دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 4. Handy Circuit Analysis Techniques

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عظیم فرقدان

مهر ۱۴۰۳

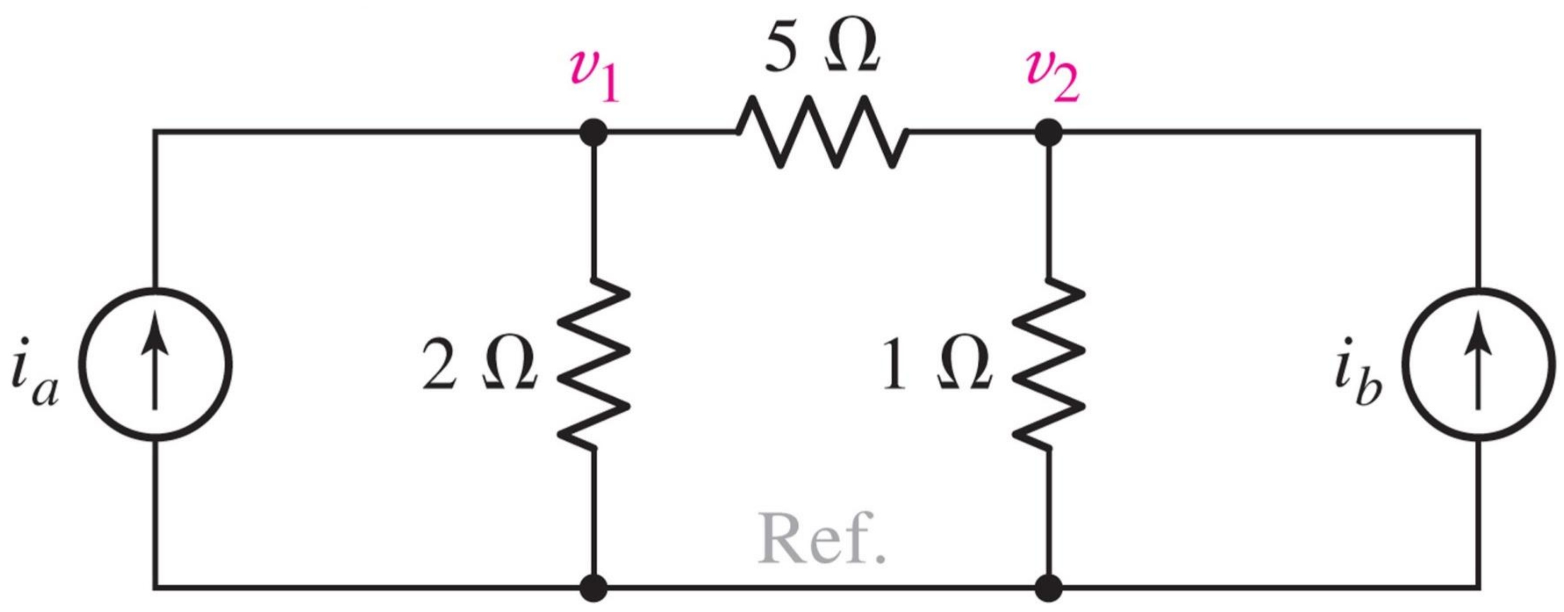
Objectives of the Lecture

- Common Circuit Analysis Methods
  - Superposition Theorem
  - Practical Sources
  - Source Transformation
  - Thevenin and Norton Circuits
  - Maximum Power Transfer Theorem
  - Star-Delta Transformation

Linear Elements and Circuits

- A linear circuit element has a linear voltage-current relationship:
  - if $i(t)$ produces $v(t)$, then $Ki(t)$ produces $Kv(t)$
  - if $i_1(t)$ produces $v_1(t)$ and $i_2(t)$ produces $v_2(t)$, then $i_1(t) + i_2(t)$ produces $v_1(t) + v_2(t)$,
- resistors, sources are linear elements
- a linear circuit is one with only linear elements

For the circuit shown, the solution can be expressed as:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{2} - i_a = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{1} - i_b = 0$$

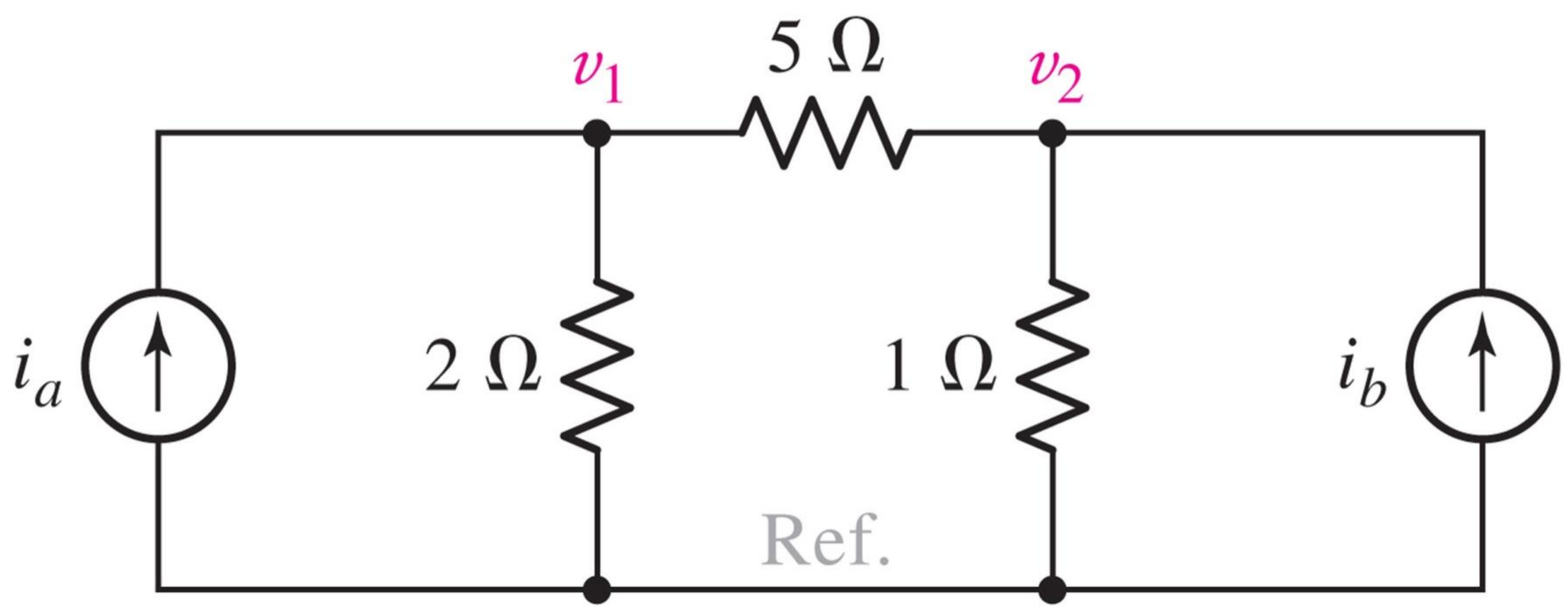
$$\rightarrow \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

$$\rightarrow A \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \end{bmatrix} \rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

Question: How much of v_1 is due to source i_a, and how much is because of source i_b?

If we define *A* as

$$A = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix}$$



then

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1}\begin{bmatrix} i_a \\ i_b \end{bmatrix} = A^{-1}\begin{bmatrix} 0 \\ i_b \end{bmatrix} + A^{-1}\begin{bmatrix} i_a \\ 0 \end{bmatrix}$$

Experiment 1

Experiment 2

Superposition:
the response is the sum of experiments 1 and 2.

The Superposition Theorem

In a linear network, the **voltage across** or the **current through** any element may be calculated by ***adding algebraically*** all the individual voltages or currents caused by the separate independent sources acting “alone”.

In other words, the effect that each independent source has on the circuit parameters is mutually independent, and the combined result of their effects is the algebraic sum of their contributions.

Applying Superposition

- Leave one source ON and turn all other sources OFF:
  - –voltage sources: *set v=0*.

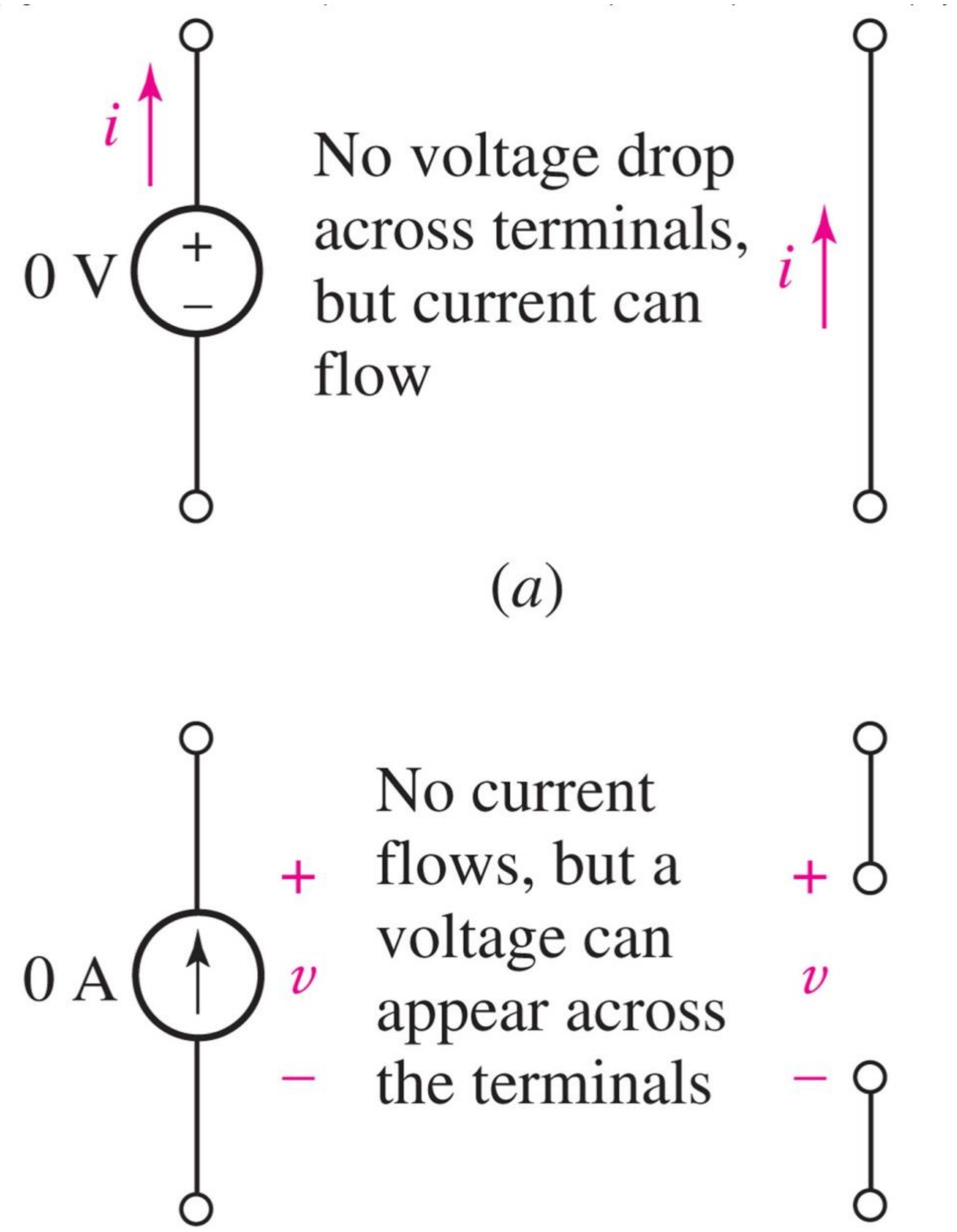
    These become *short circuits.*

  - –current sources: *set i=0*.

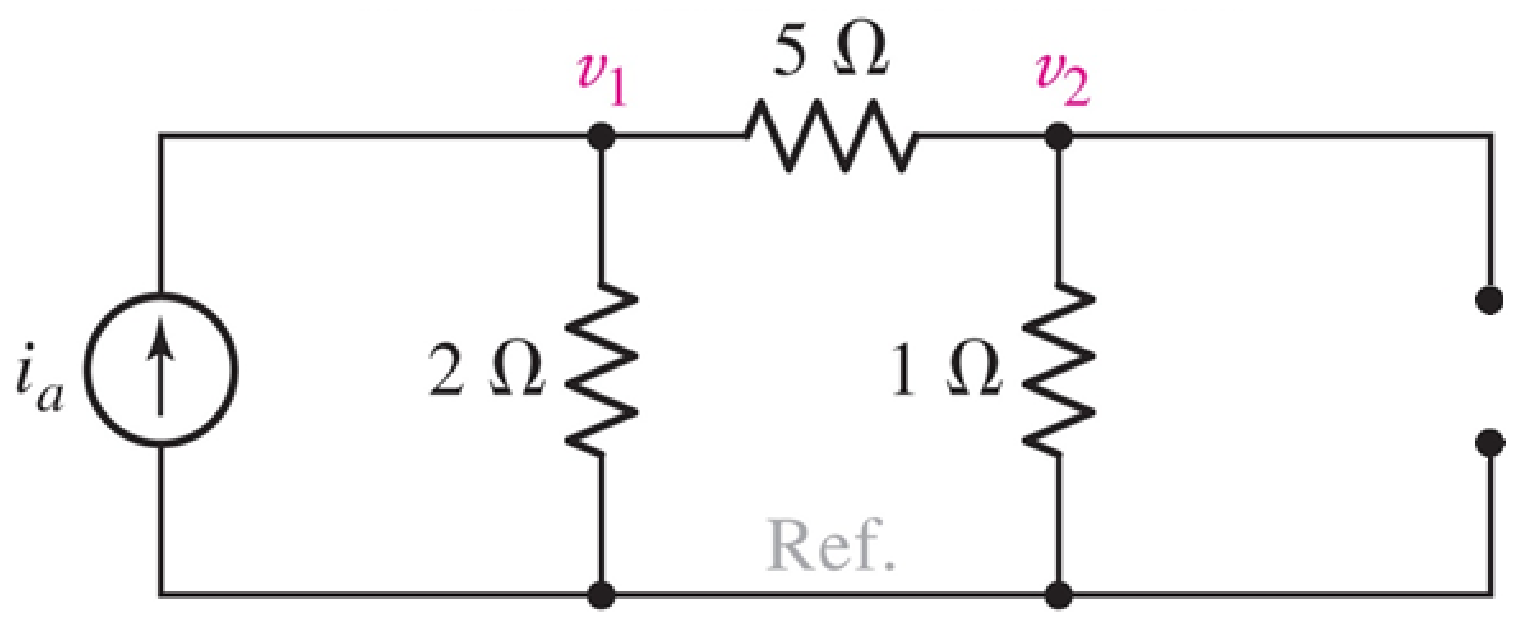
    These become *open circuits.*

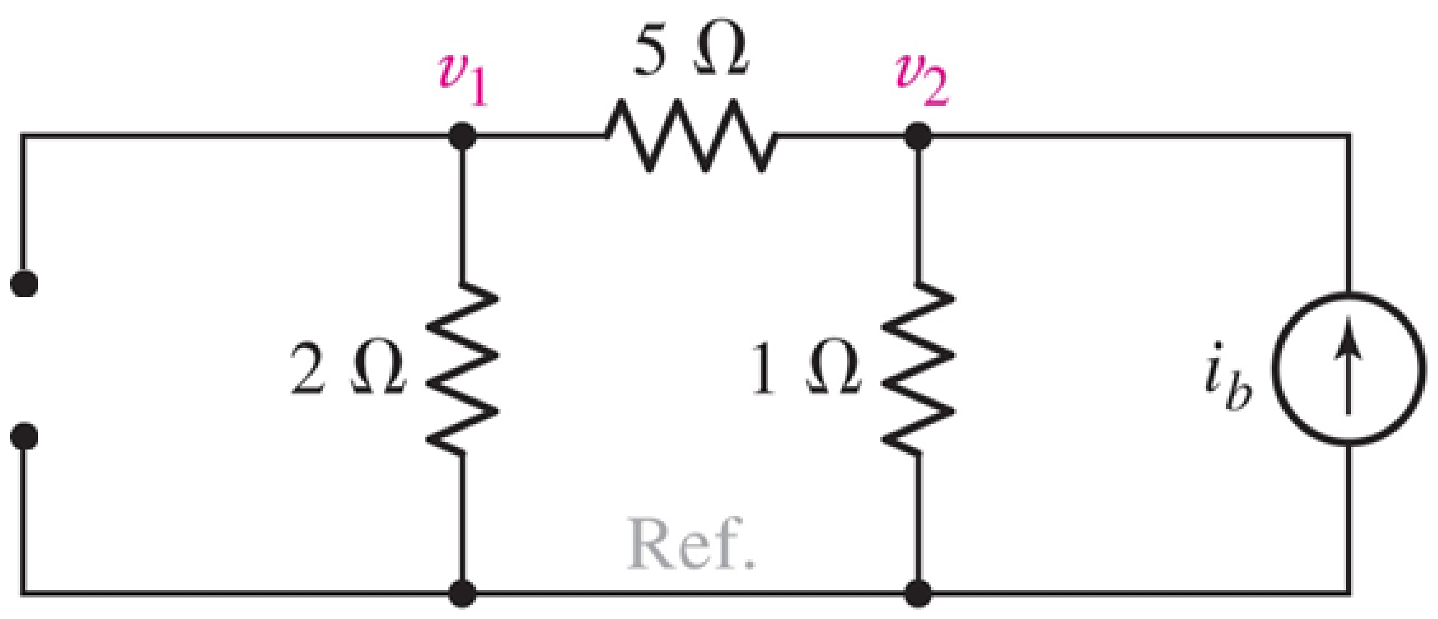
    Find the response from this source.
- Add the resulting responses

to find the total response.

