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RESEARCH ARTICLE

A new Silver–Meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing

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In a recent contribution, Teunter *et al*. [2006. Dynamic lot sizing with product returns and remanufacturing. International Journal of Production Research 44 (20), 4377–4400] adapted three well-known heuristic approaches for the single-item dynamic lot sizing problem to incorporate returning products that can be remanufactured. The Silver–Meal based approach revealed in a large numerical study the best performance for the separate setup cost setting, i.e. the replenishment options remanufacturing and manufacturing are charged separately for each order. This contribution generalises the Silver–Meal based heuristic by applying methods elaborated for the corresponding static problem and attaching two simple improvement steps. By doing this, the percentage gap to the optimal solution which has been used as a performance measure has been reduced to less than half of its initial value in almost all settings examined.

Keywords: dynamic lot sizing; Silver–Meal based heuristic; reverse logistics; remanufacturing

1. Introduction

Due to the increasing environmental awareness of firms and the public, the research field of reverse logistics has grown steadily over the past decades. By analysing not only the forward flow of products from a firm to its customers but also including the corresponding backward flow from the customers to the firm, this research area provides valuable insights on how these flows can be managed efficiently. Among many options (see, e.g., Thierry *et al*. 1995), remanufacturing has been well established in several industries as has been reported in Kumar and Putnam (2008). When including remanufactured products in their product portfolio, firms take back products from their customers, rework them to a sufficient condition in order to resell them afterwards. This saves not only a part of the value embedded in the original product but also reduces the demand for natural resources and landfill space substantially (de Brito and Dekker 2004). In industry, the process of

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remanufacturing is affected by many stochastic influences as has been depicted, for instance, by Guide (2000) as well as Inderfurth and Langella (2006). As these influences complicate the underlying problem significantly, this contribution neglects any uncertainty and presents an entirely deterministic system.

By assuming setup costs for replenishment orders and holding costs for carrying products in different conditions, a lot sizing problem arises. Such a problem has been analysed thoroughly for the case of static and continuous demand and return rates (see e.g. Minner and Lindner (2004) as well as Schulz and Ferretti (2008) for a brief literature review). However, the case of dynamic and discrete demands and returns has not achieved much attention in the recent literature. Teunter *et al.* (2006) introduce a dynamic lot sizing model with returns and remanufacturing and distinguish between the case of joint and separate setup costs for the replenishment sources remanufacturing and manufacturing. They test in a large numerical experiment three well-known heuristic approaches that were adapted from the single-item dynamic lot sizing problem without returns. In both model settings, the Silver–Meal based heuristic has been shown to be the best heuristic resulting in an average deviation of 3% from the optimal solution in the joint and 8.3% in the separate setup cost setting. Using heuristics to handle these problems has been motivated by the fact that the authors conjecture the underlying problem of the separate setup cost setting to be NP-hard.

Several other contributions have been made to this specific research field whereas only two shall be mentioned exemplarily. Richter and Sombrutzki (2000) discuss the dynamic lot sizing problem with returns and remanufacturing and analyse a situation in which a sufficiently large number of returned products are available, i.e. the entire demand could be met by solely remanufacturing returned products. They prove that the zero-inventory property known from the dynamic lot sizing problem without returns and remanufacturing must hold in such an environment. Furthermore, they apply a Silver–Meal based algorithm to illustrate the stability of its solution. This contribution employs a Silver–Meal based algorithm to the situation when the entire demand can only be met by a mix of remanufacturing and manufacturing. As illustrated by Teunter *et al.* (2006), the zero-inventory property need not be valid in such a setting. Pan *et al.* (2009) extend the analysis of Teunter *et al.* (2006) by including a disposal option for returned products and by restricting production, remanufacturing and disposal capacities. They illustrate different problem formulations and elaborate dynamic programming algorithms to solve some of these problems to optimality.

This study proposes a generalisation of the Silver–Meal based heuristic introduced by Teunter *et al.* (2006) for the separate setup cost setting (without disposal option and restricted capacities) by applying methods known from the corresponding static problem. Furthermore, a simple improvement heuristic is applied to the solution obtained to enhance the heuristic's performance. The remainder of this contribution is organised as follows: Section 2 presents the basic assumptions of the model analysed in this study and describes some solution methods for the underlying problem context. Next to a mixed-integer linear program, the Silver–Meal based heuristic introduced in Teunter *et al.* (2006) and our extension are depicted in this section. Both heuristics are tested extensively in a numerical study in Section 3. Afterwards, Section 4 points out the improvement heuristic and tests its ideas in a numerical experiment. Finally, the last section concludes this contribution and gives a short outlook on future research opportunities.

