# Improving Transient Response of Adaptive Control Systems using Multiple Models and Switching

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***Abstract*— A well-known problem in adaptive control is the poor transient response which is observed when adaptation is initiated. In this paper we develop a stable strategy for improving the transient response by using multiple models of the plant to be controlled and switching between them. The models are identical except for initial estimates of the unknown plant parameters. The control to be applied is determined at every instant by the model which best approximates the plant. Simulation results are presented to indicate the improvement in performance that can be achieved.**

## I. Introduction

Direct and indirect methods for the adaptive control of a linear time-invariant plant are currently well known [1]. Numerous globally stable algorithms have been developed over the last decade for the so-called "ideal case" that result in zero steady-state tracking error. Extensive computer simulations of these algorithms have revealed that the tracking error is quite often oscillatory, however, with large amplitudes during the transient phase. In this paper, we suggest one method of improving the transient response in the ideal case by using multiple identification models.

The unknown plant parameters are assumed to belong to a compact set $\mathcal{S}$. $N$ adaptive identification models $\{I_j\}_{j=1}^N$ with identical structures, but with initial estimates of plant parameters uniformly distributed in $\mathcal{S}$, are used. Each identification model $I_j$ is paired with a parameterized controller $C_j$, to form an indirect controller $IC_j$. The control strategy is to determine at each instant the model whose identification error is the minimum according to some criterion function and use the corresponding controller to control the plant. Consequently, the problem is to determine an appropriate criterion for the choice of the model at any instant and demonstrate that the resultant switching scheme yields a stable system with improved performance.

An important consideration in such schemes is the stability of the overall system. It is shown that any arbitrary switching scheme yields a globally stable system, provided that the interval between successive switches has an arbitrarily small but nonzero lower bound. This result provides the freedom to choose the switching scheme entirely on the basis of performance. In this sense, the nature of the problem is distinctly different from that normally encountered in the adaptive control literature.

The idea of switching between multiple adaptive models has been used in the past by Middleton *et al.* [2] to alleviate the problem of stabilizability of the estimated model in indirect control and later adapted by Morse *et al.* [3] for model reference adaptive control when the relative degree of the plant is unknown. While these results used switching techniques to solve global stability problems in adaptive control, the thrust of this paper is to demonstrate that switching between multiple models can be used to improve performance in adaptive systems, even when a single model alone is sufficient to assure stability.

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## II. Statement of the Problem

The basic problem we wish to address is closely related to the model reference adaptive control (MRAC) problem treated in standard textbooks on the subject (e.g., [1]). The plant to be controlled is linear and time-invariant, with input $u\colon \mathbb{R}_+ \to \mathbb{R}$ and output $y_p\colon \mathbb{R}_+ \to \mathbb{R}$ related by

$$y_p = W_p(s)u \tag{1}$$

where $W_p(s) = k_p(Z_p(s)/R_p(s))$ is the transfer function of the plant. Here $k_p \in \mathbb{R}$ is nonzero, and $R_p(s)$ and $Z_p(s)$ are monic coprime polynomials of degree $n$ and $m$, respectively, with $m < n$, and unknown coefficients. We make the standard assumptions [1] about $W_p(s)$ that i) the order $n$, ii) the relative degree $n^* \triangleq n - m$, iii) the sign of $k_p$ are known, and iv) $Z_p(s)$ is Hurwitz.

The reference model to be followed is linear and time-invariant, with input $r\colon \mathbb{R}_+ \to \mathbb{R}$ (which is piecewise continuous and uniformly bounded) and output $y_m\colon \mathbb{R}_+ \to \mathbb{R}$ related by

$$y_m = W_m(s)r \tag{2}$$

where $W_m(s) = k_m(Z_m(s)/R_m(s))$ is the transfer function of the reference model. Here $k_m \in \mathbb{R}$ is nonzero and $R_m(s)$ and $Z_m(s)$ are monic coprime Hurwitz polynomials of degree $n$ and $m$, respectively.

The standard MRAC problem is to determine a differentiator-free control input $u(\cdot)$ such that all signals in the overall system are bounded and the control error $e_c(t) \triangleq y_p(t) - y_m(t)$ converges to zero asymptotically. This problem was first solved in 1980 in [4]–[7] and has been widely used in the design of adaptive systems. Our additional (qualitative) requirement is that $e_c(t)$ stays within reasonable limits and settles down to an acceptably small value within a reasonable time.

## III. Description of the Adaptive Control System

The proposed control system (see Fig. 1) consists of $N$ identical indirect controllers $\{IC_j\}_{j=1}^N$. In particular, $N$ adaptive identification models $\{I_j\}_{j=1}^N$, with identical structures but different initial estimates of the plant parameters, are used. Corresponding to each $I_j$ is a controller $C_j$ and the identification and control parameters of each pair $IC_j$ are tuned simultaneously as described in this section. While all the identification models operate in parallel, only one of the controllers is connected to the plant at any instant. The decision as to which controller to connect to the plant at every instant is determined by the switching scheme, which is described in Section V.

