Genetic Algorithm for PIA

In[ ]:=
```
SetDirectory[NotebookDirectory[]];
Needs["CAGD`","CAGD.wl"];
```

Matrix Multiplication

In[ ]:=
```
GetMatrix[set_] := Module[{},
    If[set[[1]] == 0,Return[TranslationM[set[[2]],set[[3]]]];
    If[set[[1]] == 1,Return[ScalingM[set[[2]],set[[3]]]];
    (*If[set[[1]] == 3,Return[RotationM[set[[4]] Degree]];*)
    If[set[[1]] == 2,Return[ShearXM[set[[4]] Degree]];
    If[set[[1]] == 3,Return[ShearYM[set[[4]] Degree]];
];
```

In[ ]:=
```
MatMul[points_, mats_] := Module[{newPoints, k,i, midPoint, mat},
    newPoints =  Append[#, 1] & /@ points;
    midPoint = (newPoints[[1]] + newPoints[[Length[newPoints]]])/2;
    newPoints = Table[newPoints[[i]] - midPoint + {0,0,1}, {i,1,Length[points]}];
    For[i = 1, i ≤ Length[mats],i++,
        mat = Decoding[mats[[i]]];
        newPoints = Table[GetMatrix[mat]. newPoints[[k]], {k,1,Length[points]}];
    ];
    newPoints = Table[newPoints[[k]] + midPoint - {0,0,1}, {k,1,Length[points]}];
    Return[newPoints];
];
```

In[ ]:=
```
points = {{1, 2}, {3, 4}, {5, 5}};
```

In[ ]:=
```
GeneratePopulation[22, 5, 3][[1, ;; -2]]
```

Out[ ]=
```
{{0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1},
 {1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1},
 {1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1},
 {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1},
 {1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1}}
```

In[ ]:=
```
N[MatMul[points, GeneratePopulation[22, 5, 3][[1, ;; -2]]]
```

Out[ ]=
```
{{127.419, 5.02113, 1.}, {50.1937, 71.7667, 1.}, {-19.419, 97.4103, 1.}}
```

Generate Random Set

In[•]:=
```
GeneratePopulation[nBit_, nMat_, nPop_] := Module[{matL, i, temp},
    matL = {};
    For[i = 1, i ≤ nPop, i++,
        temp = RandomInteger[1, {nMat, nBit}];
        AppendTo[temp, 0];
        AppendTo[matL, temp];
    ];
    Return[matL];
]
```

In[•]:=
```
GeneratePopulation[22, 5, 3][[1]]
```

Out[•]=
```
{{0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1},
 {0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1},
 {1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1},
 {0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0}, 0}
```

Decoding

In[•]:=
```
Decoding[mat_] := Module[{typeIdxList, xIdxList, yIdxList, angIdxList, type, x, y, ang
    (*get indics of each parameter*)
    typeIdxList = {1, Length[mat]};
    xIdxList = Select[Range[Length[mat]],(# ≠ 1 && # ≠ Length[mat] && Mod[#,3] == 2)&];
    yIdxList = Select[Range[Length[mat]],(# ≠ 1 && # ≠ Length[mat] && Mod[#,3] == 0)&];
    angIdxList = Select[Range[Length[mat]],(# ≠ 1 && # ≠ Length[mat] && Mod[#,3] == 1)&

    (*decode*)
    type = FromDigits[Table[mat[[typeIdxList[[i]]],{i, 1, Length[typeIdxList]}], 2];
    x = FromDigits[Table[mat[[xIdxList[[i]]], {i, 2, Length[xIdxList]}], 2];
    y = FromDigits[Table[mat[[yIdxList[[i]]], {i, 2, Length[yIdxList]}], 2];
    ang = FromDigits[Table[mat[[angIdxList[[i]]], {i, 2, Length[angIdxList]}], 2];

    (*check sign*)
    x = If[mat[[xIdxList[[1]]] == 1, -x, x];
    y = If[mat[[yIdxList[[1]]] == 1, -y, y];
    ang = If[mat[[angIdxList[[1]]] == 1, -ang, ang];

    Return[{type, x, y, ang}];
]
```

