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Neurocomputing 69 (2006) 1645–1654

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Stability of feedback error learning method with time delay

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Received 9 January 2004; received in revised form 11 April 2005; accepted 11 April 2005
Available online 26 October 2005
Communicated by T. Heskes

Abstract

Feedback error learning method was recently proposed by Kawato et al. [A heirarchical neural-network model for control and learning of voluntary movements, Biol. Cybernet. 57 (1987) 169–185.] as a possible architecture of brain motor control which is supported by experimental results in neurophysiology. In this paper, we analyze it as a two-degree-of-freedom adaptive control for general time invariant linear plant with adaptive controller in the feedforward path. A time delay is allowed in the feedback loop as in the neuronal pathways of motor control. We derive stability condition of the feedback error learning method based on the strict positiveness of the closed loop system. The control performance of the feedback error learning method as a design strategy of adaptive control has been demonstrated by simulation results.

Keywords: Cerebellum motor control; Adaptive control; Time delay; Feedback error learning; Feedforward controller

1. Introduction

Motor control is concerned with control of all kinds of animal voluntary movements such as locomotion, grasping objects, manipulation, head rotation, eye blinking etc. Its performance is much superior to any other existing artificial control, in spite of the remarkable progress in humanoid robotics. It is much more versatile with smoother, faster and more precise response than what the present robot control can achieve. It accomplishes complex tasks with wider range of environmental changes, even with completely unknown situations.

The quality of sensors and actuators used in motor control may be far from perfect, even though they are superior to those used in current robotics research. To take an illustration, we can stretch out our arm and clutch a cup on a table in front of us and bring it to our lips with smooth movement. Such high quality of motor control is certainly ascribed to the superiority of the control strategies that brain embodies. How can brain achieve such high quality of control under limited hardware environment? This question is indeed quite natural for control engineers, as well as for neuroscientists.

Norbert Wiener was probably the first who seriously addressed the existence of common principle between machine control and human motor control. In his celebrated book, *Cybernetics* [19], he repeatedly described his observation that behaviors of a patient who had a defect in motor control was very similar to those of poorly designed control systems. Since the Wiener's era, system control theory and motor control have marked remarkable progress. In the last three decades, potential interface between control theory and motor control has been remarkably widened. One may find interesting relationship between system control theory and motor control also in [12]. A nice survey by Hatze [5] described a huge possibility of the interplay between physiology and control engineering twenty years ago. In motor control, as physiological data were compiled, the brain was understood to execute more sophisticated control than people had thought. Computational neuroscience was born which aims to understand the mechanism of the functions of neural systems and, moreover, construct artificial control systems with the same level of performance as motor control, subject to the physiological limitations of animal body.

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doi:10.1016/j.neucom.2005.04.011

Their claim is: *understanding through construction*. Thus computational neuroscience renders motor control akin to control engineering though it has wider interface with neurophysiology. Both fields have now many notions in common, such as *model*, *feedforward*, *learning*, *adaptation* and so on. Control theory is obviously benefited from motor control taking new architectures of motor control as potential new design strategies of control systems. On the other hand, motor control is benefited from control theory in getting theoretical validity of their architectures which encourages them to investigate higher level of control. Thus, the interplay between control theory and motor control seems to be quite fruitful in the current situation [4,14].

Among many control architectures proposed in motor control (e.g., [2,13]), *the feedback error learning method* (FELM) which was proposed by Kawato and his colleagues [8] seems to be most attractive from the control theoretical point of view. Because the FELM adopts two-degree-of-freedom control scheme, which has, as is well known, robustness to disturbance and perturbation. Moreover, FELM uses an inverse model of the plant as a feedforward controller in order to cope with the time delay in sensing devices.

In [15], theoretical analysis of FELM has done for delay-free case. However, existence of time delays in the sending devices, e.g., visual feedback, is the essential feature of motor control, and, at the same time, treatment of time delays in the feedback loop is a big nuisance in adaptive control. Therefore, theoretical analysis of FELM with time delays is critical to understand the mechanism of motor control as well as giving some idea of adaptation scheme in control area.

The novelties of FELM from the viewpoint of control theory are as follows:

(1) It is a new adaptive control scheme, which is a mixture of *indirect* and *direct strategies* in the context of conventional adaptive control.
(2) It provides with a method to circumvent the difficulty of sensing delay in the case that the delay time is a priori known.
(3) It accomplishes a difficult task of on-line identification of inverse models.

These novelties have not been well documented in the literature of motor control due to the lack of appropriate theoretical analysis. In this paper, we establish theoretical validity of the FELM in the context of conventional adaptive control and derive a relationship between stability of adaptation and the amount of time-delay, the choice of learning rate, and feedback gain.

2. Feedback error learning method (FELM)

It was widely recognized that biological feedback loops are unable to execute fast and smooth movements of limbs because of significant time delays in neurotransmittance and visual perception. This implies that the notion of feedback, which was brought forward by Wiener to understand the motor control turned out to be insufficient. Motor control is too rich and too complex to be reduced to a single notion.

The problems of motor control from the viewpoint of control theory are focused on:

(1) The architecture of tracking control to circumvent significant time delays of sensory organs.
(2) The on-line acquisition of models through learning.

