ImageXD
Images Across Domains

Image Registration

Maryana Alegro (UCSF)
Matt McCormick (Kitware)

BIDS
BERKELEY INSTITUTE FOR DATA SCIENCE

UCSF Bakar Computational Health Sciences Institute

BERKELEY LAB

Image Registration

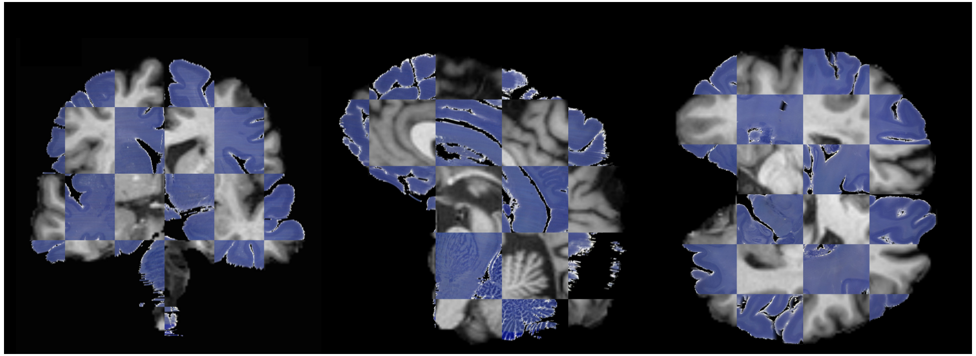
- Some background
    - What? Why?
    - Anatomy of an image registration method
    - Types of registration
    - Rigid body registration
    - Affine registration
    - Non-linear image registration
- Hands-on exercises using ITK and Python
- Github repo: https://github.com/KitwareMedical/2019-ImageXD-Registration-Tutorial

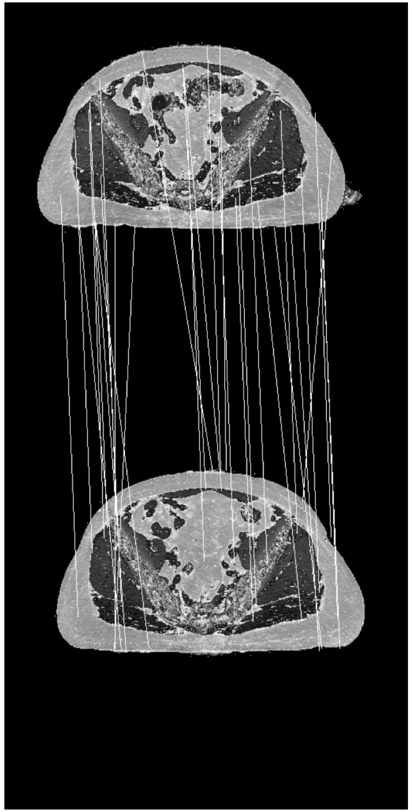
What? Why?

- Image registration is the process of *aligning* an image to a reference space.
- Formulated as a problem of estimating a *mapping* between the moving image and reference images.
- Solved as an *optimization problem* that estimates parameters of a transformation with relation to some image mismatch measurement.
- Rigid, affine and non-linear registration models.

What? Why?

- Allows analysis of data from different sensors/modalities/time points on the same coordinate system.
- Mandatory step in several image analysis problems
- Panoramic images
- Sensor fusion
- Medical imaging