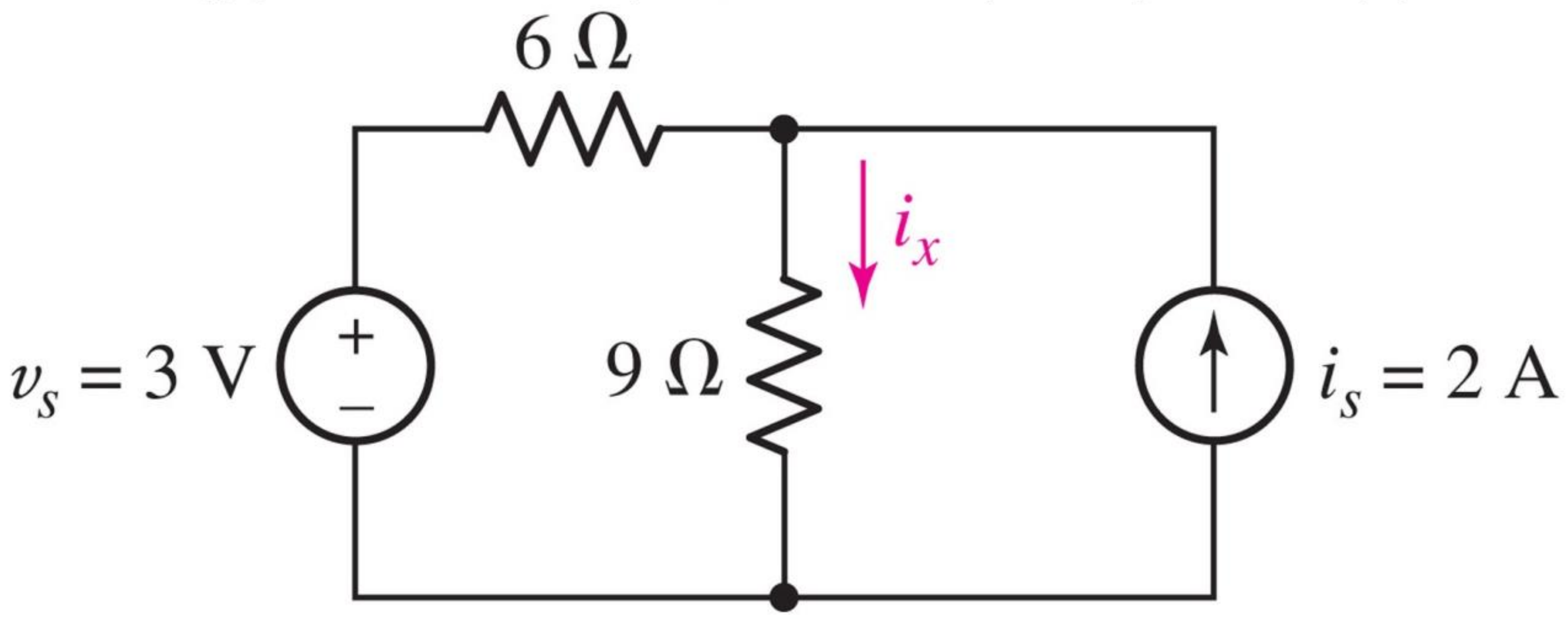


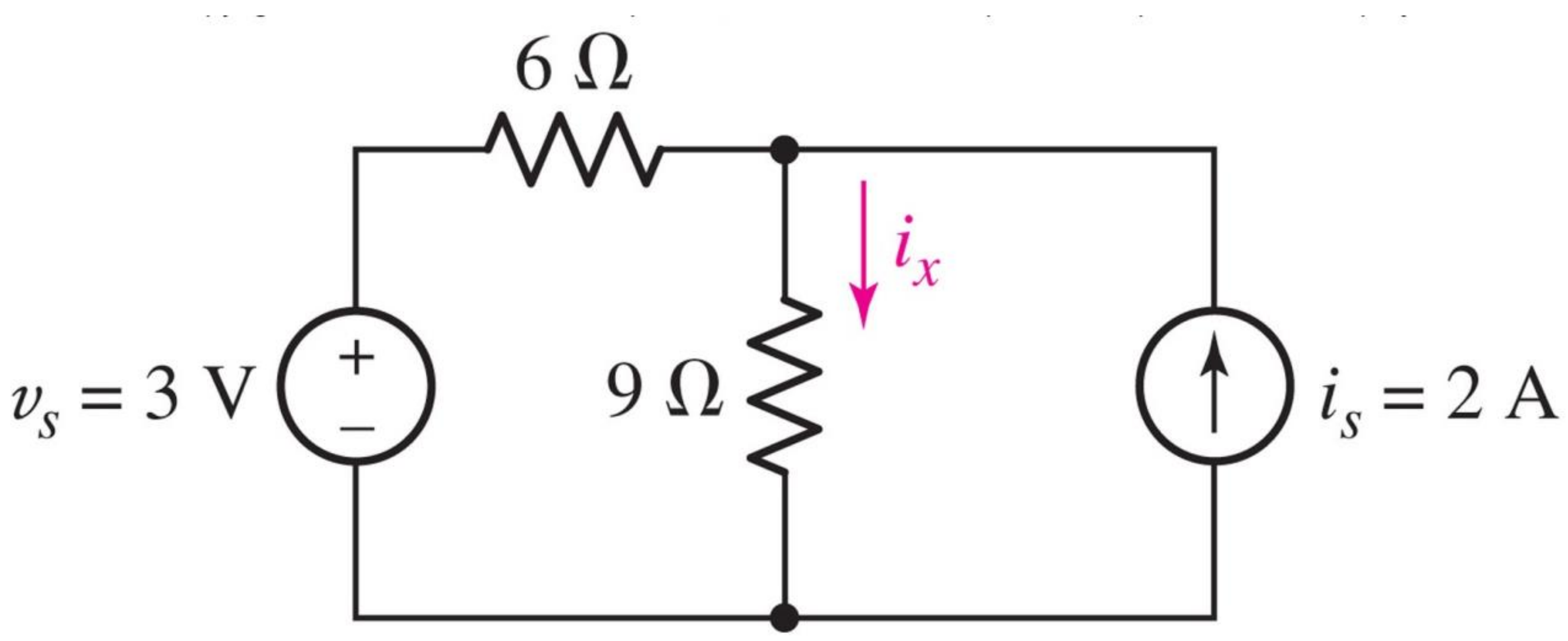
- ✓ all other independent voltage sources replaced by short circuits and
- ✓ all other independent current sources replaced by open circuits.



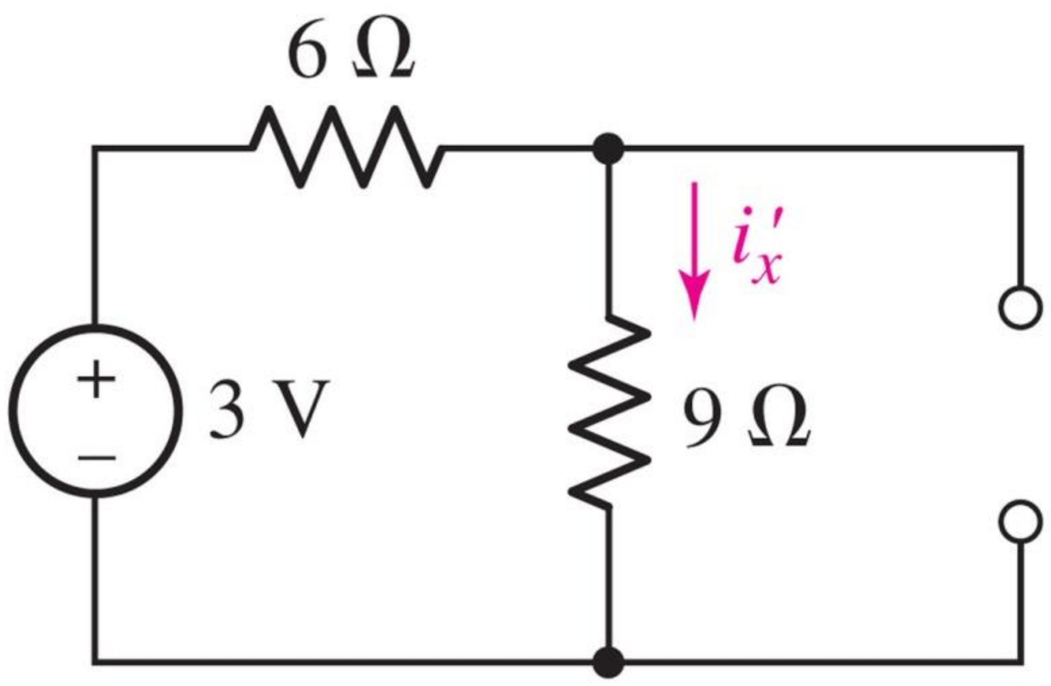


Use superposition to solve for the current i_x

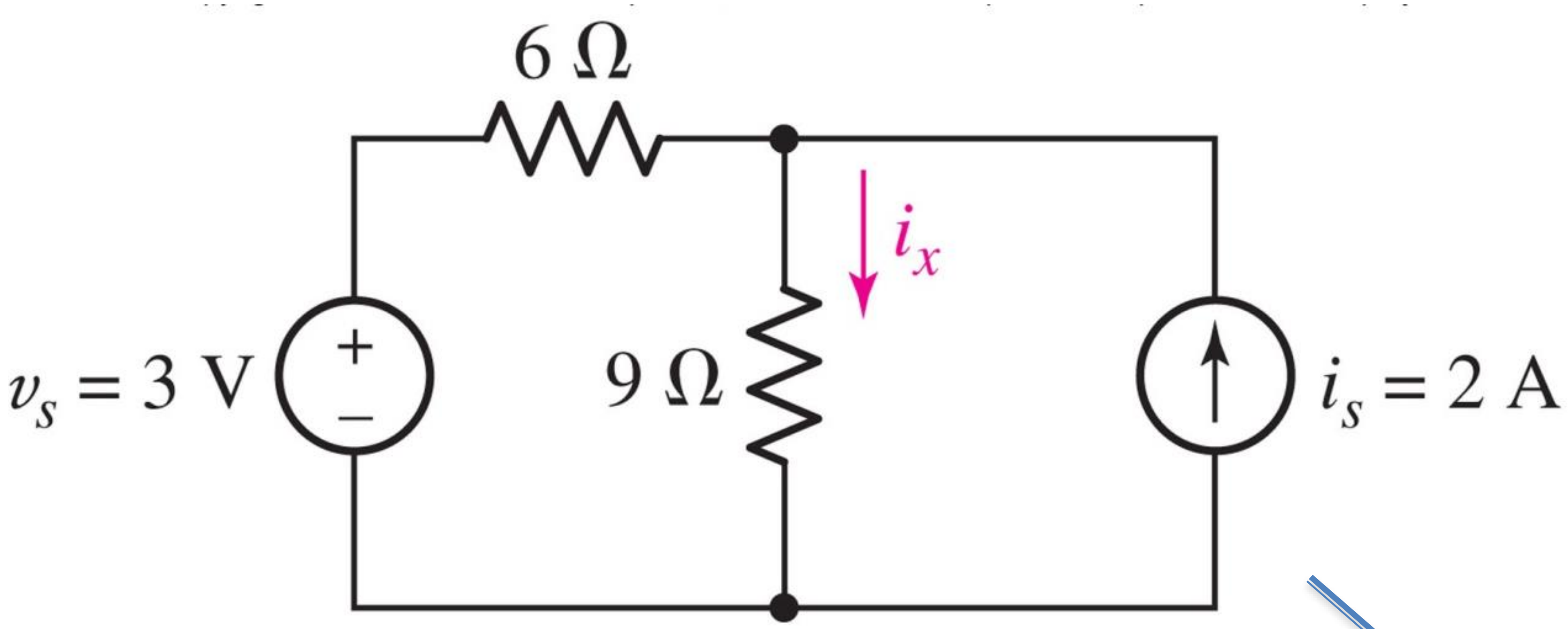




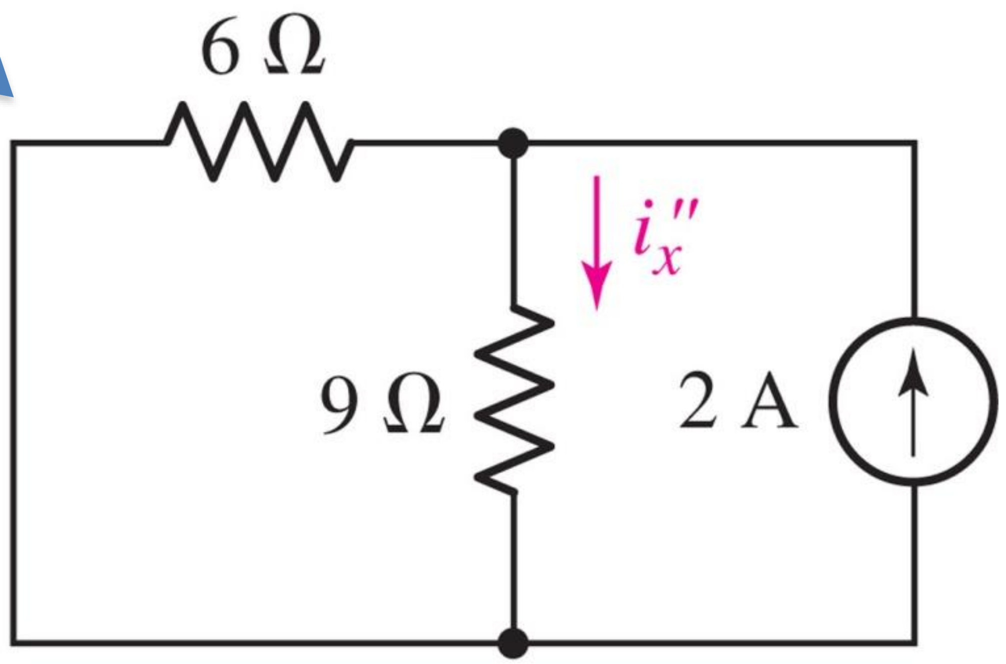
First, turn the current source off:



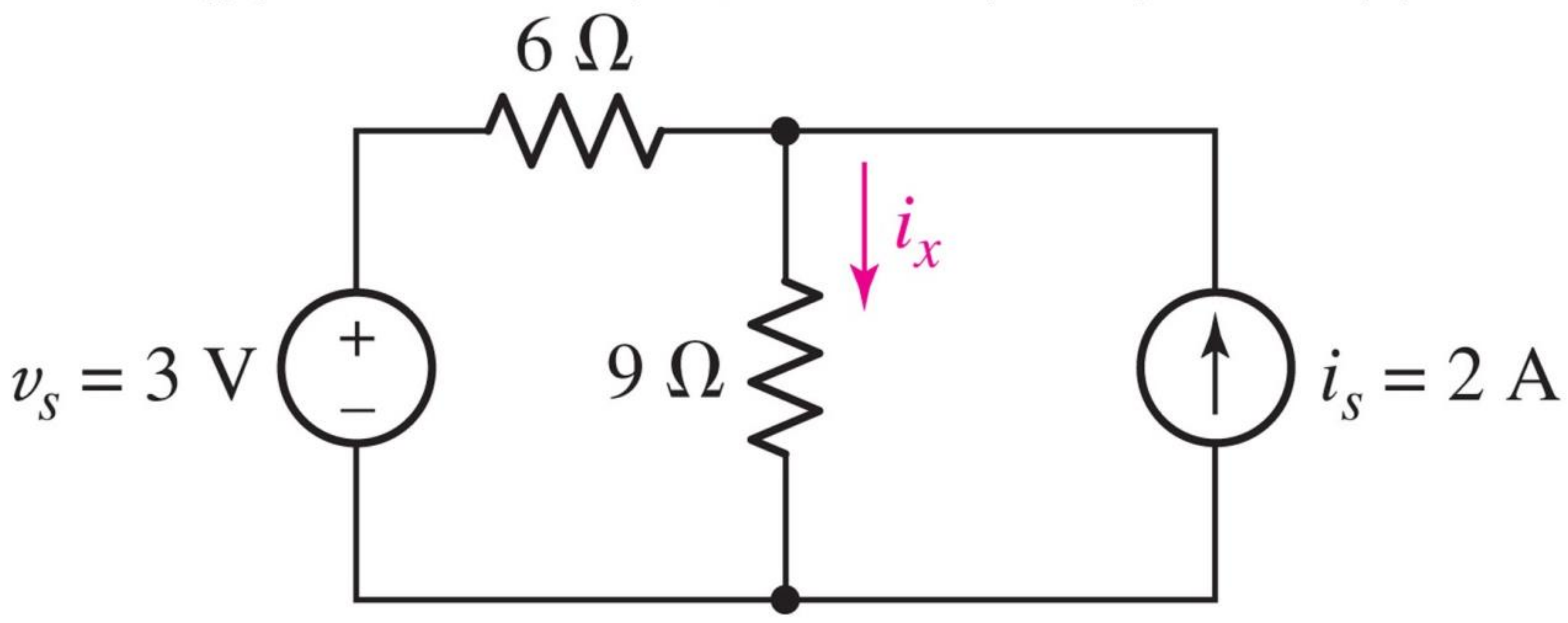
$$i'_x = \frac{3}{6+9} = 0.2$$



Then, turn the voltage source off:



$$i_x'' = \frac{6}{6+9}(2) = 0.8$$

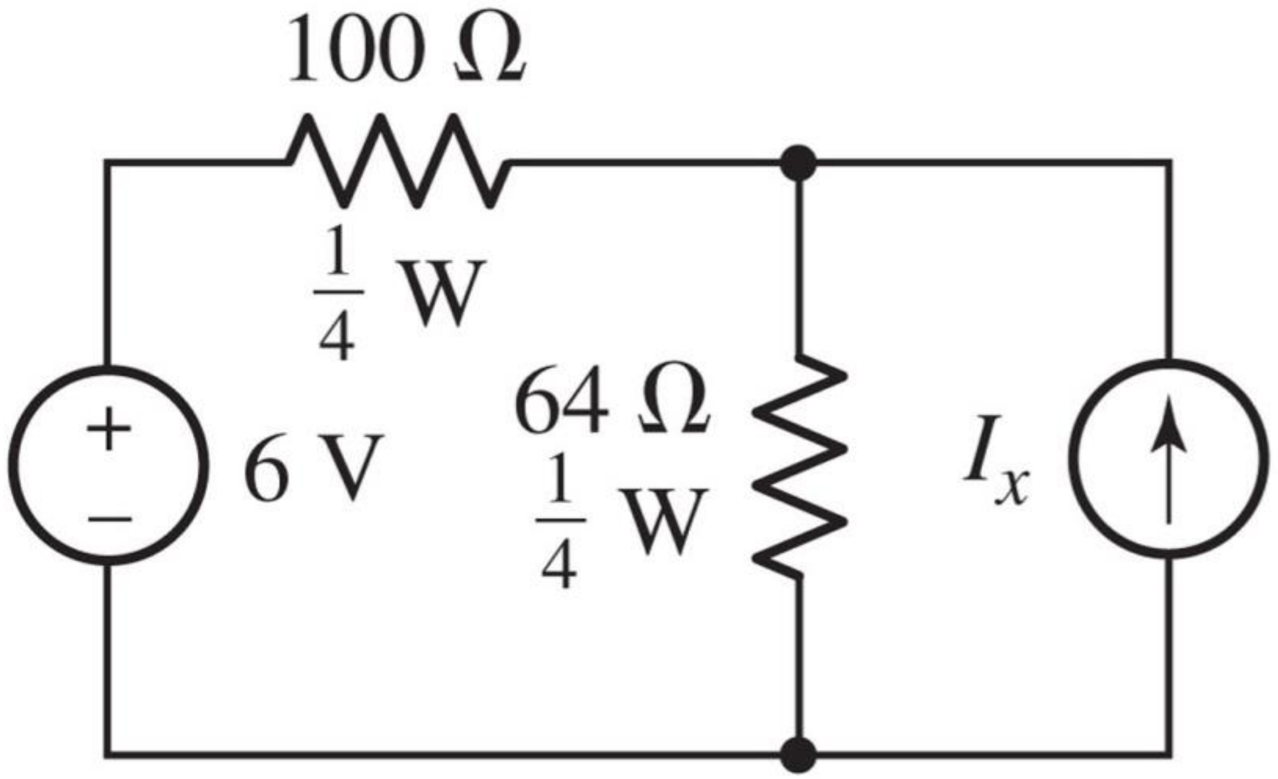


Finally, combine the results:

$$i_x = i_x' + i_x'' = 0.2 + 0.8 = 1.0$$

Example: Power Ratings

Determine the maximum *positive* current to which the source I_x can be set before any resistor exceeds its power rating.



Answer: $I_x < 42.49$ mA

Example: Power Ratings

Maximum current magnitude in 100Ω resistor is $\sqrt{0.25/100} = 50\text{mA}$

Maximum current magnitude in 64Ω resistor is $\sqrt{0.25/64} = 62.5\text{mA}$

Current from voltage source alone is 6/164 = 36.6mA flowing clockwise

Current in 100Ω from I_x alone is $\frac{64}{164} I_x$ flowing to the left.

Therefore $\left|0.0366 - \frac{64}{164} I_x\right| < 0.05$ or $-0.05 < 0.0366 - \frac{64}{164} I_x < 0.05$

$221.9\text{mA} > I_x > -34.33\text{mA}$

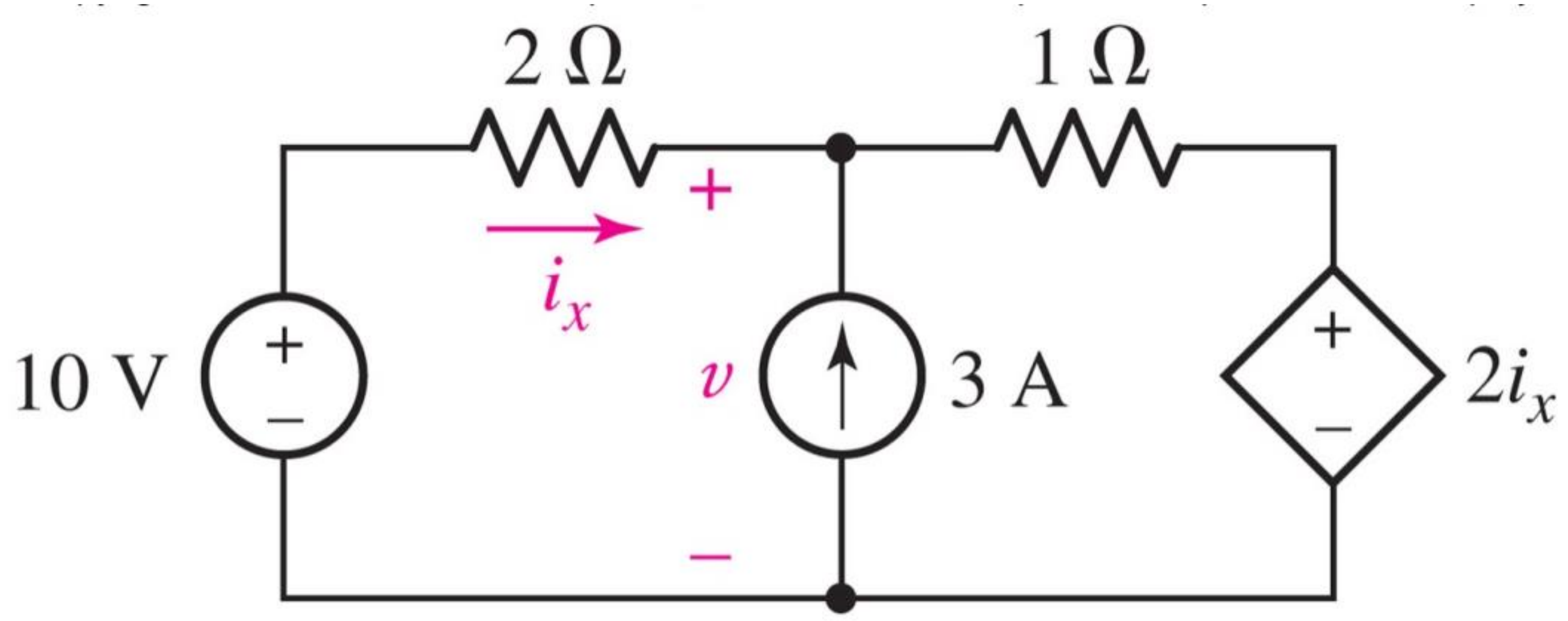
Current in 64Ω from I_x is $\frac{100}{164} I_x$ flowing downward.

Therefore $\left|0.0366 + \frac{100}{164} I_x\right| < 0.0625$ or $-0.0625 < 0.0366 + \frac{100}{164} I_x < 0.0625$

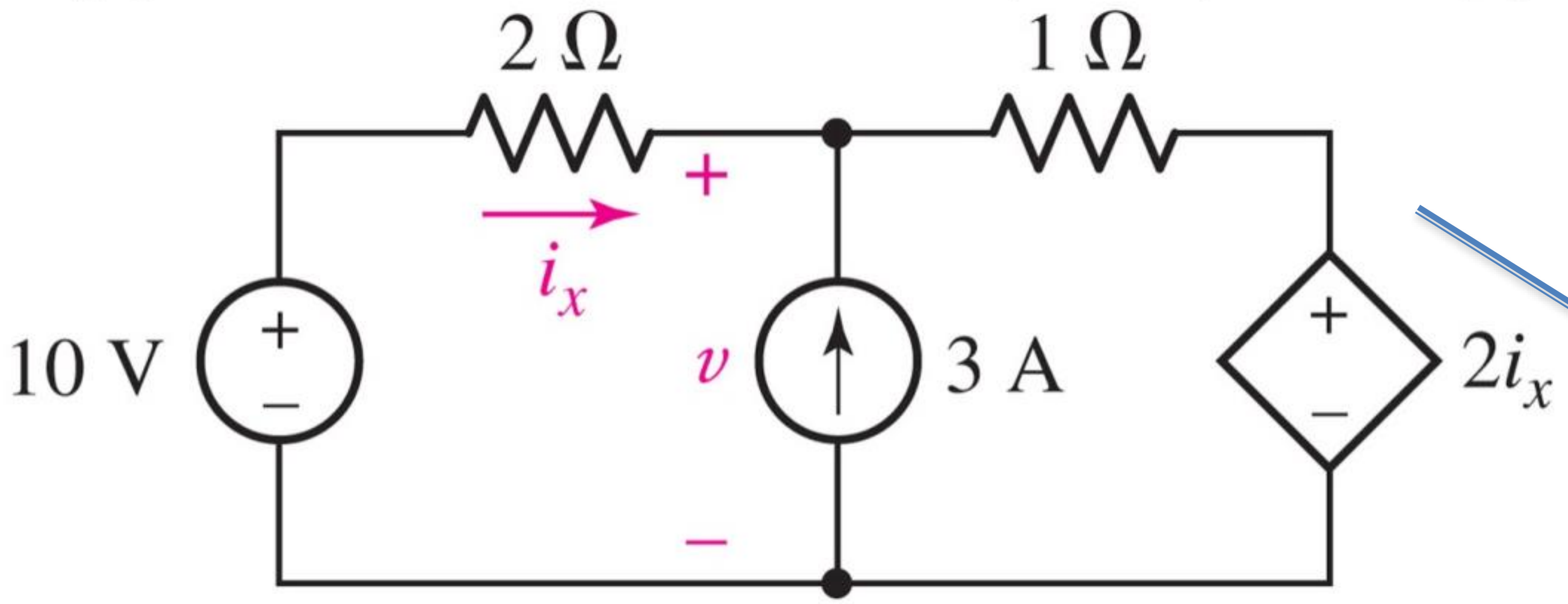
$-0.1625 < I_x < 0.04247$

Superposition with a Dependent Source

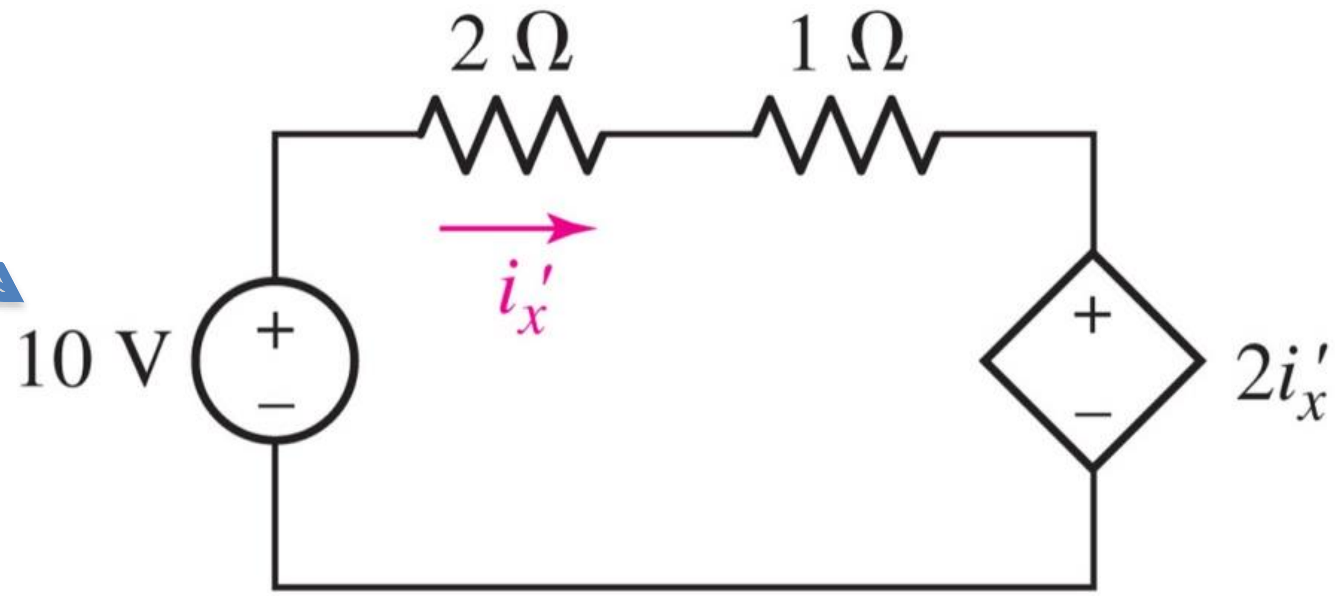
When applying superposition to circuits with *dependent* sources, these *dependent* sources are never “turned off.”