In[•]:=
```
Decoding[{0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1}]
```

Out[•]=
```
{1, -61, -48, -9}
```

Fitness Function & Termination Criteria

In[ ]:=
```
TerminationCriteria[controlPoints_, testPoints_, ϵ_ ] := Module[{n, t, poc, Δ, tol, st
    n = Length[controlPoints];
    t = Table[i/(n-1), {i, 0, n-1}];
    poc = Table[Bezier[SetC1n[controlPoints, testPoints], x] /. x→t[[i]], {i,1,n}];
    Δ = controlPoints - poc;
    tol = Max[Table[Norm[Δ[[i]]], {i, 1, n}]];  (*fitness score*)
    (*tol = Norm[Table [Norm[Δ[[i]]], {i, 1, n}]];*)
    If[tol < ϵ, status = True, status = False];
    Return[{status, tol}];
];
```

In[ ]:=
```
GetFitnessScore[controlPoints_, testSet_, nMatrix_,  ϵ_] := Module[{newPoints, newTest
    newTestSet = testSet;
    For[i = 1, i ≤ Length[testSet], i++,
        transformedPoints = N[MatMul[controlPoints, testSet[[i, ;;-2]]]][[All, ;;2]];
        newTestSet[[i, nMatrix+1]] = TerminationCriteria[controlPoints, transformedPoint
    ];
    Return[newTestSet];
];
```

In[ ]:=
```
GetFitnessScore[{{17, 71}, {1, 39}, {32, 50}, {97, 55}},
 GeneratePopulation[22, 5, 3], 5, 1]
```

Out[ ]=
```
{{{1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0},
  {0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 0, 0},
  {1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1},
  {0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1},
  {1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0}, 1457.33},
 {{1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1},
  {1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1},
  {0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1},
  {1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1},
  {1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1}, 1569.73},
 {{0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0},
  {1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0},
  {1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0},
  {1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0},
  {1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1}, 18.5851}}
```

In[ ]:=
```
(*GetFitnessScore[controlPoints_, guessSet_, testSet_, nMatrix_,  ϵ_] := Module[{newPo
    newTestSet = testSet;
    For[i = 1, i ≤ Length[newTestSet], i++,
        fitnessScore = 0;
        For[j = 1, j ≤ Length[guessSet], j++,
            transformedPoints = N[MatMul[controlPoints, testSet[[i, ;;nMatrix]]]][[All, ;
            guessSetTransformedPoints = N[MatMul[controlPoints, guessSet[[j, ;;nMatrix]]
            (*Print[testSet[[i, ;;nMatrix]]];*)
            fitnessScore = fitnessScore + Abs[TerminationCriteria[guessSetTransformedP
        ];
        newTestSet[[i, nMatrix+1]] = fitnessScore;
    ];
    Return[newTestSet];
];*)
```

In[ ]:=
```
tSet = {{{3.`, 26.`, 25.`, 13.`, -32.`},
    {35.`, 8.`, 38.`, 56.`, -21.`}, {3.`, 16.`, 16.`, -12.`, 30.`},
    {29.`, 24.`, 25.`, 84.`, -32.`}, {5.`, 7.`, 34.`, -66.`, 71.`}, 0}};
gSet = {{{2, 41, 10, -89, 11}, {4, 17, 1, -50, 3}, {4, 21, 40, 46, 66},
    {3, 15, 21, 14, 10}, {1, 49, 17, 14, -58}, 616.0332254187018`}};
```

Evolution Method

Parent Selection

Roulette Wheel Selection

In[ ]:=
```
RouletteWheelSelection[mSet_, nParent_] := Module[{n, newmSet, cumFitnessScore, selPoi
  n = Length[mSet];
  newmSet = mSet;
  invertedFitness = 1 / newmSet[[All, -1]];

  For[i = 1, i ≤ nParent, i++,
    cumFitnessScore = N[Total[invertedFitness]];
    selPoint = RandomReal[{0, cumFitnessScore}];
    cumSum = 0;

    For[j = 1, j ≤ n, j++,
      cumSum += invertedFitness[[j]];
      If[cumSum ≥ selPoint,
        AppendTo[parent, newmSet[[j]]];
        invertedFitness = Delete[invertedFitness, j];
        newmSet = Delete[newmSet, j];
        Break[];
      ];
    ];
    n--;
  ];

  Return[parent];
];
```

