Kaunas University of Technology

Faculty of Mathematics and Natural Sciences

Cryptology

2nd laboratory work report

Marius Arlauskas

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Date: 2022.12.09 16:41:32 +02'00'

Marius Arlauskas 2022-12-09

Student

Assoc. prof. dr. Kęstutis Lukšys
Lecturer

Kaunas, 2022

1. Task 1

Task.

1. Write down a set of residue classes $\mathbb{Z}p$.

Results and comments.

Liekanų klasių aibė $\mathbb{Z}_{31}$: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

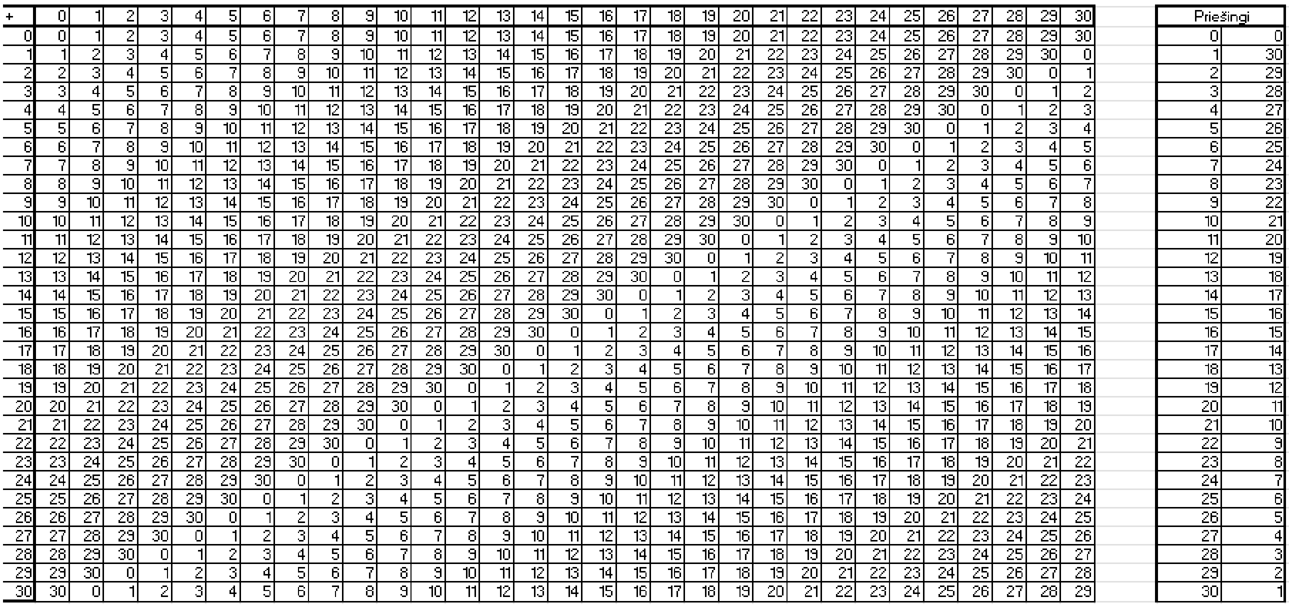
2. Task 2

Task.

Prepare addition and multiplication tables of residue ring $\langle \mathbb{Z}p; +,\cdot \rangle$. Find opposites and inverses for all elements.

Results and comments.

$\langle \mathbb{Z}_{31};\ + \rangle$, ir priešingi elementai:



