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# MATHS 340

**S2 2023**

Assignment 1
Due: 1-08-2023

## Q1

(a) We can see that the surface $S$, is defined implicitly $f(x,y,z)=0$ where $f:\mathbb{R}^3\to\mathbb{R}$ is continuously differentiable (as it is a polynomial). Thus, we can see that as $(a,b,c)=(1,2,1)$ is a surface point, the tangent plane is given by:

$$\begin{aligned}0 &= \frac{\partial f}{\partial x}(a,b,c)(x-a)+\frac{\partial f}{\partial y}(a,b,c)(y-b)+\frac{\partial f}{\partial z}(a,b,c)(z-c)\\ &= 2a(x-a)+4b(y-b)-10c(z-c)\\ &= 2(x-1)+8(y-2)-10(z-1)\\ &= 2x-2+8y-16-10z+10\\ &= 2x+8y-10z-8\end{aligned}$$

(b) MATLAB

(c) We are considering a surface defined by a function $f(x,y,z)=0$,

## Q2

(a) Consider the function $\zeta:\begin{bmatrix}u\\v\end{bmatrix}\mapsto\begin{bmatrix}x\\y\end{bmatrix}$. From the definitions given we know:

$$\zeta:\begin{bmatrix}u\\v\end{bmatrix}\mapsto\begin{bmatrix}ve^{u}\\ve^{-u}\end{bmatrix}$$

Thus we find that:

$$D\zeta=\left[\begin{array}{c|c}\frac{\partial\zeta}{\partial u} & \frac{\partial\zeta}{\partial v}\end{array}\right]=\begin{bmatrix}ve^{u} & e^{u}\\ -ve^{-u} & e^{-u}\end{bmatrix}$$

The columns of $D\zeta$ give us a moving frame of basis vectors, which we normalise to give unit vectors:

$$e_u = \frac{\begin{bmatrix} ve^u \\ -ve^{-u} \end{bmatrix}}{\sqrt{v^2e^{2u} + v^2e^{-2u}}} = \frac{\begin{bmatrix} e^u \\ -e^{-u} \end{bmatrix}}{\sqrt{e^{2u} + e^{-2u}}} = \begin{bmatrix} \sqrt{\frac{e^{2u}}{e^{2u}+e^{-2u}}} \\ -\sqrt{\frac{e^{-2u}}{e^{2u}+e^{-2u}}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{1}{\frac{e^{2u}+e^{-2u}}{e^{2u}}}} \\ -\sqrt{\frac{1}{\frac{e^{2u}+e^{-2u}}{e^{-2u}}}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{-1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

$$e_v = \frac{\begin{bmatrix} e^u \\ e^{-u} \end{bmatrix}}{\sqrt{e^{2u} + e^{-2u}}} = \begin{bmatrix} \sqrt{\frac{e^{2u}}{e^{2u}+e^{-2u}}} \\ \sqrt{\frac{e^{-2u}}{e^{2u}+e^{-2u}}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{1}{\frac{e^{2u}+e^{-2u}}{e^{2u}}}} \\ \sqrt{\frac{1}{\frac{e^{2u}+e^{-2u}}{e^{-2u}}}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

(b) See that:

$$e_u + e_v = \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{-1}{\sqrt{1+e^{4u}}} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{1}{\sqrt{1+e^{4u}}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{1+e^{-4u}}} \\ \frac{1-1}{\sqrt{1+e^{4u}}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{1+e^{-4u}}} \\ 0 \end{bmatrix} = \frac{2}{\sqrt{1 + e^{-4u}}} e_x$$

Now for $e_u - e_v$ we see that:

$$e_u - e_v = \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{-1}{\sqrt{1+e^{4u}}} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{1}{\sqrt{1+e^{4u}}} \end{bmatrix} = \begin{bmatrix} \frac{0}{\sqrt{1+e^{-4u}}} \\ \frac{-1-1}{\sqrt{1+e^{4u}}} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-2}{\sqrt{1+e^{4u}}} \end{bmatrix} = \frac{-2}{\sqrt{1 + e^{4u}}} e_y$$

(c) We find the partial derivatives of the unit vectors:

$$\frac{\partial e_u}{\partial u} = \frac{\partial}{\partial u}\begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{-1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial u}\left(1+e^{-4u}\right)^{-\frac{1}{2}} \\ -\frac{\partial}{\partial u}\left(1+e^{4u}\right)^{-\frac{1}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2}\left(1+e^{-4u}\right)^{\frac{-3}{2}}\left(0-4e^{-4u}\right) \\ -\frac{-1}{2}\left(1+e^{4u}\right)^{\frac{-3}{2}}\left(0+4e^{4u}\right) \end{bmatrix}$$

$$= 2\begin{bmatrix} e^{-4u}\left(1+e^{-4u}\right)^{\frac{-3}{2}} \\ e^{4u}\left(1+e^{4u}\right)^{\frac{-3}{2}} \end{bmatrix}$$

$$\frac{\partial e_u}{\partial v} = \frac{\partial}{\partial v}\begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{-1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

$$= 0$$

$$\frac{\partial e_v}{\partial u} = \frac{\partial}{\partial u}\begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

$$= 2\begin{bmatrix} e^{-4u}\left(1+e^{-4u}\right)^{\frac{-3}{2}} \\ -e^{4u}\left(1+e^{4u}\right)^{\frac{-3}{2}} \end{bmatrix}$$

$$\frac{\partial e_v}{\partial v} = \frac{\partial}{\partial v}\begin{bmatrix} \frac{1}{\sqrt{1+e^{-4u}}} \\ \frac{1}{\sqrt{1+e^{4u}}} \end{bmatrix}$$

$$= 0$$

$$\begin{aligned} R'(t) &= \frac{dR(t)}{dt} \\ &= \frac{d}{dt}\Big[u(t)\cdot e_u(u(t),v(t))\Big] + \frac{d}{dt}\Big[v(t)\cdot e_v(u(t),v(t))\Big] \\ &= \left[u'(t)e_u + u(t)\cdot\frac{d}{dt}\Big[e_u(u(t),v(t))\Big]\right] + \left[v'(t)e_v + v(t)\cdot\frac{d}{dt}\Big[e_v(u(t),v(t))\Big]\right] \\ &= \left[u'(t)e_u + u(t)\cdot\left[u'(t)\cdot\frac{\partial e_u}{\partial u} + v'(t)\cdot\frac{\partial e_u}{\partial v}\right]\right] + \left[v'(t)e_v + v(t)\cdot\left[u'(t)\cdot\frac{\partial e_v}{\partial u} + v'(t)\cdot\frac{\partial e_v}{\partial v}\right]\right] \end{aligned}$$

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# Q3

(a) A

(b) The mass of the wire will be given by:

$$m = \int_0^{\frac{\pi}{2}} \rho(r(t))dt$$