Among various schemes of motor control (e.g., [2,13]), the FELM proposed by Kawato and his colleagues seems to be most attractive and novel from control theoretical point of view. Fig. 1 illustrates the feedback error learning architecture. The objective of control is to minimize the error e between the command signal r and the plant output y. The input u to the plant P is composed of the sum of the output u_{ff} of feedforward controller Q and u_{fb} of feedback controller K. This is a typical two-degree-of-freedom control system. If P^{-1} exists and is stable, choosing $Q = P^{-1}$ makes the tracking perfect. The novelty of FELM lies in its way to learn the inverse model of P. In Kawato's original work [8], the feedforward controller is implemented by a neural network containing unknown parameters θ, which is denoted by $Q(\theta)$. It is assumed that a true parameter θ_0 exists such that

$$P^{-1} = Q(\theta_0).$$

The learning law is derived from the gradient method with error function

$$V = (u_{\mathrm{ff}} - u_0)^{\mathrm{T}}(u_{\mathrm{ff}} - u_0), \tag{1}$$

where u_0 denotes the correct input $u_0 = P^{-1}r$. Then, the gradient method yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{\partial V}{\partial \theta}, \tag{2}$$

which results in the tuning rule

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = u_{\mathrm{fb}}\frac{\partial u_{\mathrm{ff}}}{\partial \theta}, \tag{3}$$

assuming that the input u to the plant is always correct, i.e.,

$$u = u_0 = P^{-1}r. \tag{4}$$

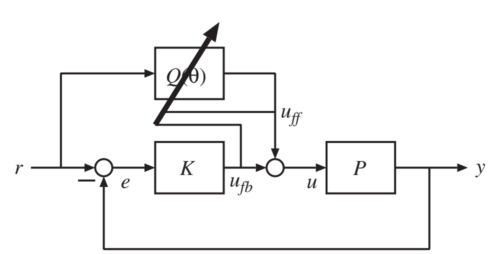

Fig. 1. Feedback error learning scheme.

The tuning rule (3) is the core of the FELM, but the underlying assumption (4) could be a serious issue of arguments, because the intrinsic difficulty of learning inverse model (see [7] for detailed arguments) is circumvented by this assumption. The precise tuning rule must be

$$\frac{d\theta}{dt} = -(u_{ff} - u_0)\frac{\partial u_{ff}}{\partial \theta}, \tag{5}$$

according to (1) and (2), but since u_0 is unknown rule (5) cannot be implemented. This difficulty is intrinsic whenever we try to obtain an inverse model adaptively, e.g., in blind deconvolution of signals [11]. The quickest way to justify assumption (4) is to prove convergence of the algorithm of (3). Kawato tried to give a convergence proof in general nonlinear framework based on the Newton method in Hilbert space [9]. But his proof was again based on assumption (4) which was a target of proof rather than an assumption. This problem in the simplest framework of linear time-invariant systems has set up and given a convergence proof of the feedback error learning algorithm (3) for delay-free case in [15].

Another important point, which was not dealt with in Kawato's work, is the problem of time delay in feedback loop, which is actually the main motivation of introducing an inverse model in the feedforward path. The treatment of time delay is a big nuisance, specially in adaptive control. No effective way of incorporating time delay in adaptive loop has been established, except some pioneering work in the context of model reference adaptive control [6,17] and internal model control [3]. In this paper, we shall show that the FELM gives a proper way of circumventing the difficulty of time delay in adaptive control, provided that the delay time is known. It is noted that theoretical analysis of FELM for nonlinear two links DD arm control systems with time delay is discussed by Ushida et al. [18] and *delay-independent* stability condition was discussed. In this paper, we will discuss about *delay-dependent* stability condition, which is less conservative than delay-independent condition, for FELM for linear delayed system.

3. Mathematical preliminaries

In this section we derive theorems and lemmas, which are important to prove the main result of this paper (Theorem 2).

Let us start to consider a time-variant-differential equation as

$$\frac{dz(t)}{dt} = -\alpha\xi(t)L(s)\xi(t)^{T}z(t), \tag{6}$$

where α is positive. The stability of differential equation (6) is obtained as

Lemma 3.1 (*Anderson et al. [1]*). *Let $L(s)$ be a strictly positive real transfer function and $\xi(t)$ be an arbitrary time-varying vector. Then, the solution $z(t)$ to the differential equation*

$$\frac{dz(t)}{dt} = -\xi(t)L(s)\xi^{T}(t)z(t) \tag{7}$$

tends to a constant vector z_0 such that $\xi^{T}(t)z_0 \to 0$. If $\xi(t)$ satisfies the so-called persistent excitation (PE) condition [16], the above z_0 is equal to 0.

This type of differential equations often appear in adaptive control context [1]. Proof of this Lemma is found in [14].

We derive now another lemma that we will use in the sequel:

Lemma 3.2. *Let $w_1(t)$ and $w_2(t)$ be differentiable functions satisfying the following differential inequalities:*

$$\frac{dw_1(t)}{dt} \leqslant -\alpha_1 w_1(t) + \beta_{11}\sup_{t-\tau\leqslant\sigma\leqslant t} w_1(\sigma) + \beta_{12}\sup_{t-\tau\leqslant\sigma\leqslant t} w_2(\sigma), \tag{8}$$

$$\frac{dw_2(t)}{dt} \leqslant -\alpha_2 w_2(t) + \beta_{21}\sup_{t-\tau\leqslant\sigma\leqslant t} w_1(\sigma) + \beta_{22}\sup_{t-\tau\leqslant\sigma\leqslant t} w_2(\sigma), \tag{9}$$

where α_1, α_2, β_{11}, β_{12}, β_{21}, and β_{22} are non-negative numbers. Then, there exist $\gamma > 0$, $k_1 > 0$, and $k_2 > 0$ such that

$$w_1(t) < k_1 e^{-\gamma t}, \quad w_2(t) < k_2 e^{-\gamma t} \tag{10}$$

if the following inequalities hold:

$$\alpha_1 > \beta_{11}, \tag{11}$$

$$\det\begin{bmatrix} \beta_{11}-\alpha_1 & \beta_{12} \\ \beta_{21} & \beta_{22}-\alpha_2 \end{bmatrix} > 0. \tag{12}$$

Proof. See Appendix A. □

Let us consider a time-variant delay differential equation described as

$$\frac{dz(t)}{dt} = -\alpha\xi(t)L(s)\xi(t)^{T}z(t-\tau(t)). \tag{13}$$

The following result is crucial for dealing with the adaptive control with delay.