2. Model formulation and proposed solution methods

2.1 *Basic assumptions and mixed-integer linear program*

In their contribution, Teunter *et al.* (2006) introduced a dynamic lot sizing model with separate setup costs for remanufacturing and manufacturing as an extension of the well-known Wagner/Whitin model (Wagner and Whitin 1958). The basic assumptions of this modelling approach are as follows: as depicted in Figure 1, we consider an original equipment manufacturer (OEM) that sells one product over a planning horizon of T periods. In each period $t = 1, \ldots, T$ customers demand a discrete and known amount of this product which will be further on denoted by D_t. The OEM provides each customer with the opportunity to return her product if it is broken or when she has no further use for it. Whenever a product is returned to the OEM, it is inspected to see whether it can be sufficiently remanufactured. All returns that pass the inspection (which will be denoted by R_t) are brought to a recoverables stock. Per unit time a recoverable product incurs holding costs of h^R and disposing it preliminarily is assumed to be prohibitively expensive. If required, the OEM can (by paying the setup cost K^R) remanufacture x_t^R recoverable products in period t in order to bring them to an as-good-as-new condition. Recovery is always successful. After remanufacturing, the recovered products are brought to a serviceables inventory from which the customer demand is satisfied. Yet, as it is not possible to serve the entire demand from remanufacturing returned products, the OEM can replenish his serviceables inventory alternatively by manufacturing x_t^M products in period t. Setting up a manufacturing lot in period t incurs fixed costs of K^M while holding a serviceable product for one period in the respective inventory costs h^M. Finally, the inventory level at the end of period t is denoted by y_t^R for the recoverables and y_t^M for the serviceables inventory.

By means of mixed-integer linear programming this model can be solved to optimality. Next to the notation introduced above two more decision variables are required. If a remanufacturing lot is initiated in period t (i.e. $x_t^R > 0$) the binary decision variable γ_t^R becomes one. However, if $x_t^R = 0$ the decision variable γ_t^R remains zero. Likewise, γ_t^M is set to one when a manufacturing lot is produced in period t and to zero if no product needs to be manufactured. The optimisation model can be formulated as:

$$\min C = \sum_{t=1}^{T} \left(K^R \cdot \gamma_t^R + K^M \cdot \gamma_t^M + h^R \cdot y_t^R + h^M \cdot y_t^M \right) \tag{1}$$

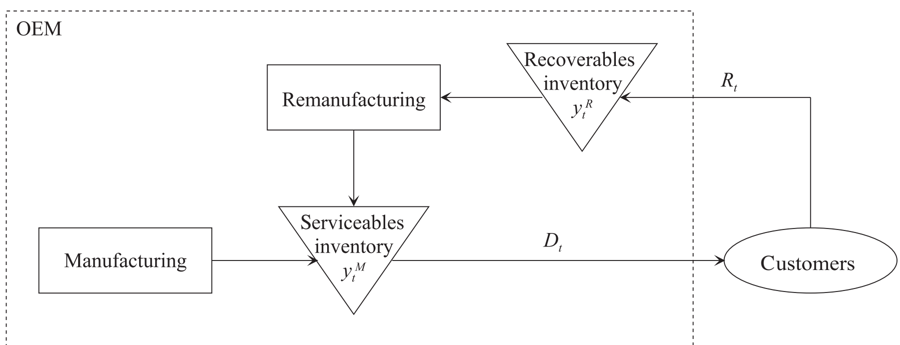

Figure 1. Dynamic lot sizing model with returns and remanufacturing.

$$s.t.: \quad y_t^R = y_{t-1}^R + R_t - x_t^R \quad \forall t = 1, \ldots, T \tag{2}$$

$$y_t^M = y_{t-1}^M + x_t^R + x_t^M - D_t \quad \forall t = 1, \ldots, T \tag{3}$$

$$x_t^R \leq Q \cdot \gamma_t^R \quad \forall t = 1, \ldots, T \tag{4}$$

$$x_t^M \leq Q \cdot \gamma_t^M \quad \forall t = 1, \ldots, T \tag{5}$$

$$y_0^R = y_0^M = 0 \tag{6}$$

$$\gamma_t^R, \gamma_t^M \in \{0, 1\} \quad \forall i = 1, \ldots, T$$
$$y_t^R, y_t^M, x_t^R, x_t^M \geq 0 \quad \forall i = 1, \ldots, T$$

