Braun - Stanley Layer
Gauss's Law, Relaxation, and Charge Bookkeeping in a Three-Terminal Water Fuel Cell

Control-volume analysis, measurement KPIs, and design implications for displacement-driven operation in a modified Stanley Meyer Water Fuel Cell system

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Abstract

We analyze a three-terminal water fuel cell (WFC) comprising two fully insulated drive plates and a single contact (extraction) electrode. The drive plates couple displacement charge into the water through solid dielectrics, while the extraction electrode is the sole Faradaic port during relaxation. Using three Gaussian control volumes—water, each insulated plate, and the external reservoir—we show how Gauss's law and global charge conservation are satisfied at all times. The analysis clarifies why the charging circuit largely 'tops up' polarization rather than supplying Faradaic coulombs directly, and how OFF-window isolation hinges on minimizing stray capacitance on the cell side while routing all chemistry through the extraction leg. We provide measurement procedures for Coulomb-accurate gas benchmarking and practical design guidance for maximal displacement charging with hard-open OFF behavior.

1. Introduction

High-field pulsed operation of insulated-electrode WFCs promises low upstream I^2R losses while confining Faradaic chemistry to well-controlled interfaces. Confusion often arises regarding how Gauss's law, displacement currents, and Faradaic currents reconcile during the OFF (relaxation) window, particularly when the power stage is isolated by a 'last-in-line' blocking diode. This paper formalizes the bookkeeping using control-volume electromagnetics and provides experimentally actionable KPIs.

2. Control Volumes and Boundary Conditions

Define three Gaussian 'boxes':

A) Water volume V_w (all liquid):

Gauss (integral form): ∯ _{∂V_w} D·da = Q_{free,in water}.

The boundary ∂V_w includes: (i) water–insulator (plates) with no free surface charge so D_n is continuous; (ii) water–metal (extraction electrode) where D_n equals the free surface charge density on the metal with opposite sign to the double layer in water.

B) Each insulated plate (metal + solid dielectric):

∮ D·da = Q_plate. With ideal insulation and no external conduction path, Q_plate is (nearly) constant during OFF; any tiny change occurs via parasitic capacitances to the world.

C) External reservoir (ground/catch capacitor):

Accumulates the charge delivered by the extraction current during relaxation.

3. OFF-Window (Relaxation) Dynamics

With the blocking SiC diode placed last on the secondary (closest to the cell), the drive leg is open in OFF.

• Extraction current flows only at the contact electrode; water loses or gains free charge with current I_ext(t) crossing that interface. • Water continuity (lumped form): d/dt Q_{free,in water}(t) + I_leak→plates(t) + I_ext(t) = 0.

For ideal insulation I_leak≈0, hence I_ext(t) = − d/dt Q_{free,in water}(t).

• Field/flux re-routing: The flux ∮ _{∂V_w} D·da must equal Q_{free,in water}. As Q_{free,in water} changes, the D-flux across the extraction interface changes by the same amount via the double layer. The flux through insulated-plate interfaces can remain essentially unchanged if the plates are well-isolated (Q_plate fixed), i.e., plate potentials ‘float’ to preserve the dielectric field.

• External neutrality: The charge leaving the water through the extraction electrode appears on an external reservoir node. d/dt Q_reservoir(t) = +I_ext(t). Global charge is conserved; a large Gaussian surface around the apparatus shows no paradox.

4. Why the Insulated Plates Do Not ‘Dump’ Charge

At a clean dielectric–water boundary there is no free surface charge; D_n is continuous and no Faradaic current crosses. Plate response is electrostatic: due to small plate-to-world capacitance C_pg, plate potential V_p can move when water potential V_w moves, but any ΔQ_p is pure displacement via C_pg.

A useful two-capacitor model for one plate: Q_p = C_pw (V_p − V_w) + C_pg V_p.

During OFF, Q_p is nearly constant. When V_w shifts (via extraction), V_p adjusts

so the dielectric field stays almost unchanged provided C_pg ≪ C_pw.