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0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	0
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21	21	22	23	24	25	26	27	28	29	30	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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Priešingi	
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4	27
5	26
6	25
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9	22
10	21
11	20
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16	15
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19	12
20	11
21	10
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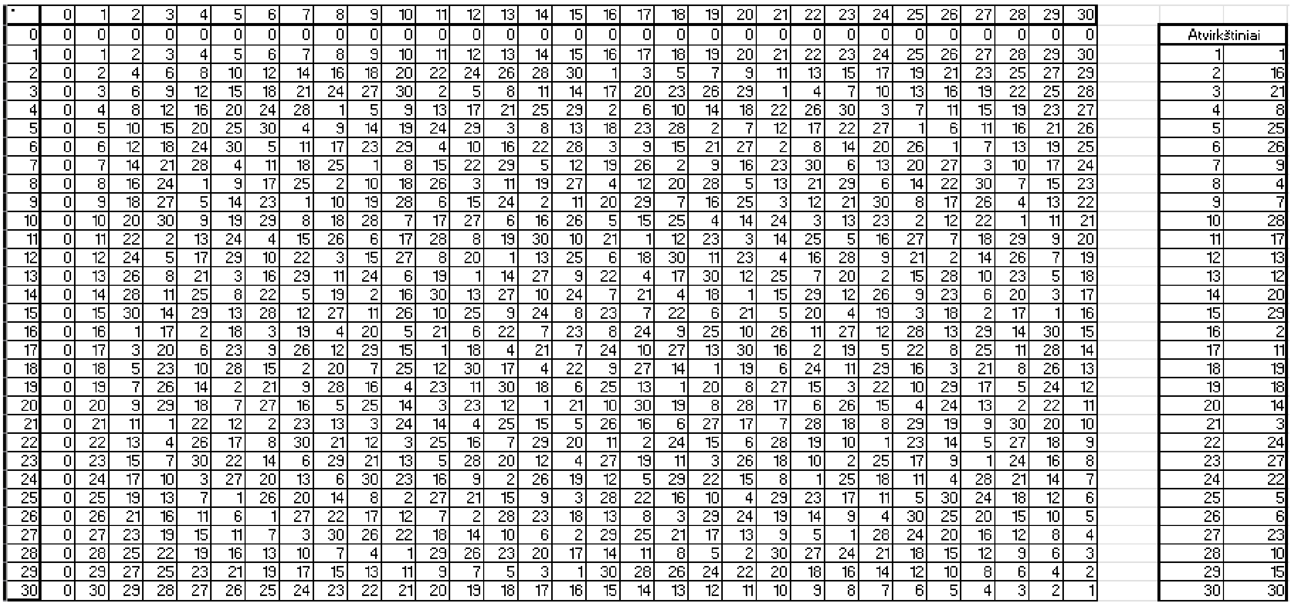
23	8
24	7
25	6
26	5
27	4
28	3
29	2
30	1

$\langle \mathbb{Z}_{31}; \cdot \rangle$, ir priešingi elementai:



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7	9
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9	7
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13	12
14	20
15	29
16	2
17	11
18	19
19	18
20	14
21	3
22	24
23	27
24	22
25	5
26	6
27	23
28	10
29	15
30	30