The objective function (1) minimises the sum of all relevant setup and holding costs. Constraints (2) and (3) represent inventory balance equations that describe the inventory at the end of period t as the inventory at the beginning of this period plus its inflows and minus its outflows. In order to ensure that fixed costs have to be paid whenever a lot is scheduled, restrictions (4) and (5) have to be established whereas Q needs to be a sufficiently large number (e.g. the sum of all demands during the planning horizon). By imposing constraint (6) the initial inventories in both stocks are set to zero. Finally, non-negativity and binary constraints have to be defined as well to assure validity of the decisions made. Interestingly, the zero-inventory property that holds for a dynamic lot sizing model without returns and remanufacturing needs not necessarily to be valid in this model setting (as has been discussed by Teunter *et al.* 2006), i.e. it can be optimal to schedule a (re)manufacturing lot in period t even when the serviceables inventory at the beginning of t is not depleted. This extends the results of Richter and Sombrutzki (2000) who proved the zero-inventory property to hold when there is a sufficiently large number of returned products in the recoverables stock at the beginning of the planning horizon. Moreover, Teunter *et al.* (2006) conjecture that the underlying optimisation problem is NP-hard, i.e. it becomes very difficult to obtain the optimal solution for a long planning horizon. Hence, they propose several heuristic algorithms on how to handle this problem. In a large numerical study, the Silver–Meal based heuristic which will be introduced subsequently revealed the best average performance when compared to the optimal solution.

2.2 *The adapted Silver–Meal heuristic*

Unfortunately, the original Silver–Meal heuristic (Silver and Meal 1973) cannot be applied to the model context presented above as the serviceables inventory from which all customer demands are satisfied can be replenished from two sources: manufacturing and remanufacturing. Thus, Teunter *et al.* (2006) adapted the original Silver–Meal heuristic to include both sources in the form of manufacturing (option 1) as well as remanufacturing and manufacturing (option 2) in the decision-making process. The basic idea of clustering the entire planning horizon into smaller time windows (starting in period τ and ending in period z) and choosing those time windows with the smallest cost per period is kept.

However, both options that will be described subsequently assume the zero-inventory property to hold.

Option 1: Manufacture only

When applying this option the entire demand in a time window is satisfied by initiating a manufacturing run in period τ. Its lot size would be

$$x_{\tau}^{M} = \sum_{i=\tau}^{z} D_i. \tag{7}$$

The associated cost per period for the entire time window (which will be denoted as $C_{\tau,z}^{1}$) contains the setup cost for scheduling a manufacturing lot in τ as well as the cost for carrying products in the serviceables inventory. Furthermore, the cost for holding the recoverable products in stock need to be taken into account as well. This gives for the cost per period for option 1 by using Equation (2) for determining y_t^R and Equation (3) for y_t^M

$$C_{\tau,z}^{1} = \frac{K^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z - \tau + 1}. \tag{8}$$

Option 2: Remanufacture (and manufacture if necessary)

The second option introduced by Teunter *et al.* (2006) seeks to remanufacture in period τ. However, as the amount of recoverable products might not be sufficient to cover the entire demand up to period z, a manufacturing lot is set up in τ if necessary. Thus, both lot sizes depend on the available number of recoverable parts which is by definition $y_{\tau-1}^R + R_\tau$. Both lot sizes are presented in the following formulae:

$$x_{\tau}^{M} = \max\left(\sum_{t=\tau}^{z} D_t - y_{\tau-1}^{R} - R_\tau, 0\right), \quad x_{\tau}^{R} = \min\left(y_{\tau-1}^{R} + R_\tau, \sum_{t=\tau}^{z} D_t\right). \tag{9}$$

Forcing the possibly required manufacturing lot to be scheduled in τ can result in an inefficient solution when there is no immediate demand for at least one of the manufactured products. Hence, all products manufactured in τ will be held in the serviceables inventory unnecessarily until they are needed. An opportunity to overcome this deficiency will be presented later in this chapter.

Next to the holding cost for the recoverables and serviceables stock, the cost per period for the second option $C_{\tau,z}^{2}$ can contain both setup costs. As a manufacturing lot is only needed when the number of recoverable parts is not sufficient, the binary variable γ_τ^M represents this fact by being one if a manufacturing run is required and zero else. Therefore, the cost per period for the second option can be formulated as

$$C_{\tau,z}^{2} = \frac{K^R + K^M \cdot \gamma_\tau^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z - \tau + 1}. \tag{10}$$

For each time window, $C_{\tau,z}^{1}$ is compared to $C_{\tau,z}^{2}$ and the smaller one is chosen. Moreover, the basic idea of the Silver–Meal heuristic is applied which means that a time window is extended as long as the smaller cost of both options does not increase. Further on, the heuristic approach introduced by Teunter *et al.* (2006) will be referred to as the SM_2 heuristic since two distinct options are evaluated. Teunter *et al.* (2006) have tested this heuristic extensively in their contribution. As a result of their numerical study a mean deviation to the optimal solution of 8.3% was observed over all instances. By generalising the SM_2 heuristic with two additional options derived from the results of the corresponding static model we will enhance the heuristic's performance. This approach (which will be further on denoted as the SM_4 heuristic) is presented in the following.