In[ ]:=
```
ppopSet = GeneratePopulation[22, 5, 10];
ppopSet =
  GetParentFitnessScore[{{17, 71}, {1, 39}, {32, 50}, {97, 55}}, ppopSet, 5, 1];
```

In[ ]:=
```
RouletteWheelSelection[ppopSet, 2]
```

Out[ ]=
```
{{{0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1},
  {0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0},
  {0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0},
  {1, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1}, 48.1827},
 {{0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0},
  {1, 1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1},
  {1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1},
  {1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1},
  {1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0}, 47.757}}
```

Stochastic Universal Sampling

In[ ]:=
```
StochasticUniversalSampling[mSet_,nParent_]:=Module[{n, newmSet, cumFitnessScore, selP
    n = Length[mSet];
    newmSet = mSet;
    invertedFitness = 1 / newmSet[[All, -1]];
    cumFitnessScore = Accumulate[invertedFitness];
    totalFitness = Last[cumFitnessScore];
    stepSize = totalFitness / nParent;
    startPoint = RandomReal[{0, stepSize}];
    points = Table[startPoint + (i - 1) stepSize, {i, nParent}];
    j = 1;
    For[i = 1, i ≤ nParent, i++,
        While[points[[i]] > cumFitnessScore[[j]],
          j++;
        ];
        AppendTo[parent, newmSet[[j]]];
    ];
    Return[parent];
];
```

In[ ]:=
```
StochasticUniversalSampling[ppopSet , 2];
```

Tournament Selection

In[ ]:=
```
TournamentSelection[mSet_, nParent_] := Module[{n ,k, min, parent={}, i, j, candIdxLis
    n = Length[mSet];
    k = 0.1;
    min = 5;
    For[i = 1, i ≤ nParent, i++,
        newmSet = mSet;
        While[Length[newmSet] > min,
            candIdxList = RandomInteger[{1, Length[newmSet]}, Floor[k*Length[newmSet]]
            newmSet = newmSet[[candIdxList]];
        ];
        AppendTo[parent, Sort[newmSet][[-1]]];
    ];
    Return[parent];
];
```

In[•]:=
```
TournamentSelection[ppopSet, 2]
```

Out[•]=
```
{{{0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1},
  {0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0},
  {0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0},
  {1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1},
  {0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1}, 16936.7},
 {{1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1},
  {0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0},
  {0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1},
  {1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0},
  {1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1}, 1892.99}}
```

Rank Selection

Crossover Method

Uniform crossover

In[•]:=
```
UniformCrossover[p_ ,q_ ] := Module[{i, temp, c1, c2, m, n, prob},
    m = Length[p];
    n = Length[p[[1]]];
    c1 = Flatten[p];
    c2 = Flatten[q];
    For[i = 1, i ≤ m*n, i++,
        prob = RandomInteger[{1,100}]/100;
        If[prob > 0.5,
            temp = c1[[i]];
            c1[[i]] = c2[[i]];
            c2[[i]] = temp;
        ];
    ];
    Return[{ArrayReshape[c1, {m,n}],ArrayReshape[c2, {m,n}]}];
]
```

In[•]:=
```
offsprings = GeneratePopulation[22, 5, 10][[ ;; 2]]
UniformCrossover[offsprings[[1,  ;; -2]], offsprings[[2,  ;; -2]]]
```

Out[•]=
```
{{{0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0},
  {1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1},
  {0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1},
  {1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0}, 0},
 {{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0},
  {1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1},
  {1, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0},
  {0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0},
  {0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 0, 1}, 0}}
```

Out[•]=
```
{{{0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0},
  {1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1},
  {1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0},
  {0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1},
  {1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0}},
 {{0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0},
  {1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1},
  {0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0},
  {0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0},
  {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1}}}
```