*Parameterization of the Plant:* The plant is parameterized as follows. Define $\omega_1, \omega_2\colon \mathbb{R}_+ \to \mathbb{R}^{n-1}$ by

$$\dot{\omega}_1 = \Lambda\omega_1 + lu$$

$$\dot{\omega}_2 = \Lambda\omega_2 + ly_p \tag{3}$$

where $(\Lambda, l)$ is an asymptotically stable system in controllable

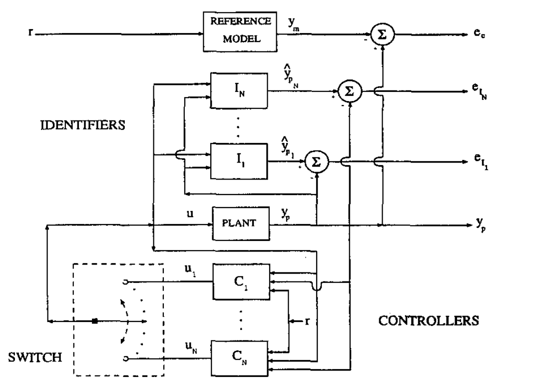


Fig. 1. Structure of the adaptive control system with $N$ identifiers $\{I_j\}_{j=1}^N$ and controllers $\{C_j\}_{j=1}^N$.

canonical form, with

$$\lambda(s) \triangleq \det(sI - \Lambda) = \lambda_1(s)Z_m(s) \tag{4}$$

for some monic Hurwitz polynomial $\lambda_1(s)$ of degree $n - m - 1$. Since $Z_p(s)$ and $R_p(s)$ are coprime, there exist unique polynomials $\alpha(s)$ and $\beta(s)$ each of degree $n-1$ satisfying the Bezout identity

$$R_p(s)\beta(s) + k_p Z_p(s)\alpha(s) = \frac{k_p}{k_m} Z_p(s)R_m(s)\lambda_1(s). \tag{5}$$

There exist unique constants $\beta_0^* \triangleq k_p/k_m \in \mathbb{R}$, $\beta_1^* \in \mathbb{R}^{n-1}$, $\alpha_0^* \in \mathbb{R}$, $\alpha_1^* \in \mathbb{R}^{n-1}$ satisfying $\beta_0^* + {\beta_1^*}^T(sI-\Lambda)^{-1}l = \beta(s)/\lambda(s)$ and $\alpha_0^* + {\alpha_1^*}^T(sI-\Lambda)^{-1}l = k_p\alpha(s)/\lambda(s)$ such that $y_p$ can be expressed as

$$y_p = W_m(s)\{\beta_0^* u + {\beta_1^*}^T\omega_1 + \alpha_0^* y_p + {\alpha_1^*}^T\omega_2\} \triangleq W_m(s)\{p^{*T}\omega\} \tag{6}$$

where $\omega \triangleq (u, \omega_1^T, y_p, \omega_2^T)^T$ and $p^* \triangleq (\beta_0^*, {\beta_1^*}^T, \alpha_0^*, {\alpha_1^*}^T)^T$. Since $p^*$ is a constant, $y_p$ can be rewritten as

$$y_p = p^{*T}\overline{\omega} \tag{7}$$

where $\overline{\omega} \triangleq W_m(s)I_{2n}\{\omega\}$.

*Structure of the Identifiers:* The $N$ identifiers $\{I_j\}_{j=1}^N$ have outputs

$$\hat{y}_{p_j} = \hat{p}_j^T\overline{\omega} \tag{8}$$

where $\hat{y}_{p_j}$ and $\hat{p}_j$ are the $j$th estimates of $y_p$ and $p^*$, respectively. Defining the $j$th identification error as $e_{I_j} = \hat{y}_{p_j} - y_p$ and the parameter error vector $\tilde{p}_j \triangleq \hat{p}_j - p^*$, we obtain

$$e_{I_j} = \tilde{p}_j^T\overline{\omega}. \tag{9}$$

*Structure of the Controllers:* If $p^*$ is known and the control input is chosen as $u^*(t) = \theta^{*T}\underline{\omega}(t)$ where $\underline{\omega} \triangleq (r, \omega_1^T, y_p, \omega_2^T)^T$ and $\theta^* \triangleq (k^*, {\theta_1^*}^T, \theta_0^*, {\theta_2^*}^T)^T \in \mathbb{R}^{2n}$, it follows from (6) that $y_p \equiv W_m(s)r = y_m$ (assuming zero initial conditions). The elements of

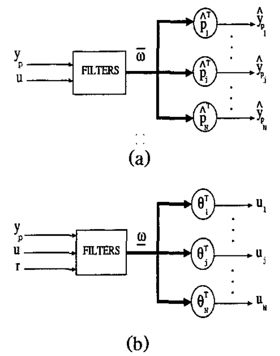


Fig. 2. Internal structure of the adaptive control system: (a) identifiers (b) controllers.

$\theta^*$ are computed from $p^*$ as

$$k^* \triangleq \frac{1}{\beta_0^*}, \qquad \theta_1^* \triangleq -\frac{\beta_1^*}{\beta_0^*}, \qquad \theta_0^* \triangleq -\frac{\alpha_0^*}{\beta_0^*}, \qquad \theta_2^* \triangleq -\frac{\alpha_1^*}{\beta_0^*}. \tag{10}$$

The output of each $C_j$ is given by

$$u_j(t) = \theta_j^T(t)\underline{\omega}(t) \tag{11}$$

where $\theta_j \triangleq (k_j, \theta_{1_j}^T, \theta_{0_j}, \theta_{2_j}^T)^T$ is the $j$th estimate of $\theta^*$ with error vector $\tilde{\theta}_j \triangleq \theta_j - \theta^*$. While all the identifiers share the same regression vector $\overline{\omega}$, all the controllers share the vector $\underline{\omega}$ (see Fig. 2).