Theorem 3.3. *Assume that the following conditions hold:*

(i) *The differential equation*

$$\frac{dz(t)}{dt} = -\alpha\xi(t)L(s)\xi(t)^{T}z(t)$$

is asymptotically stable;

(ii) *$\xi(t)$ is bounded, i.e., there exists $M_0 > 0$ such that*

$$\|\xi(t)\| \leqslant M_0 \quad \forall t \geqslant 0;$$

(iii) *$\tau(t)$ is bounded, i.e., $0 \leqslant \tau(t) \leqslant \tau_0$.*

Then, for any τ_0, there exists $\alpha > 0$ such that the delay-differential equation (13) *is asymptotically stable.*

Proof. See Appendix B. □

4. FELM with time delay

4.1. Problem formulation

Fig. 2 illustrates the feedback error learning architecture with time delay in the feedback loop. The objective of control is to minimize the error $e = r - y$ where r is the command signal and y is the plant output. The input u to the plant P is composed of the output $u_{\rm ff}$ of feedforward controller K_2 which contains tunable parameters, and $u_{\rm fb}$ of feedback controller K_1. If we disregard the learning part of the architecture, this is a typical two-degree-of-freedom control system. If P is known and P^{-1} exists and stable, choosing $K_2 = P^{-1}$ makes the tracking perfect. Indeed, from the relationship $u_{\rm ff} = P^{-1}r$, $u_{\rm fb} = K_1(r - y)$ and $y = P(u_{\rm ff} + u_{\rm fb})$, we easily see that $y = r$.

The basic premise of FELM is that the cerebellum cortex acquires a model of outer world through learning while it is engaged in actual motor control [10]. This implies that the motor control is essentially a sort of adaptive control. The crucial factor, which has been neglected in formulating motor control as an adaptive control, is the significant time delay caused by neurotransmittance and visual perception.

In this section we discuss FELM from the viewpoint of adaptive control. The feedforward controller K_2 is chosen to be identical to the inverse P^{-1} of P if P is known and P^{-1} exists. Since P is unknown, we must employ some adaptive scheme for K_2 so that K_2 converges to P^{-1}. Note that the time-delay $e^{-\tau s}$, which is identical to the time-delay contained in the feedback loop, is introduced to the reference signal to produce the feedback error consistent with adaptation law. We first make the following assumptions:

(A1) The plant P has a stable inverse P^{-1}.
(A2) An upper bound of the order of P is known.
(A3) The high frequency gain $k_0 = \lim_{s\to\infty} P(s)$ is assumed to be positive.
(A4) The time delay τ is known.

The assumption (A1) is rather restrictive in the context of control system design. This may be relaxed later.

If k_0 is negative in (A3), the subsequent results are valid by taking $-P(s)$ instead of $P(s)$. Hence, (A3) is relaxed to the assumption that *the sign* of the high frequency gain is known. For the sake of the simplicity of exposition, however, we retain (A3).

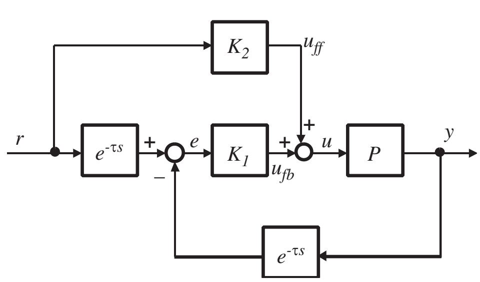

Fig. 2. Feedback error learning method (FELM) with time delay.

Remark 4.1. In practice, most real plants has $k_0 = 0$, so that the plant P has no inverse. In this case we can make a modification of the proposing method in the similar way as for delay-free case [15] (or see [14]).

4.2. Parameterization of unknown systems

Now, we describe a method of adaptive construction of a desired K_2 under the assumption that τ is known a priori. Throughout this paper, we use the following parameterization of the unknown system K_2 [15]:

$$\frac{d\xi_1(t)}{dt} = F\xi_1(t) + gr(t), \tag{14}$$

$$\frac{d\xi_2(t)}{dt} = F\xi_2(t) + gu(t), \tag{15}$$

$$u_{\rm ff}(t) = c(t)^{\rm T}\xi_1(t) + d(t)^{\rm T}\xi_2(t) + k(t)r(t), \tag{16}$$

where F is any stable matrix and g is any vector with $\{F, g\}$ being controllable. In (14)–(16), $c(t)$, $d(t)$ and $k(t)$ are unknown parameters to be estimated. This is a standard parameterization of adaptive controller used in [16]. Assume that the *true* system is written as

$$\frac{dz_1(t)}{dt} = Fz_1(t) + gr(t), \tag{17}$$

$$\frac{dz_2(t)}{dt} = Fz_2(t) + gu_d(t), \tag{18}$$

$$u_d(t) = c_0^{\rm T} z_1(t) + d_0^{\rm T} z_2(t) + k_0 r(t). \tag{19}$$

It is easy to see that taking $u(t) = u_d(t)$ and appropriate selection of parameters $c(t) = c_0$, $d(t) = d_0$ and $k(t) = k_0$ can yield an arbitrary transfer function from $r(t)$ to $u_{\rm ff}(t)$. Therefore, we can construct any transfer function of LTI system degree less than or equal to n by selecting parameters c_0, d_0 and k_0 appropriately.