N-point crossover

In[•]:=
```
Range[1, 100, 5]
```

Out[•]=
```
{1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76, 81, 86, 91, 96}
```

In[•]:=
```
NPointsCrossover[p_, q_] := Module[{m, n, c1, c2, crossoverPoints, temp1, temp2, flip,
    m = Length[p];
    n = Length[p[[1]]];
    c1 = Flatten[p];
    c2 = Flatten[q];
    crossoverPoints = Range[1, m*n, n];
    flip = True;
    j = 1;
    For[i = 1, i ≤ m*n, i++,
        If[j ≤ Length[crossoverPoints] &&i == crossoverPoints[[j]],
            flip = !flip;
            j++;
        ];
        If[flip== True,
            temp1 = c1[[i]];
            c1[[i]] = c2[[i]];
            c2[[i]] = temp1;
        ];
    ];
  Return[{Partition[c1, n], Partition[c2, n]}];
];
```

In[•]:=
```
offsprings = GeneratePopulation[22, 5, 10][[ ;; 2]]
NPointsCrossover[offsprings[[1,  ;; -2]], offsprings[[2,  ;; -2]]]
```

Out[•]=
```
{{{1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0},
  {1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1},
  {0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1},
  {0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0}, 0},
 {{0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1},
  {0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1},
  {0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0},
  {1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0}, 0}}
```

Out[•]=
```
{{{1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0},
  {0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1},
  {0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0}},
 {{0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1},
  {1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1},
  {0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1},
  {1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0}}}
```

Mutation Method

Random Mutation

In[ ]:=
```
BitFlipMutation[chro_] := Module[{m, n, i, newChro, prob},
    m = Length[chro];
    n = Length[chro[[1]]];
    newChro = Flatten[chro];
    For[i = 1, i ≤ m*n, i++,
        prob = RandomInteger[100];
        If[prob > 80,
            newChro[[i]] = 1 - newChro[[i]];
        ]
    ];
    Return[ArrayReshape[newChro, {m, n}]];

];
```

In[ ]:=
```
offsprings = GeneratePopulation[22, 5, 10][[ ;; 1]];
BitFlipMutation[offsprings[[1,  ;; -2]]]
```

Out[ ]=
```
{{0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0},
 {1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0},
 {1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1},
 {1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1}}
```

Inverse Mutation

In[ ]:=
```
InverseMutation[chro_] := Module[{m, n, start, end, newChro, temp, i},
    m = Length[chro];
    n = Length[chro[[1]]];
    newChro = Flatten[chro];
    {start, end} = Sort[RandomInteger[{1, m*n}, 2]];
    temp = Reverse[Take[newChro, {start, end}]];
    For[i = start, i ≤ end, i++,
        newChro[[i]] = temp[[i - start + 1]];
    ];

    Return[ArrayReshape[newChro, {m, n}]];

];
```

In[•]:=
```
offsprings = GeneratePopulation[22, 5, 10][[ ;; 1]];
InverseMutation[offsprings[[1,  ;; -2]]]
```

Out[•]=
```
{{1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0},
 {0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1},
 {1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0},
 {0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}
```

Scramble Mutation

In[•]:=
```
ScrambleMutation[chro_] := Module[{m, n, start, end, newChro, temp, i},
    m = Length[chro];
    n = Length[chro[[1]]];
    newChro = Flatten[chro];
    {start, end} = Sort[RandomInteger[{1, m*n}, 2]];
    temp = RandomSample[Take[newChro, {start, end}]];
    For[i = start, i ≤ end, i++,
        newChro[[i]] = temp[[i - start + 1]];
    ];

    Return[ArrayReshape[newChro, {m, n}]];

]
```

In[•]:=
```
offsprings = GeneratePopulation[22, 5, 10][[ ;; 1]]
InverseMutation[offsprings[[1,  ;; -2]]]
```

Out[•]=
```
{{{1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1},
  {1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1},
  {1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0},
  {0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1},
  {0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0}, 0}}
```

