Electron Hybrid model for R/L-waves in cold plasmas

- $$\frac{\partial \mathbf{u}_\mathrm{c}}{\partial t} + (\mathbf{u}_\mathbf{c} \cdot \nabla)\mathbf{u}_\mathrm{c} = \frac{q_\mathrm{e}}{m_\mathrm{e}}(\mathbf{E} + \mathbf{u}_\mathrm{c} \times \mathbf{B})$$
- $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
- $$\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mu_0(\mathbf{j}_\mathrm{c} + \mathbf{j}_\mathrm{h})$$
- $$\frac{\partial f_\mathrm{h}}{\partial t} + \mathbf{v} \cdot \nabla f_\mathrm{h} + \frac{q_\mathrm{e}}{m_\mathrm{e}}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_\mathbf{v} f_\mathrm{h} = 0$$
- $\nabla \cdot \mathbf{B} = 0$ at $t = 0$
- $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}[q_\mathrm{i} n_\mathrm{i} + q_\mathrm{e}(n_\mathrm{c} + n_\mathrm{h})]$ at $t = 0$

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- Linearization about an equilibrium state:
- $n_c(\mathbf{x}, t) = n_{c,0}(\mathbf{x}) + \tilde{n}_c(\mathbf{x}, t)$
- $\mathbf{u}_c(\mathbf{x}, t) = \tilde{\mathbf{u}}_c(\mathbf{x}, t)$
- $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0(\mathbf{x}) + \tilde{\mathbf{B}}(\mathbf{x}, t)$
- $\mathbf{E}(\mathbf{x}, t) = \tilde{\mathbf{E}}(\mathbf{x}, t)$

  → cold current density $\tilde{\mathbf{j}}_c = q_e \left(n_{c,0} + \tilde{n}_c\right) \tilde{\mathbf{u}}_c \approx q_e n_{c,0} \tilde{\mathbf{u}}_c$

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- $$\frac{\partial \tilde{\mathbf{j}}_\mathrm{c}}{\partial t} = \epsilon_0 \Omega_\mathrm{pe}^2 \tilde{\mathbf{E}} + \tilde{\mathbf{j}}_\mathrm{c} \times \mathbf{\Omega}_\mathrm{ce}$$
- $$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = -\nabla \times \tilde{\mathbf{E}}$$
- $$\frac{1}{c^2}\frac{\partial \tilde{\mathbf{E}}}{\partial t} = \nabla \times \tilde{\mathbf{B}} - \mu_0 (\tilde{\mathbf{j}}_\mathrm{c} + \mathbf{j}_\mathrm{h})$$
- $$\frac{\partial f_\mathrm{h}}{\partial t} + \mathbf{v} \cdot \nabla f_\mathrm{h} + \frac{q_\mathrm{e}}{m_\mathrm{e}} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_\mathbf{v} f_\mathrm{h} = 0$$

$$\Omega_\mathrm{pe}^2 = \frac{n_{\mathrm{c},0} e^2}{\epsilon_0 m_\mathrm{e}}$$

$$\mathbf{\Omega}_\mathrm{ce} = \frac{q \mathbf{B}_0}{m}$$

→ cold plasma model + energetic kinetic electrons

Electron Hybrid model for R/L-waves in cold plasmas

The dynamical equations conserve exactly the following energy:

$$\epsilon := \frac{1}{2}\left[\epsilon_0 \int \tilde{\mathbf{E}}^2 \mathrm{d}^3\mathbf{x} + \frac{1}{\mu_0}\int \tilde{\mathbf{B}}^2 \mathrm{d}^3\mathbf{x} + \frac{1}{\epsilon_0 \Omega_{\mathrm{pe}}^2}\int \tilde{\mathbf{j}}_{\mathrm{c}}^2 \mathrm{d}^3\mathbf{x} + m_{\mathrm{e}} \int f_{\mathrm{h}} v^2 \, \mathrm{d}^3\mathbf{v}\mathrm{d}^3\mathbf{x}\right]$$