The control error $e_c(t) \triangleq y_p(t) - y_m(t)$ can then be expressed as

$$e_c = W_m(s)\{\beta_0^*\tilde{\theta}^T\underline{\omega}\} \tag{12}$$

where $\tilde{\theta}(t) \triangleq \theta(t) - \theta^*$, and $\theta(t) \in \{\theta_1(t), \cdots, \theta_N(t)\}$ is the control parameter vector used at time $t$. With the above definitions, the manner in which the parameters $\hat{p}_j(t)$ and $\theta_j(t)$ are tuned can be described.

*Dynamical Adjustment of Identification and Control Parameters:* Motivated by (10), we define the following closed-loop estimation errors in terms of $\hat{p}_j$ and $\theta_j$

$$\epsilon_{k_j} \triangleq \hat{\beta}_{0_j}k_j - 1, \qquad \epsilon_{\theta_{1_j}} \triangleq \hat{\beta}_{0_j}\theta_{1_j} + \hat{\beta}_{1_j},$$

$$\epsilon_{\theta_{0_j}} \triangleq \hat{\beta}_{0_j}\theta_{0_j} + \hat{\alpha}_{0_j}, \qquad \epsilon_{\theta_{2_j}} \triangleq \hat{\beta}_{0_j}\theta_{2_j} + \hat{\alpha}_{1_j} \tag{13}$$

and $\epsilon_{\theta_j} \triangleq (\epsilon_{k_j}, \epsilon_{\theta_{1_j}}^T, \epsilon_{\theta_{0_j}}, \epsilon_{\theta_{2_j}}^T)^T$. In addition to the identification errors $e_{I_j}$, the parametric errors $\epsilon_{\theta_j}$ are also used in the adjustment of $\hat{p}_j$ and $\theta_j$ as shown in (14). When $\hat{p}_j(t) \equiv p^*$ and $\theta_j(t) \equiv \theta^*$, these errors are identically zero. Hence, they determine the extent to which $\hat{p}_j(t)$ and $\theta_j(t)$ satisfy the relation (10). In Section IV, it is shown that the following adaptive laws, together with any allowable switching scheme, force the identification errors $e_{I_j}(t)$, the closed-loop estimation errors $\epsilon_{\theta_j}(t)$ and the control error $e_c(t)$ to tend to zero asymptotically

$$\dot{\tilde{p}}_j = \dot{\hat{p}}_j = -\frac{e_{I_j}\overline{\omega}}{1+\overline{\omega}^T\overline{\omega}} - \begin{pmatrix}\theta_j^T \\ 0 \mid I_{(2n-1)\times(2n-1)}\end{pmatrix}\epsilon_{\theta_j}$$

$$\dot{\tilde{\theta}}_j = \dot{\theta}_j = -sgn(\beta_0^*)\epsilon_{\theta_j}. \tag{14}$$

## IV. Global Stability with Arbitrary Switching Schemes

The adaptive laws (14) indicate how the identification and control parameters are to be adjusted at every instant, but do not determine the sequence of controllers to connect to the plant. The principal result of the paper guarantees that, as far as stability is concerned, it does not matter what this sequence is: The overall system will be globally stable for any arbitrary switching sequence, provided that the intervals between successive switches have a nonzero lower bound $T_{\min} > 0$, which can be chosen to be arbitrarily small. This result is intuitively reasonable since it is known that each one of the $N$ controllers $\{IC_j\}$ results in global stability if used alone. The proof does not follow directly, however, since switching between stabilizing controllers need not result in a stable system.

The proof is based on the methods in [1] and [4] for the case of a single model (i.e., $N = 1$). Since all the identification models and the adaptive control laws are identical, the overall system is globally stable if the switching settles down after a finite time at any one of the indirect controllers. We prove that the same is the case with an arbitrary switching sequence between the $N$ controllers, even if switching never stops. Let $\{T_i\}_{i=0}^{\infty}$ denote the instants at which switching takes place, so that $T_i \to \infty$, and $T_{i+1} - T_i > T_{\min} > 0$. The input to the plant is $u(t) = \theta^T(t)\underline{\omega}(t)$ where $\theta(t)$ is chosen by the switching scheme from the set $\{\theta_j(t)\}_{j=1}^{N}$. The proof proceeds by contradiction. We assume that the signal $\omega_p \triangleq (\omega_1^T, y_p, \omega_2^T)^T$ is unbounded and show that this leads to a contradiction.

The proof is given in Steps 1–6. The results of Steps 1 and 2 are standard and follow the same arguments used in [1] and [4]. The results of Steps 3–5 are based on two propositions which are extensions of well-known results in adaptive control theory (involving bounded signals) to the case of unbounded signals. The propositions are stated without proof since they follow along the same lines as in the case of bounded signals.

If a signal $f \in \mathcal{L}^2$ and $\dot{f} \in \mathcal{L}^\infty$, it is well known that $\lim_{t\to\infty} f(t) = 0$. Proposition 1 is an extension of this fact.