Out[•]=
```
{{1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1},
 {1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0},
 {1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0},
 {1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1},
 {0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0}}
```

Execution Part

Parameter Declaration

ControlPoints

In[•]:=
```
cpSize = 5;
cp = RandomInteger[100, {cpSize, 2}]
ϵ = 1;
```

Out[•]=
```
{{58, 70}, {75, 27}, {61, 38}, {66, 94}, {100, 39}}
```

In[•]:=
```
cp = {{0.`, 0.`}, {1.2566370614359172`, 0.9510565162951535`},
   {2.5132741228718345`, 0.5877852522924731`},
   {3.7699111843077517`, -0.5877852522924731`},
   {5.026548245743669`, -0.9510565162951535`}, {6.283185307179586`, 0.`}} * 50
```

Out[•]=
```
{{0., 0.}, {62.8319, 47.5528}, {125.664, 29.3893},
 {188.496, -29.3893}, {251.327, -47.5528}, {314.159, 0.}}
```

GA Parameter

In[•]:=
```
populationSize = 1000;
generationSize = 100;
nMatrix = 7;
```

Genetic Algorithm

In[•]:=
```
GeneticAlgorithm[controlPoints_, ϵ_ , populationSize_, generationSize_, nMatrix_]:= Mo
    (*generate population set*)
    populationSet = GeneratePopulation[22, nMatrix, populationSize];
    populationSet = GetFitnessScore[controlPoints, populationSet, nMatrix, ϵ];
    gen = 1;
    eligibleSet = {};
    Print["new"];
    Δ = 100;
    While[gen ≤ generationSize && Δ > ϵ,
        newGen = {};
        Print[gen];
        gen = gen+1;
        While[Length[newGen] ≤ populationSize,
            (*parent selection*)
            parentSelType = 3;
            If[parentSelType == 1, parentSet = TournamentSelection[populationSet, 2]];
            If[parentSelType == 2, parentSet = StochasticUniversalSampling[populationSe
            If[parentSelType == 3, parentSet = RouletteWheelSelection[populationSet, 2
```

```
            offspring = {};
            crossoverType = 2;
            If[crossoverType == 1, {offS1, offS2} = UniformCrossover[parentSet[[1, ;;-2]
            If[crossoverType == 2, {offS1, offS2} = NPointsCrossover[parentSet[[1, ;;-2]
            AppendTo[offspring, offS1];
            AppendTo[offspring, offS2];


            For[i = 1, i ≤ Length[offspring], i++,
                mutateRate = 0.1;
                mutateProb = N[RandomInteger[{1,100}]/100];
                If[mutateProb ≥ mutateRate,
                        mutateType = 1;
                        If[mutateType == 1, mutated = BitFlipMutation[offspring[[i]]];
                        If[mutateType == 2, mutated = ScrambleMutation[offspring[[i]]];
                        If[mutateType == 3, mutated = InverseMutation[offspring[[i]]];
                        AppendTo[mutated, 0];
                        AppendTo[newGen, mutated];
                    ,
                        mutated = offspring[[i]];
                        AppendTo[mutated, 0];
                        AppendTo[newGen, mutated];

                ];
                (*Print[newGen[[-1]];*)
            ];

        ];

        newGen = GetFitnessScore[controlPoints, newGen, nMatrix,  ϵ];
        Table[AppendTo[eligibleSet, newGen[[i]], {i, 1, Length[newGen]}];
        Δ = Min[newGen[[All, -1]];
        Print[Δ];
        avgNorm = Mean[newGen[[All, -1]];
        Print[Δ, " ", avgNorm, " ", Max[newGen[[All, -1]]];
        populationSet = newGen;
        (*For[j = 1, j ≤ Length[newGen], j++,
            If[newGen[[j ,-1]] > avgNorm,
                AppendTo[populationSet, newGen[[j]];,
                AppendTo[populationSet, GeneratePopulation[22, nMatrix, 1][[1]];
            ]

        ];*)
    ];
    candidate = SortBy[eligibleSet, Last][[1]];
    Print[candidate];
    Return[candidate];

];
```