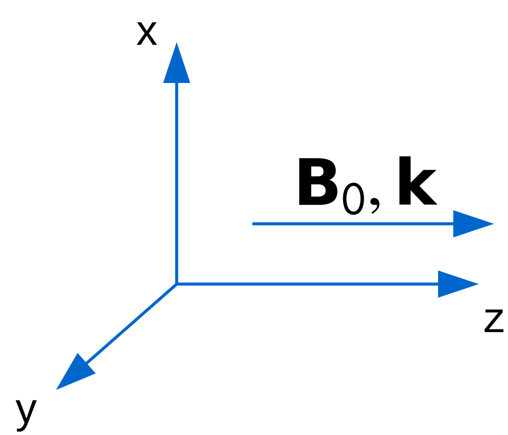
$$= \epsilon_{\mathrm{em}} + \epsilon_{\mathrm{c}} + \epsilon_{\mathrm{h}}$$

Electron Hybrid model for R/L-waves in cold plasmas

Dispersion relation for ...

- ... a homogeneous background plasma: $n_{c,0} = const.$
- ... a uniform magnetic background field: $\mathbf{B}_0 = B_0 \mathbf{e}_z$
- ... parallel wave propagation: $\mathbf{k} \parallel \mathbf{B}_0 \parallel \mathbf{e}_z$
- ... a uniform equilibrium hot electron distribution:

$$f_h(\mathbf{x}, \mathbf{v}, t) = f_h^0(\mathbf{v}) + \tilde{f}_h(\mathbf{x}, \mathbf{v}, t)$$



Electron Hybrid model for R/L-waves in cold plasmas

- Solution of the fully linearized problem by plane wave ansatz:
  - $\nabla \to ik\mathbf{e}_z$
  - $\frac{\partial}{\partial t} \to -i\omega$
- 3 types of solutions of which
  - 2 correspond to electromagnetic waves with $\tilde{\mathbf{E}}, \tilde{\mathbf{B}} \perp \mathbf{k}$
  - 1 corresponds to electrostatic waves with $\tilde{\mathbf{E}} \parallel \mathbf{k}, \quad \tilde{\mathbf{B}} = 0$

Electron Hybrid model for R/L-waves in cold plasmas

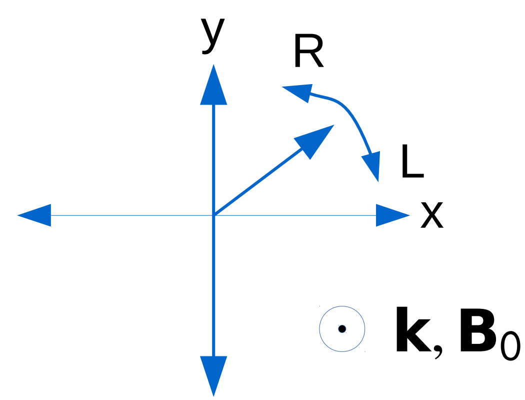
- Dispersion relation for electromagnetic waves: $\tilde{\mathbf{E}}, \tilde{\mathbf{B}} \perp \mathbf{k}$

$$D_{R/L}(k,\omega) = 1 - \frac{c^2 k^2}{\omega^2} - \frac{\Omega_{pe}^2}{\omega(\omega \pm \Omega_{ce})} + \nu_h \frac{\Omega_{pe}^2}{\omega} \int d^3\mathbf{v} \frac{v_\perp}{2} \frac{\hat{G} f_h^0(v_\parallel, v_\perp)}{\omega \pm \Omega_{ce} - k v_\parallel} = 0$$

$$\hat{G} = \frac{\partial}{\partial v_\perp} + \frac{k}{\omega}\left(v_\perp \frac{\partial}{\partial v_\parallel} - v_\parallel \frac{\partial}{\partial v_\perp}\right), \qquad \nu_h = n_{h,0}/n_{c,0}, \quad d^3\mathbf{v} = dv_\parallel dv_\perp v_\perp 2\pi$$