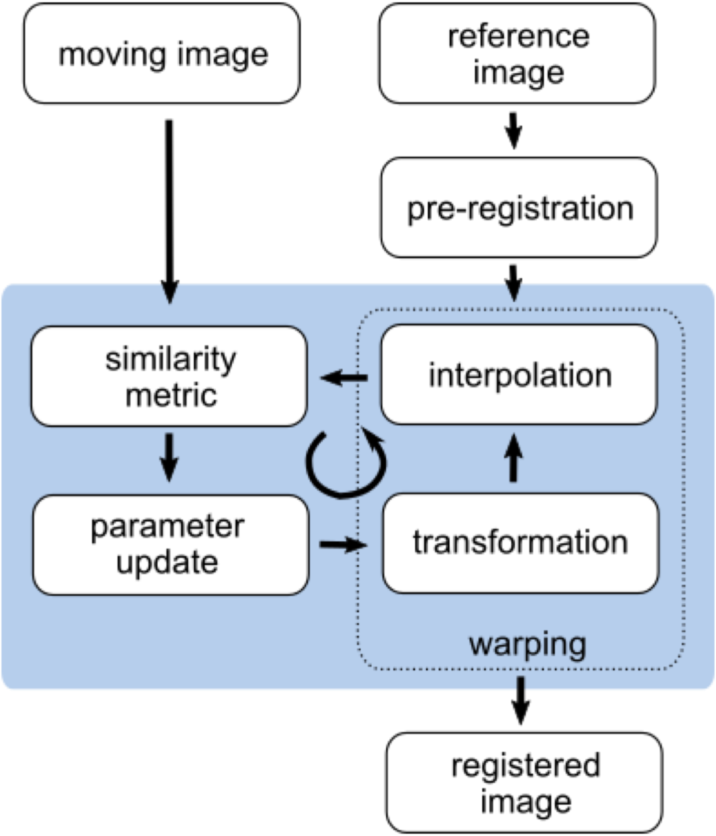




http://medpure.blogspot.com/2011/12/deformable-registration-with-sift.html

Anatomy of a Registration Algorithm

- Iterative process
- Pre-registration: centering, helps optimization convergence
- **Interpolation**: re-compute pixel values
- **Similarity metric**: measure how similar new image is to reference image
- **Transformation**: registration mathematical model, rigid, affine, diffeomorphic, so on…
- **Optimization algorithm** updates model parameters



Anatomy of a Registration Algorithm

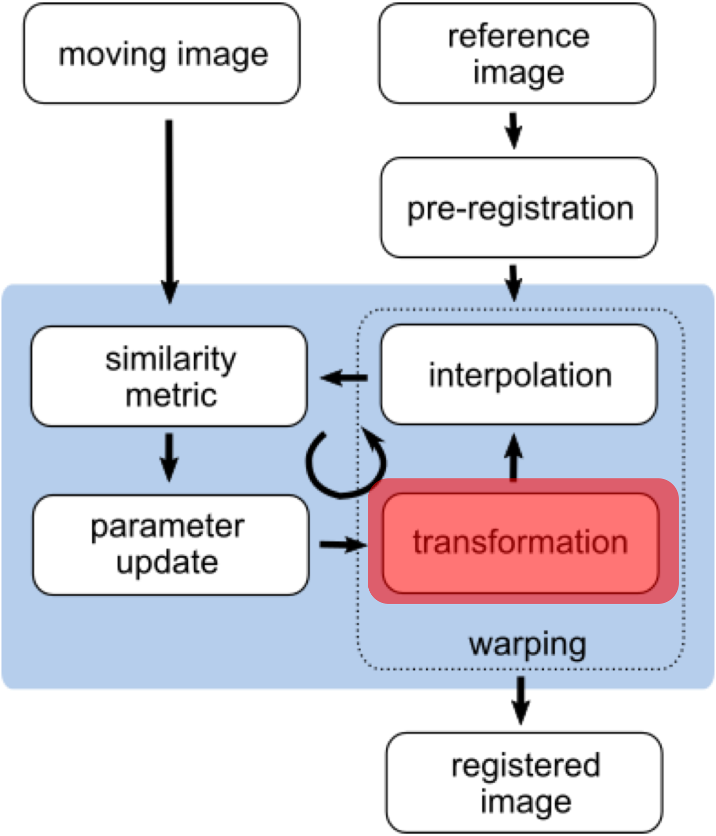
- Types of registration
- Feature-based registration
  - Transform parameters are computed based on a few landmarks
  - Manual landmarks
  - Points of interest. I.e. SIFT
  - Example application: panorama images, stitching
- Intensity-based registration
  - Transformation parameters are computed using pixel intensity information
  - Use of similarity metrics
  - Example application: medical imaging