2.3 *The SM_4 heuristic*

Although the dynamic lot sizing model with returns and remanufacturing has not been analysed extensively in the literature so far, the corresponding static model (with constant demand and return rates) has received much more attention. Among many contributions, two shall be mentioned explicitly. In his study, Schrady (1967) was the first author who examined this model context. His option on how to handle this problem effectively was to create a cyclic pattern that is repeated over the entire infinite planning horizon. This cyclic pattern begins with a manufacturing lot and is always followed by a constant number of remanufacturing lots R. Teunter (2001) generalises these findings by introducing cycles that commence with one remanufacturing lot which is always succeeded by a constant number of manufacturing lots M. He argues as well that in order to be efficient each cycle should have either one remanufacturing or one manufacturing lot. As this provides very good solutions to the static problem, both cyclic patterns can be incorporated into the dynamic lot sizing model with returns and remanufacturing which will be presented subsequently. While the third option analyses time windows with a manufacturing lot in τ that is followed by remanufacturing lots in later periods, a time window in the fourth option commences with a remanufacturing lot in τ that is succeeded by a number of manufacturing lots. Two promising effects can be observed when applying both additional options. At first, by considering more than one lot in each time window the recoverables inventory, which is a critical cost factor, can be controlled more accurately. Furthermore, contrary to the first two options the zero-inventory property is only presumed to hold for the first period of a time window but not within each time window any more. Hence, a (re)manufacturing lot can be scheduled although the initial serviceables inventory of the period under consideration is not zero.

Option 3: Manufacture first, remanufacture (in multiple lots) later

When applying this option, a manufacturing lot is scheduled in τ that is followed by one or more remanufacturing lots in the consecutive periods $\tau+1$ to z. As the amount of products available in the recoverables stock needs not to be sufficient, the manufacturing lot in period τ must replenish the unavailable products. The number of unavailable products in each period t ranging from $\tau+1$ to z (which will be referred to as the net requirement NR_t) can be determined as

$$NR_t = \sum_{i=\tau}^{t} (D_i - R_i) - y_{\tau-1}^R \quad \forall t = \tau+1, \ldots, z. \tag{11}$$

As a manufacturing lot has to be scheduled in period τ and no remanufacturing takes place in that period, the lot size x_τ^M cannot be smaller than D_τ as the entire demand in the first period of the time window has to be met. On the other hand, this lot must be able to complement all unavailable products and corresponds therefore at least to the maximum of all net requirements. Calculating the manufacturing lot size for period τ differs from Equation (9) as the timing of all returns and demands has to be taken into account for this option. Thus,

$$x_\tau^M = \max\left(D_\tau, \max_{t=\tau+1,\ldots,z}(NR_t)\right), \quad x_\tau^R = 0. \tag{12}$$

In all consecutive periods of the time window under consideration, no manufacturing lot will be set up. However, as the amount manufactured in period τ can be sufficient to satisfy the customer demand at least partly between period $\tau+1$ and z, only the actually required products are remanufactured in these periods in order to avoid unnecessary holding cost for the serviceables inventory. By establishing Equation (12), it is ensured that in every period between $\tau+1$ and z enough products are available in the recoverables stock to be remanufactured. The resulting lot sizes can be visualised as in Equation (13). After determining the relevant inventories y_t^R and y_t^M using Equations (2) and (3), the total cost per period can be calculated as in (14).

$$x_t^M = 0, \quad x_t^R = \max\left(\sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x_i^R - x_\tau^M, 0\right) \quad \forall t = \tau+1, \ldots, z. \tag{13}$$

$$C_{\tau,z}^3 = \frac{\sum_{t=\tau}^{z} \gamma_t^R \cdot K^R + K^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z-\tau+1}. \tag{14}$$

After creating a first initial solution for option 3 using formulae (12) and (13) it must be noticed that the total cost per period of this option (denoted by C_{ini}) can be very high. This is especially the case when a long time window is examined and the setup cost K^R is large. Therefore, a greedy algorithm has been formulated in addition that commences in $\tau+1$ and checks two possible improvement opportunities for each remanufacturing lot. Common to both opportunities I and II is that all products obtained in the remanufacturing lot under consideration (which has been originally scheduled in period k and contains x_k^R products) are replenished alternatively. Therefore, no remanufacturing lot is scheduled in period k in order to save the setup costs incurred. Firstly, the potential cost saving is evaluated if the manufacturing lot in τ is increased by x_k^R. Since this decision affects all remanufacturing lots between $\tau+1$ and k, formula (13) is applied to update the corresponding lot sizes. The second opportunity comprises the option to increase the last remanufacturing lot before period k by x_k^R products which is scheduled in period l. In order to do that, at least x_k^R products need to be available in the recoverables stock in period l. On the other hand, if the recoverables stock does not contain enough recoverable products the difference is manufactured additionally in period τ and again all remanufacturing lots that are affected by this decision are determined using formula (13). Obviously, this option cannot be evaluated for the first remanufacturing lot in a time window. Both improvement opportunities are checked for each remanufacturing lot between $\tau+1$ and z, i.e. at most $2 \cdot (z-\tau)$ different schedules are examined. For each schedule, the total cost is calculated by using formula (14) and afterwards compared to C_{ini}. The schedule yielding the largest cost saving is chosen and the entire proceeding is repeated until no further improvement can be achieved. Finally, after the greedy local search has been applied, a number of remanufacturing lots R succeeds one manufacturing lot. To illustrate the third option to be analysed, the following pseudocode can be implemented:

Step 1: Find initial schedule

Determine net requirements using Equation (11)
Determine x_τ^M and x_τ^R using Equation (12)
Determine x_t^M and x_t^R using Equation (13)
Determine y_t^M and y_t^R using Equations (2) and (3)
$C_{ini} = C_{\tau,z}^3$

Step 2: Improve the initial schedule for periods τ to z

For $k = \tau + 1$ to z
  If $x_k^R > 0$ then
    $x_\tau^M = x_\tau^M + x_k^R, \quad x_k^R = 0$
    For $i = \tau + 1$ to k
      Update x_i^R using Equation (13)
      Update y_t^M and y_t^R using Equations (2) and (3)
    Next i
    Determine $\Delta C_I(k) = C_{\tau,z}^3 - C_{ini}$
    Reset initial schedule
    Find period l (period of the last remanufacturing lot before period k)
    If $y_l^R \geq x_k^R$ then
      $x_l^R = x_l^R + x_k^R, \quad x_k^R = 0$
    Else
      $x_\tau^M = x_\tau^M + \left(x_k^R - y_l^R\right), \quad x_l^R = x_l^R + y_l^R, \quad x_k^R = 0$
    End If
    Update y_t^M and y_t^R using Equations (2) and (3)
    Determine $\Delta C_{II}(k) = C_{\tau,z}^3 - C_{ini}$
  End If
Next k

Step 3: Implement the best option

If $\min_{k \in \{\tau+1,...,z\}} (\Delta C_I(k), \Delta C_{II}(k)) < 0$ then
  Implement the best schedule which becomes the updated initial schedule
  $C_{ini} = C_{ini} + \min_{k \in \{\tau+1,...,z\}} (\Delta C_I(k), \Delta C_{II}(k))$, Goto Step 2:
End If

Option 4: Remanufacture first, manufacture (in multiple lots) later

This option seeks to establish a time window in which a remanufacturing run is started in period τ which is followed by at least one manufacturing lot in the consecutive periods. By assumption, the entire recoverables stock is remanufactured in the first period of the time window τ and no manufacturing lot is set up. Obviously, if the number of available recoverable products in period τ is not sufficient to meet the demand of that period D_τ, option 4 cannot be applied and option 2 provides the only solution incorporating a remanufacturing lot in τ. On the other hand, whenever at least one manufacturing lot is required to satisfy the demand up to period z and $x_\tau^R > D_\tau$, option 2 will always be dominated by option 4 because the holding cost for the serviceables inventory is smaller. This gives for period τ

$$x_\tau^M = 0, \quad x_\tau^R = y_\tau^R + R_\tau. \tag{15}$$

In order to create an initial solution to this option the lot sizes of the remaining periods have to be determined as well. In each period from $\tau+1$ to z all missing parts are manufactured as there are no further remanufacturing lots allowed in this time window. The respective formulae are:

$$x_t^M = \max\left(\sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x_i^M - x_\tau^R, 0\right), \quad x_t^R = 0 \quad \forall t = \tau+1, \dots, z. \tag{16}$$

Similar to option 3, the initial solution can be quite expensive if a large setup cost K^M prevails. Therefore, a greedy algorithm can be used again to search for possible cost reductions. In contrast to the third option, this algorithm reviews all manufacturing lots. It begins by checking whether it would be less expensive to combine the second and the third manufacturing lot of the time window and proceeds in this manner (merging the third and the fourth manufacturing lot, …) to the end of the corresponding time window. The alternative revealing the largest cost reduction is implemented and the proceeding is restarted until no further cost reductions are possible. We omit the presentation of this algorithm as its general structure is similar to the one presented for Option 3. After applying this algorithm, one remanufacturing lot is followed by a number of manufacturing lots M which can be used to determine the associated cost per period of the fourth option:

$$C_{\tau,z}^4 = \frac{K^R + \sum_{t=\tau}^{z} \gamma_t^M \cdot K^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z-\tau+1}. \tag{17}$$

Including options 3 and 4 into the decision-making process extends the original Silver–Meal based heuristic introduced by Teunter *et al.* (2006). We will refer to this heuristic as the SM_4 heuristic as the decision to extend the time window will be made by comparing the resulting costs per period of all four options. The following chapter tests both heuristics extensively in a numerical experiment to assess their performance.