- Wave-particle interaction due to electrons with $k v_\parallel = \omega \pm \Omega_{ce}$

- Solutions correspond to
  - Right-hand circularly polarized waves (R)
  - Left-hand circularly polarized waves (L)

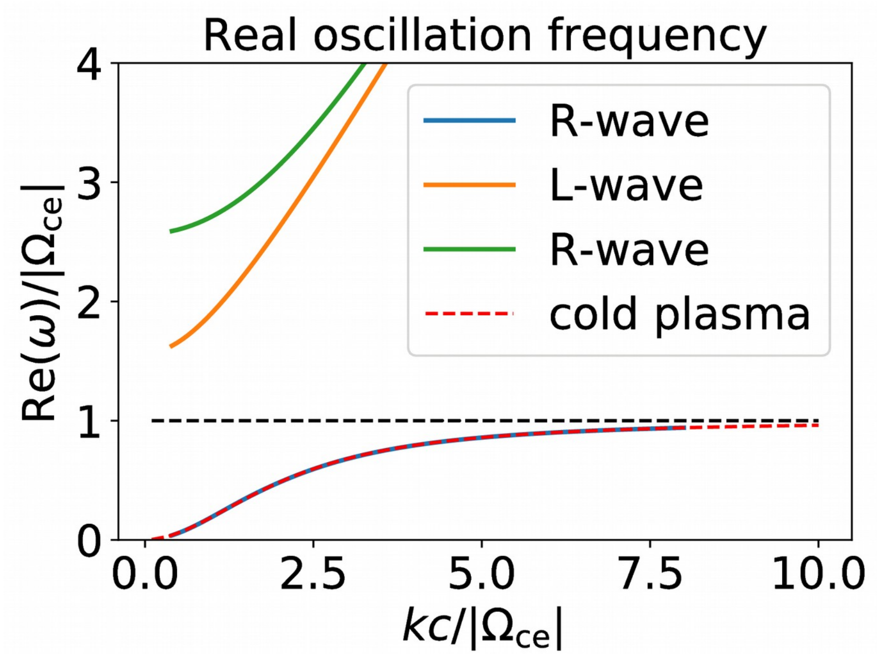


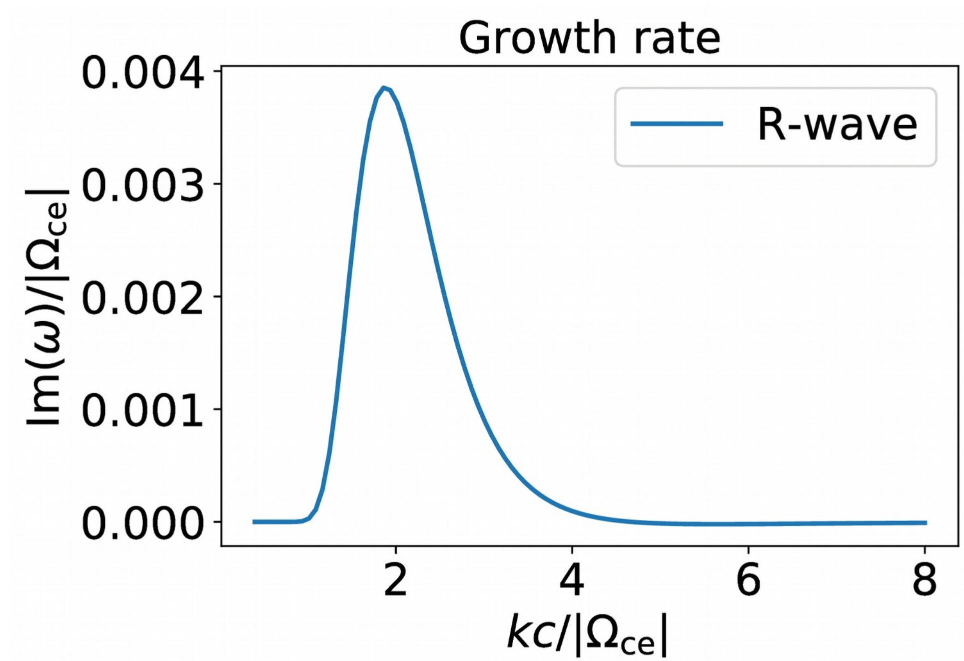
Electron Hybrid model for R/L-waves in cold plasmas