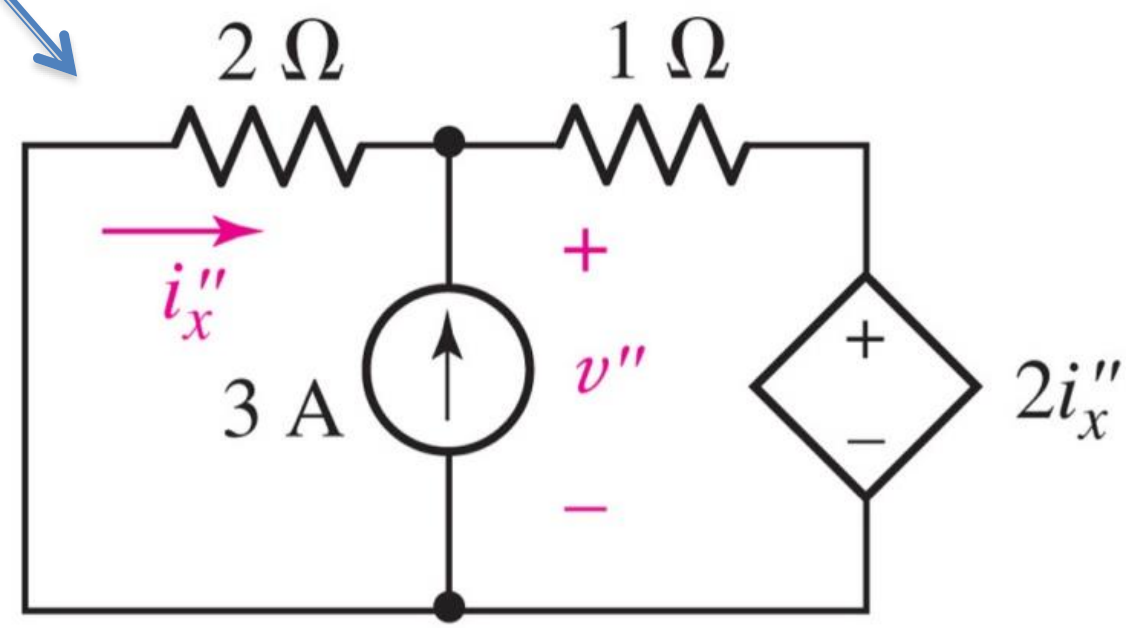
Superposition with a Dependent Source



current source off



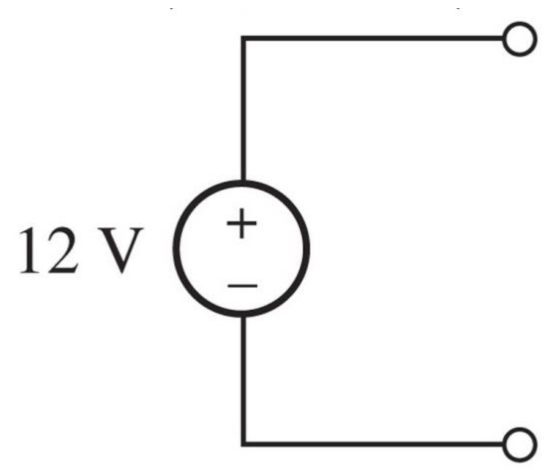
voltage source off



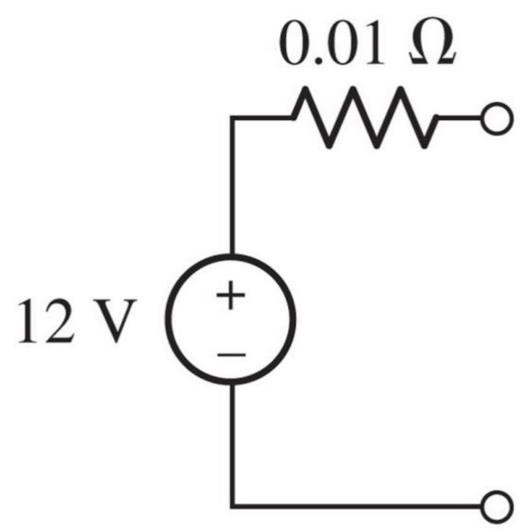
$$i_x = i_x' + i_x'' = 2 + (-0.6) = 1.4 \text{ A}$$

Practical Voltage Sources

- Ideal voltage sources: a first approximation model for a battery.

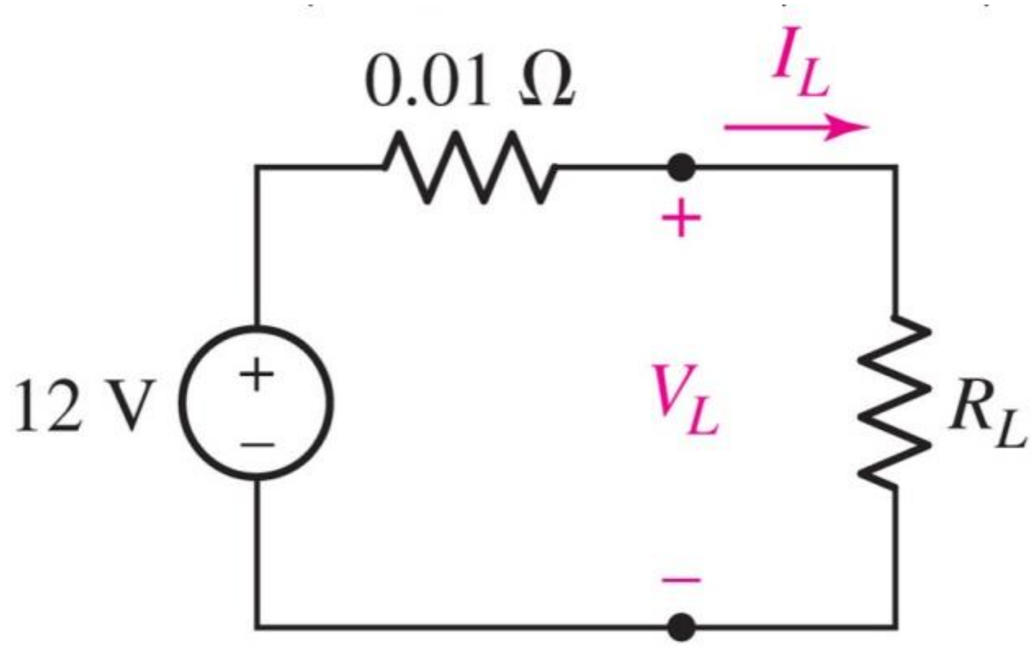


- Why do real batteries have a current limit and experience voltage drop as current increases?

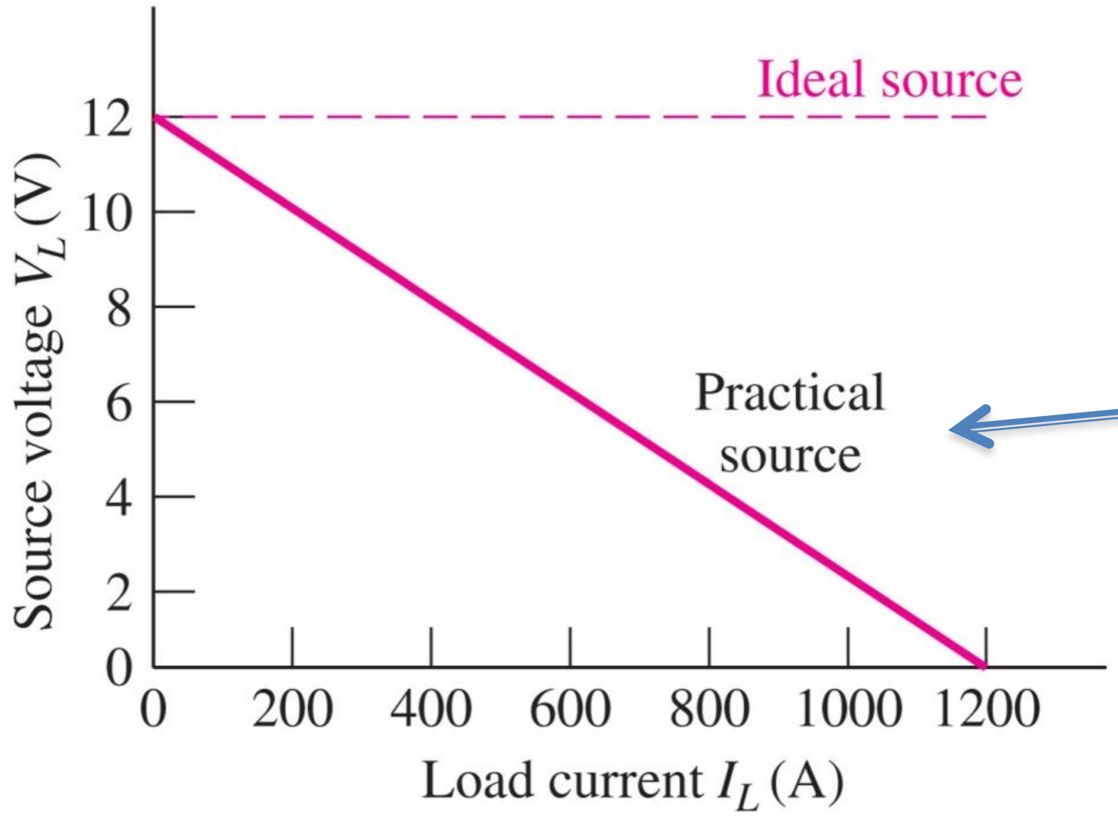


Practical Source: Effect of Connecting a Load

For the car battery example:



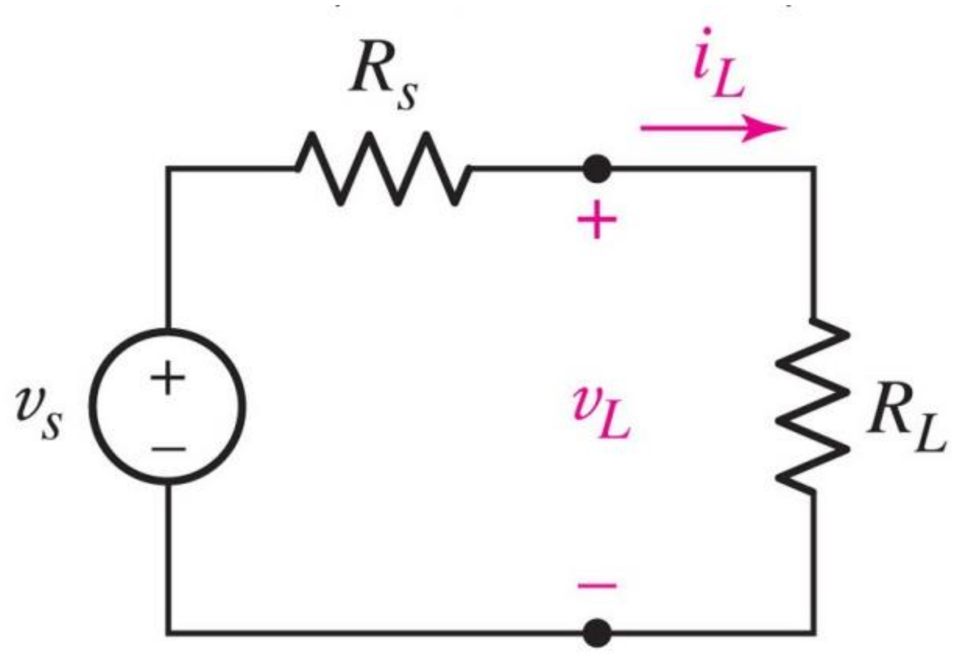
$$V_L = 12 - 0.01\, I_L$$

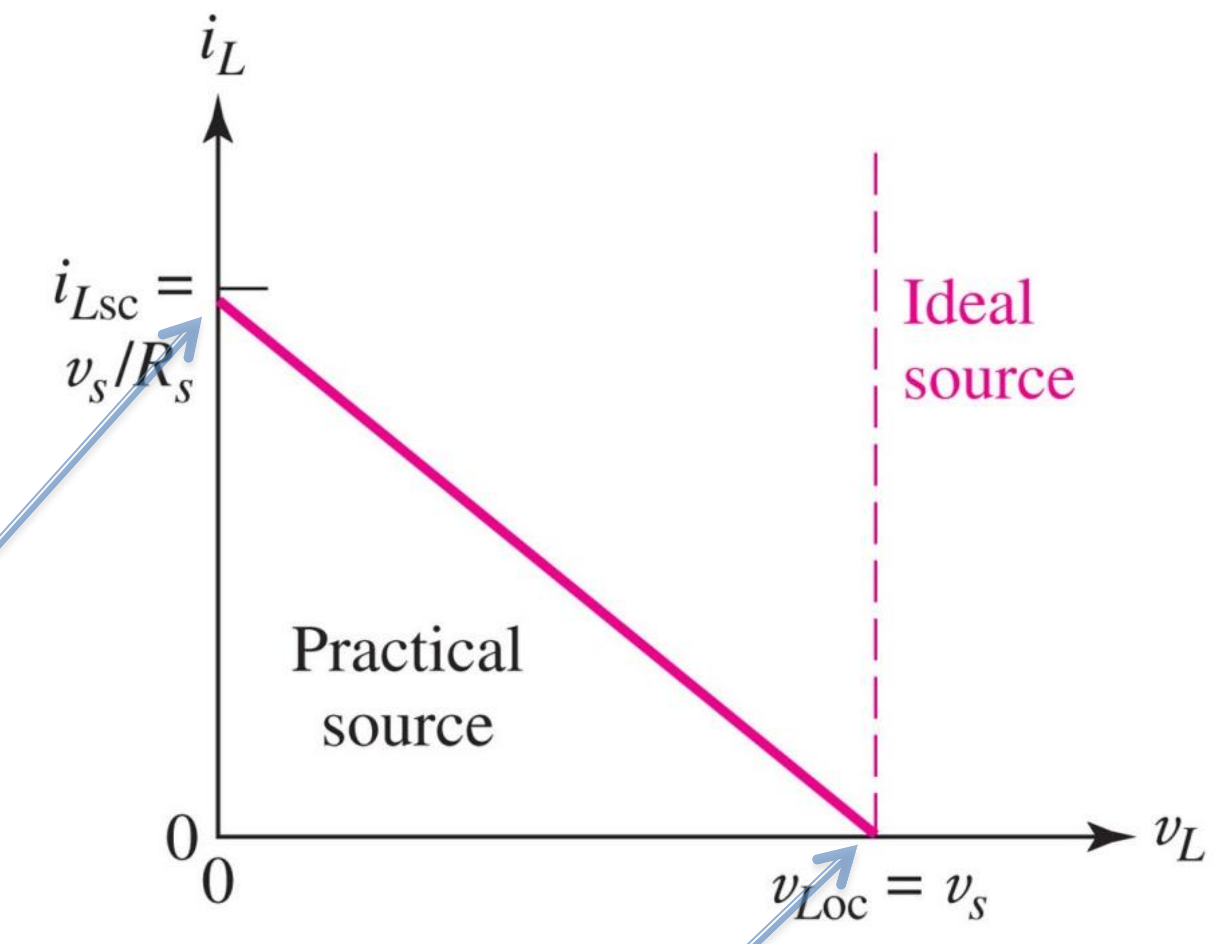


This line represents all possible R_L

Practical Voltage Source

The source has an internal resistance or output resistance, which is modeled as R_s



