{1}{2\mu\sqrt{\mu}}\left[\int_0^{\sqrt{\mu}(1+x_D)} K_0(z)dz + \int_0^{\sqrt{\mu}(1-x_D)} K_0(z)dz\right]$
Classic Semi-log	$\Delta p_{pws} = \frac{70.6\mu_o q_o}{kh}\left[\ln(\Delta t_{sa}) + \left(\ln\left(\frac{1}{1688}\frac{k}{\phi\mu c_t r_w^2}\right)+2S\right)\right]$
Finite Cond. Fracture soln. (real-domain)	$\Delta p_{pws} = \frac{44.1\mu_o q_o B_o}{h\sqrt{k_f w_f}}\sqrt{\frac{\Delta t_{sa}}{\phi\mu c_t k}}$
Phase redistribution (Check Fair's paper: $P_{\phi Df} = C_{\phi Df}(1-exp(-t_{Df}/\alpha_{Df}))$)	

Challenge & Issues:
- [De-superposition] Blasingame and Poe added "Finite-conductivity" element of trilinear solution to Okzan's solution which includes the pseudo radial flow at the late time. This de-superposition technique requires multiple solutions and longer computational time.
- Use $s_f \approx 0$ and $\eta_{fD} \approx 200 \times C_{fD}$

Discussion of Results:

Diagnostics:
- Wellbore storage is changing, as the hump in the early time is seen. (= phase redistribution). The early portion of data has slope close to 1.
- The bilinear flow is clear. The well test derivative p'_D has slope of 1/4.
- The IARF data is still 1 log cycle away from the last data point, but the match is excellent.

Analyses:
- Permeability from PTA is identical to the one from RTA (k = 0.016 md).
- Fracture half-length from PTA (xf = 119 ft) agrees extremely well with the one from RTA (xf = 112 ft)
- IARF period is expected after bilinear flow.
- According to PTA, fracture is finite fracture-conductive (FcD = 4) In RTA, FcD = 27 (still finite-conductive). Bilinear flow regime is more pronounced in PBU.

Assessment:
- IARF inferred, permeability estimate is good.
- Xf is solved implicitly from tDf matching
- Use L = 0.1-0.15 to smooth the data provides even better-quality well test derivative

Recommendations/Extra work:

How can the methodology be improved?
- The β-derivative is also computed. It is helped to spot out the flow behavior (1/4 for bilinear flow)

Technical developments that would help?
- The augmented plot for β-derivative
- The Δp_{pws} vs $\sqrt[4]{\Delta t_{sa}}$ plot or "Bi-linear flow" specialized plot could also be used to illustrate "Finite-conductivity" quantitatively by having the priori knowledge of permeability.

Rate Transient Analysis (RTA)

SPE 114947 - East Texas Gas Well

Problem Description/Data/Reference: Final Project (RTA) - SPE 114947 (Ilk)

Problem Description:

Elements:
- This is a production analysis with variable rate + changing flowing bottom hole pressure (FBHP)
- This is a gas well with finite conductivity fracture (bilinear flow).
- Require the use of material balance pseudo time $(\bar{t_a})$
- Require the use of pseudo-pressure integration

Challenges:
- Need to guess initial gas-in-place (IGIP) and compute average reservoir pressure, pseudo time (t_a), and material balance pseudo time $(\bar{t_a})$, using SPE 17708.
- Need to plot flowing material balance equation, correct for the transient flow production and iterate for initial gas-in-place (IGIP), using the procedure in SPE 17708.
- Need to use 'De-superposition technique to incorporate finite conductivity element from Lee-Brockenbrough trilinear flow model into Ozkan's fracture well solution. Then use the convolution theorem to convert it to the rate solution to obtain Pratikno's type curve.

Results from publication/reference:
- The time-rate-pressure for the entire history is given
- The preliminary analysis is provided in SPE 84287.
- The permeability and fracture half-length are obtained previously from pressure build up test (PBU)

Data Description:
- There are 5,039 data points (t, q, P_{wf}) with noise.
- Need to manually clean the data and take out some outliers from the shut-in events. The edited data set has only 4,416 data point.

Reference:

Pratikno, H., Rushing, J. A., and Blasingame, T. A. (2003) Decline Curve Analysis Using Type Curves - Fractured Wells. Society of Petroleum Engineers. doi:10.2118/84287-MS

Table of Properties

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Fluid Properties:
 REF. GAS FVF, MSCF/STB                   = 0.5483
 REF. GAS VISCOSITY, cp                   = 0.03605
 REF. TOTAL COMPRESSIBILITY, 1/psia       = 5.0975E-5
 TEMPERATURE, deg F                       = 300

Reservoir Properties:
 RESERVOIR THICKNESS, ft                  = 170
 WELLBORE RADIUS, ft                      = 0.333
 POROSITY, fraction                       = 0.088
 EQUIVALENT POROSITY, fraction            = 0.07647 (*)

 (*) Adjusted to the irreducible water saturation
     Swi = 0.131

Production Properties:
 INITIAL PRESSURE (Pi), psia              = 9330
 PSEUDO INITIAL PRESSURE (Ppi),psia       = 7540.13
 INITIAL GAS-IN-PLACE (G), BSCF           = 3.23 (*)

 (*) INTIAL GUESS FROM SPE 17708 METHOD


Properties to be Solved: (Final Estimates)

 (from Python Program)

 (Finite-conductivity fracture in a bounded
  reservoir model)

 PERMEABILITY (k), md                     = 0.016
 FRACTURE HALF-LENGTH (xf), ft            = 112
 EQUIVALENT SKIN (S), Dim-Less            = -5.12 (*)
 FRACTURE COND.(FcD), Dim-less            = 27
 RESERVOIR RADIUS (re), ft                = 491
 INITIAL GAS-IN-PLACE (G), BSCF           = 3.20 (**)

 (*) Calculated from S = -ln(xf/2rw)
 (**) Use volumetric calculation from reservoir
 properties and matched reservoir radius (re)
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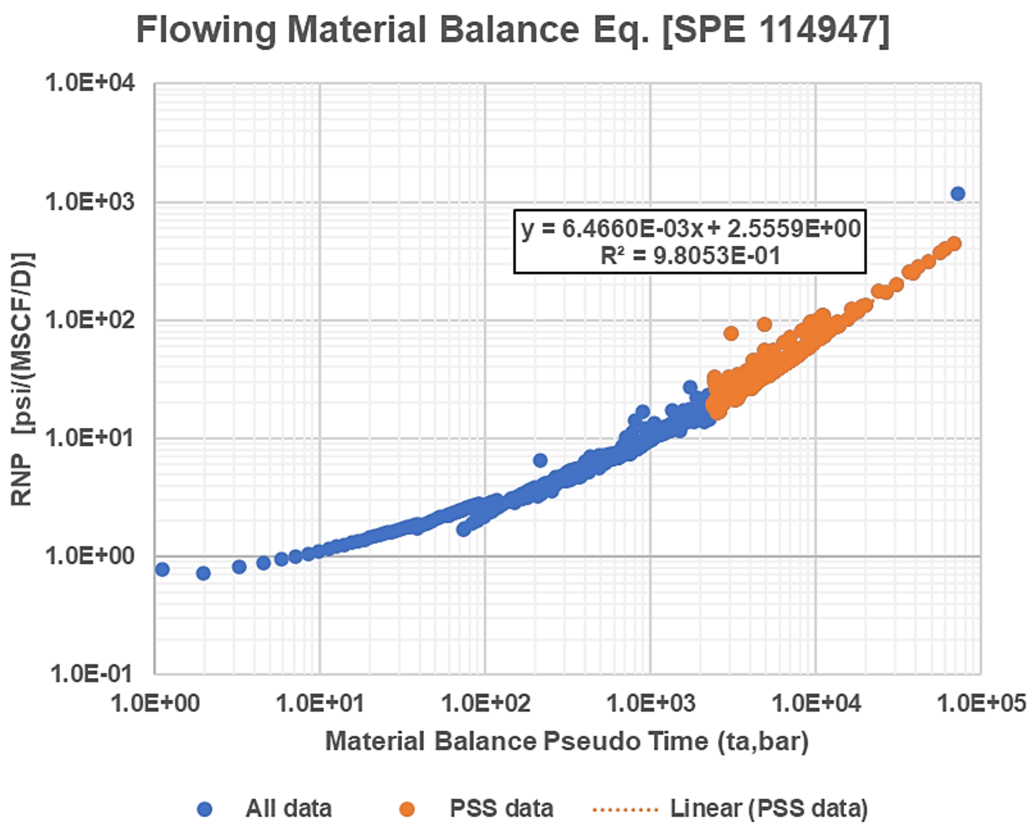
Material Balance Pseudo Time $(\bar{t_a})$ and Initial Gas-In-Place Iterations (G)

Diagnostic Plot [log-log]:



IGIP Iterations (BSCF)	
G (guess)	G (cal)
3.2	3.2085
3.21	3.2185
3.22	3.2220
3.23	3.2293
3.24	3.2387
3.3	3.2734