Example: anisotropic Maxwellian

$$f_h^0(v_\|, v_\perp) = \frac{n_{h,0}}{(2\pi)^{3/2} w_\| w_\perp^2} \exp\left(-\frac{v_\|^2}{2w_\|^2} - \frac{v_\perp^2}{2w_\perp^2}\right)$$

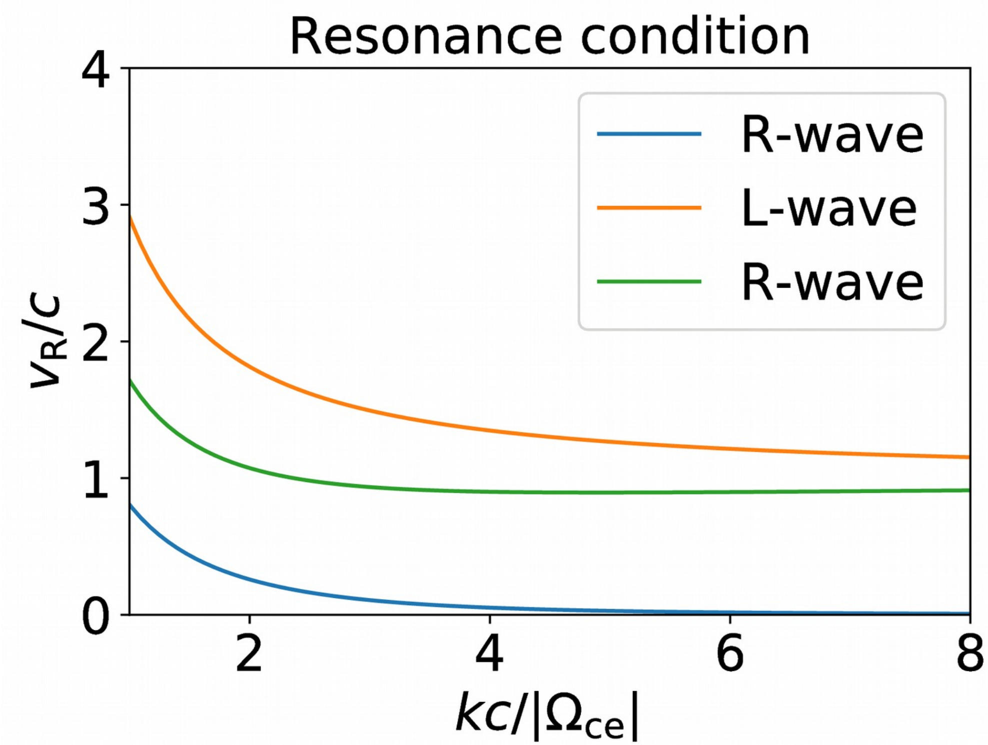
$$w_\| = 0.2\,c$$
$$w_\perp = 0.5\,c$$
$$\nu_h = 5 \cdot 10^{-3}$$
$$\Omega_{pe} = 2|\Omega_{ce}|$$

