## DEPARTMENT OF MATHEMATICS

MATHS 326 Assignment 4 Due: 24/05/2024

Give appropriate justifications for your answers.

- 1. For each of the following parameter values either determine the remaining parameters and give an example of a  $(v, b, r, k, \lambda)$ -BIBD with these parameters, or show that no such BIBD exists.
  - (a)  $r = 4, k = 11, \lambda = 2.$
  - (b) b = 30, r = 6, k = 5.
  - (c)  $v = 46, r = 10, \lambda = 2.$
- 2. Consider a symmetric balanced design with parameters  $(v, b, r, k, \lambda)$  with v = 40. Show that  $\lambda \in \{4, 18, 38\}$ . (You do not have to construct BIBDs with these parameters.)
- **3.** Consider a BIBD with parameters  $(v, b, r, k, \lambda)$ .
  - (a) Show that if  $\lambda = 2$ , then  $v \leq {r \choose 2} + 1$ .
  - (b) Show that if  $\lambda=2,\,r=7,$  and k>1, then v=15. (Again, you do not have to construct BIBDs with these parameters.)
- **4.** Prove that Construction 4.31 in the course notes works; in other words, show that if a line  $\ell$  and all points incident to  $\ell$  are removed from a projective plane, the result is an affine plane.
- **5.** Construct a set of 6 MOLS of order 7 (you do not need to write them out in full, it suffices to give a formula for the entry in position i, j and show that this is a set of MOLS).
- **6.** Consider the following partially filled latin square

1	2	3	4	5
2	1	5	3	4

- (a) Find a completion.
- (b) Show that there is no completion which has an orthogonal mate.

Hint: use Theorem 4.63.