4.3. Adaptation law

In the ideal situation, K_2 is identical to P^{-1}. In that case, $e(t) = 0$, $u(t) = u_{\rm ff}(t) = u_d(t) = P^{-1}(s)r(t)$. The true values c_0, d_0 and k_0 of $c(t)$, $d(t)$ and $k(t)$, respectively, satisfy

$$\frac{k_0 + c_0^{\rm T}(sI - F)^{-1}g}{1 - d_0^{\rm T}(sI - F)^{-1}g} = P^{-1}(s). \tag{20}$$

The error signal $e(t)$ is defined as

$$e(t - \tau) = r(t - \tau) - y(t - \tau).$$

The cost function is defined as

$$J(t) = \frac{1}{2}\int_0^t e^2(\sigma)\,d\sigma. \tag{21}$$

Since the unknown parameters $c(t), d(t)$ and $k(t)$ must be updated so that the error signal $e(t)$ decreases, the adaptation law is given as

$$\theta(t)=\left[c(t)^{\mathrm{T}}\quad d(t)^{\mathrm{T}}\quad k(t)^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$\begin{aligned}\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}&=-\alpha\frac{\partial\dot{J}(t-\tau)}{\partial\theta}\\&=-\alpha\frac{\partial e(t-\tau)}{\partial\theta}\cdot e(t-\tau).\end{aligned}\tag{22}$$

We choose $K_1=\text{const}$. Then, it follows that

$$e(t-\tau)=\frac{u(t)-u_{\mathrm{ff}}(t)}{K_1}.$$

Using the approximation $u(t)\sim u_d(t)$, we have

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\frac{\alpha}{K_1}\frac{\partial u_{\mathrm{ff}}(t-\tau)}{\partial\theta}e(t-\tau).\tag{23}$$

From (16),

$$u_{\mathrm{ff}}(t)=\theta(t)^{\mathrm{T}}\xi(t),$$

where

$$\xi(t):=[\xi_1(t)^{\mathrm{T}}\ \ \xi_2(t)^{\mathrm{T}}\ \ r(t)]^{\mathrm{T}}.$$

Then, (23) can be written as

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\frac{\alpha}{K_1}e(t-\tau)\xi(t-\tau).\tag{24}$$

We delayed the time of adaptation in accordance with the time-delay in the feedback loop, rather than taking the real time signal.

4.4. Convergence of algorithm

Now $u(t)$ and u_d are written as

$$u(t)=u_d(t)-P^{-1}(s)e(t-\tau),$$
$$u_d(t):=P^{-1}(s)r(t).$$

Then, if we define $\Delta u(t)=u_{\mathrm{ff}}(t)-u_d(t)$, we have

$$\begin{aligned}\Delta u(t)=&\,(c(t)-c_0)^{\mathrm{T}}\xi_1(t)+(d(t)-d_0)^{\mathrm{T}}\xi_2(t)\\&-d_0^{\mathrm{T}}(sI-F)^{-1}gP^{-1}(s)e(t-\tau).\end{aligned}\tag{25}$$

Since F is stable, we use the asymptotic relationship

$$\xi_1(t)\to z_1(t),$$
$$\xi_2(t)\to z_2(t)-d_0^{\mathrm{T}}(sI-F)^{-1}gP^{-1}(s)e(t-\tau).$$

The relationship (25) is written as

$$\Delta u(t)=\psi(t)^{\mathrm{T}}\xi(t)-d_0^{\mathrm{T}}(sI-F)^{-1}gP^{-1}(s)e(t-\tau),$$

where

$$\psi(t):=\theta(t)-\theta_0=\begin{bmatrix}c(t)-c_0\\d(t)-d_0\\k(t)-k_0\end{bmatrix}.\tag{26}$$

From the relationship $u(t)=u_{\mathrm{ff}}(t)+K_1e(t-\tau)$, we have

$$e(t)=-(G(s)+K_1\mathrm{e}^{-\tau s})^{-1}\psi(t)^{\mathrm{T}}\xi(t),\tag{27}$$

where $G(s)$ is given by

$$G(s)=k_0+c_0^{\mathrm{T}}(sI-F)^{-1}g.\tag{28}$$

On the other hand, from (24), we have

$$\frac{\mathrm{d}\psi(t)}{\mathrm{d}t}=\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}=\frac{\alpha}{K_1}e(t-\tau)\xi(t-\tau).\tag{29}$$

Then, combining the above relationship with (27) yields

$$\frac{\mathrm{d}\psi(t)}{\mathrm{d}t}=-\alpha\xi(t-\tau)L_\tau(s)\xi(t-\tau)^{\mathrm{T}}\psi(t-\tau),\tag{30}$$

where

$$L_\tau(s)=\frac{1}{K_1}(G(s)+K_1\mathrm{e}^{-\tau s})^{-1}.\tag{31}$$

Theorem 4.2. *Delay-differential equation* (30) *is asymptotically stable if*

(i) L_τ *is strictly positive real* (*s.p.r*).
(ii) $\xi(t)$ *satisfies PE condition.*
(iii) $\alpha>0$ *satisfies the following condition*

$$\alpha<\frac{\lambda}{\tau M}\left(\frac{1}{\mathrm{d}PR+(M_\delta/\delta)SPQ}\right),$$

where M_δ, δ *and* λ *are*

$$\|\mathrm{e}^{At}\|\leqslant M_\delta\mathrm{e}^{-\delta t},\quad M_\delta>0,\ \ \delta>0,$$
$$\|U(t,s)\|\leqslant M\mathrm{e}^{-\lambda(t-s)},\quad M>0,\ \ \lambda>0,$$

where $U(t,s)$ *is a transition matrix of* $\mathrm{d}\psi(t)/\mathrm{d}t=-\alpha\xi(t-\tau)L_\tau(s)\xi(t-\tau)^{\mathrm{T}}\psi(t)$, *and*

$$P=\sup_t\left\|\begin{bmatrix}-b\xi^{\mathrm{T}}(t)\\\alpha\,\mathrm{d}\xi(t)\xi(t)^{\mathrm{T}}\end{bmatrix}\right\|,$$

$$Q=\sup_t\ \|\xi(t)c^{\mathrm{T}}\|,\quad R=\sup_t\ \|\xi(t)\xi^{\mathrm{T}}(t)\|,$$

$$S=\sup_t\ \|b\xi^{\mathrm{T}}(t)\|.$$

Proof. From the proof of Theorem 3.3 (Appendix B), these conditions are derived easily. □

Consequently, when we can select K_1 as L_τ being s.p.r. and the adaptive parameter α satisfies the condition of Theorem 4.2(ii), FELM scheme with time delay case converges and the feedforward controller K_2 converges to inverse of P.