In[•]:= `GeneticAlgorithm[cp, ϵ , populationSize, generationSize, nMatrix];`

new

1

26.4012

26.4012 648 812. 5.23929×10^{8}

2

24.4897

24.4897 254 915. 1.05941×10^{8}

3

24.1391

24.1391 67 624.1 2.11211×10^{7}

4

22.3663

22.3663 1.52436×10^{6} 1.20514×10^{9}

5

22.4611

22.4611 256 403. 9.29524×10^{7}

6

22.408

22.408 190 050. 5.09166×10^{7}

7

22.7709

22.7709 118 706. 4.27106×10^{7}

8

20.1987

20.1987 23 734.7 3.31585×10^{6}

9

21.2254

21.2254 300 829. 1.96041×10^{8}

10

21.6877

21.6877 357 304. 1.013×10^{8}

11

20.9496

20.9496 41 607.3 1.39121×10^{7}

12

23.3624

23.3624 179 137. 5.96229×10^{7}

13

20.6715

20.6715 546 272. 3.45421×10^{8}

14

24.4298

24.4298 115 879. 3.41416×10^{7}

15

21.9928

21.9928 52 645.8 1.91201×10^{7}

16

21.5935

21.5935 1.10018×10^{6} 1.04618×10^{9}

17

21.3424

21.3424 4.15868×10^{6} 2.6196×10^{9}

18

21.8359

21.8359 163 220. 3.76323×10^{7}

19

23.3061

23.3061 64 106.4 1.47452×10^{7}

20

22.1292

22.1292 67 716.7 4.50307×10^{7}

21

20.7615

20.7615 58 612.9 1.58698×10^{7}

22

23.4561

23.4561 65 494.1 3.37629×10^{7}

23

17.0256

17.0256 99 304. 3.05631×10^{7}

24

20.6024

20.6024 67 716.5 2.41558×10^{7}

25

21.8406

21.8406 83 739.8 5.74746×10^{7}

26

21.8094

21.8094 25 374. 4.98065×10^6

27

22.2709

22.2709 61 097.7 2.0804×10^7

28

22.4967

22.4967 92 263.4 6.11262×10^7

29

21.2865

21.2865 208 395. 1.37225×10^8

30

22.6253

22.6253 83 806. 3.56991×10^7

31

21.4956

21.4956 112 720. 6.85629×10^7

32

21.8707

21.8707 30 011.4 2.98089×10^6

33

21.5864

21.5864 36 655.8 1.0321×10^7

34

20.5122

20.5122 43 163.5 1.09385×10^7

35

22.6957

22.6957 75 800.6 4.48357×10^7

36

21.8485

21.8485 1.4153×10^6 1.31439×10^9

37

22.2228

22.2228 575 928. 5.42629×10^8

38

20.7707

20.7707 98 310.3 2.69941×10^7

39

21.1515

21.1515 340 859. 2.43598×10^8

40

23.4943

23.4943 89 730.6 3.29445×10^7

41

23.8609

23.8609 77 002.3 3.46872×10^7

42

23.7374

23.7374 389 730. 3.16529×10^8

43

24.0218

24.0218 1.10503×10^6 8.92383×10^8