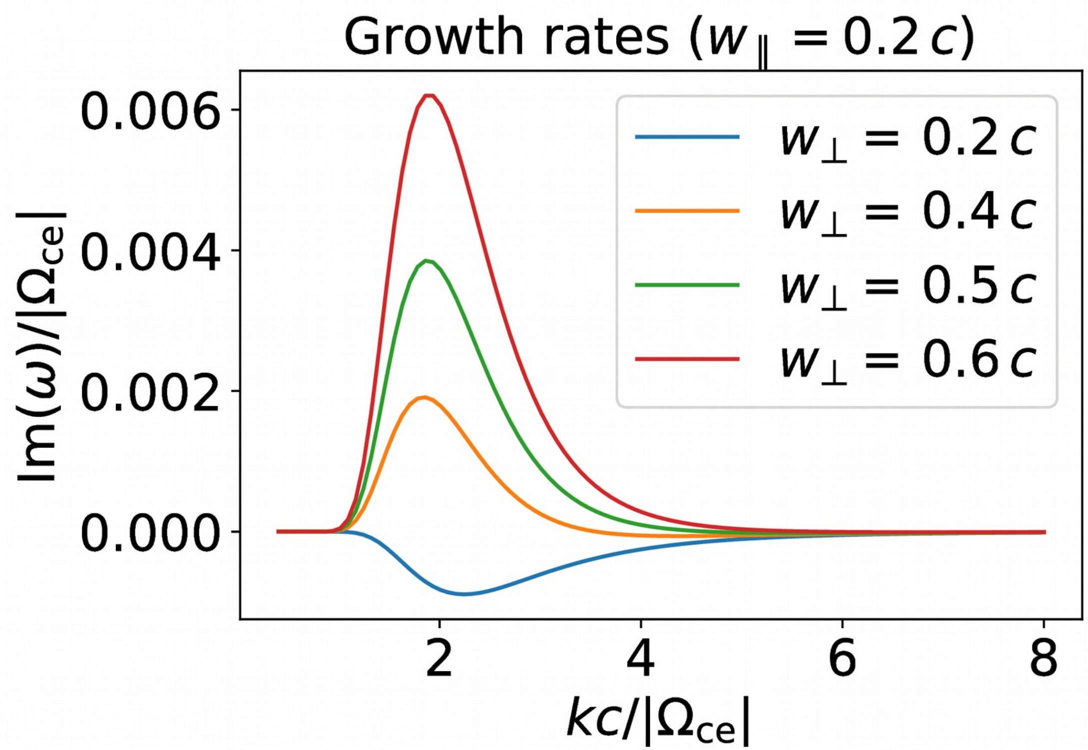




Electron Hybrid model for R/L-waves in cold plasmas

Example: anisotropic Maxwellian



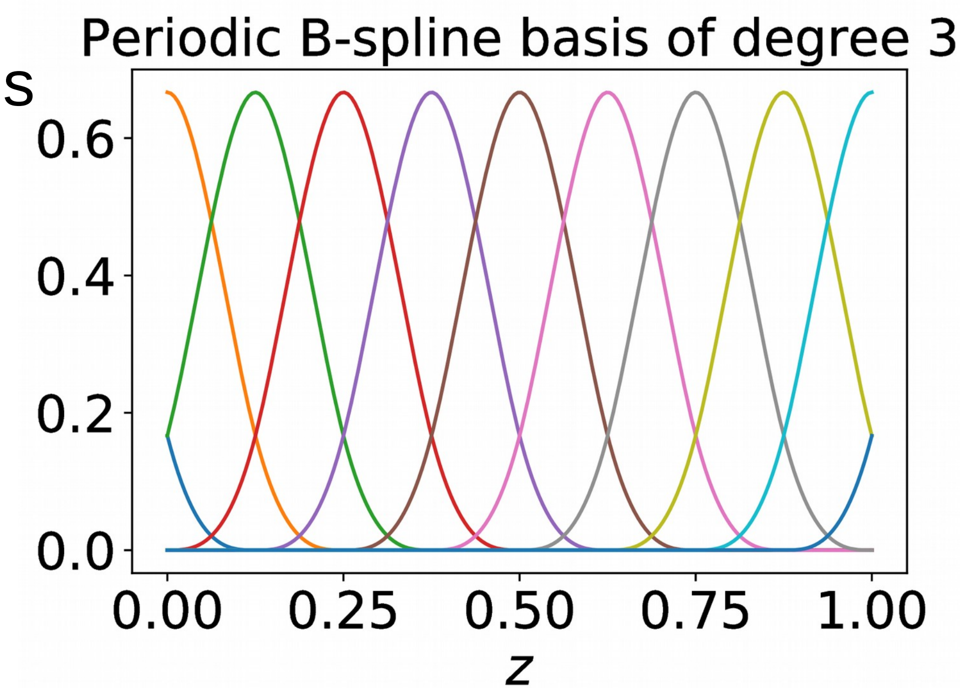


→ no wave-particle interaction for R/L-waves with $\omega \gg |\Omega_{ce}|$

Electron Hybrid model for R/L-waves in cold plasmas

Implementation of the model in Python:

- 1d B-spline Finite Elements for fields
  $\mathbf{U} = \left(E_x, E_y, B_x, B_y, j_{\mathrm{c},x}, j_{\mathrm{c},y}\right)$



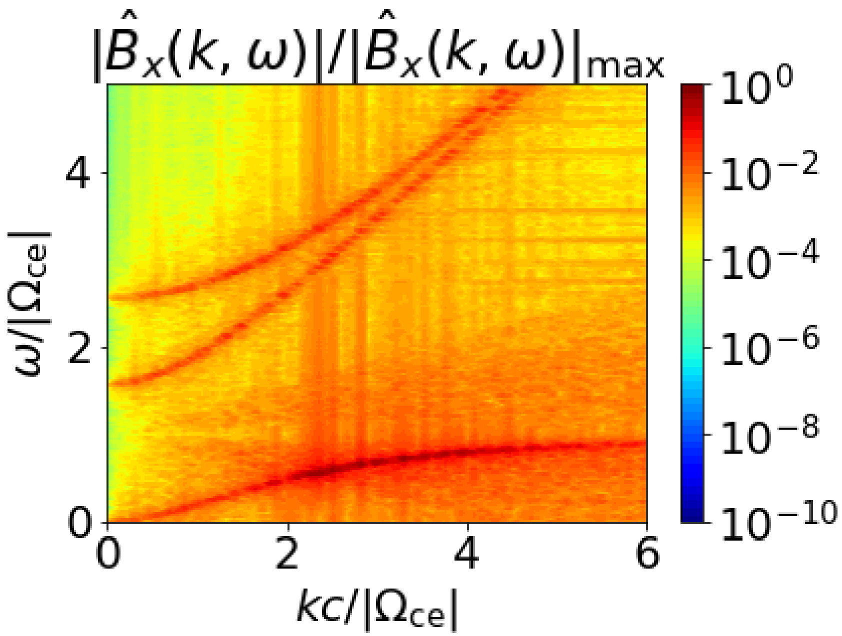
- 1d3v Particle-In-Cell with control variate and Boris particle pusher

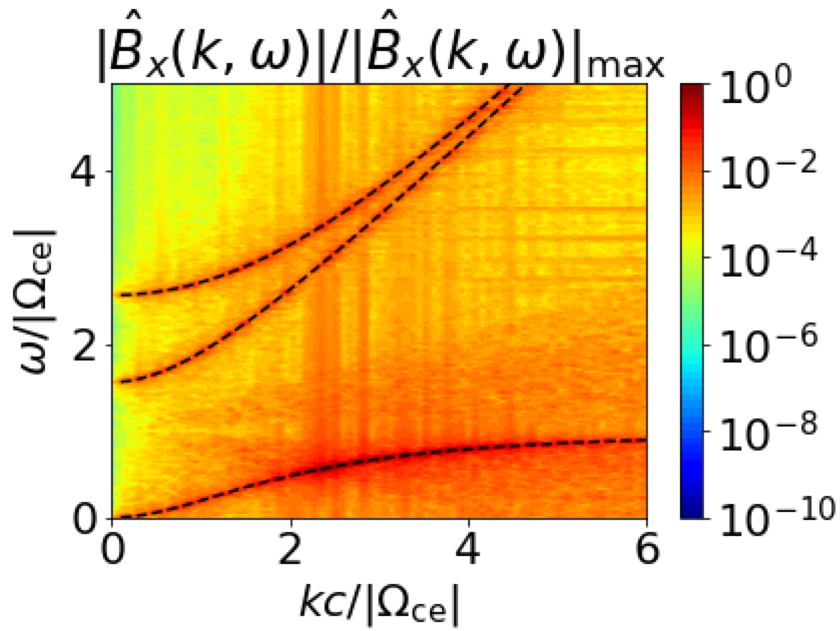
- Implicit Crank-Nicolson time stepping scheme