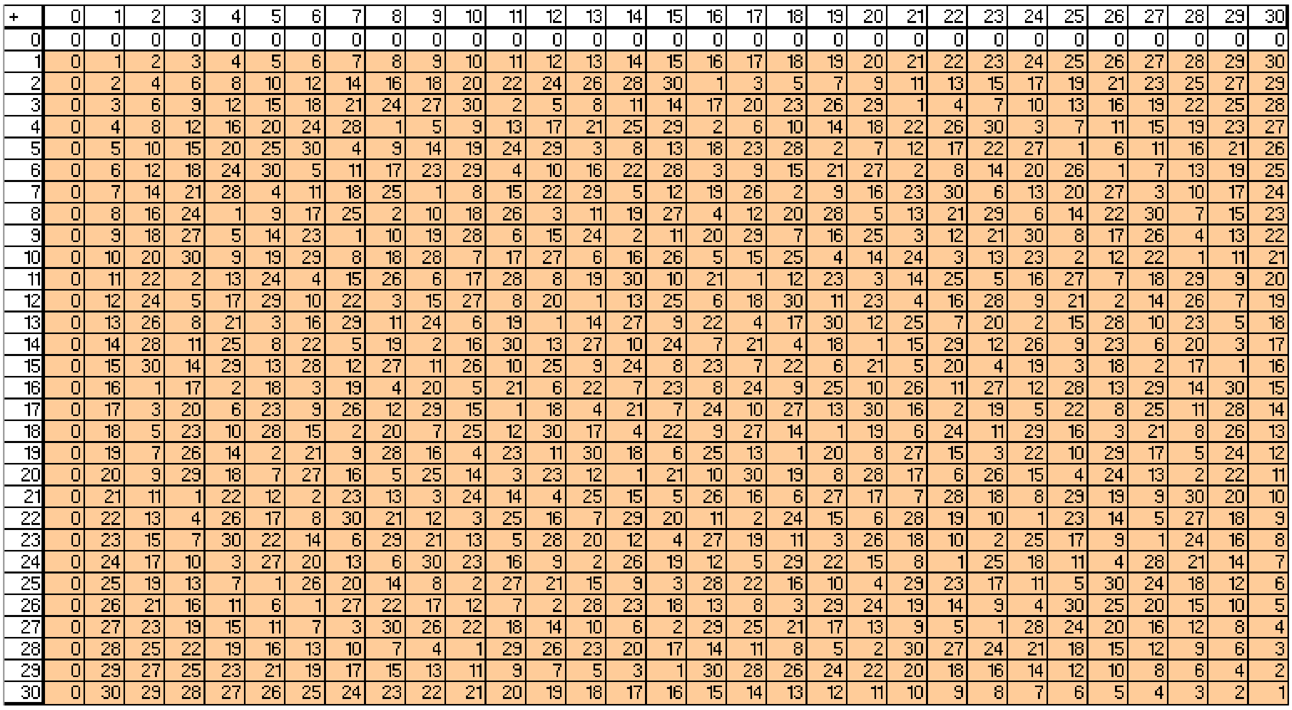
3. Task 2

Task.

How many generators are in group $\langle \mathbb{Z}p; +\rangle$? Find them.

Results and comments.

Sudėties operacijos laipsnių lentelė:



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30	0	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Generatoriai nustatomi iš laipsnių lentelės, kurioje reikšmės apskaičiuojamos moduliu 31 pvz: $2^4\,mod(31) = 2 + 2 + 2 + 2\,mod(31) = 2 \cdot 4\,mod(31)$. Visi sudietės aibės elementai yra generatoriai, nes sugeneruoja visus aibės elementus. Išskyrus 0, nes $0^n\ = 0 \cdot n\,mod(31)\ =\ 0$