3. Numerical experiments

In order to guarantee a fair comparison to the original heuristic of Teunter *et al.* (2006) the experimental design that has been used to conduct the numerical study presented in this section corresponds mostly to their design. A full factorial study has been chosen in which all instances examined have a planning horizon T of 12 periods in common. Both setup cost parameters K^M and K^R can take on values of 200, 500 and 2000. While the rate of keeping a serviceable product for one period in stock (h^M) is set to one holding a recoverable product for one period (h^R) can cost 0.2, 0.5 and 0.8. All customer demands D_t have been drawn randomly from a normal distribution with a mean of 100 units per period. Likewise, the amount of returned products per period R_t has been drawn from a normal distribution with a mean of 30 (i.e. a return ratio of 30% prevails), 50 and 70. Both normal distributions were further distinguished into a small and a large variance setting. While the coefficient of variation in the small variance setting has always been set to 10% it takes on the value of 20% in the large variance setting. Contrary to the experiment conducted in Teunter *et al.* (2006), we omit the use of different demand and return patterns such as positive/negative trends and seasonal patterns. For each demand and return setting

20 instances (instead of 4 in their study) were drawn randomly. Therefore, the full factorial study considers in total $3^4 \cdot 2^2 \cdot 20 = 6480$ different examples.

For all examples both heuristic results have been calculated whereas CPLEX 11 has been used to determine the optimal solution. Both heuristics are evaluated by using the percentage gap to the optimal solution as a performance measure. The results of the numerical experiments are presented in Table 1.

By including two additional options in the decision-making process, the average performance of the SM_2 heuristic improves slightly from 7.5% to 6.1% over all instances. Comparing the performance of the SM_2 heuristic to the original numerical study in Teunter *et al.* (2006), it must be noticed that the performance in our study is slightly better which can be attributed to the differences in the experimental design. Although the SM_4 heuristic reduces the average percentage gap in almost all settings, an improvement of more than 2% can only be observed for a small setup cost for remanufacturing ($K^R = 200$) and a large holding cost for the recoverables inventory ($h^R = 0.8$). Both heuristics seem to perform well when the return ratio or the setup cost for manufacturing K^M is low and when the setup cost for remanufacturing K^R is high. On the contrary, for the opposite directions the performance of both heuristics is not sufficient with average errors of more

Table 1. Performance of the SM_2 and SM_4 heuristic.

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2	SM_4	SM_2	SM_4	SM_2	SM_4
All instances	7.5%	6.1%	7.9%	7.6%	49.2%	47.3%
Demand						
Small variance	7.2%	6.0%	7.9%	7.6%	43.6%	47.3%
Large variance	7.8%	6.1%	8.0%	7.5%	49.2%	43.9%
Returns						
Small variance	7.3%	6.1%	7.8%	7.6%	47.2%	47.3%
Large variance	7.7%	6.1%	8.0%	7.5%	49.2%	46.3%
Return ratio						
30%	5.5%	3.7%	5.5%	4.5%	31.3%	28.5%
50%	8.5%	7.3%	9.4%	8.2%	40.1%	41.8%
70%	8.4%	7.2%	8.0%	8.7%	49.2%	47.3%
K^M						
200	4.3%	3.4%	4.5%	3.6%	20.2%	17.6%
500	5.4%	3.9%	5.2%	3.9%	25.1%	19.3%
2000	12.8%	10.9%	9.9%	10.4%	49.2%	47.3%
K^R						
200	10.9%	6.6%	9.1%	7.8%	49.2%	40.2%
500	7.9%	8.1%	6.6%	8.2%	34.7%	47.3%
2000	3.7%	3.5%	6.0%	5.7%	29.4%	25.7%
h^R						
0.2	5.9%	5.3%	8.0%	8.0%	42.9%	47.3%
0.5	7.5%	6.5%	7.7%	7.6%	49.2%	42.4%
0.8	9.1%	6.3%	7.7%	7.0%	44.4%	40.3%

than 7%. In the next section, the heuristic solutions are examined to determine whether small modifications can be made to the initially obtained solution in order to reduce the total cost significantly.

4. Improvement phase

A commonly applied methodology to improve the performance of lot sizing heuristics is to use metaheuristics (see, for instance, Jans and Degraeve 2007, for an overview). However, metaheuristics rely on an appropriate selection of parameter values which itself might be hard to determine. Therefore, this contribution omits the use of metaheuristics and tries to enhance the solutions found by the SM_2 and SM_4 heuristic by examining two possible improvement opportunities.

Improvement 1: Check whether two consecutive time windows can be combined

A first improvement to the initial solution can be found by checking whether a cost reduction can be achieved if two consecutive time windows are combined. Hence, it is examined whether one of the four (two) options introduced in chapter 2 for the SM_4 (SM_2) heuristic could improve the solution for an integrated time window that comprises both initial time windows.