Anatomy of a Registration Algorithm

- Desirable properties
- Symmetry
  - transformations are invertible functions
- Topology preservation
  - mapping is smooth, one-to-one and invertible
- Inverse consistency:
  - Correspondence between moving and reference images does not depend on the order of selection

Rigid Body Transformation

- Simplest registration model
- 2d case: 3 degrees of freedom: 2 translations + 1 rotation (3DOF)
- 3d case: 3 translations + 3 rotations (6 DOF)



Rigid Body Transformation

- Translations:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{T}{\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rigid Body Transformation

- Rotation along X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{M_x}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\omega) & -\sin(\omega) & 0 \\ 0 & \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rigid Body Transformation

- Rotation along Y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{M_y}{\begin{bmatrix} \cos(\varphi) & 0 & -\sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rigid Body Transformation

- Rotation along Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{M_z} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Affine Transformation

- Rigid body transformations + 3 scales + 3 shearing
- Scaling:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{\textsf{S}}{\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

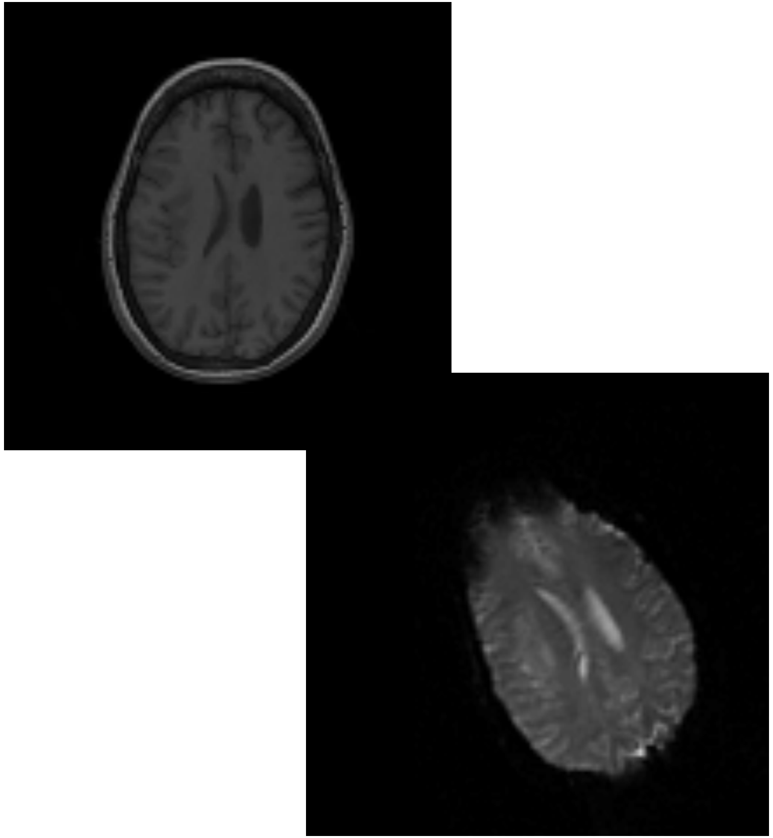
Affine Transformation

- Shear:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{\text{K}}{\begin{bmatrix} 1 & c & e & 0 \\ a & 1 & f & 0 \\ b & d & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