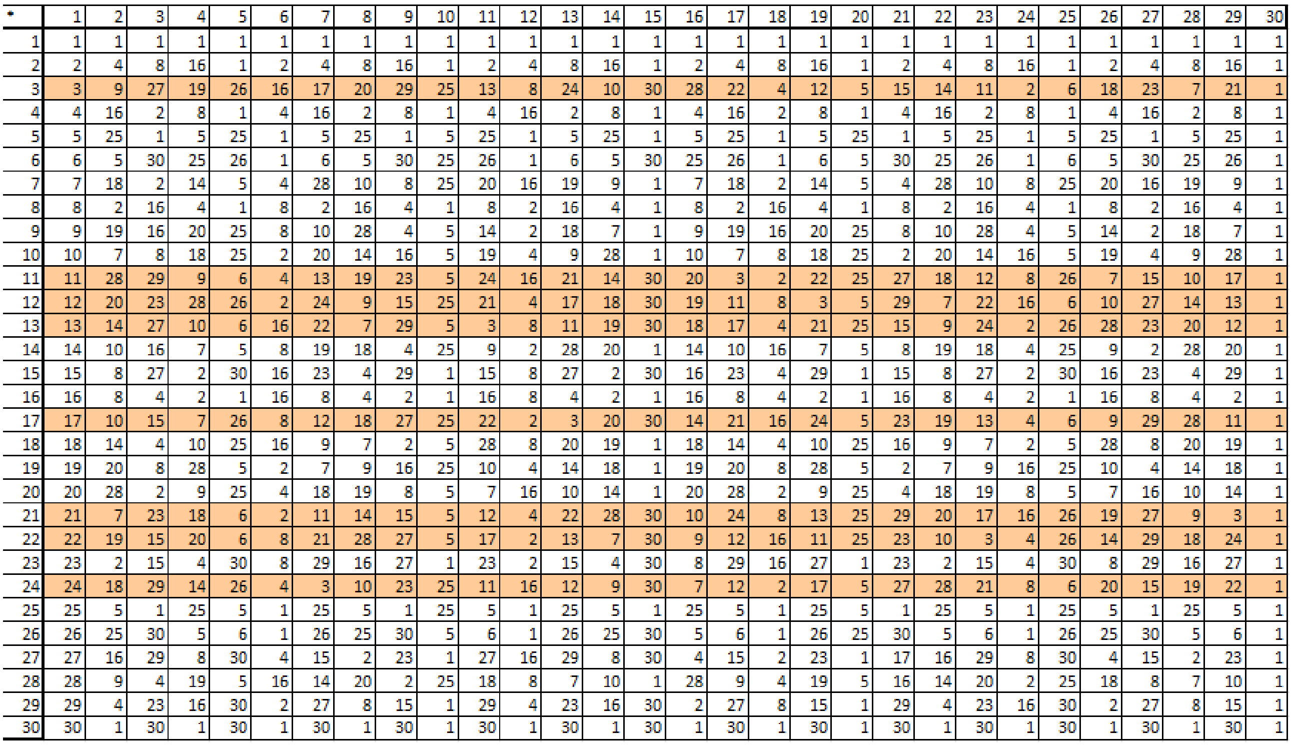
4. Task 2

Task.

How many generators are in group $\langle \mathbb{Z}p * ;\cdot\rangle$? Find them

Results and comments.

Daugybos operacijos laipsnių lentelė:



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7	7	18	2	14	5	4	28	10	8	25	20	16	19	9	1	7	18	2	14	5	4	28	10	8	25	20	16	19	9	1
8	8	2	16	4	1	8	2	16	4	1	8	2	16	4	1	8	2	16	4	1	8	2	16	4	1	8	2	16	4	1
9	9	19	16	20	25	8	10	28	4	5	14	2	18	7	1	9	19	16	20	25	8	10	28	4	5	14	2	18	7	1
10	10	7	8	18	25	2	20	14	16	5	19	4	9	28	1	10	7	8	18	25	2	20	14	16	5	19	4	9	28	1
11	11	28	29	9	6	4	13	19	23	5	24	16	21	14	30	20	3	2	22	25	27	18	12	8	26	7	15	10	17	1
12	12	20	23	28	26	2	24	9	15	25	21	4	17	18	30	19	11	8	3	5	29	7	22	16	6	10	27	14	13	1
13	13	14	27	10	6	16	22	7	29	5	3	8	11	19	30	18	17	4	21	25	15	9	24	2	26	28	23	20	12	1
14	14	10	16	7	5	8	19	18	4	25	9	2	28	20	1	14	10	16	7	5	8	19	18	4	25	9	2	28	20	1
15	15	8	27	2	30	16	23	4	29	1	15	8	27	2	30	16	23	4	29	1	15	8	27	2	30	16	23	4	29	1
16	16	8	4	2	1	16	8	4	2	1	16	8	4	2	1	16	8	4	2	1	16	8	4	2	1	16	8	4	2	1
17	17	10	15	7	26	8	12	18	27	25	22	2	3	20	30	14	21	16	24	5	23	19	13	4	6	9	29	28	11	1
18	18	14	4	10	25	16	9	7	2	5	28	8	20	19	1	18	14	4	10	25	16	9	7	2	5	28	8	20	19	1
19	19	20	8	28	5	2	7	9	16	25	10	4	14	18	1	19	20	8	28	5	2	7	9	16	25	10	4	14	18	1
20	20	28	2	9	25	4	18	19	8	5	7	16	10	14	1	20	28	2	9	25	4	18	19	8	5	7	16	10	14	1
21	21	7	23	18	6	2	11	14	15	5	12	4	22	28	30	10	24	8	13	25	29	20	17	16	26	19	27	9	3	1
22	22	19	15	20	6	8	21	28	27	5	17	2	13	7	30	9	12	16	11	25	23	10	3	4	26	14	29	18	24	1
23	23	2	15	4	30	8	29	16	27	1	23	2	15	4	30	8	29	16	27	1	23	2	15	4	30	8	29	16	27	1
24	24	18	29	14	26	4	3	10	23	25	11	16	12	9	30	7	12	2	17	5	27	28	21	8	6	20	15	19	22	1
25	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1	25	5	1
26	26	25	30	5	6	1	26	25	30	5	6	1	26	25	30	5	6	1	26	25	30	5	6	1	26	25	30	5	6	1
27	27	16	29	8	30	4	15	2	23	1	27	16	29	8	30	4	15	2	23	1	17	16	29	8	30	4	15	2	23	1
28	28	9	4	19	5	16	14	20	2	25	18	8	7	10	1	28	9	4	19	5	16	14	20	2	25	18	8	7	10	1
29	29	4	23	16	30	2	27	8	15	1	29	4	23	16	30	2	27	8	15	1	29	4	23	16	30	2	27	8	15	1
30	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1	30	1

Generatorius, tai elementas, kuris sugeneruoja visus kitus grupės elementus, tai matome, kad šio atveju generatoriai yra 3, 11, 12, 13, 17, 21, 22, 24.