Improvement 2: Check whether a remanufacturing lot can be increased

Being a myopic heuristic approach, the SM_2 and SM_4 heuristics neglect all decisions beyond the time window currently examined. Thus, some solutions revealed that recoverable products are held in stock until the end of the planning horizon although they could have been used instead of manufacturing them later. Starting in the first period of the planning horizon, the algorithm checks for the current period i whether a remanufacturing lot has been initiated. The basic idea of the second improvement is to examine if the total cost can be reduced by enlarging the remanufacturing lot in period i and simultaneously decreasing the first manufacturing lot scheduled after period i by the same amount. In order to do that, the algorithm needs to determine at first period n which represents the period of the first manufacturing lot scheduled after period i. Without changing the number and sequence of lots determined initially by the SM_2 or SM_4 heuristic, the maximum number of recoverable products that can be remanufactured additionally in period i is restricted by two values. First, the number of recoverable products available needs to be taken into account. This number can be determined by the minimum of all subsequent recoverable stock levels, i.e. $\min_{k \in \{i,\ldots,T\}}\left(y_k^R\right)$. On the other hand, since the manufacturing lot in period n must be non-negative, the number of additionally remanufactured products must not exceed x_n^M. After adapting the corresponding lot sizes x_i^R and x_n^M, the algorithm checks whether these changes increase or decrease the total cost determined by Equation (1). If the total cost can be decreased, the modified lot sizes are kept and the algorithm proceeds with the next period. Contrary, if changing both lot sizes leads to an increase in the total cost, all changes made are reversed and the next period is analysed.

When approaching the end of the planning horizon, it might be the case that no manufacturing lot is scheduled after period i. In this case, the algorithm examines whether it is possible to reduce the preceding manufacturing lot set up in period l. However, a positive serviceables stock in period $i-1$ (y_{i-1}^M) must prevail in order to follow this idea.

This value restricts the possible change of the remanufacturing lot in period i as well as the manufacturing lot x_l^M. Furthermore, x_i^R is constrained by the maximum number of recoverable products available for remanufacturing in period i which has been depicted above. Again, when the change in lot sizes increases the total cost, the responsible changes are reversed. Only when it leads to a decrease in costs, the changes are kept and the algorithm proceeds with the next period. To clarify the second improvement in greater detail, the following pseudocode has been elaborated:

For $i = 1$ to T
  If $x_i^R > 0$ then
    Find period n (period of the next manufacturing lot after period i)
    If $i + 1 \leq n \leq T$ then
      $x_i^R = x_i^R + \min\left(x_n^M, \min_{k\in\{i,\ldots,T\}}\left(y_k^R\right)\right)$
      $x_n^M = \max\left(x_n^M - \min_{k\in\{i,\ldots,T\}}\left(y_k^R\right), 0\right)$
      Update y_t^M and y_t^R using Equations (2) and (3)
      If total cost determined by Equation (1) cannot be reduced then
        Reverse decisions made regarding x_i^R and x_n^M
        Update y_t^M and y_t^R using Equations (2) and (3)
      End If
    Else If $y_i^M > 0$ then
      Find period l (period of the last manufacturing lot before period i)
      $x_i^R = x_i^R + \min\left(y_{i-1}^M, x_l^M, \min_{j\in\{i,\ldots,T\}}\left(y_j^R\right)\right)$
      $x_l^M = \max\left(x_l^M - \min\left(y_{i-1}^M, \min_{j\in\{i,\ldots,T\}}\left(y_j^R\right)\right), 0\right)$
      Update y_t^M and y_t^R using Equations (2) and (3)
      If total cost determined by Equation (1) cannot be reduced then
        Reverse decisions made regarding x_i^R and x_n^M
        Update y_t^M and y_t^R using Equations (2) and (3)
      End If
    End If
  End If
Next i

As mentioned above, both improvements can be applied to the solutions obtained by the SM_2 and SM_4 heuristic. Table 2 summarises the results of the numerical study in which the superscript$^+$ indicates that the initial solution has been examined for both improvements.

It can be seen that the performance of the SM_4 heuristic could be enhanced substantially from 6.1% to 2.2% by applying both improvements. The larger influence on the solution improvement can be credited to improvement 1 which was able to affect the SM_4 heuristic especially (around 85% of the improvement). That is because by analysing all four options introduced in Section 2, a larger flexibility in satisfying the customer demand is established in comparison to the SM_2 heuristic. Regarding the zero-inventory property, 61.4% of all heuristic solutions obtained by the SM_4 heuristic revealed at least one period in which the zero-inventory property did not hold. In contrast to the original results of the SM_2 heuristic, the SM_4^+ heuristic could reduce the percentage gap to less than half of its original value in almost all settings examined. When comparing the median of all instances the improvement is even more noticeable. While the median of all instances has been 5.6% for the SM_2 heuristic the SM_4^+ heuristic could reduce it to around 1.0%. Interestingly, the SM_4^+ heuristic is able to stabilise the average performance of all settings

Table 2. Performance of the SM_2^+ and SM_4^+ heuristic.