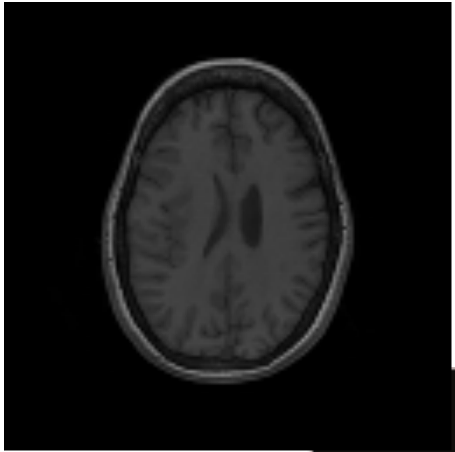
Affine Transformation

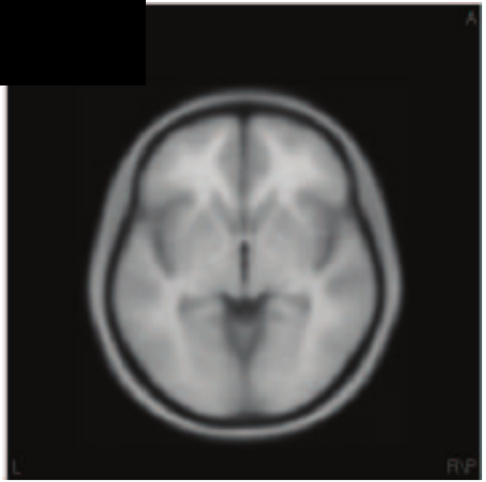
- Transformation matrices are combined together to form a *global* transformation.
- All pixels are mapped the same way
- Affine registration is sometimes referred as global registration as opposed to *non-linear*, local registration methods.
- Commonly used for *intra-patient* registration, initial registration alignment.



Non-linear Transformation

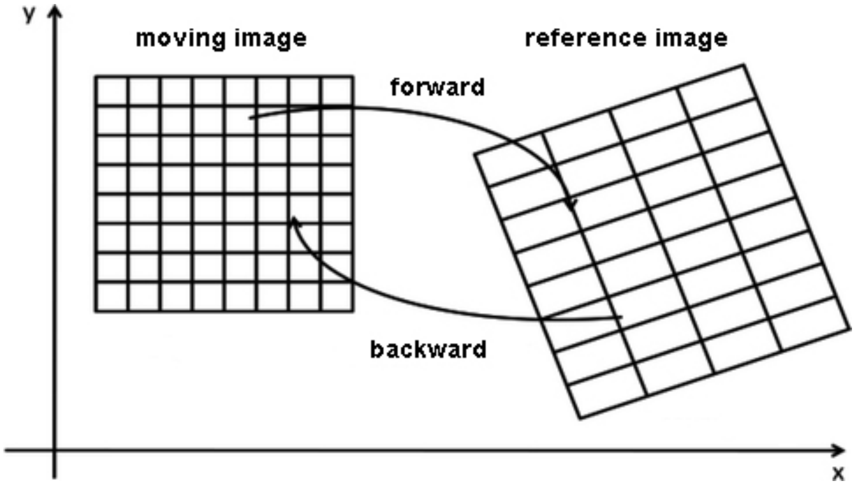
- Allows local deformation
- Higher computational complexity and memory footprint
- Models based on mechanics
  - Elastic bodies
  - Fluid mechanics
  - Diffeomorphisms
- Models based on interpolation theory
  - Thin-plate Splines
- Applications: inter-patient registration, template registration, histology registration