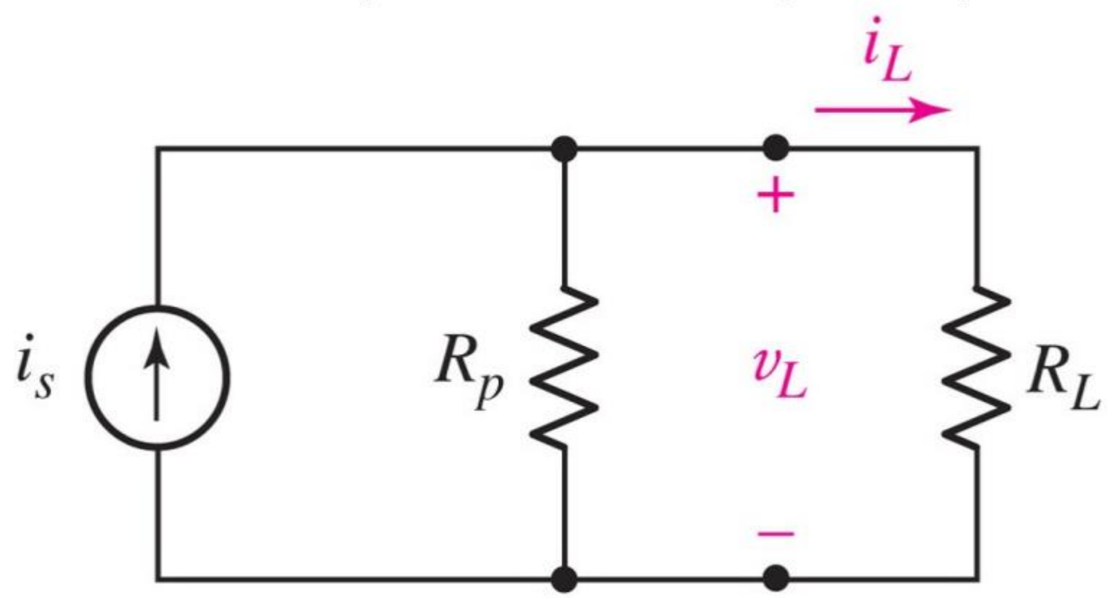


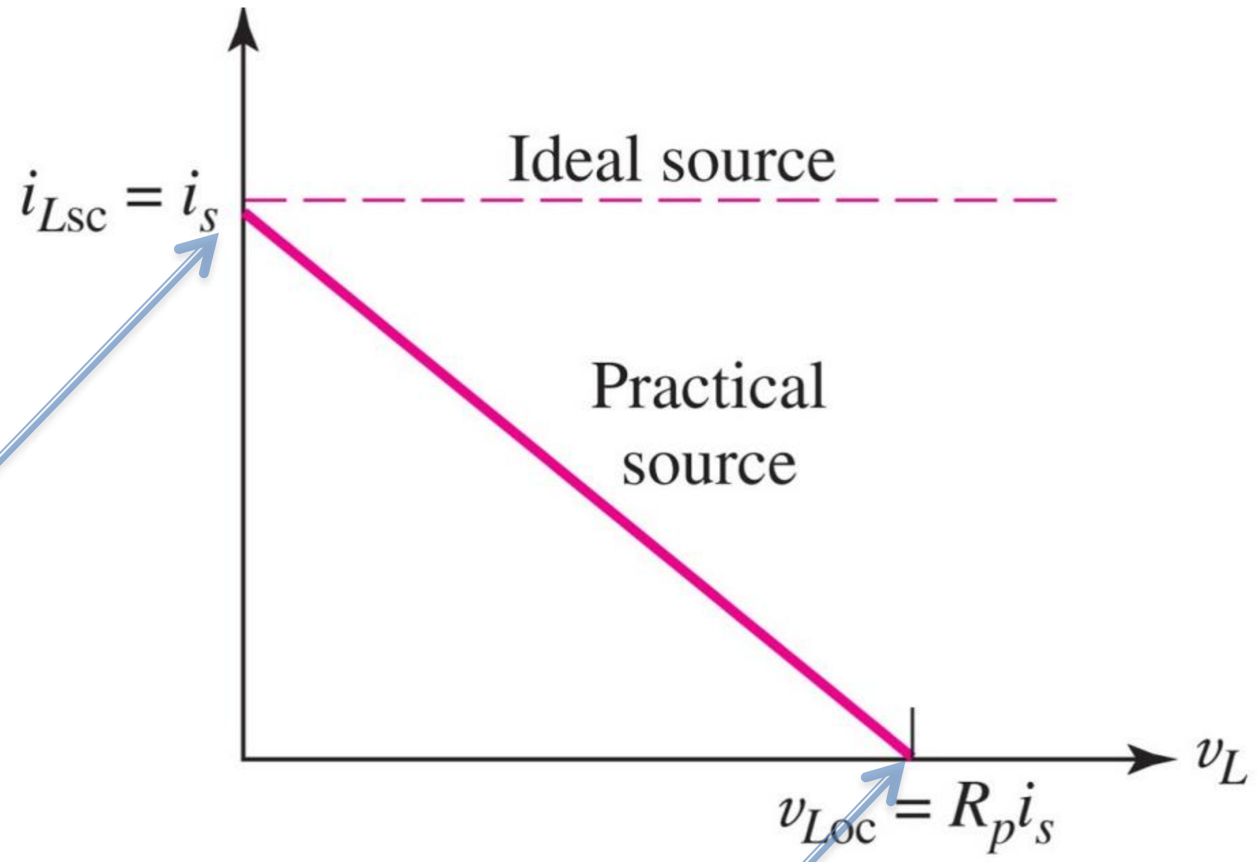
short circuit current (when R_L=0)

open circuit voltage (when $R_L=\infty$)

Practical Voltage Source

The source has an internal *parallel* resistance which is modeled as R_p





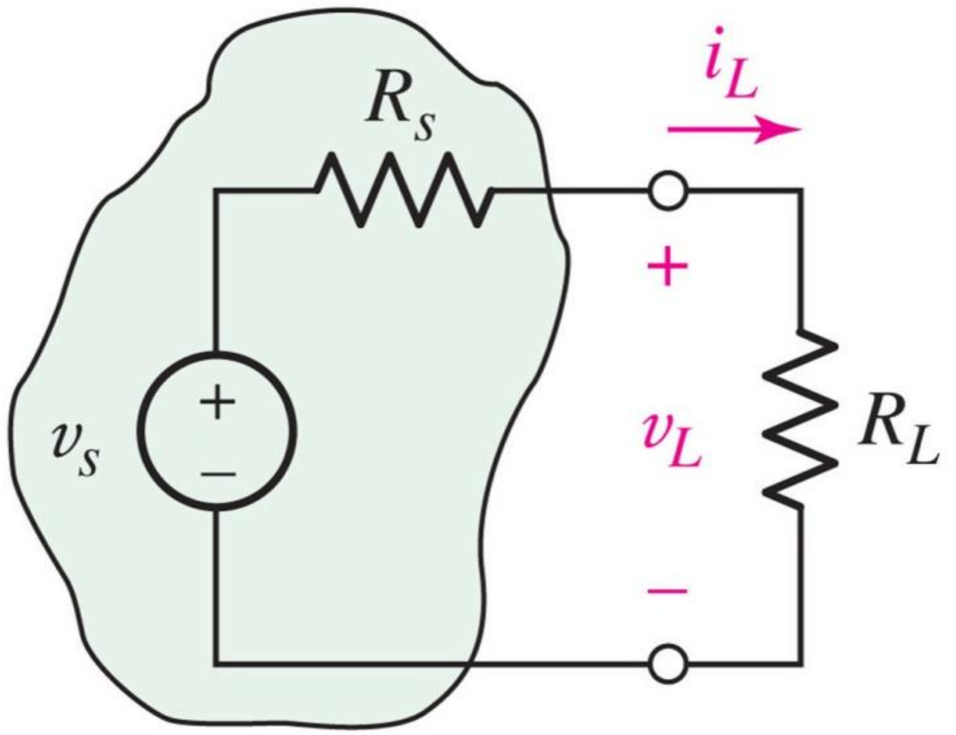
short circuit current (when R_L=0)

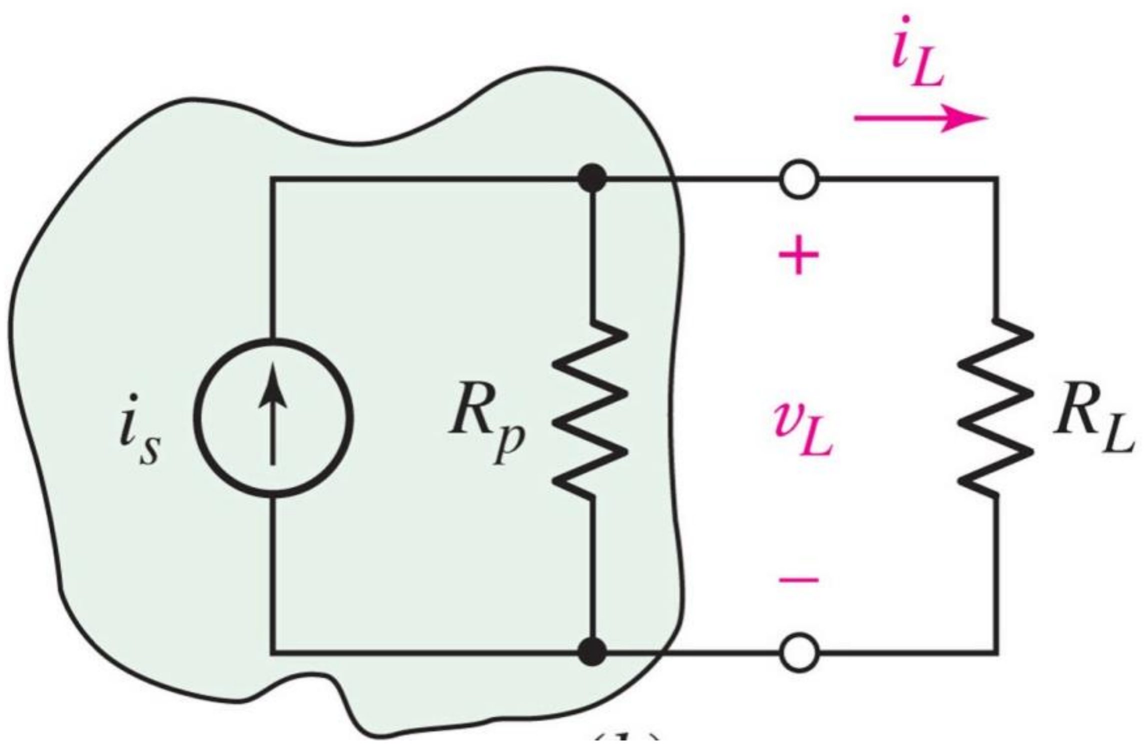
open circuit voltage (when R_L=∞)

Source Transformation and Equivalent Sources

The sources are equivalent if

$$R_s = R_p \text{ and } v_s = i_s R_s$$



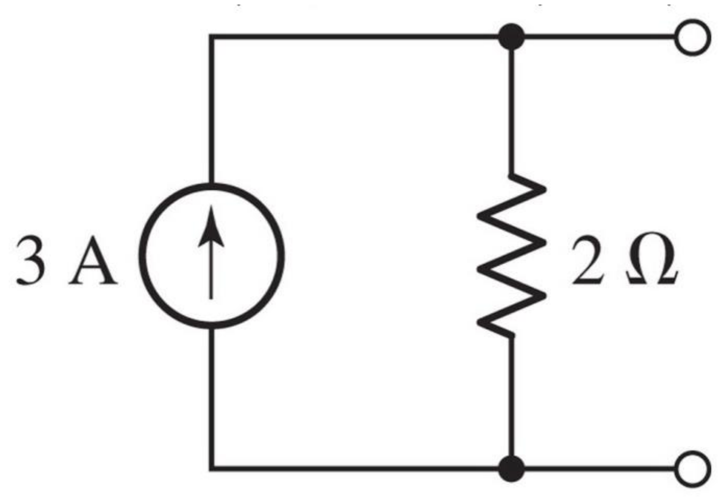


- The circuits (a) and (b) are equivalent at the terminals.

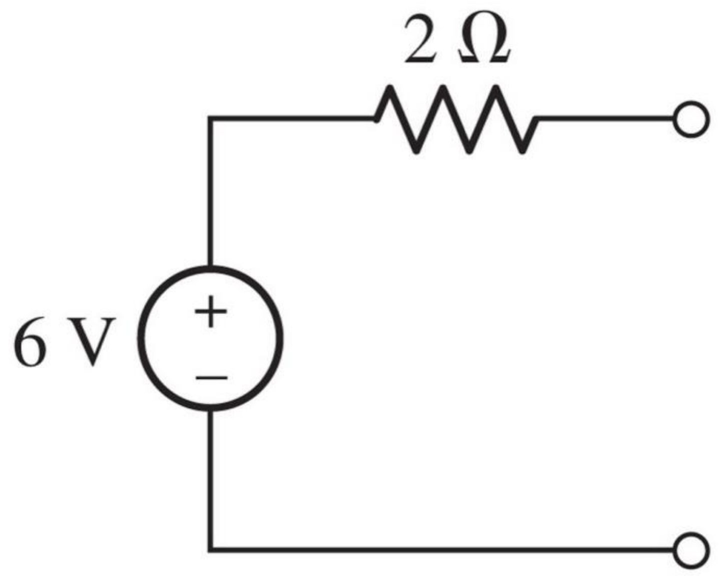
- If given circuit (a), but circuit (b) is more convenient, switch them!

- This process is called

source transformation.