*Proposition 1:* Let $f$: $\mathbb{R}_+ \to \mathbb{R}$ and $\omega$: $\mathbb{R}_+ \to \mathbb{R}^n$ be continuous signals. If $f(t) = \beta(t)O(\sup_{\tau\le t}\|\omega(\tau)\|)$ where $\beta \in \mathcal{L}^2$ is continuous, and $\dot{f}(t) = O(\sup_{\tau\le t}\|\omega(\tau)\|)$, then $f(t) = o(\sup_{\tau\le t}\|\omega(\tau)\|)$.

If the input $u(t)$ to a strictly proper, asymptotically stable, minimum-phase system $W_m(s)$ is bounded with bounded derivative, and if the output $y(t)$ tends to zero, it is well known that the input must also tend to zero. This fact is generalized in Proposition 2.

*Proposition 2:* Let $u(t)$ be the input to a strictly proper, asymptotically stable, minimum-phase single-input/single-output (SISO) system $W_m(s)$, with output $y(t)$. Let $\omega$: $\mathbb{R}_+ \to \mathbb{R}^n$ be a given signal. If $u(t)$ is piecewise differentiable with $u(t) = O(\sup_{\tau\le t}\|\omega(\tau)\|)$ and $\dot{u}(t) = O(\sup_{\tau\le t}\|\omega(\tau)\|)$, and if $y(t) = o(\sup_{\tau\le t}\|\omega(\tau)\|)$, it follows that $u(t) = o(\sup_{\tau\le t}\|\omega(\tau)\|)$.

### *Proof of Global Stability*

*Step 1:* By using Lyapunov function candidates $V_j \triangleq \|\tilde{p}_j\|^2 + |\beta_0^*|\,\|\tilde{\theta}_j\|^2$, it follows that $(d/dt)V_j(t) = -e_{I_j}^2(t)/(1+\overline{\omega}^T(t)\overline{\omega}(t)) - \|\epsilon_{\theta_j}(t)\|^2 \le 0$ along the trajectories of the system described by (1)–(14). From this it follows that for each $j$, i) the parameters $\hat{p}_j$, $\theta_j \in \mathcal{L}^\infty$, ii) their time derivatives $\dot{\hat{p}}_j$, $\dot{\theta}_j \in \mathcal{L}^\infty \cap \mathcal{L}^2$, iii) the identification errors satisfy $e_{I_j}(t) = \beta_j(t)\sqrt{1+\overline{\omega}^T(t)\overline{\omega}(t)}$, where $\beta_j \in \mathcal{L}^2$ is continuous, and iv) the parametric errors $\epsilon_{\theta_j}(t) \to 0$, as $t \to \infty$. Since all the control parameters $\theta_j$ are bounded, the parameter vector $\theta(t)$ is also bounded. Hence, the states of the overall system can grow at most exponentially. This in turn ensures the existence of a unique solution on $[0, \infty)$.

*Step 2:* Since the signals $\omega = (u, \omega_p^T)^T$, $\overline{\omega} = W_m(s)I_{2n}\{\omega\}$ and $\underline{\omega} = (r, \omega_p^T)^T$ are common to all the indirect controllers, it follows using standard arguments that $\dot{\omega}_p(t) = O(\sup_{\tau\le t}\|\omega_p(\tau)\|)$ and $\sup_{\tau\le t}|y_p(\tau)| \sim \sup_{\tau\le t}\|\omega_p(\tau)\|$, i.e., $y_p$ and $\omega_p$ grow at the same rate. Moreover, no signal can grow faster than $\omega_p$.

*Step 3:* In this step we show that the $N$ identification errors satisfy $e_{I_j}(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$. Since $e_{I_j} = \tilde{p}_j^T\overline{\omega} = \beta_j(t)\sqrt{1+\overline{\omega}^T(t)\overline{\omega}(t)}$ where $\beta_j \in \mathcal{L}^2$, taking derivatives and using the facts that $\tilde{p}_j$, $\dot{\tilde{p}}_j \in \mathcal{L}^\infty$, and $\overline{\omega}(t)$ and $\dot{\overline{\omega}}(t)$ are $O(\sup_{\tau\le t}\|\omega_p(\tau)\|)$, it follows that $\dot{e}_{I_j}(t) = O(\sup_{\tau\le t}\|\omega_p(\tau)\|)$. Since the conditions of Proposition 1 are satisfied, the result follows, i.e., $e_{I_j}(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$.