Calculations:

1) Estimate IGIP = 3.23 BSCG

2) Estimate productivity index = 1 / 2.5559 = 0.391 psi / (MSCF/D)

3) IGIP is sensitive to curve-fitting, depending on how the time to stabilized flow (t_{PSS}) is selected. I will cross-check this IGIP with the one obtained from RTA type curve matching

Workflow Plot (SPE 17708 - Blasingame):

1) Estimate (guess) initial gas-in-place (G)
2) Calculate average reservoir pressure, using the material balance equation. For Gp, use the trapezoid rule of qg. Note that z-factor is in a function of reservoir pressure $\overline{p_r}$

$$\frac{\overline{p_r}}{\bar{z}} = \frac{p_i}{z_i}\left(1 - \frac{G}{G_p}\right), \qquad G_p = \int_0^t q_g dt$$

3) Calculate pseudo time (t_a) based on $\overline{p_r}$

$$\mathrm{t_a} = \mu_{gi} c_{ti} \int_0^t \frac{1}{\mu_g(\overline{p_r}) c_t(\overline{p_r})} d(\tau)$$

4) Calculate material balance pseudo time $(\bar{t_a})$ as follows;

$$\bar{t_a} = \frac{1}{q_g} \int_0^{t_a} q_g(\tau_a) d(\tau_a)$$

5) Calculate pseudo-pressure (P_{pwf}) as follows;

$$P_{pwf} = \frac{\mu_{gi} z_{gi}}{P_i} \int_0^t \frac{P}{\mu_g(\overline{p_r}) z_g(\overline{p_r})} d(P)$$

6) Plot the flowing material balance equation (between rate-normalized pseudo-pressure and material balance pseudo time to obtain initial gas-in-place and the reciprocal of productivity index from slope and intercept, respectively.

$$RNP = \frac{P_{pi} - P_{pwf}}{q_g} = \frac{1}{\mathrm{J}} + \frac{\bar{t_a}}{c_{ti} G}$$

$$J = \frac{kh}{70.6 \mu_{gi} B_{gi} \left[ln\left(\frac{4A}{e^{\gamma} C_A r_w^2}\right) + 2S\right]}$$

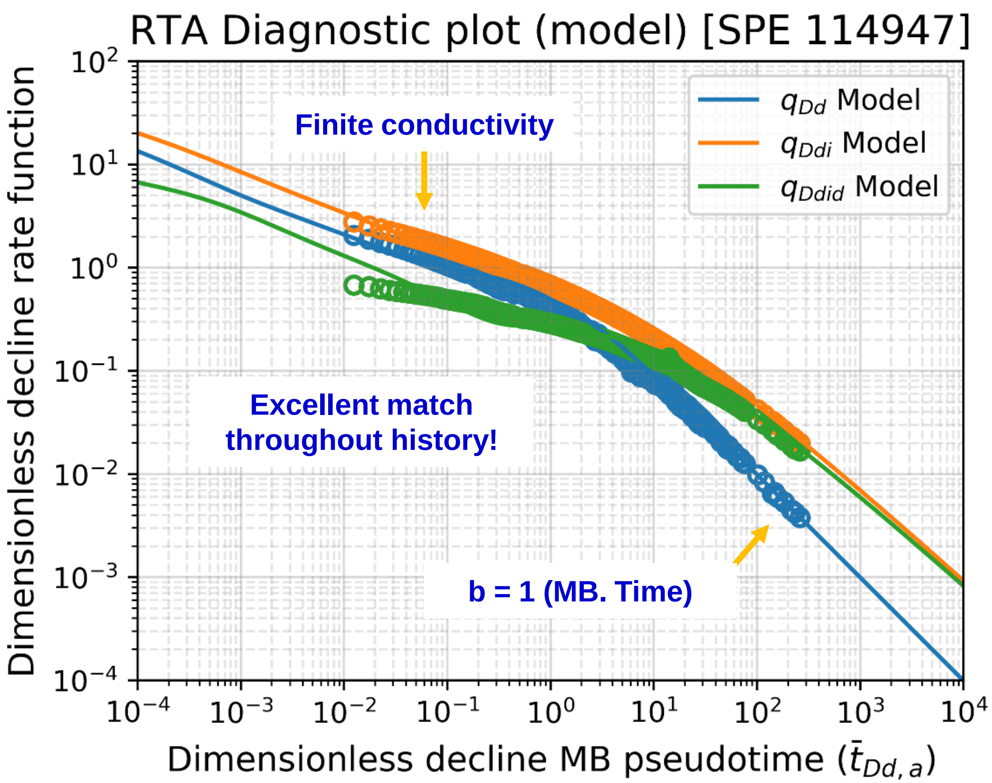
7) Need to correct extra production from a transient flow

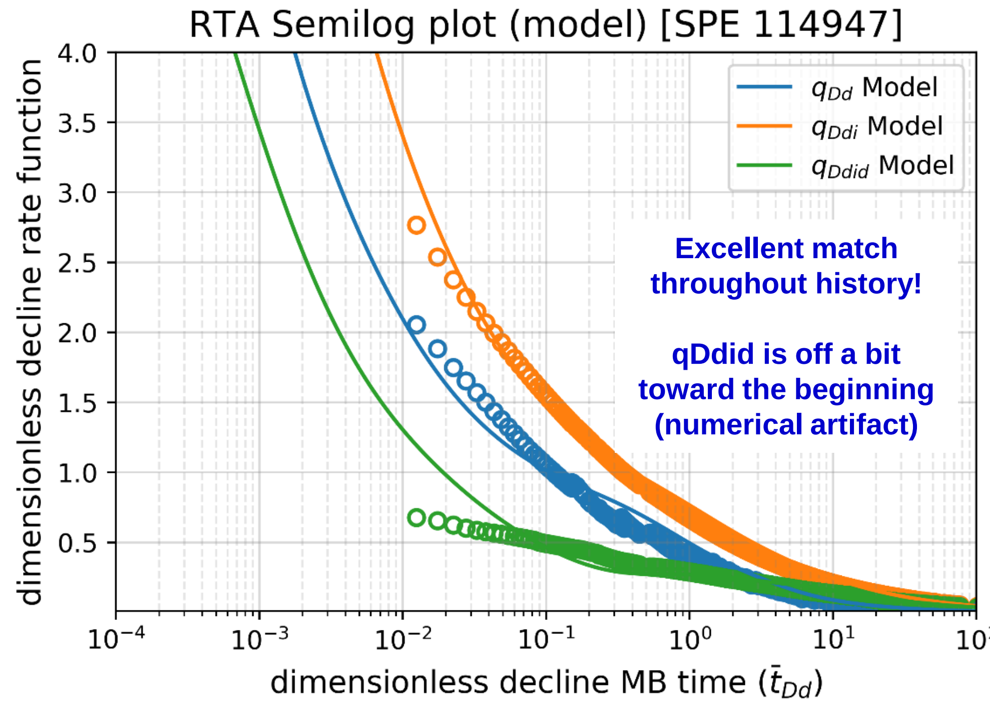
$$q_{trn} = \frac{\Delta \mathrm{p_{obs}}}{b + m\bar{t_a}}, \qquad G_p(trn) = \int_0^{t_{PSS}} q_{trn}(\tau) d(\tau)$$

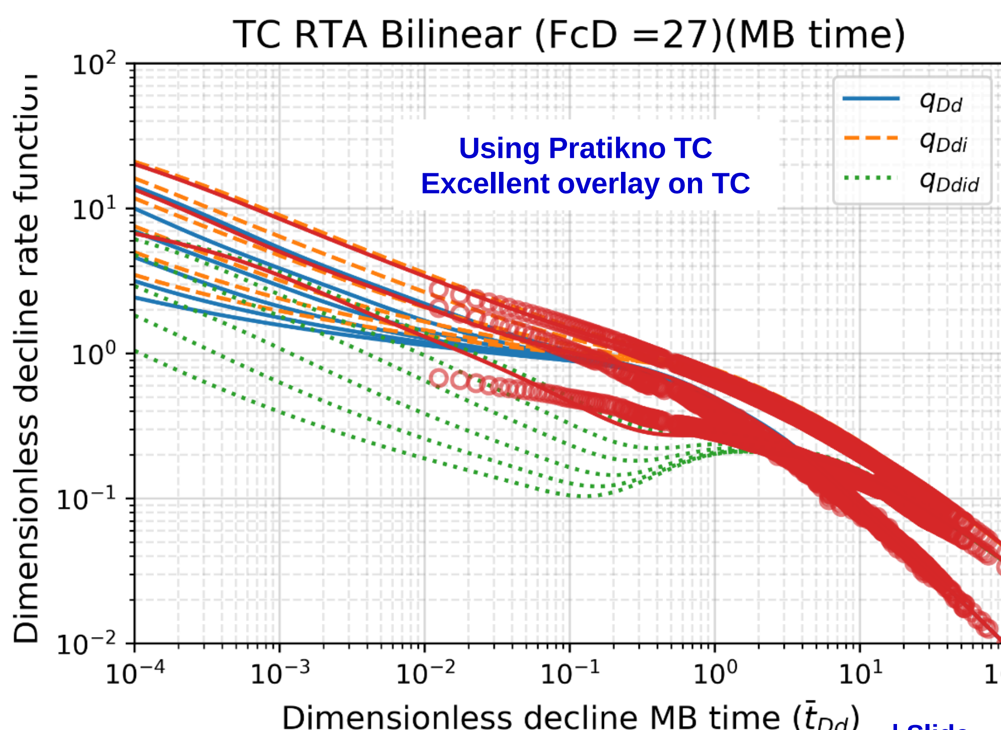
$$G = \frac{1}{mc_{ti}} + G_p(t_{PSS}) - G_p(trn)$$