44

21.2663

21.2663 152 653. 5.40423×10^7

45

23.7351

23.7351 55 339.3 1.7469×10^7

46

22.3655

22.3655 374 319. 2.81198×10^8

47

25.1133

25.1133 158 053. 1.00654×10^8

48

23.7215

23.7215 123 251. 5.87602×10^7

49

21.2447

21.2447 192 768. 8.63145×10^7

50

22.8795

22.8795 146 077. 6.16586×10^7

51

22.981

22.981 156 366. 8.78477×10^7

52

24.7578

24.7578 115 277. 2.98406×10^7

53

25.5492

25.5492 1.63813×10^6 1.6035×10^9

54

23.4336

23.4336 114 073. 2.66692×10^7

55

21.4958

21.4958 102 353. 2.95833×10^7

56

21.99

21.99 34 326.8 5.9811×10^6

57

21.1276

21.1276 351 259. 1.83661×10^8

58

21.6475

21.6475 1.17364×10^6 1.15653×10^9

59

21.1906

21.1906 120 214. 3.15258×10^7

60

23.3631

23.3631 95 110.7 4.17125×10^7

61

21.8901

21.8901 117 373. 5.57584×10^7

62

21.8205

21.8205 163 888. 4.59952×10^7

63

21.0095

21.0095 121 069. 4.96074×10^7

64

21.1311

21.1311 69 822.2 1.57261×10^7

65

20.5943

20.5943 129 418. 6.81952×10^7

66

22.3364

22.3364 337 419. 2.22128×10^8

67

20.6979

20.6979 92 501.1 4.59849×10^7

68

22.1818

22.1818 983 937. 9.26655×10^8

69

22.7979

22.7979 92 749.5 2.86839×10^7

70

21.0896

21.0896 174 234. 4.78866×10^7

71

21.2773

21.2773 51 074.7 2.36889×10^7

72

21.5741

21.5741 32 632.8 9.55637×10^6

73

22.6572

22.6572 132 949. 5.87606×10^7

74

21.0889

21.0889 103 873. 3.25818×10^7

75

16.5474

16.5474 121 864. 7.48721×10^7

76

22.0428

22.0428 138 182. 5.89564×10^7

77

22.3205

22.3205 1.19864×10^6 1.14355×10^9

78

22.1397

22.1397 178 210. 8.14907×10^7

79

21.9563

21.9563 2.44579×10^6 1.12468×10^9

80

24.757

24.757 415 481. 1.32245×10^8

81

21.9588

21.9588 186 300. 1.11488×10^8

82

21.6323

21.6323 162 531. 1.00932×10^8

83

22.8606

22.8606 67 437.2 2.01744×10^7

84

22.7618

22.7618 2.01393×10^6 1.96582×10^9

85

21.2709

21.2709 77 926.3 2.7223×10^7

86

23.8431

23.8431 22 841.6 4.75122×10^6

87

22.2756

22.2756 32 123.3 4.08995×10^6

88

23.7283

23.7283 47 925.9 1.34521×10^7

89

21.2652