*Step 4:* Since $\dot{\tilde{p}}_j \in \mathcal{L}^2$, it is known [1] that

$$W_m(s)\{\tilde{p}_j^T\omega\} - \tilde{p}_j^T W_m(s)I_{2n}\{\omega\} = o\left(\sup_{\tau\le t}\|\omega(\tau)\|\right).$$

Since from Step 2, $\omega(t) = O(\sup_{\tau\le t}\|\omega_p(\tau)\|)$ and from Step 3, $e_{I_j}(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$, this implies

$$W_m(s)\{\tilde{p}_j^T\omega\}(t) = o\left(\sup_{\tau\le t}\|\omega_p(\tau)\|\right).$$

*Step 5:* From Step 4, $W_m(s)\{\tilde{p}_j^T\omega\}(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$. In this step, we show that $\tilde{p}_j^T(t)\omega(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$. We first express $\tilde{p}_j^T\omega$ as

$$\tilde{p}_j^T(t)\omega(t) = \tilde{\beta}_{0_j}(t)k(t)r(t) + v_j(t) \tag{15}$$

where the first term in the right-hand side is bounded and contains the reference input, and the second term contains only piecewise differentiable signals (since there is a minimum time $T_{\min} > 0$ between the discontinuities due to switching). Further, $v_j(t)$ and $\dot{v}_j(t)$ are both $O(\sup_{\tau\le t}\|\omega_p(\tau)\|)$. Applying Proposition 2 to $v_j$ and $W_m(s)\{v_j\}$, noting that the first term in (15) is bounded and using the result of Step 4, it follows that $\tilde{p}_j^T(t)\omega(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$.

*Step 6:* Since $\tilde{\theta}_j \triangleq \theta_j - \theta^*$, it follows from (13) that the signals $\beta_0^*\tilde{\theta}_j^T\underline{\omega}$ satisfy the error equation

$$\beta_0^*\tilde{\theta}_j^T\underline{\omega} = \epsilon_{\theta_j}^T\underline{\omega} - \tilde{p}_j^T\omega - \tilde{\beta}_{0_j}(u_j - u)$$

where $u(t)$ is the actual input to the plant and $u_j(t)$ is the input generated by controller $C_j$, at time $t$. For the choice of any arbitrary sequence of the controllers (with switching at instants $\{T_i\}$), let the control input over any arbitrary interval between switches be $u_j$. Over that interval, $u_j(t) - u(t) \equiv 0$ and $\beta_0^*\tilde{\theta}_j^T(t)\underline{\omega}(t) = \epsilon_{\theta_j}^T(t)\underline{\omega}(t) - \tilde{p}_j^T(t)\omega(t)$. This holds over any arbitrary interval $[T_i, T_{i+1})$. Using the results of Steps 1 and 5, it then follows that $\beta_0^*\tilde{\theta}^T(t)\underline{\omega}(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$, $t \ge 0$. Hence, from (12), it follows that $y_p(t) = e_c(t) + y_m(t) = o(\sup_{\tau\le t}\|\omega_p(\tau)\|)$, or $y_p$ grows at a slower rate than $\omega_p$, which is a contradiction. This implies that (refer to Step. 2) all signals in the overall system are bounded. Further, $e_{I_j} \in \mathcal{L}^2$ and has a bounded derivative, implying that $e_{I_j}(t) \to 0$, $j = 1, \cdots, N$. Finally, it follows that $\tilde{\theta}^T(t)\underline{\omega}(t) \to 0$ and so the control error $e_c(t)$ also tends to zero asymptotically.

## V. A Switching Scheme for Improving Transient Response

From the result given in Section IV, it can be seen that the questions of stability and performance are decoupled. Hence, one can choose any switching scheme which one would expect to result in improved transient performance. The switching scheme that we propose is