Analysis Summary Plots (Model)

"Model" matching: (by Python)







Type curve matching results:

$$k = 0.016\, md, \qquad x_f = 112\, ft$$
$$F_{cD} = 27, \qquad r_e = 491\, ft$$

Calculation check for IGIP:

$$G = \frac{\pi r_e^2 h \phi (1 - S_{wi})}{B_{gi}\left[\frac{rb}{MSCF}\right] \times 5.6146} = \frac{\pi (491^2)(170)(0.088)(1 - 0.131)}{0.5483 \times 5.6146}$$
$$G = 3.198 \times 10^6\ MSCF \ \approx 3.20\ BSCF$$

This result agrees well with G obtained from pseudo time calculation, using the method in SPE 17708 (3.23 BSCF)

| LIMCHUCHUA | kittipong2541@tamu.edu |

Method of Work / Discussion of Results

Method of Work:

Starting Point:

- Type curve matching requires Stehfest's algorithm.
- Need to follow IGIP and $\overline{t_a}$ calculation shown in slide 2

Governing Equations:

Blasingame-Poe Desuperposition "Trilinear Pseudo radial"	$\bar{p}_{TPR,inf}(C_{fD}, s_f) = \bar{p}_{LBD,inf}(C_{fD}, s_f) - \bar{p}_{LBD,inf}(C_{fD} = \infty, s_f = 0) + \bar{p}_{ORD,inf}(\lvert x_D(C_{fD})\rvert \le 1, y_D = 0)$
Lee-Brockenbrough Trilinear flow equation	$\bar{p}_{LBD,inf}(C_{fD}, s_f) = \frac{\pi}{\mu C_{fD}} \frac{1}{\psi \tanh(\psi)}$, $\psi = \sqrt{\frac{2}{C_{fD}} \frac{\alpha}{1+\alpha s_f} + \frac{\mu}{\eta_{fD}}}, \alpha = \sqrt{\mu + \sqrt{\mu}}$
Infinite Cond. Fracture soln. (Okzan/Laplace)	$\bar{p}_{sD}(x_D = 0.732, y_D = 0, \mu) = \frac{1}{2\mu\sqrt{\mu}}\left[\int_0^{\sqrt{\mu}(1+x_D)} K_0(z)dz + \int_0^{\sqrt{\mu}(1-x_D)} K_0(z)dz\right] + \frac{1}{2\mu\sqrt{\mu}} \frac{K_1(\sqrt{\mu}r_{eD})}{I_1(\sqrt{\mu}r_{eD})}\left[\int_0^{\sqrt{\mu}(1+x_D)} I_0(z)dz + \int_0^{\sqrt{\mu}(1-x_D)} I_0(z)dz\right]$
$b_{D,PSS}$ Correlation	$b_{D,PSS} = f(r_{eD}, F_{cD})$ (check SPE 84287 Eq.5)
Dimensionless rate from q_g and P_{pwf}	$q_D = \frac{141.2\mu_{gi}B_{gi}}{kh}\left(\frac{q_g}{P_{pi} - P_{pwf}}\right) = \frac{141.2\mu_{gi}B_{gi}}{kh}(PNR)$
Dimensionless *decline* function	$\begin{Bmatrix} q_{Dd} \\ q_{Ddi} \\ q_{Ddid} \end{Bmatrix} = \begin{Bmatrix} q_D \\ q_{Di} \\ q_{Did} \end{Bmatrix} [b_{D,PSS}]$
Dimensionless *decline* material balance pseudo time	$\overline{t_{Dd,a}} = 2t_{Df}\left\{b_{D,PSS}\left[\left(\frac{r_e}{x_f}\right)^2 - 1\right]\right\}^{-1}$

Challenge & Issues:

- [De-superposition] Blasingame and Poe added "Finite-conductivity" element of trilinear solution to Okzan's solution which includes the pseudo radial flow at the late time. Use $s_f \approx 0$ and $\eta_{fD} \approx 200 \times C_{fD}$

Discussion of Results:

Diagnostics:

- The bilinear flow at early time is clear. The q_{Dd} function in the transient stem has slope of ¼.
- The depletion stem follows the harmonic solution because the material balance pseudo time is used.
- Excellent match on qDd and qDdi
- Some numerical artifact in qDdid toward the beginning

Analyses:

- Permeability from RTA is identical to the one from PTA (k = 0.016 md).
- Fracture half-length from RTA (xf = 112 ft) agrees extremely well with the one from PTA (xf = 119 ft)
- The well is stimulated (negative skin = -5.12)
- According to RTA, fracture is finite fracture-conductive (FcD = 27) In PTA, FcD = 4. Bilinear flow regime is more pronounced in PBU.
- Gas in-place estimates from SPE 17708 iteration and type curve match agree within 0.03 BSCF.

Assessment:

- Use L = 0.1-0.15 to smooth the data provides even better-quality rate-integral derivative (qDdid)

Recommendations/Extra work:

Technical developments that would help?

- The augmented plot for β-derivative
- Try log-log plot by 'flipping' TC upside down and work with "rate-normalized pressure (RNP)" instead of "pressure-normalized rate (PNR)"
- The p_D vs $\sqrt[4]{t_D}$ plot or "Bi-linear flow" specialized plot could yield the straight-line in case of finite-conductivity fracture.

Decline Curve Analysis (DCA)

SPE 114947 - East Texas Gas Well

Problem Description/Data/Reference: Final Project (DCA) - SPE 114947 (Ilk)

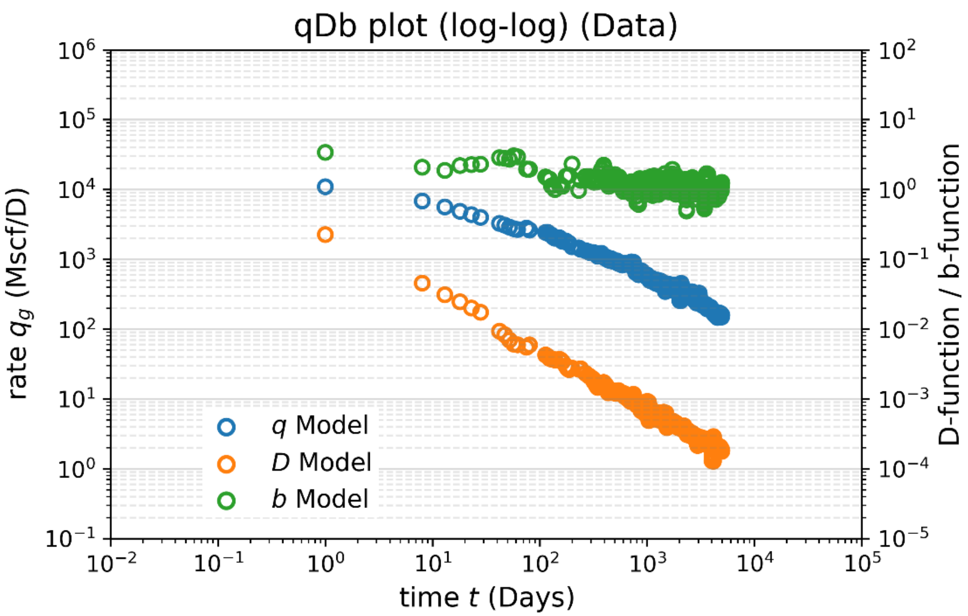
Problem Description:

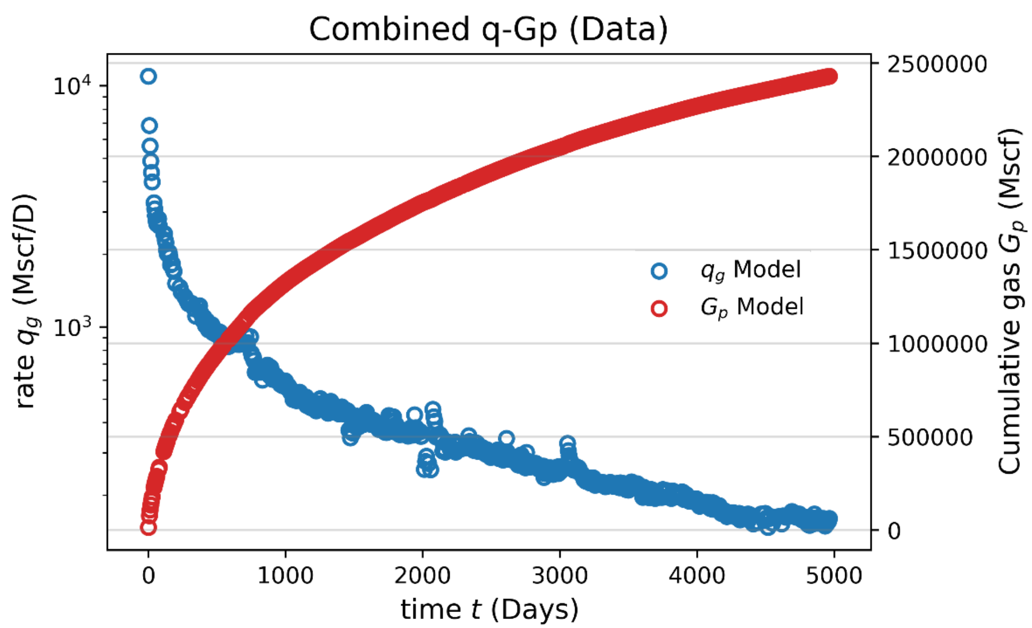
Elements:
- This is a modern decline curve analysis. All models requested in the syllabus are attempted.
- qDb [log-log] plots are computed for data and all models
- qGp [log-log] plots are computed for data and all models
- qGp combined semi-log plots are computed for data and all models to assess the quality of the match

Data Description:
- There are 5,039 data points (t, q, P_{wf}) with noise.
- Need to manually clean the data and take out some outliers from the shut-in events. The edited data set has only 698 data point.

Challenges:
- Because the matching algorithm is programmed in Python module. I need to compute D and b functions analytically for all models.
- Need to remove some noise before calculating D and b functions because they are the point-to-point derivatives from the production data
- Cumulative gas production at 30 years from all models will be compared.

Results from publication/reference:
- The time-rate-for the entire history is given
- b and D functions (with smoothing) are provided.



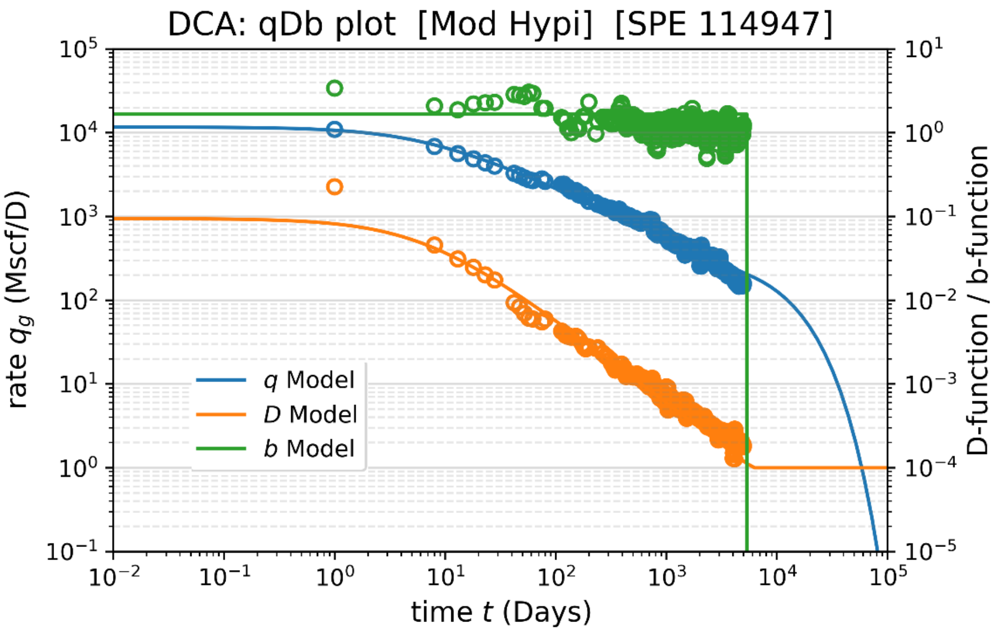


Reference:
Ilk, D., Perego, A. D., Rushing, J. A., and Blasingame, T. A. (2008) Integrating Multiple Production Analysis Techniques to Assess Tight Gas Sand Reserves: Defining a New Paradigm for Industry Best Practices. Society of Petroleum Engineers. doi:10.2118/114947-MS

DCA model: Modified Hyperbolic (SPE 119369)

Equation box:

Rate	$q(t) = \begin{cases} \dfrac{q_i}{(1+bD_it)^{1/b}} & for\ t < t_{lim} \\ q_{i,lim}\ exp\left(-D_{i,lim}(t-t_{lim})\right) & for\ t \geq t_{lim} \end{cases}$
D-function	$D(t) = \begin{cases} \dfrac{D_i}{(1+bD_it)} & for\ t < t_{lim} \\ D_{lim} & for\ t \geq t_{lim} \end{cases}$
b-function	$b(t) = \begin{cases} b & for\ t < t_{lim} \\ 0 & for\ t \geq t_{lim} \end{cases}$



Comment:

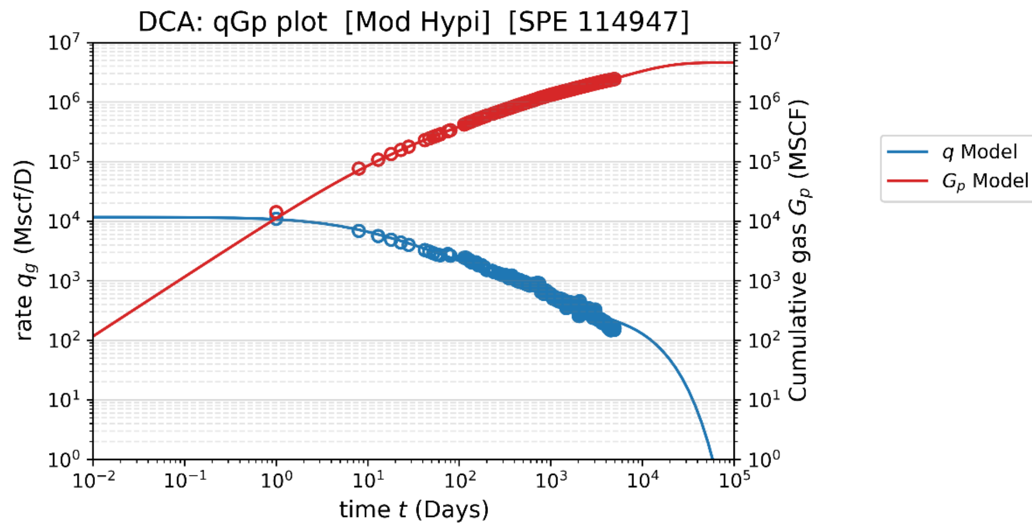
- I applied the terminal decline rate, so the hyperbolic decline will be switched to the exponential decline.
- B-function is <u>not</u> as constant as imposed by hyperbolic model. But the approximation is OK.
- Hyperbolic model misses the decline rate (D) toward the early time
- Hyperbolic decline is more optimistic compared to the other methods (3.42 BSCF is quite high)

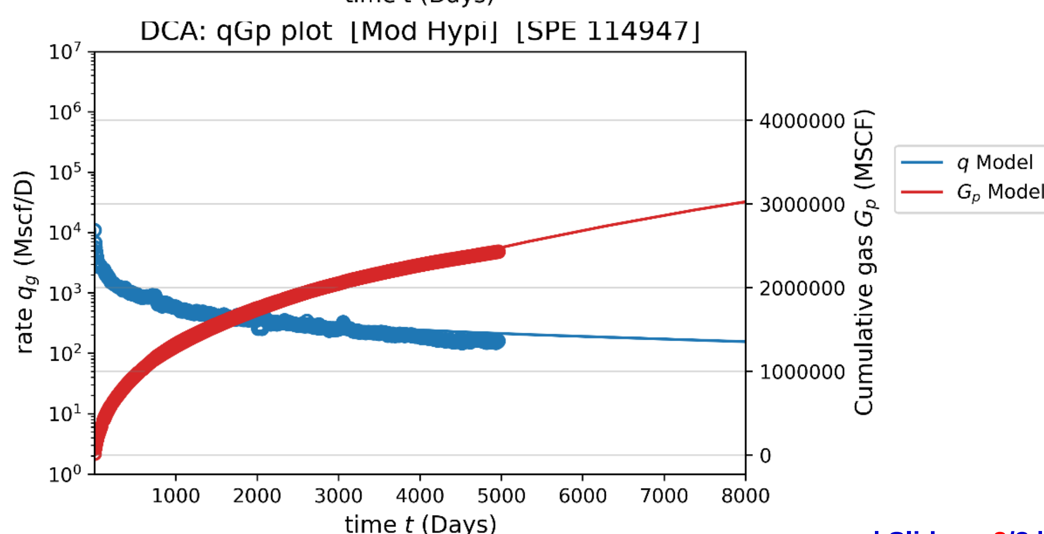
```
Parameters to be Solved: (Final Estimates)

 (from Python Program)

 INITIAL RATE (qi), MSCF/D             = 11677
 Di-CONSTANT (Di), 1/Day               = 0.09456
 b-CONSTANT (b), Dim-less              = 1.6657
 LIMIT Di (Di-limit), 1/Day            = 1.0E-4  (*)
 CUM. GAS @ 30 YRS (Gp), BSCF          = 3.416

 (*) Switch to exponential decline
```