Remark 4.3. If the relative degree of P is zero and P is invertible, to make $G(s)+K_1\mathrm{e}^{-\tau s}$ be s.p.r., $P(s)$ must be a system having a high positive gain at high frequency area.

Remark 4.4. To satisfy the inequality in Theorem 2(ii), we should select a small value for α, i.e., we make the adaptation speed slow enough to overcome the existence of time-delay in the feedback pathway.

Remark 4.5. The selection of the adaptation law (23) makes it possible to formulate the dynamics of convergence as (30), which has a interesting structure from the control view point and stability analysis of delayed differential equation as (30) or (13), e.g., Theorem 1 is an interesting result in itself.

Remark 4.6. This result can be extended to FELM for non-invertible plants as the same manner in [15].

5. Simulation result

We show a simulation result for a plant, $P(s) = (s^2 + 12s + 20)/(s^2 + 7s + 12)$. This plant has stable inverse, We use F and g as

$$F = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Therefore, we can calculate that $G(s) = (s^2 + 3.5s + 2.1)/(s^2 + 2s + 1)$. The Nyquist diagram of $G(s)$ is shown in Fig. 3. From Fig. 3, we can conclude that L_τ is s.p.r. by selecting K_1 less than 1. It is noted that for higher order systems, it may be difficult to guarantee this s.p.r. condition by the same manner. In such case, sufficiently small K_1 would be selected at the beginning of adaptation and make it grater gradually as adaptation proceeds. Fig. 4 shows the result of simulation with time delay $\tau = 1.0(s)$. It is noted that we derive the condition for α in Theorem 2, however, since we cannot know the values of P, Q, R, and S a priori, we may start the value α from a small one. For feedback gain $K_1 = 0.5$ is selected. And we set the adaptation coefficient $\alpha = 50$, which satisfies the condition of Theorem 2. In Fig. 4 a simulation result is sown, where (a)–(e) illustrate time change of parameter $c(t)$, time change of parameter $d(t)$, time change of parameter $k(t)k$, time series of $y(t)$ and $r(t)$ after starting adaptation, and time change of error signal $e(t)$, respectively. A chirp signal was used for input signal $r(t)$ in this simulation. A chirp signal is a signal sinusoid with a time varying frequency:

$$r(t) = \cos(2\pi f(t)t),$$

where the time varying frequency part is $f(t) = 2t$. The advantage of chirp signal is that the system can be excited over all specified frequency band, i.e., $r(t)$ satisfies persistent excitation condition. From Fig. 4(a)–(c), we can see that tuned parameters $c(t)$, $d(t)$, and $k(t)$ converges to constant values. Fig. 4(d) illustrates the improvement of tracking performance of $y(t)$ for $r(t)$ after adaptation. It is noted that adaptation for $c(t)$ and $d(t)$ starts at $t = 1.0(s)$ and for $k(t)$ at $t = 0.0(s)$. Fig. 4(e) illustrates the error signal $e(t)$ converges into sufficiently small value. It is noted this residual error comes from the lack of simulation time and selection of input signal.

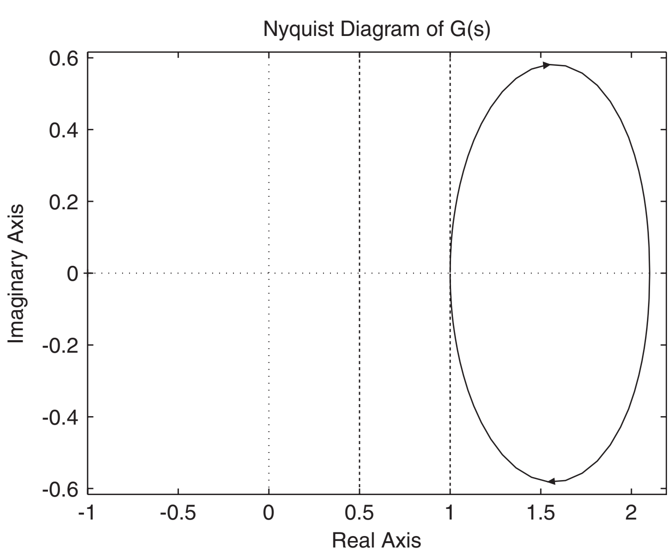

Fig. 3. Nyquist diagram of $G(s)$.

6. Conclusion

In this paper FELM with time delay in the feedback loop proposed as architecture of brain motor control has been investigated from the viewpoint of two-degree-of-freedom adaptive control. One of the advantages of FELM is its ability to surmount the difficulty of time delay. We have investigated its capability of compensating time delay through feedforward control. It is our intuition that adaptive control can overcome time delay by slowing down the speed of adaptation and it has been shown that the convergence of FELM for the plant with time delay by making the updating rate small. It may be interesting to exploit physiological ground of these equations, as well as to investigate mathematical properties of these equations in depth to obtain less conservative stability conditions of these equations.