21.2652 108 824. 3.40618×10^7

90

19.6621

19.6621 31 908. 3.92089×10^6

91

22.3857

22.3857 90 897. 4.34031×10^7

92

21.3514

21.3514 274 953. 7.25936×10^7

93

23.1281

23.1281 93 399.6 4.30339×10^7

94

20.7491

20.7491 366 293. 3.33817×10^8

95

21.9776

21.9776 173 218. 6.5493×10^7

96

21.2461

21.2461 154 624. 7.24946×10^7

97

20.8122

20.8122 29 976.7 1.01383×10^7

98

22.238

22.238 90 496.5 3.22094×10^7

99

22.0766

22.0766 131 130. 6.88153×10^7

100

22.5423

22.5423 26 566.1 2.19758×10^6

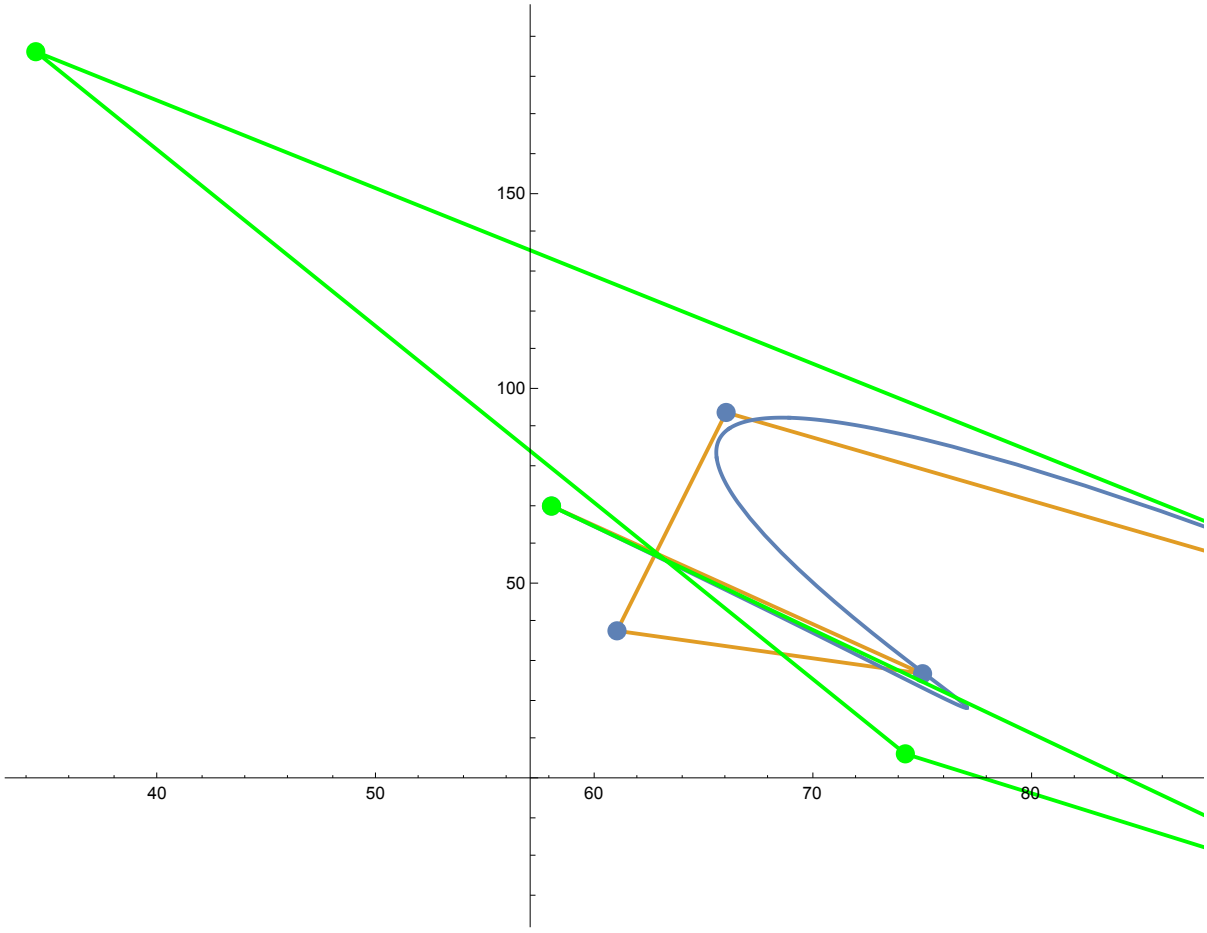
```
{{0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0},
 {1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1},
 {1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 1, 1},
 {1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0},
 {1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0},
 {1, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1}, 16.5474}
```

In[•]:=
```
cp
nn = {{1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0},
   {1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0},
   {1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0},
   {0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1},
   {1, 1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0},
   {1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1},
   {1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1}};
nn = N[MatMul[cp, nn]];
nn = SetC1n[cp , nn[[All,  ;; -2]]]
newPlot = ParametricPlot[Bezier[nn, t], {t, 0, 1}];
nnPlot = ListPlot[{nn, nn}, Joined → {False, True}, PlotStyle → Green];
cpPlot =  ListPlot[{cp, cp}, Joined → {False, True}];
Show[cpPlot, newPlot, nnPlot, PlotRange → All]
TerminationCriteria[cp, nn, 1 ]
```

Out[•]=
```
{{58, 70}, {75, 27}, {61, 38}, {66, 94}, {100, 39}}
```

Out[•]=
```
{{58, 70}, {97.0176, -33.457}, {74.2106, 6.45058}, {34.3747, 186.516}, {100, 39}}
```

Out[•]=




```
{{0., 0.}, {1.25664, 0.951057}, {2.51327, 0.587785},
 {3.76991, -0.587785}, {5.02655, -0.951057}, {6.28319, 0.}}
```