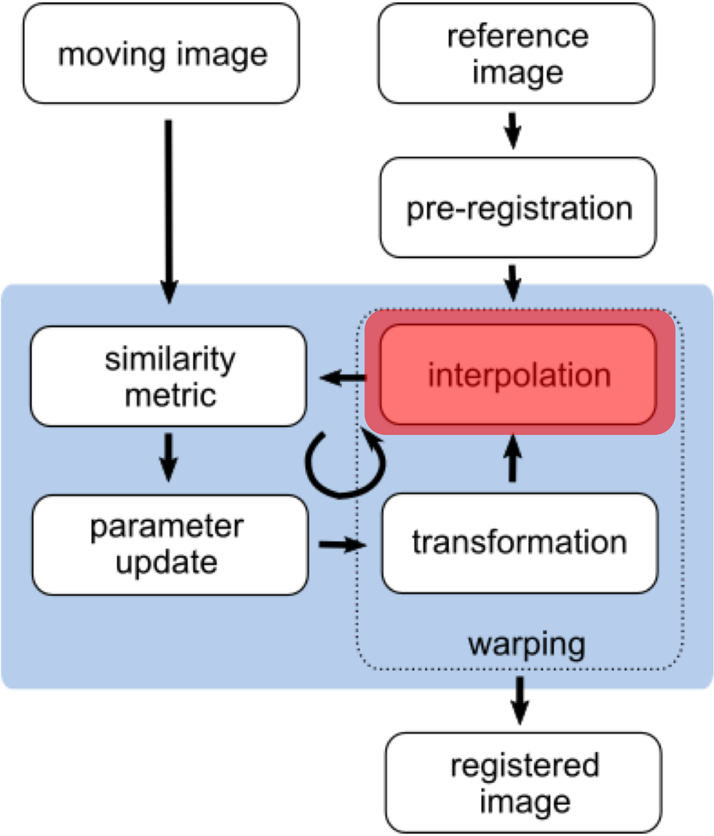
Mapping

- Forward mapping
  - Moving image <u>pixels</u> are mapped to reference image space
  - Multiple pixels can get mapped to a single location
  - Holes can occur (empty coordinates)
- Backward mapping
  - Reference image <u>coordinates</u> are mapped to moving image space
  - All reference image pixels are mapped, no holes
  - Transformation need to be invertible



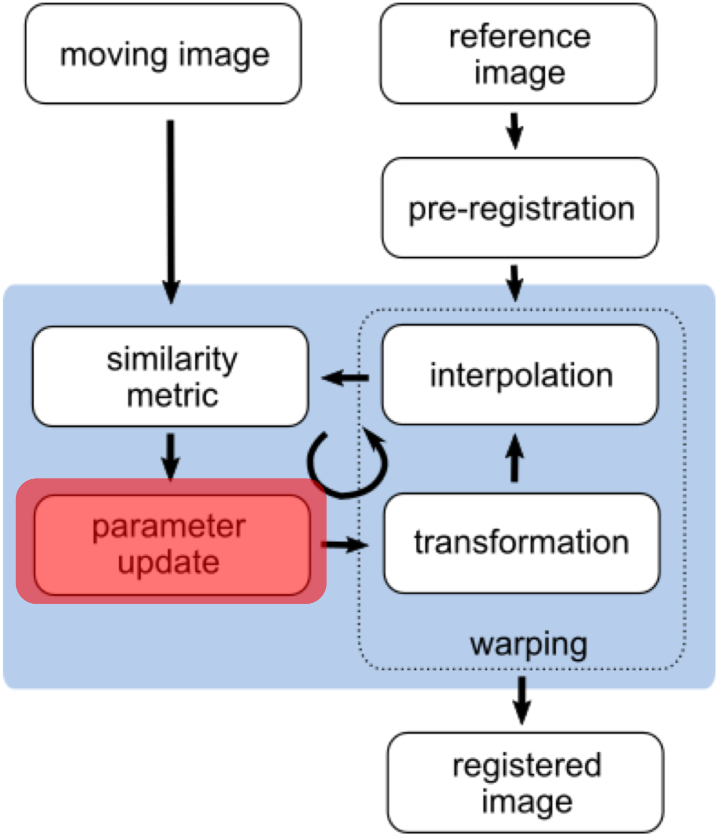
Intensity Interpolation

- Linear
  - Uses a linear function to model intensity transition from one pixel to the other
  - Allows non-integer coordinates
- Neared neighbors
  - Images are considered to be composed of tiles of same intensity values
  - Noise and block artifacts
- Cubic
  - 3rd order polynomial is fit to pixel in a neighborhood and used to infer new intensity values



Similarity Metrics and Optimization

- Optimizer help finding an good set of transformation parameters
  - Gradient descent
  - Simplex
  - etc
- Iterative process
- Similarity metric is used as objective function



Similarity Metrics and Optimization

- Quantifies how similar reference and registered images are
- Some metrics are suitable for same modality registration while other are good for multi-modality.
- Cross-correlation coefficient (same modality)
- Mean squared error (same modality)
  - Simple measure of how pixels line up.
- Sum of absolute value (same modality)
  - Simple measure of how pixels line up.
- Mutual information (multi-modality)
  - Measure how histogram match-up based on Entropy