DCA model: Power-law Exponential (SPE 116731)

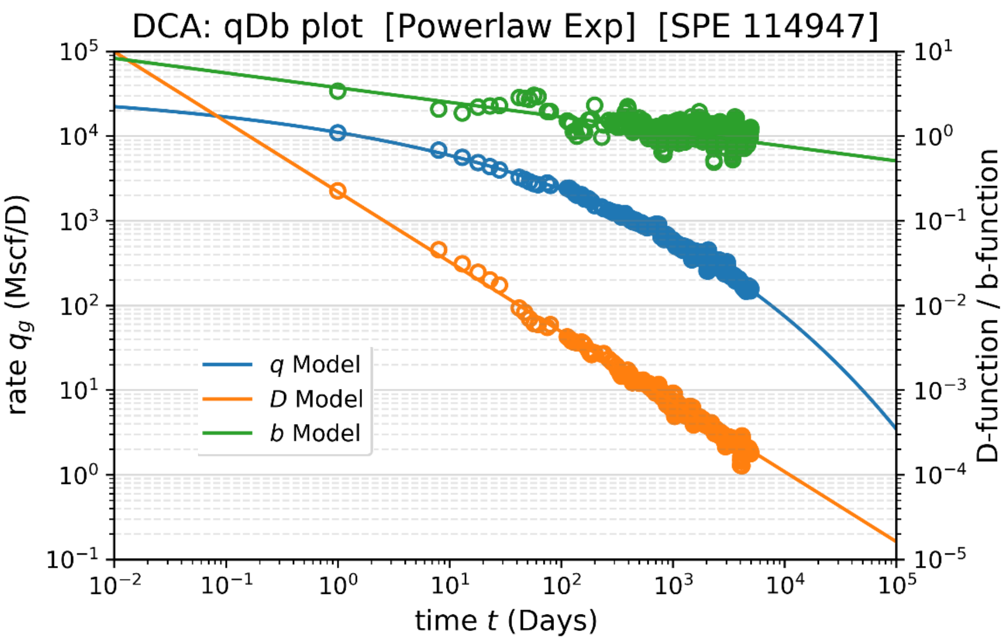
Equation box:

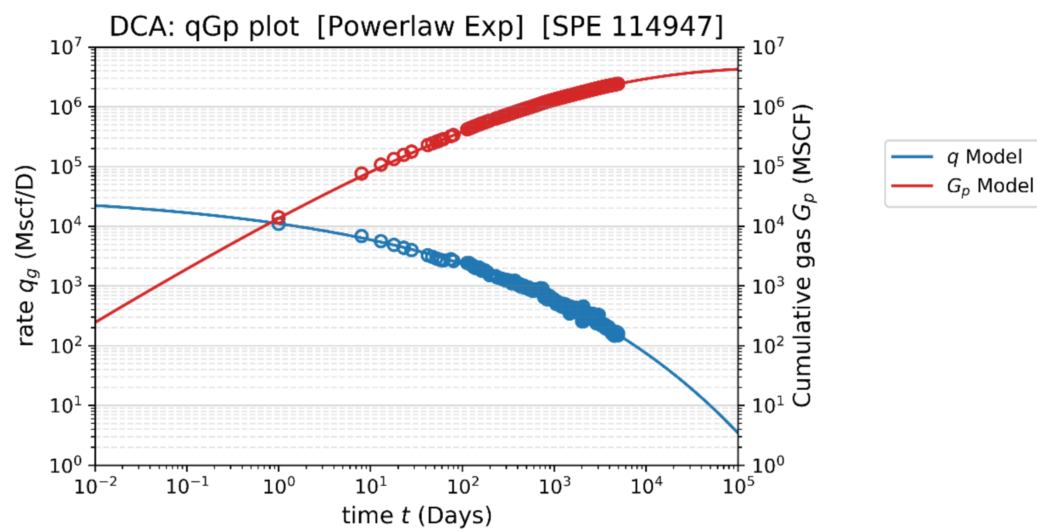
Rate	$q(t) = \hat{q}_i \, exp\left[-D_\infty t - \widehat{D}_i t^n\right]$
D-function	$D(t) = D_\infty + n\widehat{D}_i t^{n-1}$
b-function	$b(t) = \dfrac{n\widehat{D}_i(1-n)t^{-n}}{\left[n\widehat{D}_i + D_\infty t^{1-n}\right]^2}$

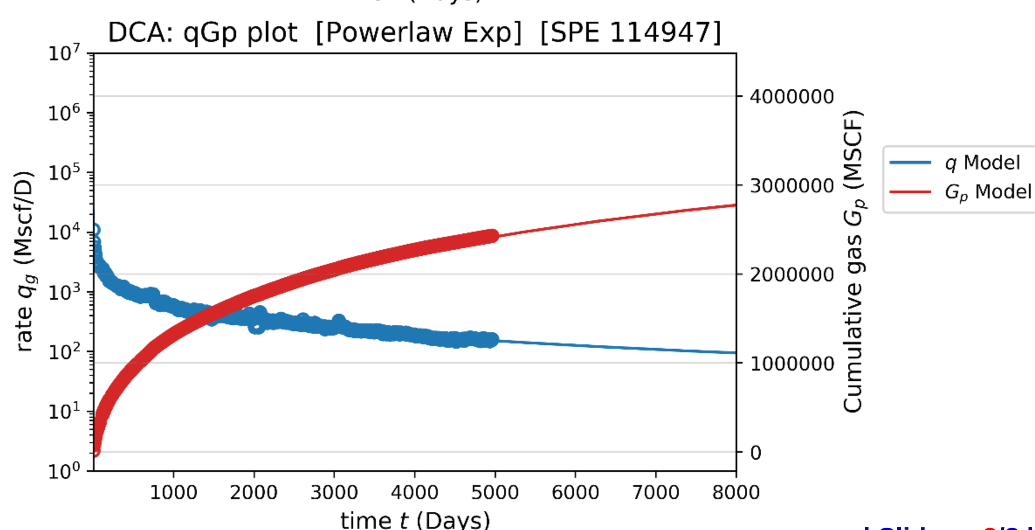
Parameters to be Solved: (*Final Estimates*)

(from Python Program)

```
INITIAL RATE (qi), MSCF/D          = 39693
Di-CONSTANT (Di), Vary             = 1.27511
n-CONSTANT (n), constant           = 0.17306
Di-infinity (Di-inf), 1/Day        = 0
CUM. GAS @ 30 YRS (Gp), BSCF       = 3.005
```







Comment:

- This model fits extremely well with q, D-function and b-function compared to the other models.
- Power-law exponential provides more conservative cumulative production (3.01 BSCF) than modified hyperbolic (3.42 BSCF) but more optimistic than logistic (2.82 BSCF) and Weibull (2.65 BSCF)

DCA model: Duong's model (SPE 137748)

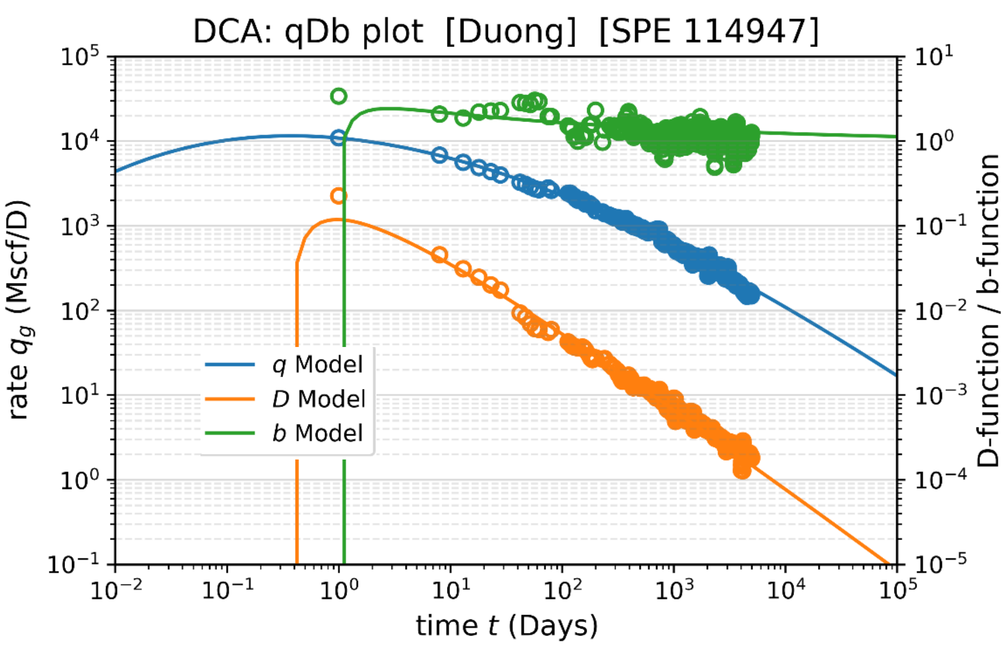
Equation box:

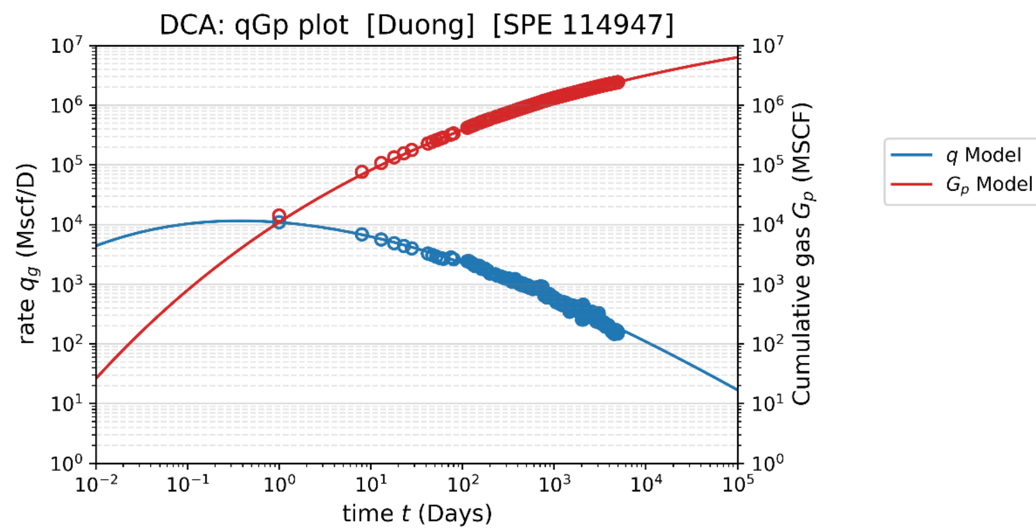
Rate	$q(t) = q_i t^{-m} exp\left[\frac{a}{1-m}\left(t^{1-m}-1\right)\right]$
D-function	$D(t) = mt^{-1} - at^{-m}$
b-function	$b(t) = \frac{mt^m(t^m - at)}{(at - mt^m)^2}$

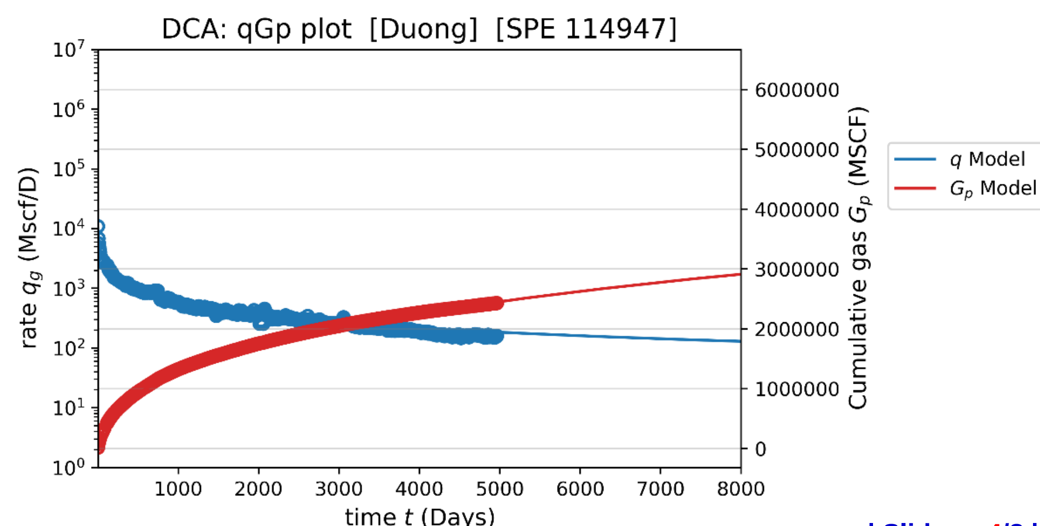
Parameters to be Solved: (*Final Estimates*)

(from Python Program)

```
INITIAL RATE (qi), MSCF/D          = 10895
a-CONSTANT (a), constant           = 0.99596
m-CONSTANT (m), constant           = 1.1147
CUM. GAS @ 30 YRS (Gp), BSCF       = 3.241
```







Comment:

- This model has some peculiar behaviors at the early time. However, it is a model artifact that will not be used for reserve prediction in the future
- Duong's model assumes power law relation between q/Gp ratio and time. It tends to give more optimistic cumulative production compared to other models. (3.24 BSCF in Duong is higher than 3.01 BSCF in Power-law exponential and 2.82 BSCF in logistic model)

DCA model: Logistic model (SPE 144790)

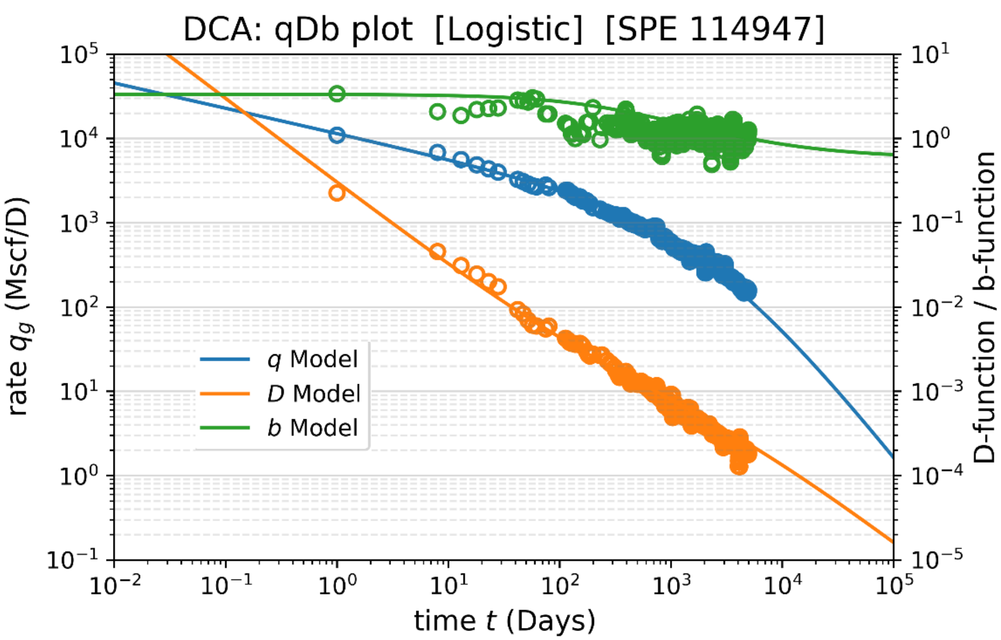
Equation box:

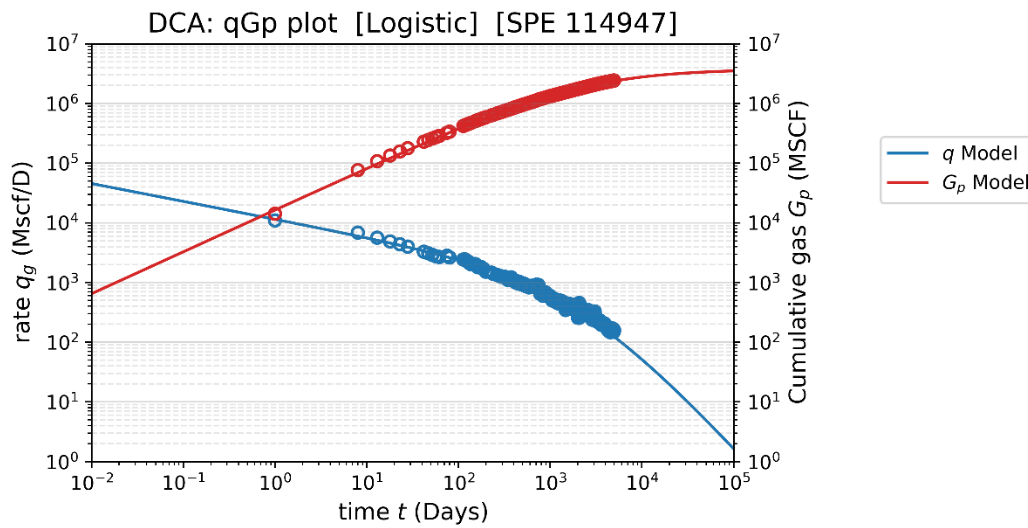
Rate	$q(t)=\dfrac{\hat{a}K\hat{n}t^{\hat{n}-1}}{(\hat{a}+\hat{t}^{n})^{2}}$
D-function	$D(t)=\dfrac{1}{t}\left[1-\hat{n}\left(\dfrac{2\hat{a}}{\hat{a}+\hat{t}^{n}}-1\right)\right]$
b-function	$b(t)=\dfrac{1}{1-\hat{n}\left[\dfrac{2\hat{a}}{\hat{a}+\hat{t}^{n}}-1\right]}-\dfrac{2\hat{a}\hat{n}^{2}t^{\hat{n}}}{(a+\hat{t}^{n})^{2}\left[1-\hat{n}\left(\dfrac{2\hat{a}}{\hat{a}+\hat{t}^{n}}-1\right)\right]^{2}}$