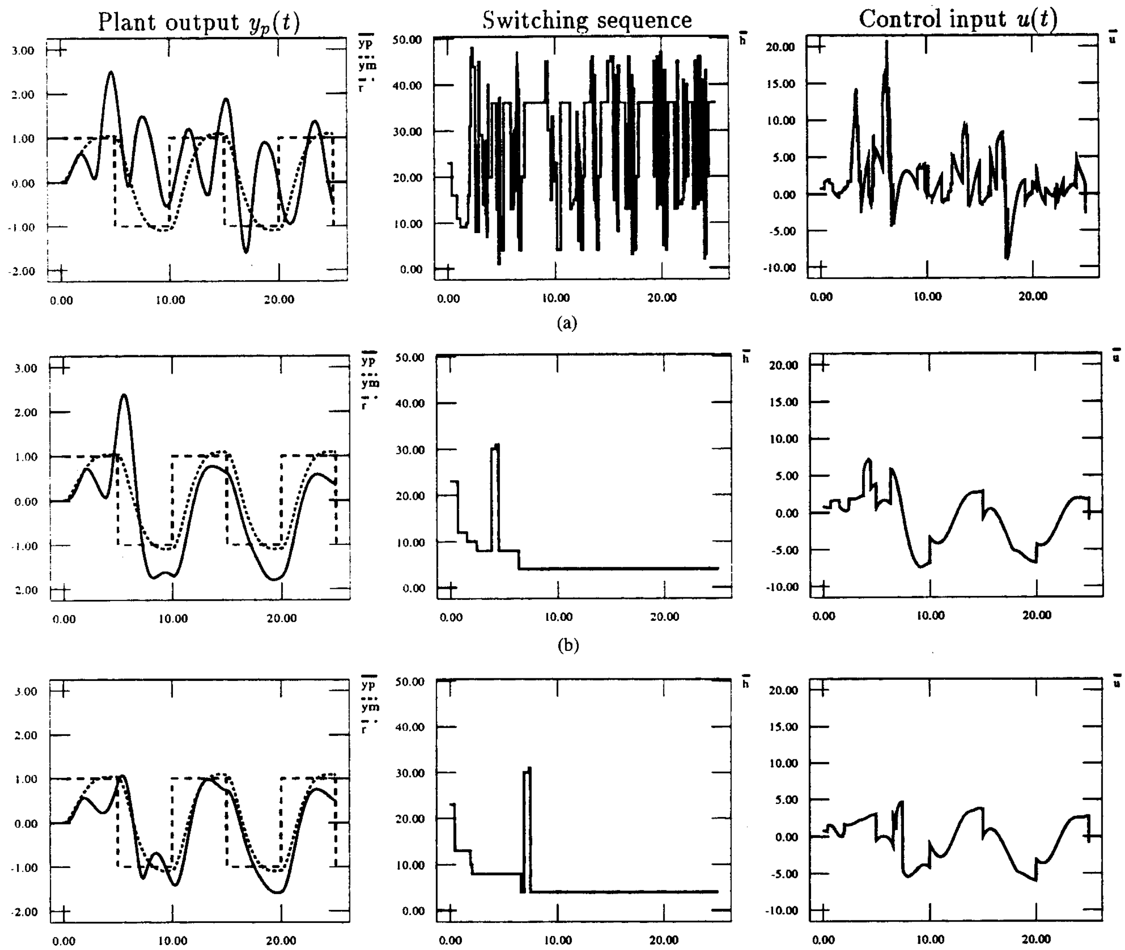


Fig. 3. Response of an unstable oscillatory plant $W_p(s) = 0.5/(s^2 - 0.35s + 2)$, with $N = 48$ models, using three different switching criteria: (a) $J_j(t) = e^2_{I_j}(t)$, (b) $J_j(t) = \int_0^t e^2_{I_j}(\tau)\,d\tau$, and (c) $J_j(t) = e^2_{I_j}(t) + \int_0^t e^2_{I_j}(\tau)\,d\tau$.

based on monitoring a criterion function for each of the identification errors $\{e_{I_j}\}_{j=1}^N$ and choosing the controller which corresponds to the minimum at every instant. The rationale for choosing such a scheme is presented in what follows.

The motivation for the indirect control method is that stable control of the identification model on-line will lead to stable control of the plant. Extending this idea, it seems reasonable to assume that if several identification models are used, the one with the "best" performance would yield the best control performance as well. Since our interest is in transient response, we need to determine a reliable index of identifier performance at any instant. While many such indices can be chosen, motivated by quadratic optimal control, we choose the performance index for $I_j$ as

$$J_j(t) = \alpha e^2_{I_j}(t) + \beta \int_0^t e^2_{I_j}(\tau)\,d\tau, \qquad \alpha,\, \beta > 0 \qquad (16)$$

where $\alpha$ and $\beta$ are design parameters. If $J_j(t) \triangleq e^2_{I_j}(t)$, transient peaks in $e^2_{I_j}(t)$ (and hence in $e_c(t)$) would be quickly detected, but switching may be extremely rapid, resulting in poor control. On the other hand, $J_j(t) \triangleq \int_0^t e^2_{I_j}(\tau)\,d\tau$ is a good indicator of steady-state identifier accuracy, but results in infrequent switching and sluggish response to transient peaks. The criterion (16) incorporates both instantaneous and long-term measures of accuracy and appears to have the advantages of both terms. Given the set $S$, the design parameters $\alpha$ and $\beta$ can be chosen off line. Fig. 3 shows the performance of an unstable plant with oscillatory impulse response (refer to Section VI), when the three criteria (i.e., $J_j(t) = e^2_{I_j}(t)$, $J_j(t) = \int_0^t e^2_{I_j}(\tau)\,d\tau$, and $J_j(t) = \alpha e^2_{I_j}(t) + \beta \int_0^t e^2_{I_j}(\tau)\,d\tau$) are used in the switching scheme and capture the essence of the above qualitative discussions.

With the instantaneous criterion, switching occurs with high frequency and the response is not satisfactory; with the integral criterion, switching is infrequent but the error is still too large. Using the combination (16) with $\alpha = \beta = 1$ (determined off line), the response is seen to be better than either of the two.

The proposed switching scheme thus consists of monitoring the performance index $J_j(t)$ at every instant, for $j = 1, \cdots, N$. After every switch, an interval $T_{\min} > 0$ is allowed to elapse, and then the controller corresponding to the identifier with the minimum $J_j(t)$ is switched in series with the plant. Simulation results are presented in the next section which conclusively demonstrate that the use