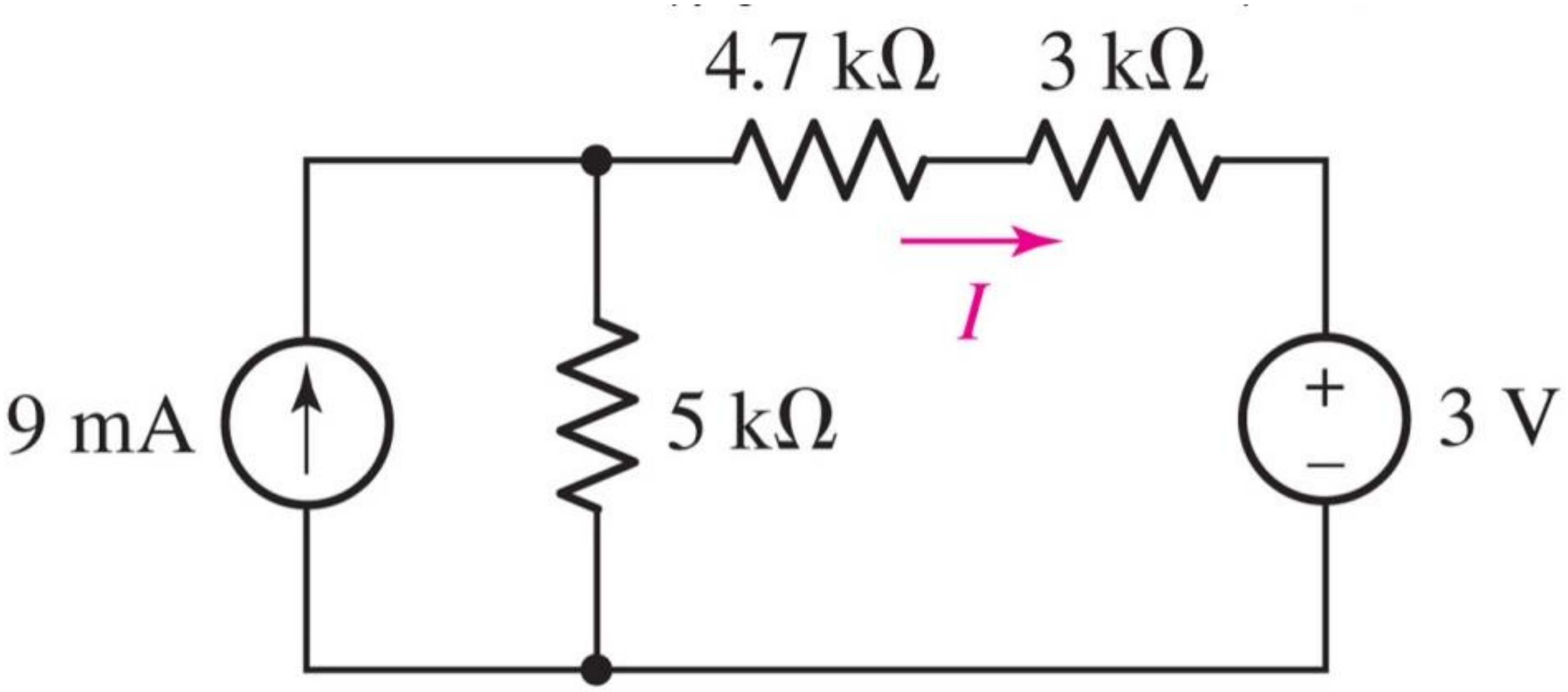
(a)

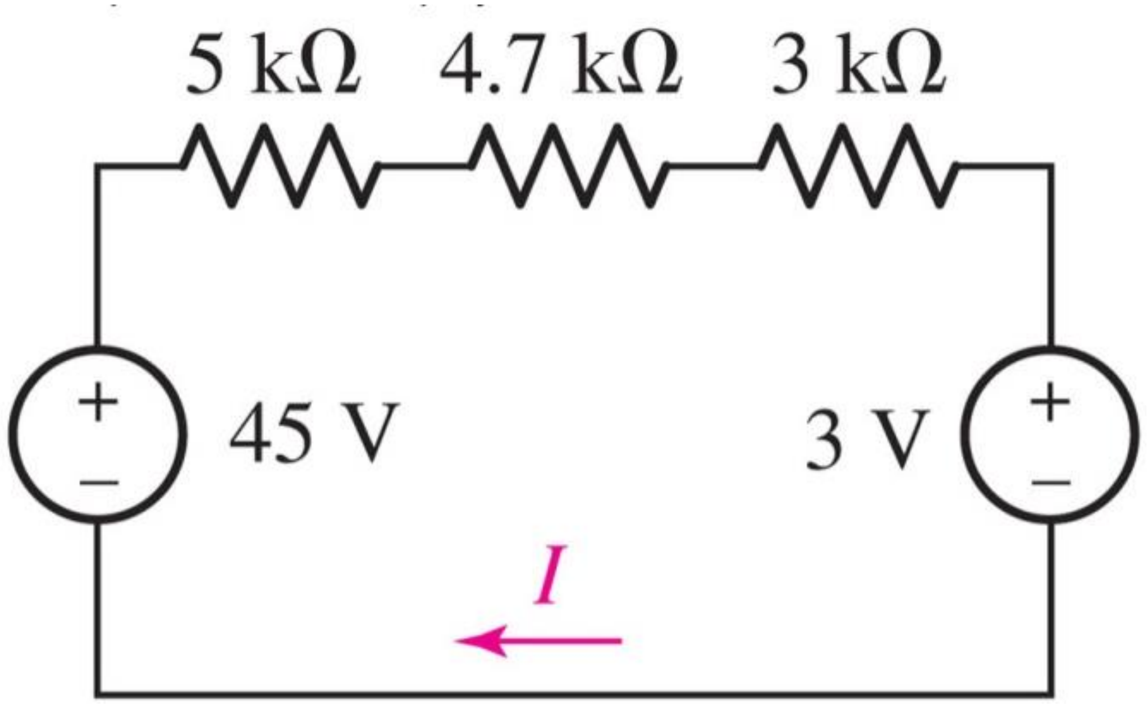


(b)

Example: Source Transformation

We can find the current *I* in the circuit below using source transformation, as shown.



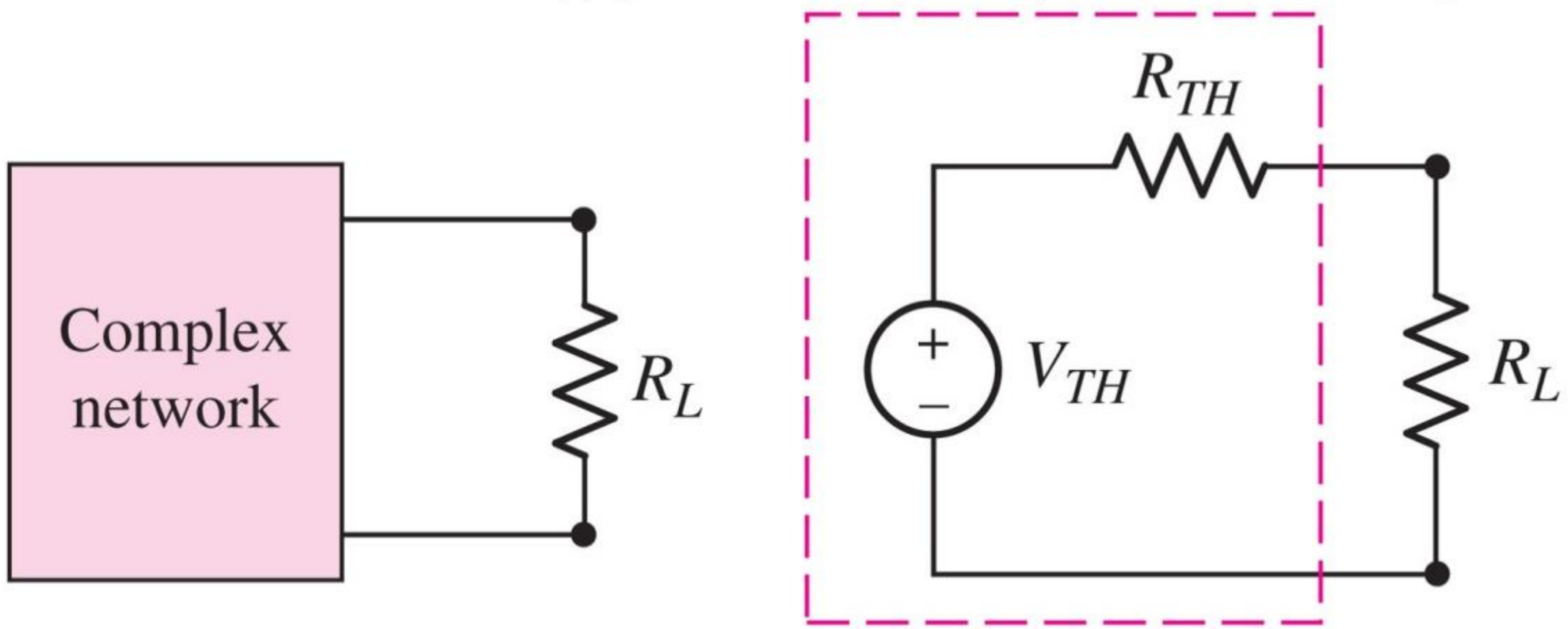


$I = (45\text{-}3)/(5+4.7+3) = 3.307\text{ mA}$

Thévenin Equivalent Circuits

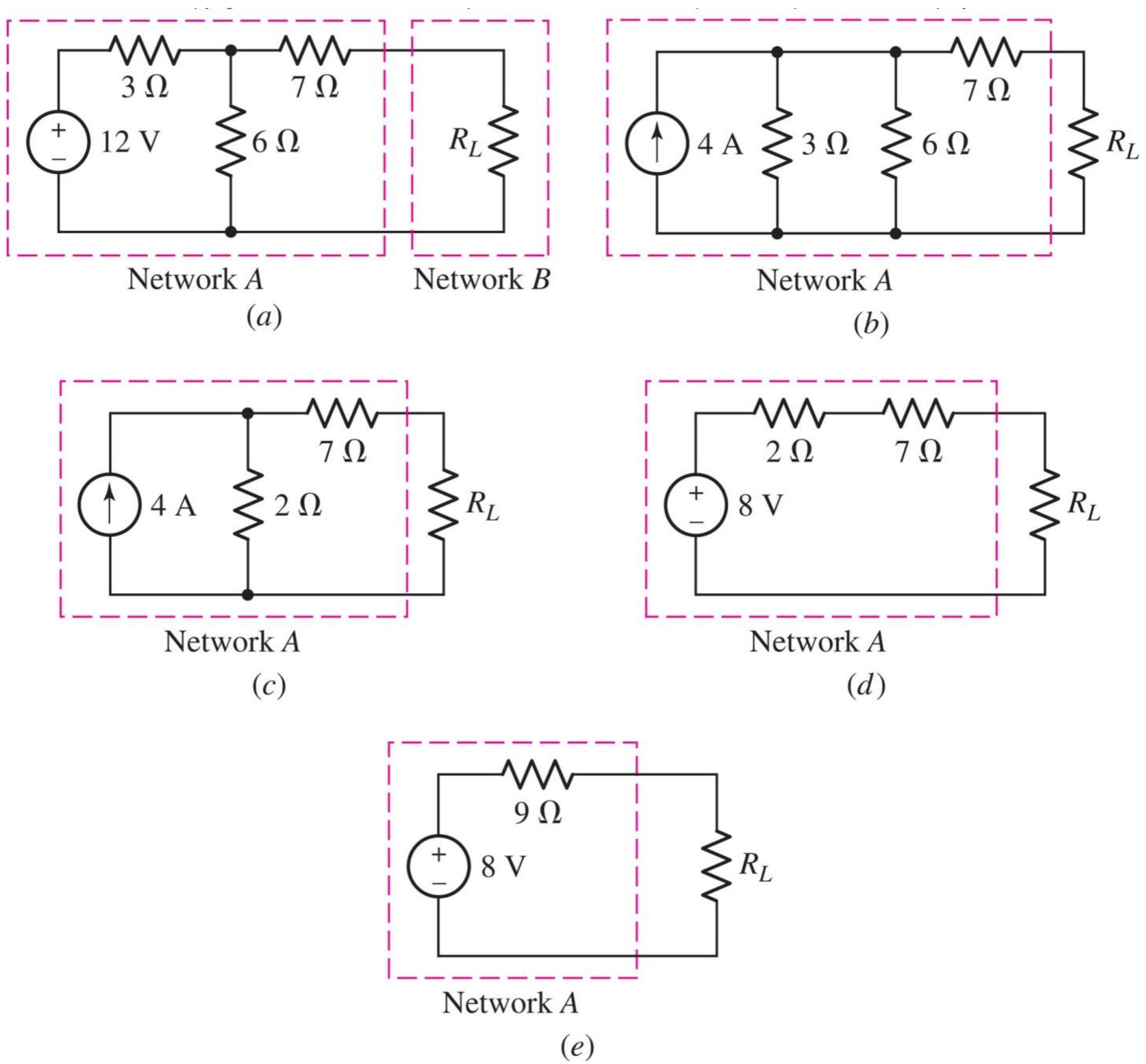
- ✓ Thévenin's theorem: a linear network can be replaced by its Thévenin equivalent circuit, In such a way that the current and voltage of the resistor R_L remain unchanged.

- ✓ This new circuit is called the Thevenin equivalent circuit. V_{TH} is referred to as the Thevenin voltage, and R_{TH} as the Thevenin resistance.



- We can repeatedly apply source transformation on network A to find its Thévenin equivalent circuit.

- This method has limitations- not all circuits can be source transformed.



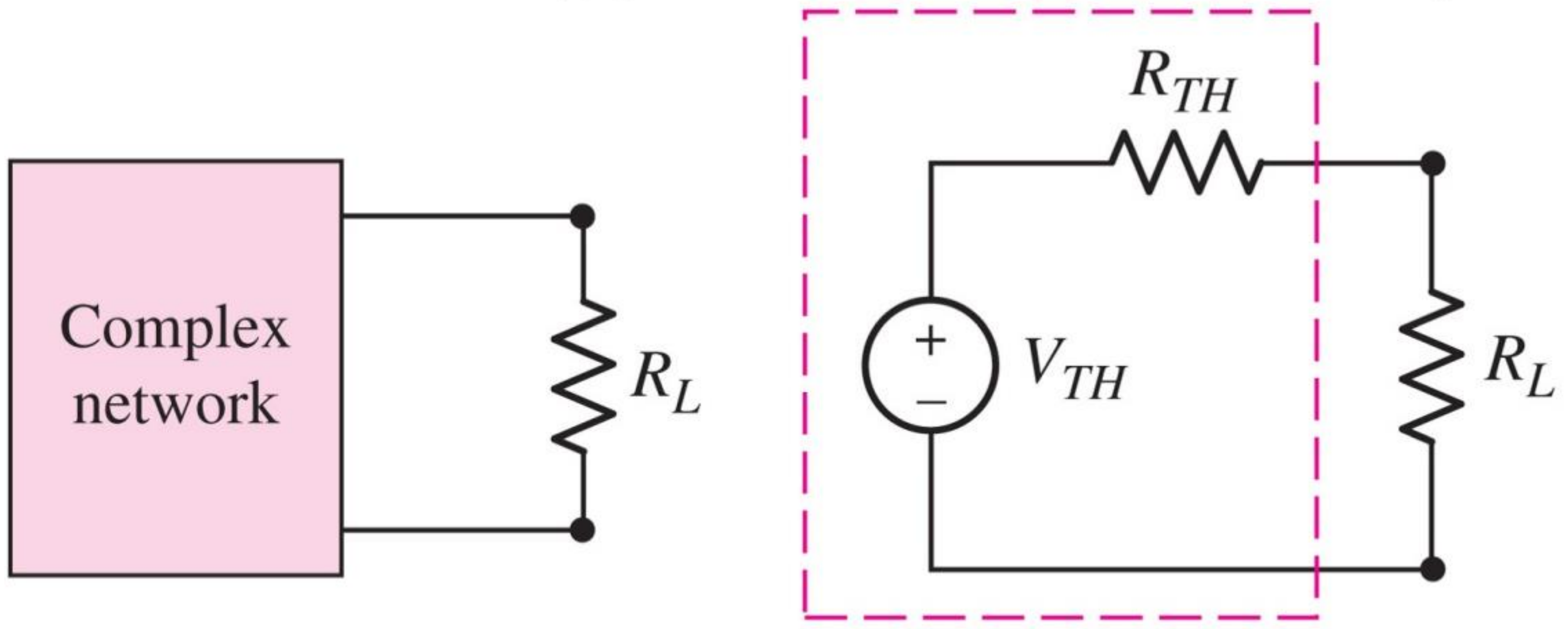
Finding the Thévenin Equivalent

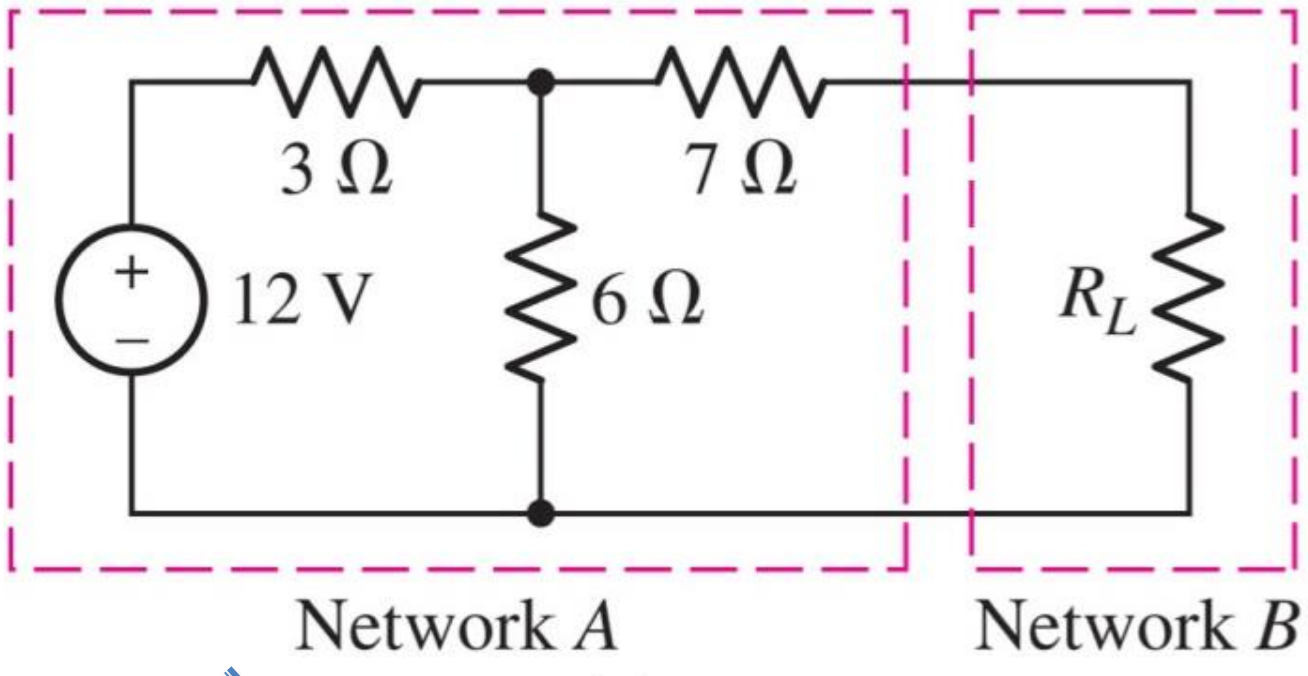
- Disconnect the load.
- Find the open circuit voltage v_{oc}
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off.

