Robert Christie

# MATHS 340

**S2 2023**

Assignment 3

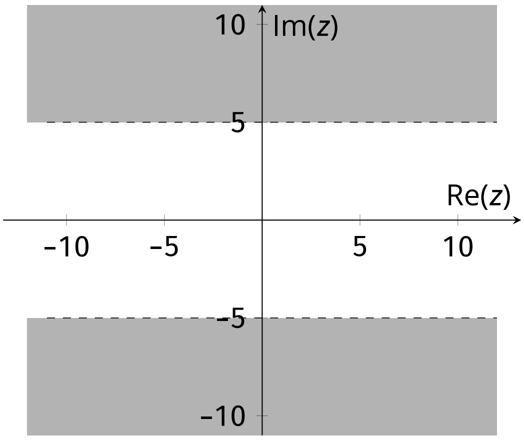
Due: 26-08-2023

## Q1

(a) Let $z = x + iy$, then:

$$|z + \bar{z}| = |2x|$$

Hence the region corresponds to $|x| > 5$ (Due to the strict inequality boundary points are not included.)




(b) Let $z = re^{i\theta}$. Note that at $z = 0$, the region's existence is not well-defined. However, in the limit as $z \to 0$, $\frac{1}{|z|} \to +\infty$, while $|\bar{z}| \to 0$, so we redefine the region $R$ as:

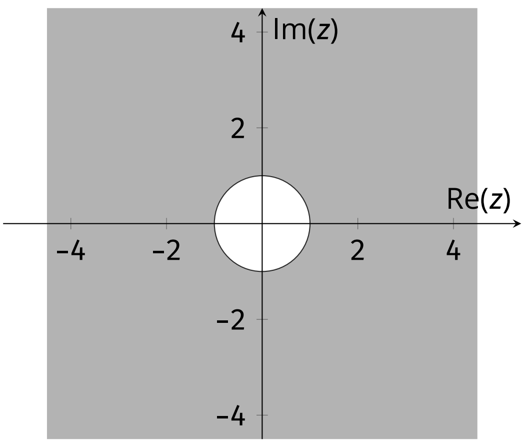
$$R = \left\{ z \in \mathbb{C} \setminus \{0\} \;\middle|\; \frac{1}{|z|} \leq |\bar{z}| \right\} \subset \mathbb{C}$$

Since $\bar{z} = re^{-i\theta}$, $|z| = |\bar{z}| = r$, and:

$$\frac{1}{r} \leq r \iff 1 \leq r^2 \iff 1 \leq r$$

Thus the region is given by:

$$R = \left\{ re^{i\theta} \in \mathbb{C} \setminus \{0\} \;\middle|\; 1 \leq r \right\}$$

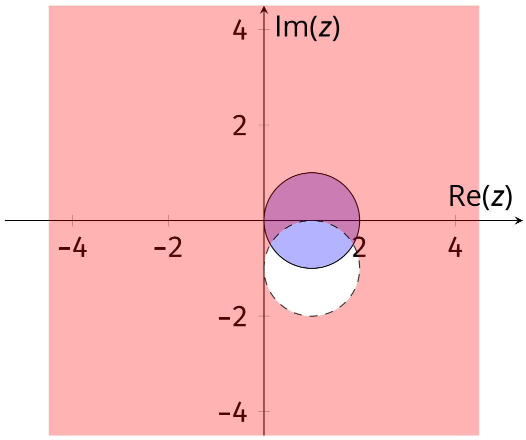



(c) Let $z = x + yi$, then:

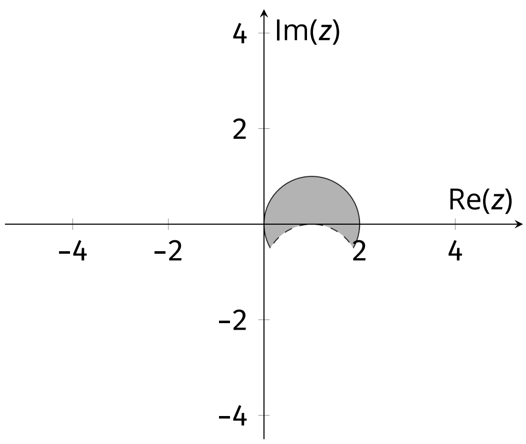
$$R_1 = \left\{ x + yi \in \mathbb{C} \;\middle|\; \sqrt{(x-1)^2 + y^2} \leq 1 \right\}$$
$$R_2 = \left\{ x + yi \in \mathbb{C} \;\middle|\; \sqrt{(x-1)^2 + (y+1)^2} > 1 \right\}$$

Thus we see that $R_1$, is a closed circle of radius 1 about $z = 1 + 0i$ and $R_2$ is all of $\mathbb{C}$ excluding a circle of radius 1 around $z = 1 - 1i$, the boundary of $R_2$ is open.

Plotting $R_1$ (blue) and $R_2$ (red):




Thus, plotting the intersection of the regions:




Note that the points where the two boundaries meet are not included in the region as they are closer that 1 to $1 - i$ and thus not part of $R_2$ and hence not present in $R_1 \cap R_2$.