Acknowledgments

The study was started in Prof. Kimura's lab at the University of Tokyo. It has been continued in Prof. Aihara's lab at the University of Tokyo. The author would like to thank both professors for their helpful comments and suggestions. The author also would like to express her gratitude to referees and the editor for their valuable suggestions and comments.

Appendix A. Proof of Lemma 3.2

Let

$$g_i(z) := z - \alpha_i + \beta_{ii} e^{\tau z}, \quad i = 1, 2.$$

From (11), it follows that $g_1(0) < 0$. Since $g_1(z)$ goes to $+\infty$ as $z \to +\infty$, there exists $z_1 > 0$ such that $g_1(z_1) = 0$. The same argument applies to $g_2(z)$. Let

$$f(z) := \det \begin{bmatrix} g_1(z) & \beta_{12} e^{\tau z} \\ \beta_{21} e^{\tau z} & g_2(z) \end{bmatrix}.$$

From (12), it follows that $f(0) > 0$. Since $f(z_1) = -\beta_{12}\beta_{21} e^{2\tau z_1} < 0$, there exists $0 < \gamma < z_1$ such that

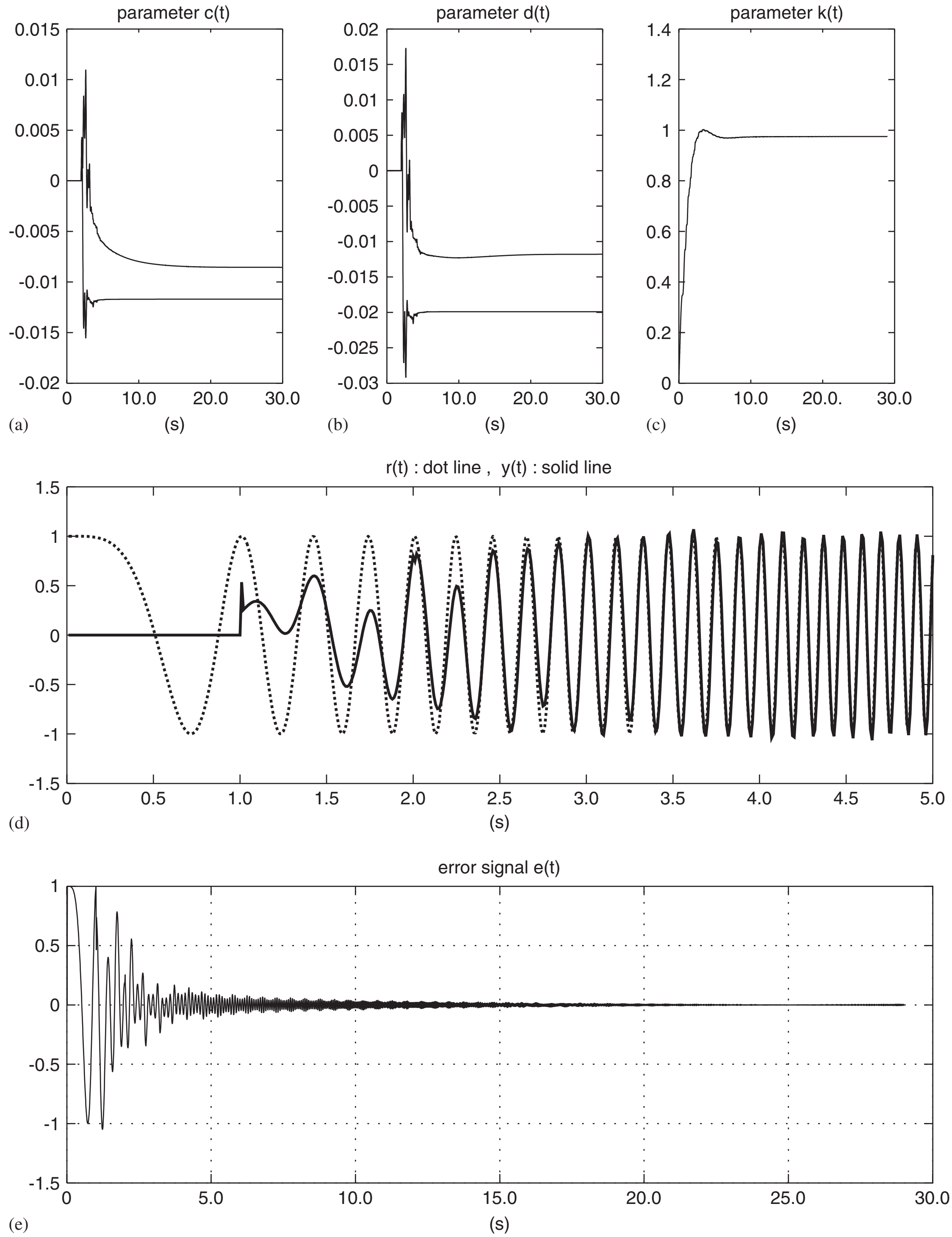

Fig. 4. A simulation result: (a) time change of parameter $c(t)$, (b) time change of parameter $d(t)$, (c) time change of parameter $k(t)k$, (d) improvement of tracking performance of $y(t)$ for $r(t)$ after starting adaptation, (e) time change of error signal $e(t)$.

$f(\gamma)=0$. Then, there exist k_1 and k_2 such that

$$\begin{bmatrix} g_1(\gamma) & \beta_{12}\mathrm{e}^{\tau\gamma} \\ \beta_{21}\mathrm{e}^{\tau\gamma} & g_2(\gamma) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0. \tag{A.1}$$

Since $f(\gamma)=0$ and $g_1(\gamma)<0$, there holds $g_2(\gamma)<0$. Therefore, we can take $k_1>0$, $k_2>0$.