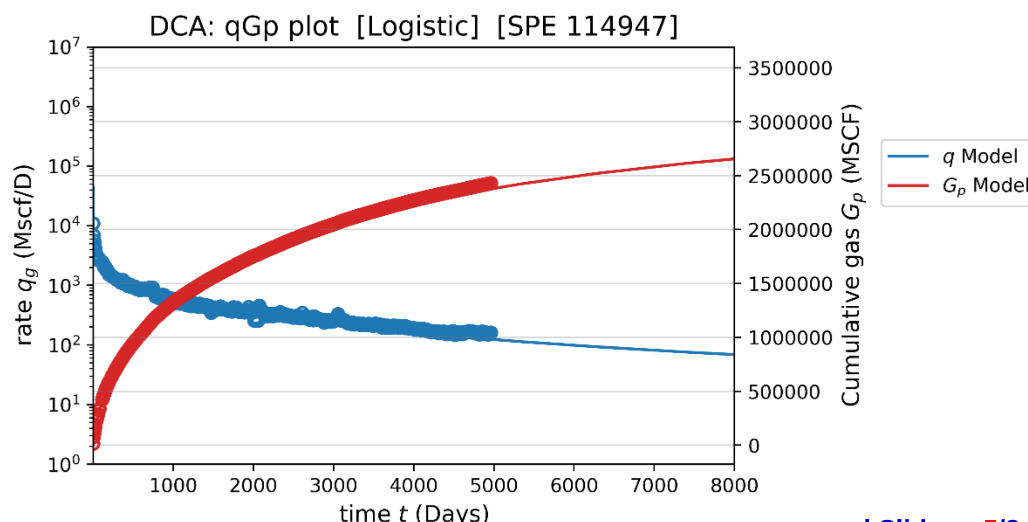
Parameters to be Solved: (*Final Estimates*)

(from Python Program)

```
a-CONSTANT (a), constant             = 229.06
n-CONSTANT (n), constant             = 0.70196
K-CONSTANT (n), MSCF                 = 3.7659E+6
CUM. GAS @ 30 YRS (Gp), BSCF         = 2.817
```







Comment:

- This model is built on the logistic growth model and carrying capacity (K).
- The logistic growth model does not extrapolate to non-physical values.
- The cumulative production from this model is more conservative (2.82 BSCF) than the other methods. (e.g. Duong = 3.24 BSCF, Power-law Exp. = 3.01 BSCF)

DCA model: Weibull model (SPE 161092)

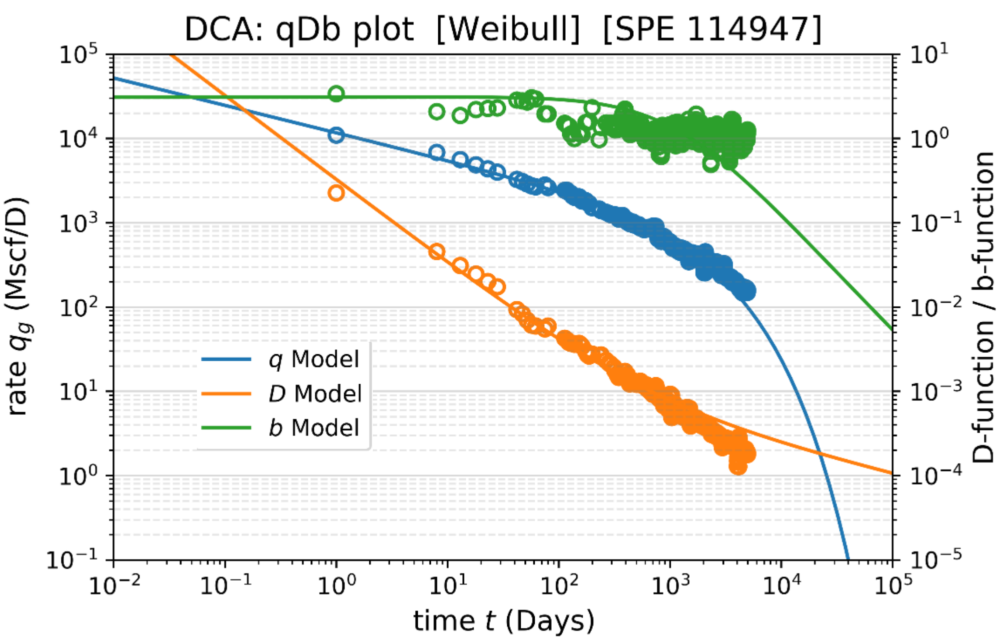
Equation box:

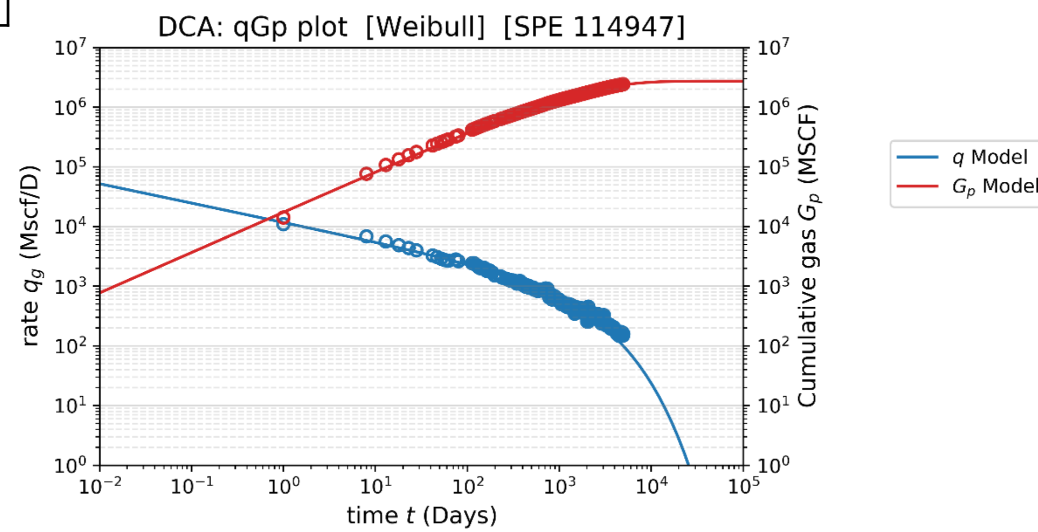
Rate	$q(t) = M\frac{\gamma}{\alpha}\left[\frac{t}{\alpha}\right]^{\gamma-1} exp\left(-\left[\frac{t}{\alpha}\right]^{\gamma}\right)$
D-function	$D(t) = \frac{1}{t}\left[\gamma\left[\frac{t}{\alpha}\right]^{\gamma} - (\gamma - 1)\right]$
b-function	$b(t) = \frac{(1-\gamma)\left[\gamma\left[\frac{t}{\alpha}\right]^{\gamma} + 1\right]}{\left[\gamma\left(\left[\frac{t}{\alpha}\right]^{\gamma} - 1\right) + 1\right]^{2}}$

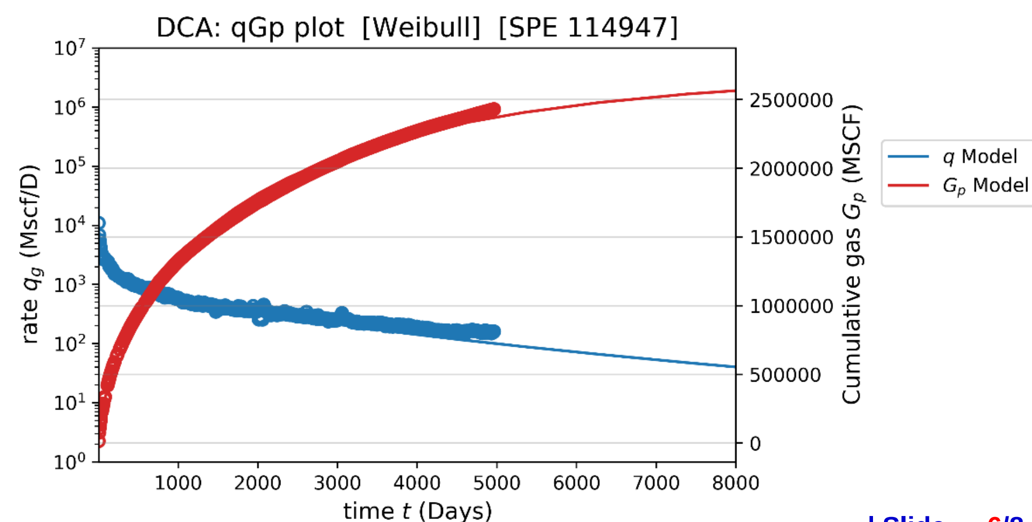
Parameters to be Solved: (*Final Estimates*)