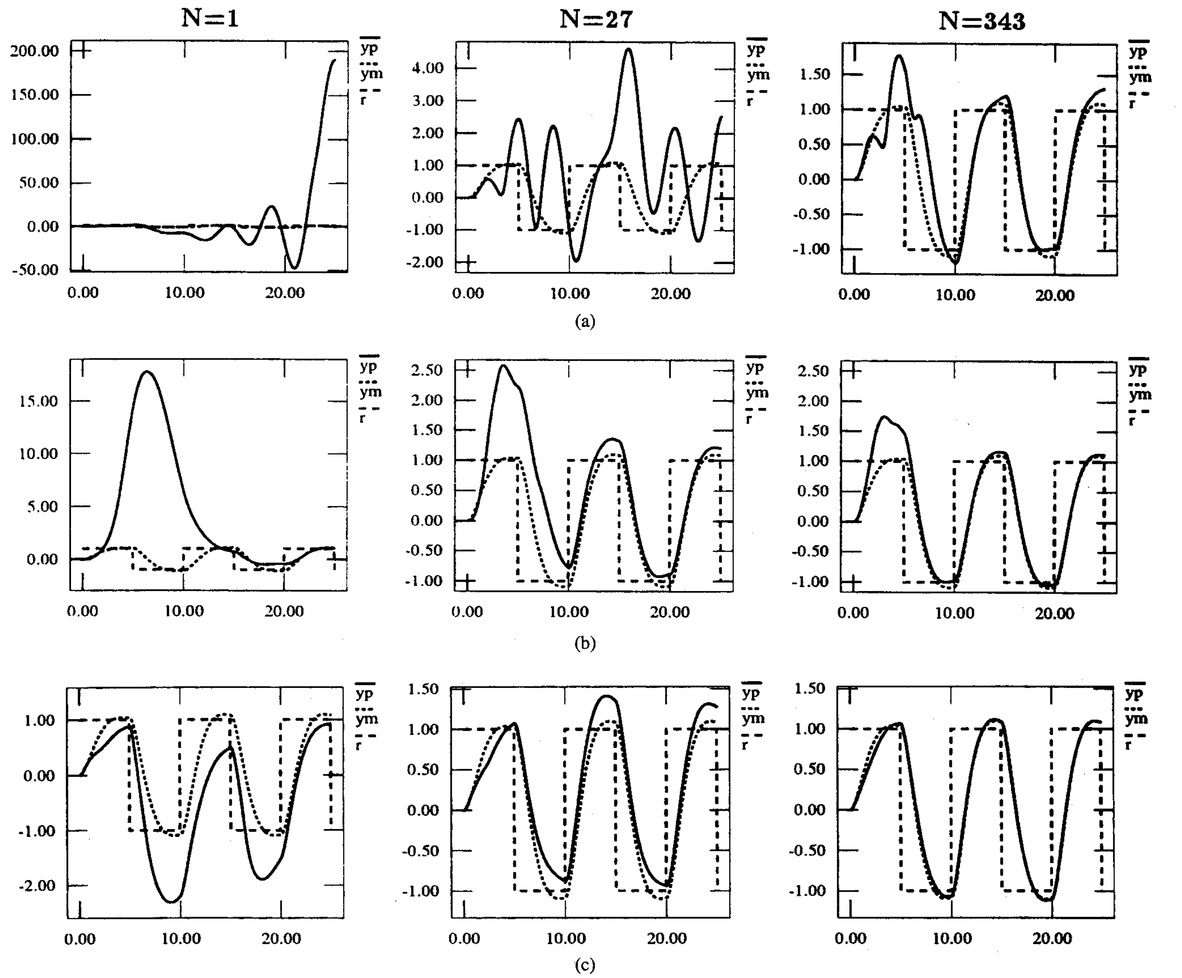


Fig. 4. Response of typical plants in $\mathcal{S}$ using the proposed switching scheme with $J_j(t) = e_{I_j}^2(t) + \int_0^t e_{I_j}^2(\tau)\,d\tau$, and $N = 1$, 27, and 343 models: (a) $k_p = 0.5, a_1 = -0.5, a_0 = 2$— unstable and oscillatory plant; (b) $k_p = 0.5, a_1 = 0.5, a_0 = -2$— unstable and nonoscillatory plant; (c) $k_p = 2, a_1 = 3.4, a_0 = 2$— asymptotically stable and nonoscillatory plant.

of multiple models together with the proposed switching scheme improves the transient performance dramatically over that of a single adaptive controller.

## VI. Simulation Results

*Problem 1 (Zero Initial Conditions):* The plant to be controlled is known to have a transfer function of the form $W_p(s) = k_p/(s^2 + a_1 s + a_0)$, where the unknown parameters lie in the known compact set

$$\mathcal{S} = \{0.5 \le k_p \le 2,\ -0.6 \le a_1 \le 3.4,\ -2 \le a_0 \le 2\}.$$

The initial conditions of the plant are known to be zero. The control objective is to track the output of a reference model $W_m(s) = 1/(s^2 + 1.4s + 1)$ to a reference input $r$. The set $\mathcal{S}$ was chosen so that wide variations in the adaptive system responses are obtained for different choices of the plant parameters.

The method described in the paper was successively applied to three plants in $\mathcal{S}$. In each case, adaptive control was carried out using 1, 27, and 343 models. The results obtained are shown in Figs. 4(a)–(c), which show the responses $y_p(t)$ and $y_m(t)$ to a square wave reference input $r(t)$ with unit amplitude and period 10 units. Each row corresponds to the same plant. In Fig. 4(a), the plant parameters are $(k_p, a_1, a_0) = (0.5, -0.5, 2)$, corresponding to an unstable plant with oscillatory impulse response. In Fig. 4(b), the parameters are $(0.5, 0.5, -2)$, corresponding to an unstable nonoscillatory plant. In Fig. 4(c), the parameters are $(2, 3.4, 2)$, and the plant is asymptotically stable and nonoscillatory.

In all three cases, the monotonic improvement in performance with the number of models is evident. Also, as would be expected, the responses in all cases improve as we proceed from plants in Fig. 4(a) to Fig. 4(c).

