- Your assignment must be submitted on Canvas.
- Late assignments will **not** be marked.
- Some marks for this assignment will be awarded for the quality of explanation you give. Make sure your results are presented in a logical manner, using clear English, and that your ideas are easy to follow. You must show your working to obtain full marks.
- Unless otherwise noted, you should work out the problems by hand.
- In some problems, we do allow (or require) you to use MATLAB to solve them. In this case, you must **include your MATLAB code** with your submission. Please make sure that your code is commented to make it clear what you are doing (you can use the % symbol to add comments to your code).
- If you are prevented from completing your assignment by circumstances beyond your control (e.g, illness) speak to or email the course coordinator as soon as possible.

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1. (10 marks) Consider the surface defined implicitly by the equation $f(x, y, z) = 0$ where f is given by
$$f(x, y, z) = x^2 + 2y^2 - 5z^2 - 4$$
   (a) Find the tangent plane to the surface at the point $(x, y, z) = (1, 2, 1)$.
   (b) Plot both the surface and the tangent plane from (a) using MATLAB. Make sure to rotate your plot suitably so it can be seen that the plane is indeed tangent to the surface.
   (c) Why does it not make sense to ask for the equation of the tangent plane at $(x, y, z) = (0, 0, 1)$?

   Include your code and all your resulting plots with your solutions.

2. (17 marks) Let x and y be the usual Cartesian co-ordinates, and consider the hyperbolic coordinates defined by
$$u = \ln\sqrt{\frac{x}{y}}, \quad v = \sqrt{xy}$$
   or alternatively,
$$x = v\mathrm{e}^{u}, \quad y = v\mathrm{e}^{-u}$$
   (a) Find the unit bases vectors $\mathbf{e}_u$ and $\mathbf{e}_v$ in terms of u, v and the standard Cartesian basis vectors $\mathbf{e}_x$ and $\mathbf{e}_y$.
   (b) Show that
$$\mathbf{e}_u + \mathbf{e}_v = 2(1 + \mathrm{e}^{-4u})^{-1/2}\mathbf{e}_x$$
   and find a similar expression for $\mathbf{e}_u - \mathbf{e}_v$.

(c) Suppose that a particle moves along a curve defined by $\mathbf{R}(t) = u(t)\mathbf{e}_u + v(t)\mathbf{e}_v$. Find an expression for the velocity $\mathbf{R}'(t)$ in terms of u, v, their derivatives with respect to time, $\mathbf{e}_u$ and $\mathbf{e}_v$.
You will find the expressions you derived in part (b) useful.
You are welcome to use MATLAB to help simplify the algebraic expressions if you wish; please include any MATLAB code you use if you do.

3. (8 marks) Consider a wire bent into the shape of a quarter of the ellipse with equation $x^2 + 2y^2 = 4$ which lies in the positive quadrant. The linear density of the wire is given by $\rho(x, y) = xy$.

(a) Show that the wire can written as a parametrised curve in the form:

$$\mathbf{r}(t) = \begin{pmatrix} \alpha \cos t \\ \beta \sin t \end{pmatrix}$$

and find appropriate values for α and β.

(b) Find the mass of the wire.
If you use MATLAB or any other computational algebra software to help you compute the integral, include that information here and the code you used to generate your solutions. You should still include all the steps required to solve the integrals by hand.