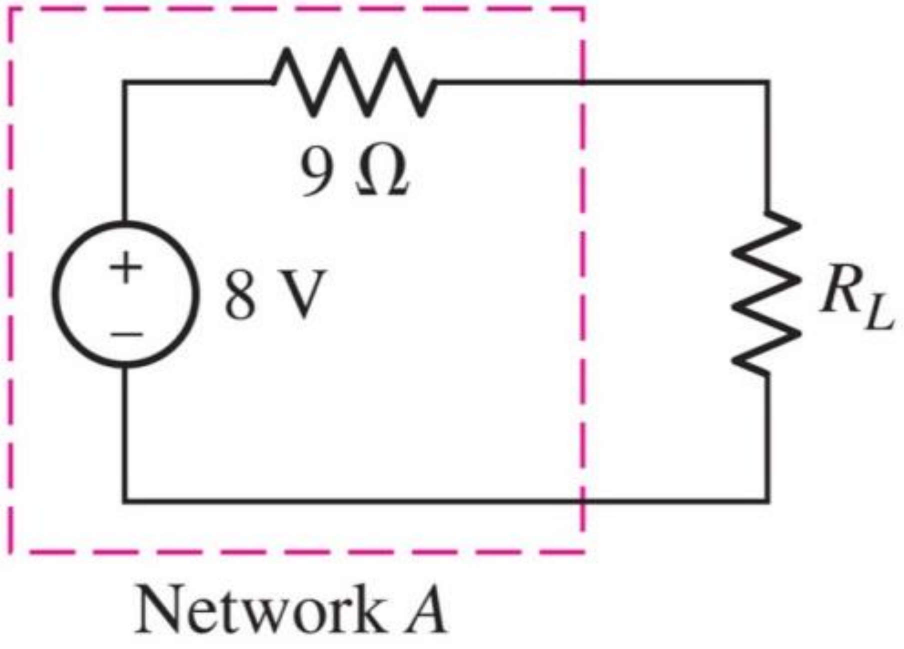
Then:

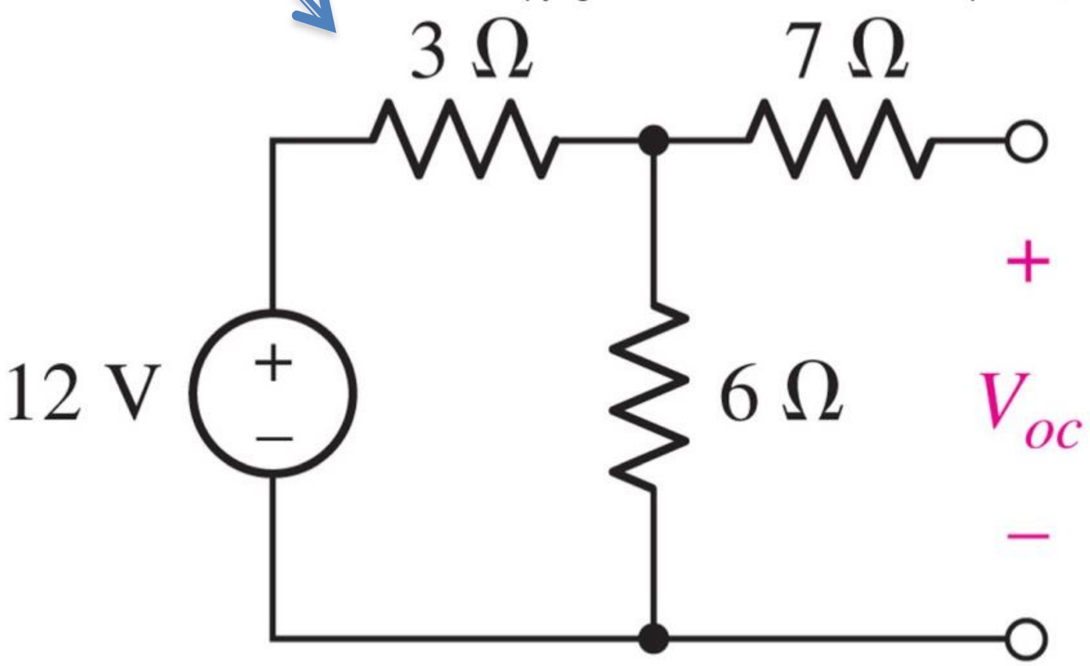
$V_{TH}=v_{oc}$ and

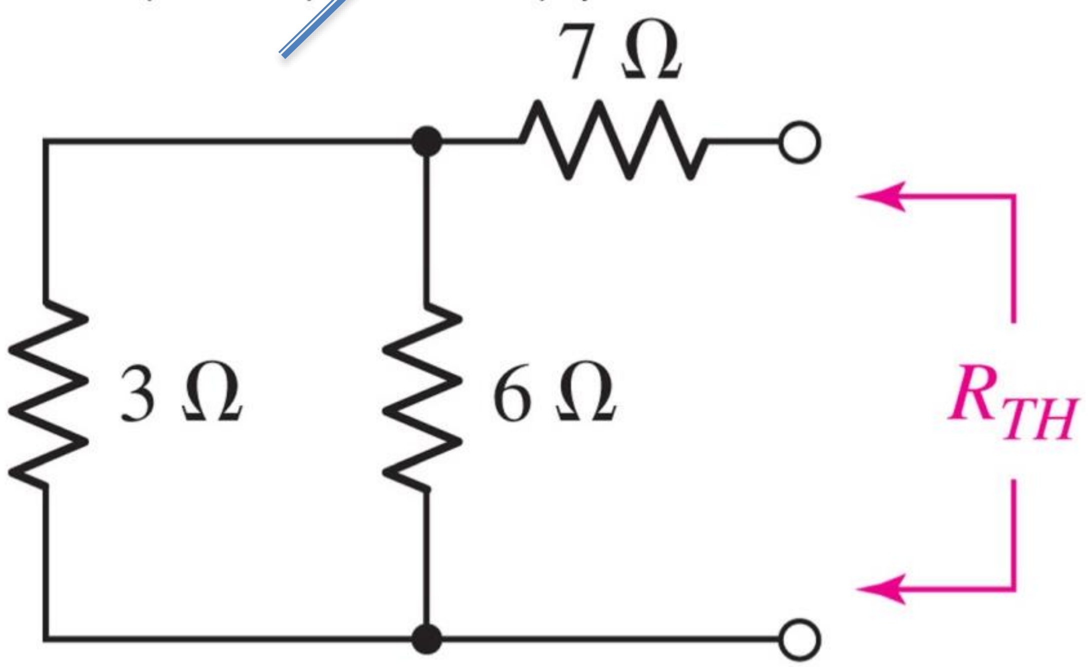
$R_{TH}=R_{eq}$




3 Ω
7 Ω
12 V
6 Ω
R_L
Network A
Network B

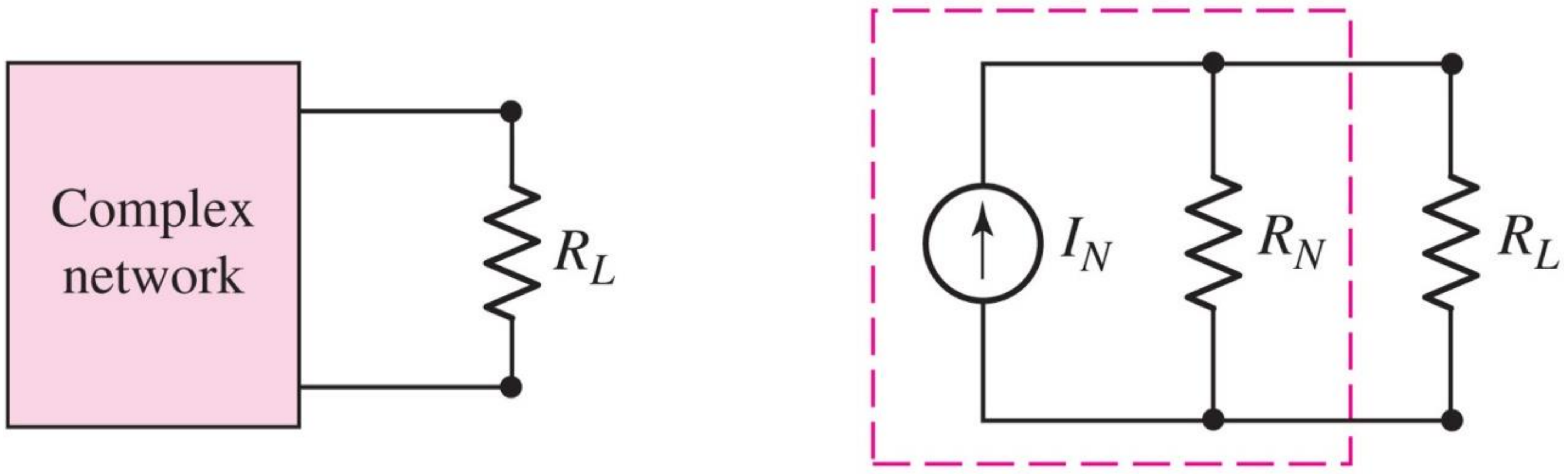

9 Ω
8 V
R_L
Network A


3 Ω
7 Ω
12 V
6 Ω
V_{oc}


7 Ω
3 Ω
6 Ω
R_{TH}

Norton Equivalent Circuits

- ✓ Norton’s theorem: a linear network can be replaced by its Norton equivalent circuit, in such a way that the current and voltage of the resistor R_L remain unchanged.
- ✓ This new circuit is called the Norton equivalent circuit. I_N is referred to as the Norton current, and R_N as the Norton resistance.

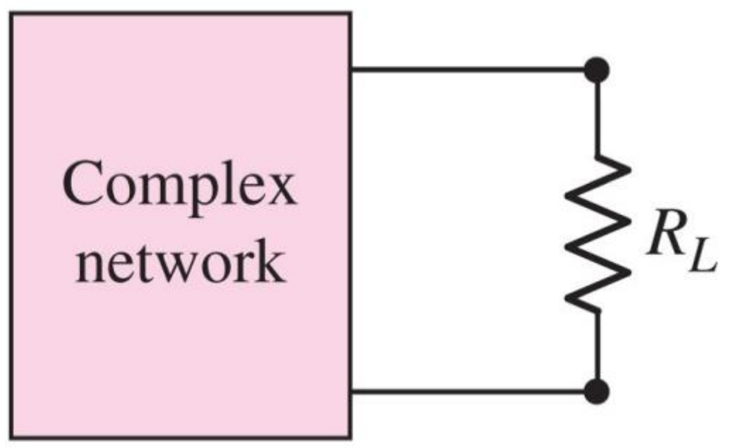


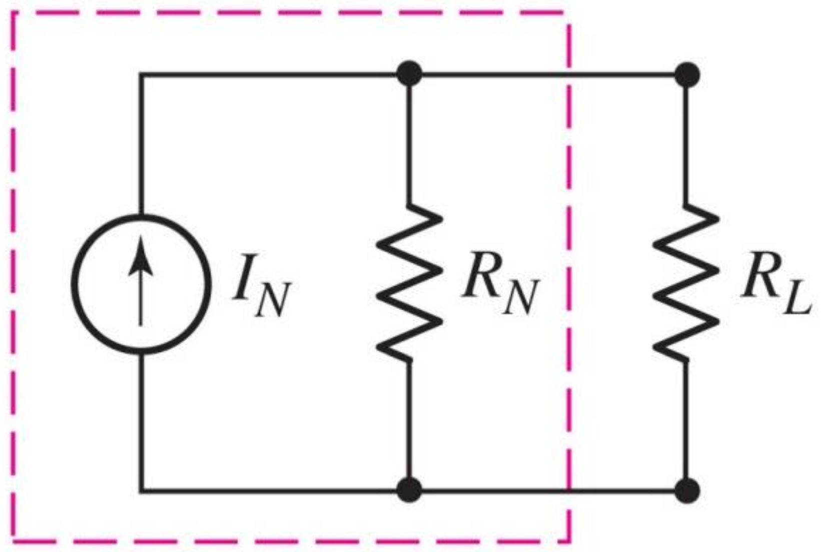
Finding the Norton Equivalent

- Replace the load with a short circuit.
- Find the short circuit current i_{sc}
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off.