5. Measurement That Proves the Bookkeeping

• Extraction leg shunt: measure I_ext(t). The Faradaic tally is Q_Faradaic = ∫ I_ext dt, which must map to gas via Faraday.
• Drive leg shunt (cell side of the blocking diode): should collapse to diode leakage in OFF; otherwise the drive still participates in relaxation.
• Optional potential probe: a high-Z probe to water (vs. ground) will show water potential drifting in OFF; insulated plates drift with it, confirming 'plates follow, they don't feed'.

6. Practical Design Implications

• Put the **blocking SiC diode last** and physically closest to the cell; select low C_j at your reverse bias.
• Keep HV copper/planes away from the post-diode node; maximize air gaps, use low-ε_r supports (PTFE/PEEK/PP).
• Avoid interleaved windings; use sectional/split bobbin to minimize primary↔secondary capacitance C_ps.
• No snubbers/TVS across the cell; clamp and recover energy only on the primary side. • Count coulombs at the extraction leg; tune bursts for maximum Wh/L improvement while staying below streamer onset.

7. Useful Relations and KPIs

• Faraday link: ṅ_{H2} = I/(2F). Practical: V̇_{H2}[L/min] ≈ 0.00697·I[A]. • Energy density (field storage in water): u = ½ ε_0 ε_r E^2. • OFF-state bleed (cell HV node): Q_bleed ≈ C_eq ΔV, I_avg ≈ Q_bleed f. With last-in-line diode, C_eq ≈ (C_j · C_ps)/(C_j + C_ps) + C_direct. • Plate tracking (parasitics): Δ(V_p−V_w) = −(C_pg/(C_pw + C_pg)) ΔV_w.

Appendix A — Diode-Limited OFF-State Capacitance

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Let the HV node after the blocking diode see the world only through: the diode junction capacitance C_j, the transformer primary–secondary capacitance C_ps (in series with C_j), and any direct parasitic C_direct to ground.
Series combination gives C_series = (C_j · C_ps) / (C_j + C_ps). The effective OFF-state capacitance is then C_eq ≈ C_series + C_direct. If you choose C_j ≪ C_ps, then C_eq ≈ C_j + C_direct — the diode sets the bleed.

Appendix B — Faraday Conversion (A → L/min H_2 at STP)

Appendix B — Faraday Conversion (A → L/min H_2 at STP) Hydrogen production rate from current I is $\dot{n}_{H2} = I / (2F)$ (two electrons per H_2).
Converting mol/s to L/min at STP (22.414 L/mol):
$\dot{V}_{H2} = (I / (2F)) \cdot 22.414 \cdot 60 \approx 0.006969 \cdot I$ (L/min). Example: 1.00 A → 6.97 mL/s ≈ 0.418 L/min H_2 (theoretical, 100% Faraday).

Appendix C — Nomenclature

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V_w: water potential; V_p: plate potential; Q_p: plate charge; D: electric flux density;
C_pw: plate-to-water capacitance through solid dielectric; C_pg: plate-to-world parasitic;
C_j: diode junction capacitance; C_ps: primary–secondary capacitance;
C_direct: direct parasitic to ground;
F: Faraday constant; I_ext: extraction-leg current; Q_{free,in water}: net free charge in water.

Appendix D — Measurement Checklist (Burst Mode)

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1) Extraction shunt: log V(t), I_ext(t), and integrate Coulombs per burst and per minute.
2) Drive-leg shunt (post-diode): verify OFF-window current collapses to diode leakage; no opposite-polarity tails.
3) Gas flow: calibrate to STP; track humidity/temperature; compute Faraday yield η_F = (measured $\dot{n}_{H2}$) / (I/(2F)).
4) Sweep duty/field below streamer onset; compare Wh/L vs a DC benchmark at the same Coulombs/min.

References

[1] Maxwell's Equations in Integral Form; standard EM texts.
[2] Butler–Volmer relation; standard electrochemistry texts.
[3] Prior discussion notes and practical derivations (this preprint series).

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