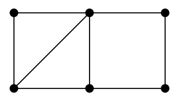
DEPARTMENT OF MATHEMATICS

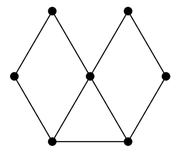
MATHS 326 | Assignment 1 | Due: 22/03/2024

All working should be complete and **your own work, written in your own words**.

1. Determine the chromatic polynomials of the graphs drawn below:



(a)



(b)

2. Let G be a graph and let $P_G(x)$ be its chromatic polynomial.

(a) Prove that the coefficient of x in $P_G(x)$ is non-zero if and only if G is connected.

(b) Use part (a) to show that the word 'connected' can be ommited in the statement of Corollary 1.15. In other words, prove that a simple graph on n vertices is a tree if and only if its chromatic polynomial is $x(x-1)^{n-1}$.

3. Prove that none of the following polynomials is the chromatic polynomial of a graph.

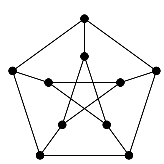
(a) $(x-1)^4$

(b) $x^6 - 6x^5 + 7x^3 - 2x$

(c) $x^4 - 3x^3 + 4x^2 - 2x$

4. Prove that every planar graph with less then 12 vertices has a vertex of degree at most 4. Show that this is sharp by finiding a planar graph on 12 vertices which has no such vertex.

5. Let P be the Petersen graph (drawn below).



(a) Determine the chromatic number and the edge chromatic number of P.

(b) Using Wagner's theorem or otherwise, show that P is not planar.

6. Prove that a graph G is bipartite if and only if every subgraph H of G satisfies $\alpha(H) \geq \frac{1}{2}m_H$, where m_H denotes the number of vertices of H.