Fig. 5 shows the sequence of controllers which were used, for the case $N = 343$ (corresponding to the last column in Fig. 4). In all cases, there is an initial period of rapid switching. In the first two cases [Figs. 5(a) and 5(b)], the switching process settles down quickly at the model those initial estimates were closest to the parameters of the plant, while in the third case [Fig. 5(c)], switching has slowed down but not ceased in 25 units of time. Further, less than a quarter (84, 67, and 72 for the three cases, respectively) of the 343 available controllers were used in all cases.

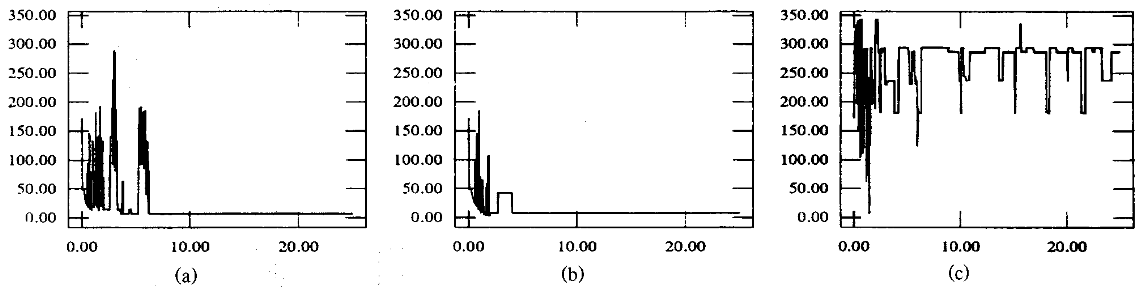


Fig. 5. Switching sequence of controllers used for the case $N = 343$ shown in Fig. 4: (a) $k_p = 0.5, a_1 = -0.5, a_0 = 2$; (b) $k_p = 0.5, a_1 = 0.5, a_0 = -2$; (c) $k_p = 2, a_1 = 3.4, a_0 = 2$.

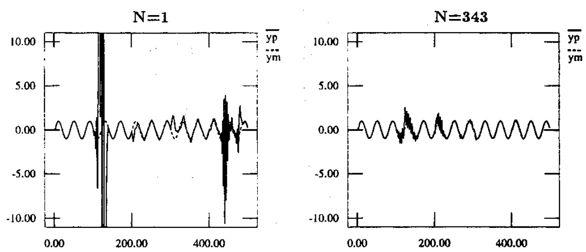


Fig. 6. Response of a plant with parameters switching every 100 units of time, using the proposed switching scheme with $J_j(t) = e_{I_j}^2(t) + \int_0^t e_{I_j}^2(\tau)\, d\tau$. The transfer functions of the plant during the intervals [0, 100), [100, 200), [200, 300), [300, 400), [400, 500] are $2/(s^2 + 3.4s + 2)$, $0.75/(s^2 - 0.6s + 1.15)$, $1.5/(s^2)$, $0.6/(s^2 + 1.5s - 1.5)$, $0.5/(s^2 + s + 2)$, respectively.

*Problem 2 (Nonzero Initial Conditions):* This case, in which the method is applied to situations where plant initial conditions are not zero, provides compelling evidence for the efficacy of multiple models. The problem statement is the same as in Problem 1, but the plant parameters ($k_p$, $a_1$, $a_0$) are time-varying. They switch to a new value in $S$ at the end of every interval of length $T$ (=100 units), in the sequence $(2, 3.4, 2) \rightarrow (0.75, -0.56, 1.5) \rightarrow (1.5, 0, 0) \rightarrow (0.6, 1.5, -1.5) \rightarrow (0.5, 1, 2)$. $T$ was chosen so that all the identification errors, as well as the control error, have settled down below 0.01 in a period of time less than $T$, and the switching procedure has also settled down at one of the controllers. When the plant parameters change, it is automatically detected by the increase in the magnitude of the identification error corresponding to the controller in place, and then the parameter estimates of all the models are re-initialized as in Problem 1. Hence, the problem can be considered as a concatenation of problems of the class discussed in Problem 1, but with arbitrary initial conditions.

The responses $y_p(t)$ and $y_m(t)$ to a sinusoidal reference input $r(t)$ with unit amplitude and period 40 units is shown in Fig. 6, using $N = 1$ and $N = 343$. In the figure for $N = 1$, the plot of the plant output is truncated; its peak value was 58. For $N = 343$, the peak value was 2.55.

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# Control of A Class of Nonlinear Uncertain Systems Via Compensated Inverse Dynamics Approach

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***Abstract*—The tracking control problem of a class of nonlinear systems via inverse dynamics method is addressed. The motivation for the study stems from the need to control practical systems arising from aerospace and mechanical engineering. Because modeling uncertainties and external disturbances are always present in these systems, inverse dynamics technique is not applicable directly. A compensated inverse dynamics approach is proposed to account for the effect of uncertainties. The compensation is achieved by adaptive and robust schemes. Application of the proposed strategy to the vibration suppression of a two-bay flexible truss structure is presented.**

## I. INTRODUCTION

This note is concerned with adaptive and robust inverse dynamics control of a class of complex nonlinear systems. The initial motivation behind the development of adaptive and robust control is the need to account for uncertain effects associated with unknown system

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