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2^+	SM_4^+	SM_2^+	SM_4^+	SM_2^+	SM_4^+
All instances	6.9%	2.2%	7.9%	2.9%	49.2%	24.3%
Demand						
Small variance	6.6%	2.1%	7.9%	2.8%	43.5%	18.9%
Large variance	7.2%	2.4%	8.0%	3.0%	49.2%	24.3%
Returns						
Small variance	6.8%	2.2%	7.8%	2.9%	47.2%	21.1%
Large variance	7.1%	2.3%	8.0%	2.9%	49.2%	24.3%
Return ratio						
30%	4.9%	1.2%	5.4%	1.8%	31.3%	12.1%
50%	8.0%	2.3%	9.3%	2.7%	39.8%	16.2%
70%	8.0%	3.3%	8.0%	3.5%	49.2%	24.3%
K^M						
200	3.5%	2.3%	4.0%	2.6%	20.2%	13.5%
500	4.8%	2.1%	4.9%	2.5%	23.7%	12.8%
2000	12.6%	2.3%	9.9%	3.4%	49.2%	24.3%
K^R						
200	10.0%	1.9%	9.4%	2.1%	49.2%	11.8%
500	7.3%	3.4%	6.6%	3.2%	34.7%	19.1%
2000	3.6%	1.4%	5.9%	2.9%	29.4%	24.3%
h^R						
0.2	5.8%	1.7%	8.0%	2.5%	42.9%	21.1%
0.5	7.0%	2.3%	7.7%	3.0%	49.2%	24.3%
0.8	8.1%	2.8%	7.8%	3.0%	44.4%	20.6%

to lie between 1.2% and 3.4%. Although the SM_4^+ heuristic has reduced the maximum deviation from the optimal solution considerably (as can be observed in the right hand side of both Tables 1 and 2) there are still instances which perform poorly. Nevertheless, the SM_4^+ heuristic was able to achieve that in only 2% of all instances the percentage gap was larger than 10%. On the contrary, 18% of all instances exhibited a percentage gap of more than 10% when using the SM_2 heuristic.

As depicted above, the performance of the original SM_2 heuristic has been enhanced substantially. In their study, Teunter *et al.* (2006) examined not only a Silver–Meal based criterion but also an adapted Least-Unit-Cost and Part-Period approach. These heuristics have been adapted as well by including the additional options and improvement steps. Unfortunately, their performance improved only slightly in comparison to the original work and was not able to outperform the performance gain of the Silver–Meal based approach. However, it must be mentioned that this improvement in performance is accompanied by an increase in computational complexity. Although the complexity of the proposed algorithms increased, only a small rise in the computation time for each instance could be observed. Therefore, implementing both the additional options and the improvement steps provides a fast and well performing heuristic algorithm for the dynamic lot sizing problem with returns and remanufacturing.

5. Conclusion and outlook

This contribution extends the seminal work of Teunter *et al*. (2006) in the area of simple heuristics for the dynamic lot sizing problem with returns and remanufacturing. In their study, the authors introduced a Silver–Meal based heuristic that analyses two options to meet the customer demand. This study included two more options to be analysed that are well-known from the corresponding static lot sizing problem. By doing this, the percentage gap to the optimal solution that has been used as a performance measure could be reduced slightly from 7.5% to 6.1% (mean over all instances). Afterwards, two simple procedures were applied to the initial solutions found by the SM_2 and SM_4 heuristic to improve the results they created. The average percentage gap to the optimal solution could be reduced over all instances to 2.2% when using the SM_4 heuristic's solution as the initial one. Comparing this result with the heuristic introduced by Teunter *et al*. (2006), the average percentage gap has thus been reduced to less than half of its original value.

Future research efforts can be directed to a more detailed modelling of the remanufacturing process. While in this contribution all remanufacturing operations have been subsumed to a single stage, in industry the process of remanufacturing contains next to the disassembly of returned products also the cleaning and rework of the parts obtained, and finally the re-assembly into as-good-as-new products. Furthermore, including the option to dispose of recoverable parts when they are not required and variable unit cost for remanufacturing and manufacturing alter the decision-making process. Another promising research opportunity would be to test the heuristics in a rolling planning horizon environment. As has been shown by Blackburn and Millen (1980) the heuristic might outperform even the optimal solution because of its schedule stability. Another interesting aspect of rolling planning horizon environments that can be analysed in this context is the uncertainty of demand and return realisations at the end of each planning roll which becomes more accurate the closer one gets to this period.

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