5. Task 2

Task.

Find subgroups of group $\langle \mathbb{Z}p * ;\cdot\rangle$ where all elements except 1 are generators and prove it (subgroup, generators).

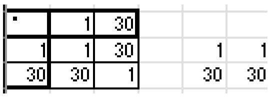
Results and comments.

Kad pogrupis būtų grupės $\langle \mathbb{Z}_{31} * ;\cdot\rangle$ pogrupiu, jis turi tenkinti šias sąlygas:

- Yra tenkinamas uždarumas (Dviejų pogrupio elementų sandauga priklauso pogrupiui).
- Pogrupio kiekvieno elemento atvirkštinis elementas priklauso pogrupiui. Taipogi pagal sąlygą visi elementai turi būti pogrupio generatoriais.

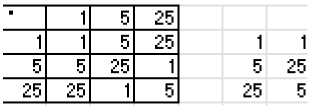
Rasti pogrupiai:

1) *{1, 30} – tenkina abi sąlygas.*



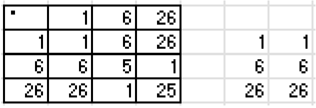
·	1	30			
1	1	30		1	1
30	30	1		30	30

2) {1, 5, 25} – *tenkina abi sąlygas.*



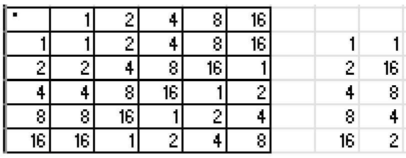
·	1	5	25			
1	1	5	25		1	1
5	5	25	1		5	25
25	25	1	5		25	5

3) {1, 6, 26} – *tenkina abi sąlygas.*



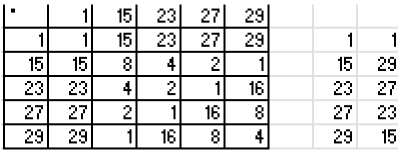
·	1	6	26			
1	1	6	26		1	1
6	6	5	1		6	6
26	26	1	25		26	26

4) {1, 2, 4, 8, 16} – *tenkina abi sąlygas.*



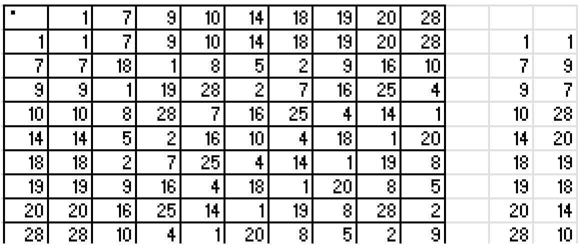
·	1	2	4	8	16			
1	1	2	4	8	16		1	1
2	2	4	8	16	1		2	16
4	4	8	16	1	2		4	8
8	8	16	1	2	4		8	4
16	16	1	2	4	8		16	2

5) {1, 15, 23, 27, 29} – *netenkina sąlygų.*



·	1	15	23	27	29			
1	1	15	23	27	29		1	1
15	15	8	4	2	1		15	29
23	23	4	2	1	16		23	27
27	27	2	1	16	8		27	23
29	29	1	16	8	4		29	15

6) {1, 7, 9, 10, 14, 18, 19, 20, 28} – *netenkina sąlygų.*



·	1	7	9	10	14	18	19	20	28			
1	1	7	9	10	14	18	19	20	28		1	1
7	7	18	1	8	5	2	9	16	10		7	9
9	9	1	19	28	2	7	16	25	4		9	7
10	10	8	28	7	16	25	4	14	1		10	28
14	14	5	2	16	10	4	18	1	20		14	20
18	18	2	7	25	4	14	1	19	8		18	19
19	19	9	16	4	18	1	20	8	5		19	18
20	20	16	25	14	1	19	8	28	2		20	14
28	28	10	4	1	20	8	5	2	9		28	10