Give appropriate justifications for your answers.

1. For each of the following parameter values either determine the remaining parameters and give an example of a (v, b, r, k, λ)-BIBD with these parameters, or show that no such BIBD exists.

(a) $r = 4$, $k = 11$, $\lambda = 2$.

(b) $b = 30$, $r = 6$, $k = 5$.

(c) $v = 46$, $b = 10$, $\lambda = 2$.

2. Consider a symmetric balanced design with parameters (v, b, r, k, λ) with $v = 40$. Show that $\lambda \in \{4, 18, 38\}$.
(You do not have to construct BIBDs with these parameters.)

3. Consider a BIBD with parameters (v, b, r, k, λ).

(a) Show that if $\lambda = 2$, then $v \leq \binom{r}{2} + 1$.

(b) Show that if $\lambda = 2$, $r = 7$, and $k > 1$, then $v = 15$.
(Again, you do not have to construct BIBDs with these parameters.)

4. Prove that Construction 4.31 in the course notes works; in other words, show that if a line ℓ and all points incident to ℓ are removed from a projective plane, the result is an affine plane.

5. Construct a set of 6 MOLS of order 7 (you do not need to write them out in full, it suffices to give a formula for the entry in position i, j and show that this is a set of MOLS).

6. Consider the following partially filled latin square

1	2	3	4	5
2	1	5	3	4

(a) Find a completion.

(b) Show that there is no completion which has an orthogonal mate.
Hint: use Theorem 4.63.