Let

$$v_1(t) = k_1\mathrm{e}^{-\gamma t}, \quad v_2(t) = k_2\mathrm{e}^{-\gamma t}. \tag{A.2}$$

Since $k_1>0$ and $k_2>0$, we see that

$$\sup_{t-\tau\leqslant\sigma\leqslant t} v_1(\sigma) = k_1\mathrm{e}^{-\gamma(t-\tau)}, \qquad \sup_{t-\tau\leqslant\sigma\leqslant t} v_2(\sigma) = k_2\mathrm{e}^{-\gamma(t-\tau)}.$$

Using these relationships, it is not difficult to see that $v_1(t)$ and $v_2(t)$ defined by (A.2) satisfy

$$\frac{dv_1(t)}{dt} = -\alpha_1 v_1(t) + \beta_{11} \sup_{t-\tau \leqslant \sigma \leqslant t} v_1(\sigma) + \beta_{12} \sup_{t-\tau \leqslant \sigma \leqslant t} v_2(\sigma), \tag{A.3}$$

$$\frac{dv_2(t)}{dt} = -\alpha_2 v_2(t) + \beta_{21} \sup_{t-\tau \leqslant \sigma \leqslant t} v_1(\sigma) + \beta_{22} \sup_{t-\tau \leqslant \sigma \leqslant t} v_2(\sigma). \tag{A.4}$$

Since $k_1 > 0$ and $k_2 > 0$ in (A.1) can be taken arbitrarily large, we can assume, for $-\tau \leqslant t \leqslant 0$,

$$v_1(t) > w_1(t), \quad v_2(t) > w_2(t). \tag{A.5}$$

We shall show that the inequalities (A.5) hold for all t.

Assume, contrary to the assertion, that one of the inequalities (A.5) is first violated at $t = t_1$. Without loss of generality, we assume that $v_1(t_1) = w_1(t_1)$. Since $v_1(t) > w_1(t)$ and $v_2(t) > w_2(t)$ for $-\tau < t < t_1$, we see that

$$\sup_{t_1-\tau \leqslant \sigma \leqslant t_1} w_1(\sigma) < \sup_{t_1-\tau \leqslant \sigma \leqslant t_1} v_1(\sigma),$$
$$\sup_{t_1-\tau \leqslant \sigma \leqslant t_1} w_2(\sigma) < \sup_{t_1-\tau \leqslant \sigma \leqslant t_1} v_2(\sigma).$$

Since β_{ij} are positive, (8) and (A.3) imply that $dv_1(t_1)/dt \geqslant dw_1(t_1)/dt$. From this and $v_1(t) > w_1(t)$ for $t < t_1$, we conclude that $v_1(t_1) > w_1(t_1)$, which is a contradiction. Thus, we have established the inequalities (10).

Appendix B. Proof of Theorem 3.3

Using state space representation of $L(s) := c^T(sI - A)^{-1} b + d$, we can rewrite (13) as

$$\frac{dx(t)}{dt} = Ax(t) + b\xi(t)^T z(t - \tau(t)), \tag{B.1}$$

$$\frac{dz(t)}{dt} = -\alpha\xi(t)c^T x(t) - \alpha\, d\xi(t)\xi(t)^T z(t - \tau(t)). \tag{B.2}$$

From (B.1), we derive a similar inequality as in Lemma 3.2 for $x(t)$. Since A is stable, we have

$$\|e^{At}\| \leqslant M_\delta e^{-\delta t}, \quad M_\delta > 0, \ \delta > 0.$$

Note that δ is independent of α. From the representation

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\sigma)} b\xi(\sigma)^T z(\sigma - \tau(\sigma))\, d\sigma$$

we have

$$\|x(t)\| \leqslant M_1 e^{-\delta t} + M_2 \int_0^t e^{-\delta(t-\sigma)} \|z(\sigma - \tau(\sigma))\|\, d\sigma,$$

which can be written as

$$\|x(t)\| \leqslant M_1 e^{-\delta t} + M_2 \int_0^t e^{-\delta(t-\sigma)} \sup_{\sigma - 2\tau_0 \leqslant \sigma' \leqslant \sigma} \|z(\sigma')\|\, d\sigma. \tag{B.3}$$

Here,

$$M_1 = M_\delta \|x(0)\|, \quad M_2 = M_\delta \sup_t \|b\xi(t)\|.$$

On the other hand, (B.1) and (B.2) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = F(t)\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} -b\xi^T(t) \\ \alpha\, d\xi(t)\xi^T(t) \end{bmatrix} \times (z(t) - z(t - \tau(t))), \tag{B.4}$$

where

$$F(t) = \begin{bmatrix} A & b\xi^T(t) \\ -\alpha\xi(t)c^T & -\alpha\, d\xi(t)\xi^T(t) \end{bmatrix}. \tag{B.5}$$

From the assumptions (i) and (ii), the fundamental matrix $U(t,s)$ of $A(t)$ defined by

$$\frac{dU(t,s)}{dt} = F(t)U(t,s), \quad U(s,s) = I,$$

satisfies

$$\|U(t,s)\| \leqslant M e^{-\lambda(t-s)} \tag{B.6}$$

for some $M > 0$ and $\lambda > 0$. It should be noted that the decaying exponent of $U(t,s)$ goes to 0 as α tend to 0, from the form (B.5) of $F(t)$. Using $U(t,s)$, we have

$$\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = U(t,0)\begin{bmatrix} x(0) \\ z(0) \end{bmatrix} + \alpha \int_0^t U(t,\sigma)\begin{bmatrix} -b\xi(\sigma)^T \\ \xi(\sigma)\xi(\sigma)^T \end{bmatrix} \times (z(\sigma) - z(\sigma - \tau(\sigma)))\, d\sigma,$$

from which we obtain the inequality

$$\|z(t)\| \leqslant M_3 e^{-\lambda t} + M_4 \int_0^t e^{-\alpha\mu(t-\sigma)} \|z(\sigma) - z(\sigma - \tau(\sigma))\|\, d\sigma, \tag{B.7}$$

where, M_3 and M_4 are given as,

$$M_3 := M\left\|\begin{bmatrix} x(0) \\ z(0) \end{bmatrix}\right\|, \quad M_4 := M \sup_t \left\|\begin{bmatrix} -b\xi(t) \\ \alpha\, d\xi(t)\xi^T(t) \end{bmatrix}\right\|.$$