(from Python Program)

```
M-CONSTANT (a), MSCF              = 2.7369E+6
γ-CONSTANT (γ), constant          = 0.6764
α-CONSTANT (α), constant          = 1.7686E+3
CUM. GAS @ 30 YRS (Gp), BSCF      = 2.646
```



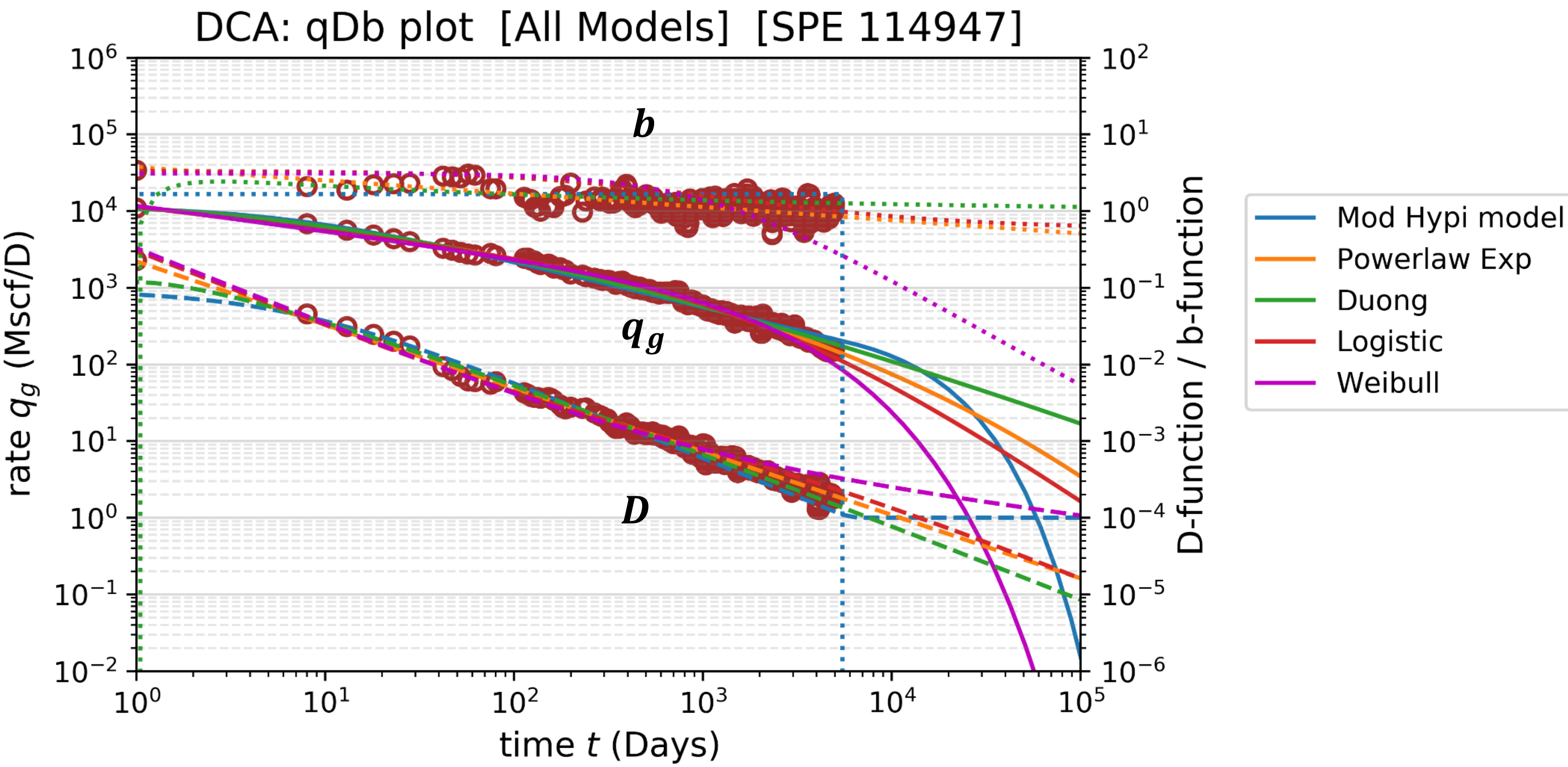




Comment:

- This model is the most conservative decline curve model and yields the least cumulative production at 2.65 BSCF.
- The flow rate is underestimated during the late time, and the decline constant (D) is overestimated during the late time as well. The overall match is acceptable

DCA model: All models



<u>Cumulative production @ 30 years</u>:

(from optimistic to conservative)

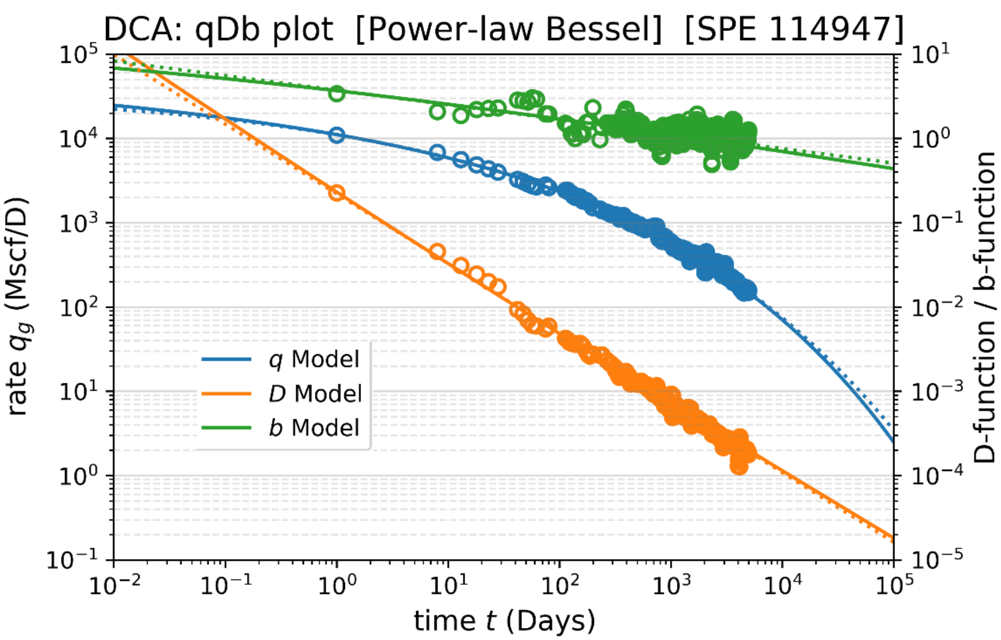
- **Modified hyperbolic = 3.42 BSCF**
- **Duong's model = 3.24 BSCF**
- **Power-law exponential = 3.01 BSCF**
- **Logistic model = 2.82 BSCF**
- **Weibull model = 2.65 BSCF**

DCA model: [NEW model] Power-law Bessel – by K.P.

Equation box:

Rate	$q(t) = q_i K_0(at^n)$
D-function	$D(t) = ant^{n-1}\dfrac{K_1(at^n)}{K_0(at^n)}$
b-function	$b(t) = \dfrac{[K_0(at^n)]^2}{[K_1(at^n)]^2} + \dfrac{t^{-n}K_0(at^n)}{anK} - 1$

(Solid line = Power-law Bessel, dash line = Power-law Exp.)



Comment:

- **This model is the "near-twin" of power-law exponential model. The predicted cumulative production at 30 years = 2.978 MSCF, which is very close to that from power-law exp. (3.005 MSCF)**
- **This empirical model is derived on the radial flow equation when K0(x) can be approximated to be ln(x) and the constant rate solution will be approximately the reciprocal of pressure solution.**

Parameters to be Solved: (*Final Estimates*)

```
(from Python Program)

INITIAL RATE (qi), MSCF/D          = 15985
a-CONSTANT (a), constant           = 0.6664
n-CONSTANT (n), constant           = 0.2152
CUM. GAS @ 30 YRS (Gp), BSCF       = 2.978
```

This model is slightly conservative than power-law exponential: (lower q + higher D toward the end)