Electron Hybrid model for R/L-waves in cold plasmas

Simulation Results

Test run with low density, isotropic Maxwellian to create thermal noise
→ no growth expected

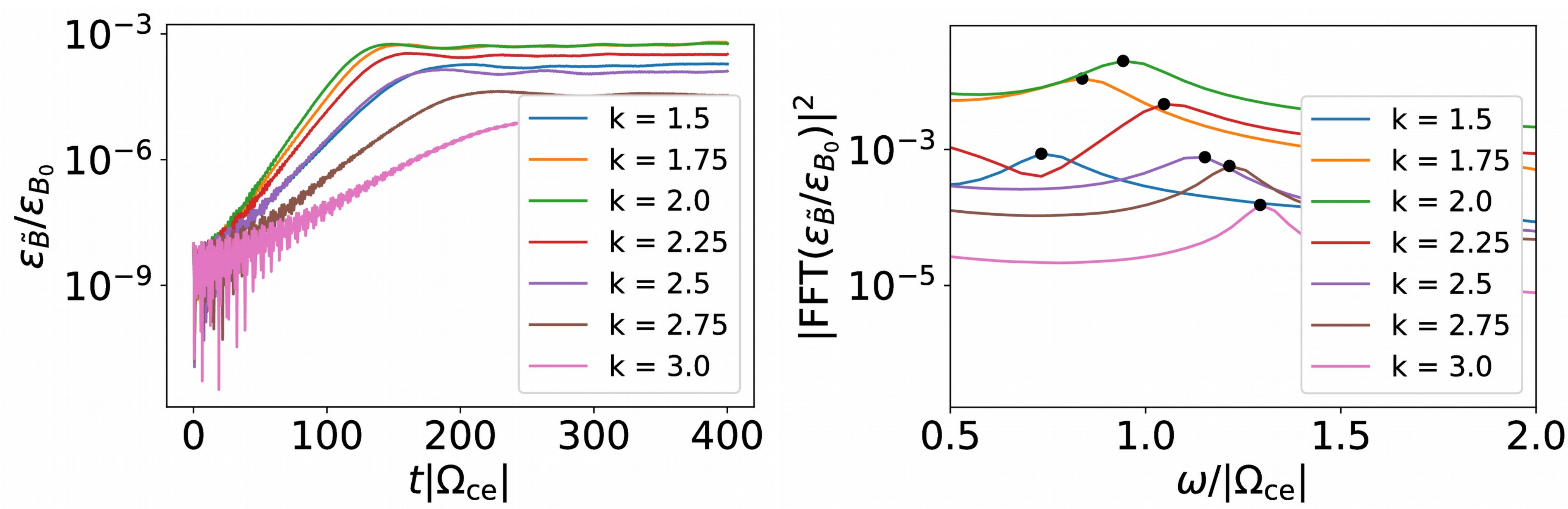




Electron Hybrid model for R/L-waves in cold plasmas

Simulation Results:

Runs with perturbed magnetic field of the form $B_x^0(z) = a \cdot \sin(kz)$ for different wavenumbers



Electron Hybrid model for R/L-waves in cold plasmas

Simulation Results:

Runs with perturbed magnetic field of the form $B_x^0(z) = a \cdot \sin(kz)$ for different wavenumbers