Now, we have

$$z(\sigma) - z(\sigma - \tau(\sigma)) = \int_{\sigma-\tau(\sigma)}^{\sigma} \frac{dz(\sigma')}{d\sigma'}\, d\sigma'.$$

Hence,

$$\begin{aligned} &\|z(\sigma) - z(\sigma - \tau(\sigma))\| \\ &\quad \leqslant \int_{\sigma-\tau(\sigma)}^{\sigma} \left\|\frac{dz(\sigma')}{d\sigma'}\right\| d\sigma' \\ &\quad \leqslant \int_{\sigma-\tau_0}^{\sigma} \|\alpha\xi(\sigma')c^T x(\sigma') + \alpha\, d\xi(\sigma')\xi(\sigma')^T z(\sigma' - \tau_0)\|\, d\sigma' \\ &\quad \leqslant \tau_0\alpha\Bigg(M_5 \sup_{\sigma-\tau_0 \leqslant \sigma' \leqslant \sigma} \|x(\sigma')\| \\ &\qquad + M_6 \sup_{\sigma-\tau_0 \leqslant \sigma' \leqslant \sigma} \|z(\sigma' - \tau_0)\|\Bigg), \end{aligned}$$

which can be rewritten as

$$\|z(\sigma)-z(\sigma-\tau(\sigma))\| \leqslant \tau_0\alpha\left(M_5 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|x(\sigma')\| + M_6 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|z(\sigma')\|\right), \tag{B.8}$$

where M_5 and M_6 are given as

$$M_5 := \sup_t \|\xi(t)c^{\mathrm{T}}\|, \quad M_6 := \sup_t \mathrm{d}\|\xi(t)\xi^{\mathrm{T}}(t)\|.$$

Substituting (B.8) in (B.7) yields

$$\|z(t)\| \leqslant M_3\mathrm{e}^{-\alpha\mu t} + \alpha\tau_0 M_4\int_0^t \mathrm{e}^{-\alpha\mu(t-\sigma)}\left(M_5 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|x(\sigma')\| + M_6 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|z(\sigma')\|\right)\mathrm{d}\sigma. \tag{B.9}$$

The inequalities (B.9) and (B.3) represent the fundamental structure of the delay-differential equation (13). Let

$$w_1(t) := M_1\mathrm{e}^{-\delta t} + M_2\int_0^t \mathrm{e}^{-\delta(t-\sigma)} \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|z(\sigma')\|\,\mathrm{d}\sigma,$$

$$w_2(t) := M_3\mathrm{e}^{-\lambda t} + \alpha\tau_0 M_4\int_0^t \mathrm{e}^{-\alpha\mu(t-\sigma)}\left(M_5 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|x(\sigma')\| + M_6 \sup_{\sigma-2\tau_0\leqslant\sigma'\leqslant\sigma}\|z(\sigma')\|\right)\mathrm{d}\sigma,$$

Due to (B.9) and (B.3), we have

$$\|x(t)\| \leqslant w_1(t), \quad \|z(t)\| \leqslant w_2(t). \tag{B.10}$$

It is easy to see that

$$\frac{\mathrm{d}w_1(t)}{\mathrm{d}t} = -\delta w_1(t) + \alpha M_2 \sup_{t-2\tau_0\leqslant\sigma\leqslant t}\|z(\sigma)\|,$$

$$\frac{\mathrm{d}w_2(t)}{\mathrm{d}t} = -\lambda w_2(t) + \alpha\tau_0 M_4\left(M_5 \sup_{t-2\tau_0\leqslant\sigma\leqslant t}\|x(\sigma)\| + M_6 \sup_{t-2\tau_0\leqslant\sigma\leqslant t}\|z(\sigma)\|\right).$$

From (B.10), it follows that

$$\frac{\mathrm{d}w_1(t)}{\mathrm{d}t} \leqslant -\delta w_1(t) + \alpha M_{10} \sup_{t-2\tau_0\leqslant\sigma\leqslant t} w_2(\sigma),$$

$$\frac{\mathrm{d}w_2(t)}{\mathrm{d}t} \leqslant -\lambda w_2(t) + \alpha\tau_0 M_4\left(M_5 \sup_{t-2\tau_0\leqslant\sigma\leqslant t} w_1(\sigma) + M_6 \sup_{t-2\tau_0\leqslant\sigma\leqslant t} w_2(\sigma)\right).$$

Since the above differential inequalities are in the same form as in (8) and (9), we can apply Lemma 3.2. The condition (11) is always satisfied since $\alpha_1 = \delta$ and $\beta_{11} = 0$. The condition (12) holds if

$$-\delta\alpha\tau_0 M_4 M_6 + \delta\lambda - \alpha\tau_0 M_4 M_5 M_2 > 0,$$

or equivalently if,

$$\alpha < \frac{\lambda\delta}{\tau_0(\delta M_4 M_6 + M_4 M_5 M_2)}. \tag{B.11}$$

If (B.11) holds, Lemma 3.2 implies that

$$w_1(t) \leqslant k_1\mathrm{e}^{-\gamma t}, \quad w_2(t) \leqslant k_2\mathrm{e}^{-\gamma t},$$

for some $k_1 > 0$, $k_2 > 0$, and $\gamma > 0$. In view of the inequalities (B.10), the assertion has been proved.

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