Then:

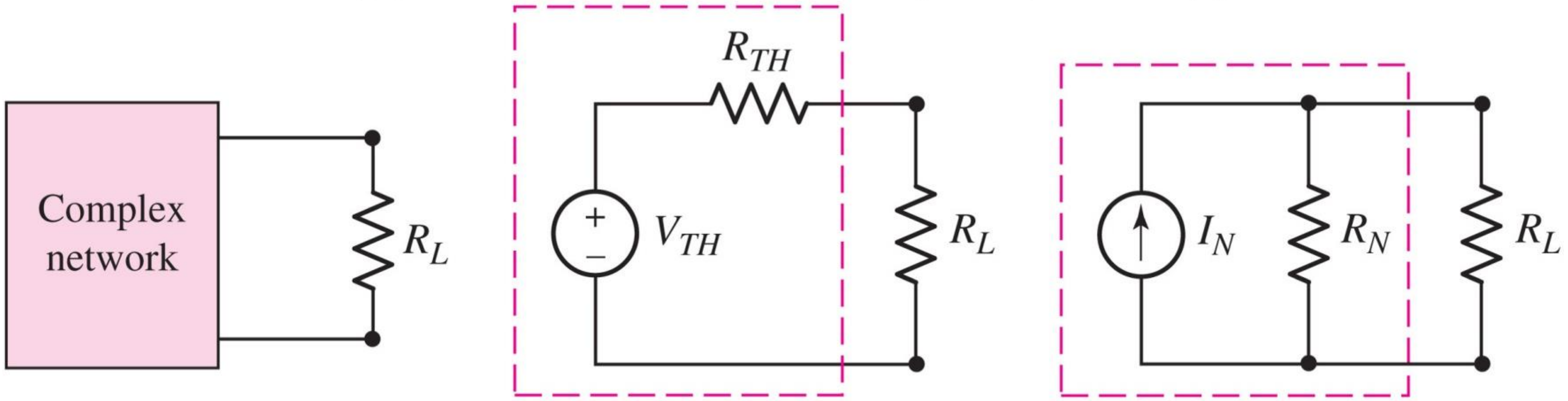
$I_N = i_{sc}$ and $R_N = R_{eq}$





Source Transformation: Norton and Thévenin

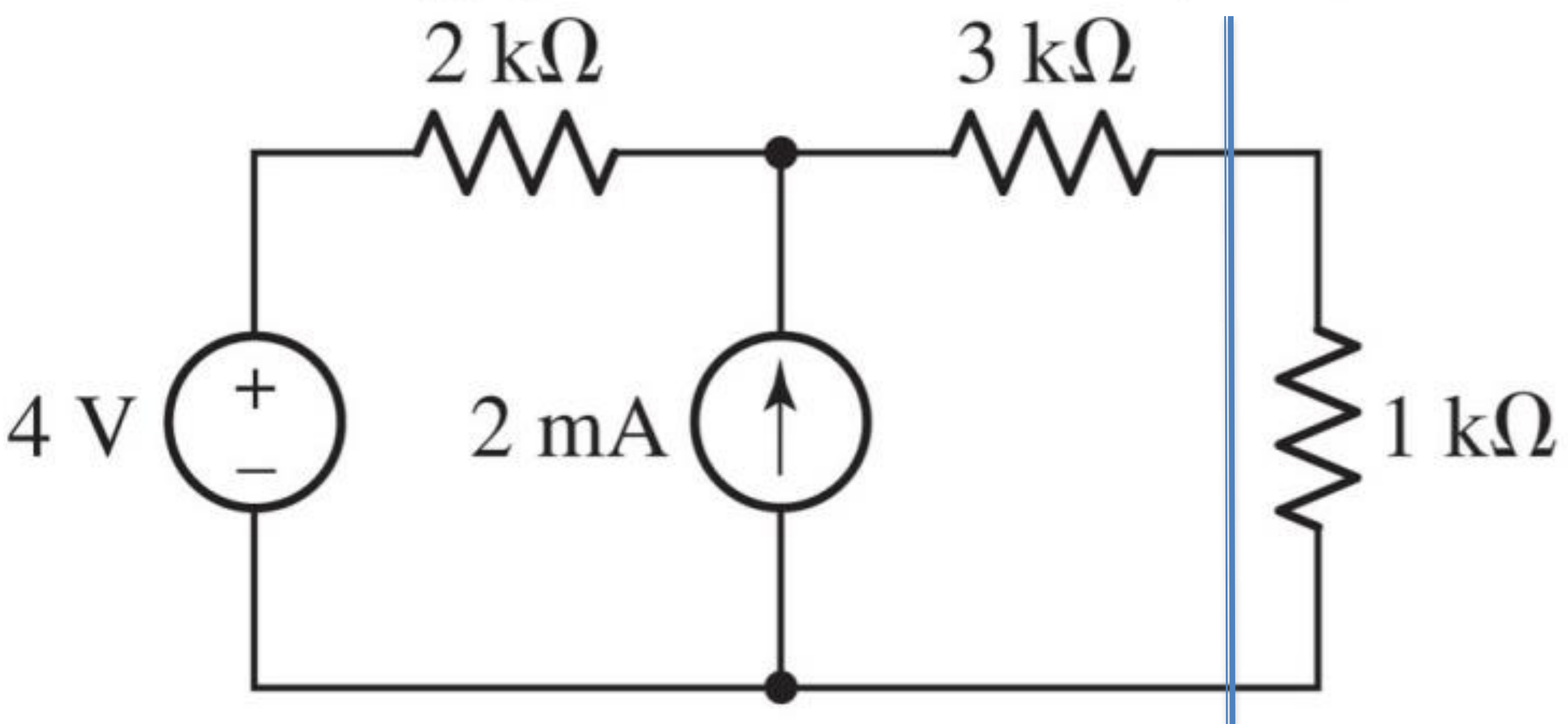
The **Thévenin** and **Norton** equivalents are **source transformations** of each other!



$$R_{TH}=R_N=R_{eq} \text{ and } v_{TH}=i_N R_{eq}$$

Example: Norton and Thévenin

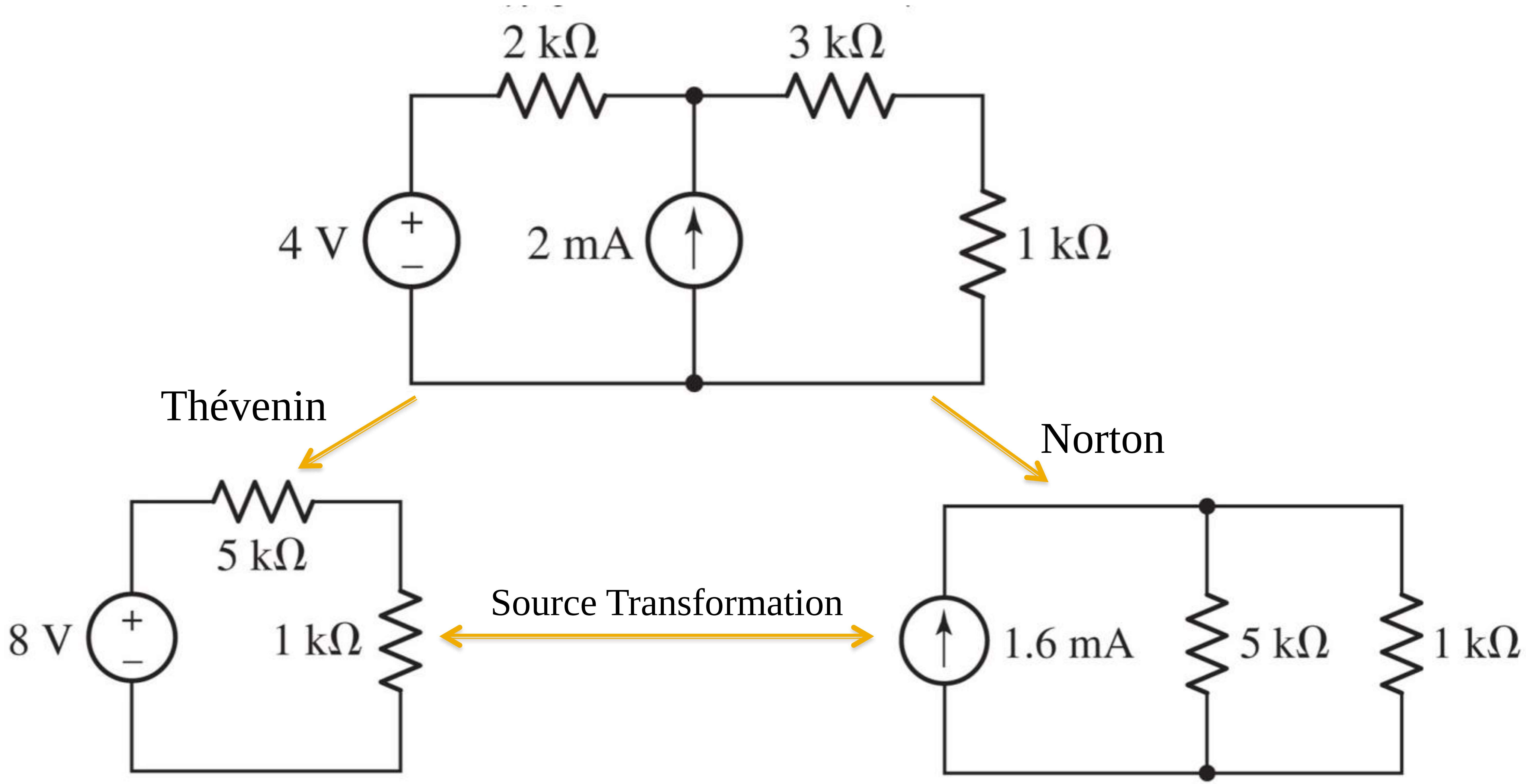
Find the Thévenin and Norton equivalents for the network faced by the 1-kΩ resistor.



the load resistor

this is the circuit we will simplify

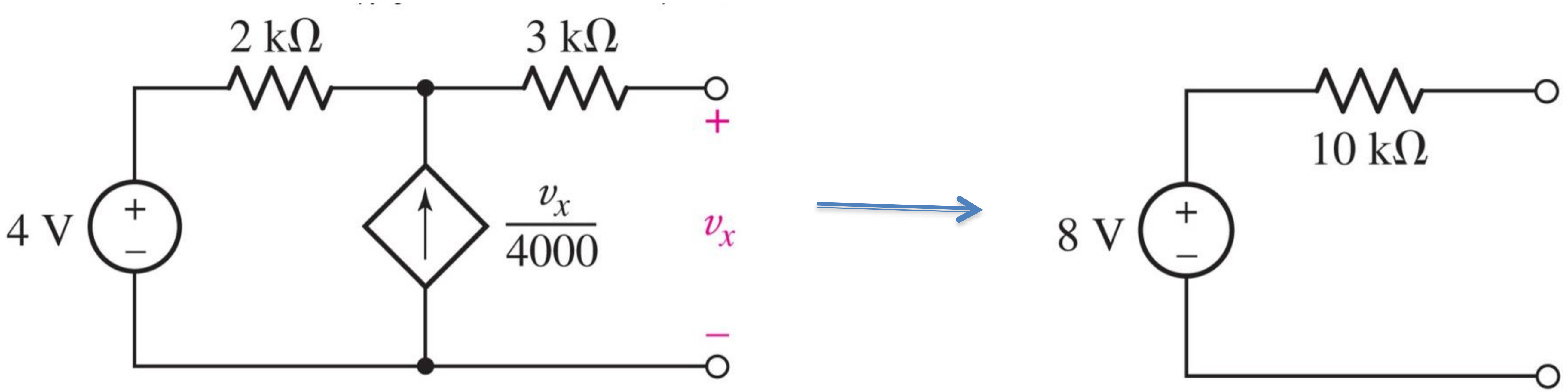
Example: Norton and Thévenin



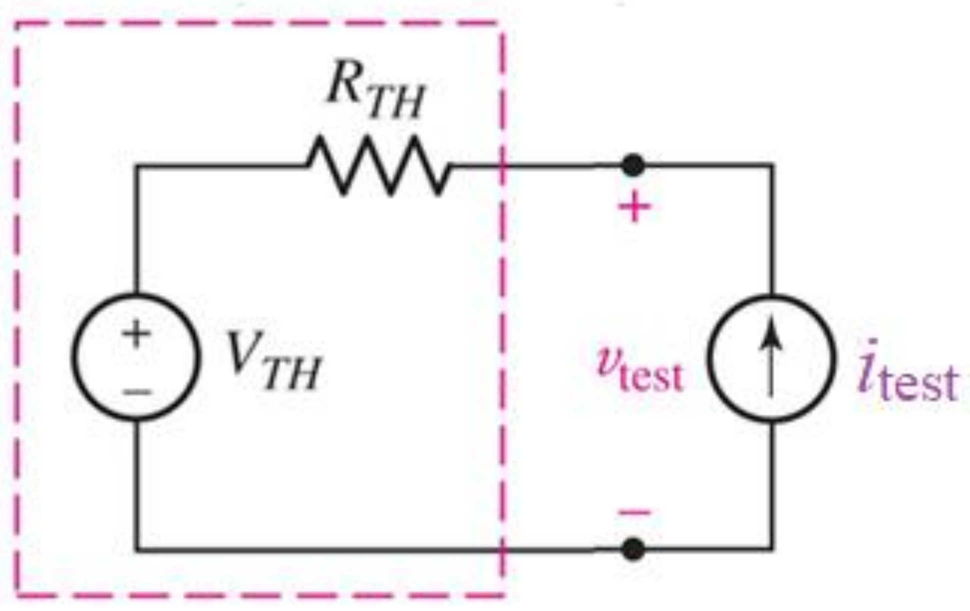
One method to find the Thévenin equivalent of a circuit with a dependent source: find V_{TH} and I_N and solve for

$$R_{TH}=V_{TH}/I_N$$

Example:



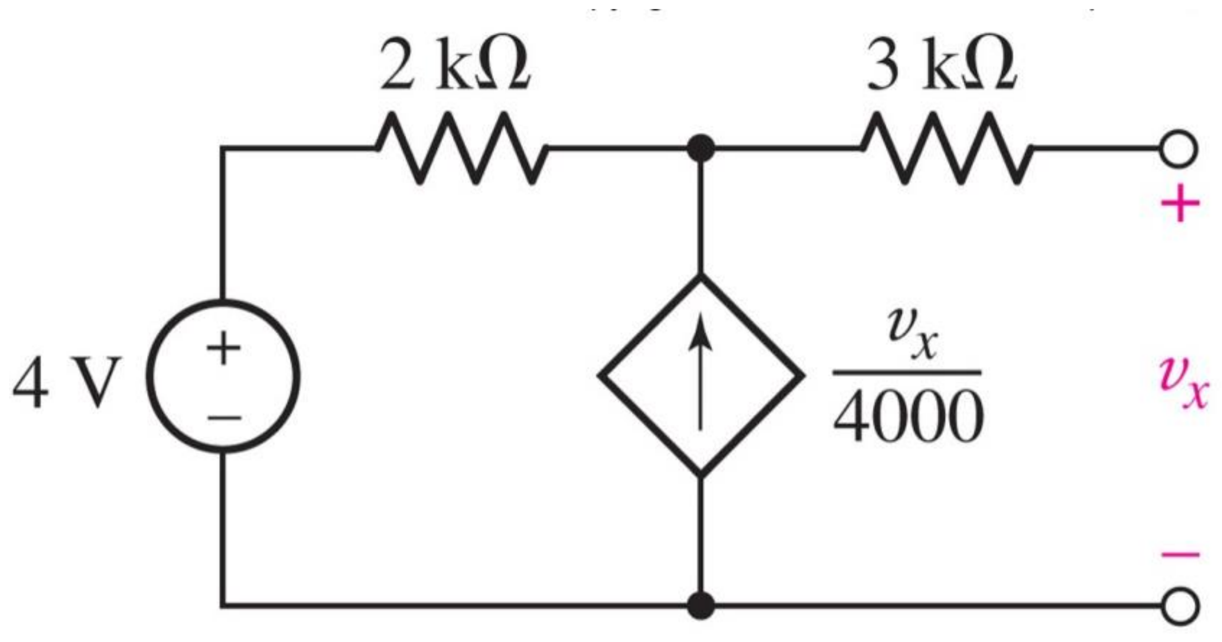
In general, a test source can be used to find the Thevenin and Norton equivalent circuits of any circuit.



$$v_{test} = R_{TH} i_{test} + V_{TH}$$

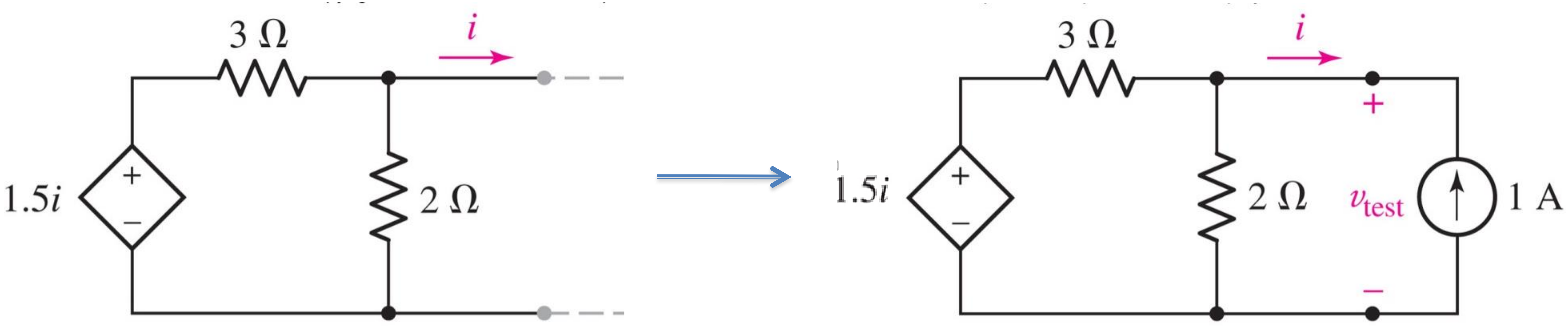
By placing the test source, an equation is obtained in which the coefficient i_{test} represents the Thevenin resistance, and the accumulated value with it is the Thevenin voltage.

Using a test source to find the equivalent circuit

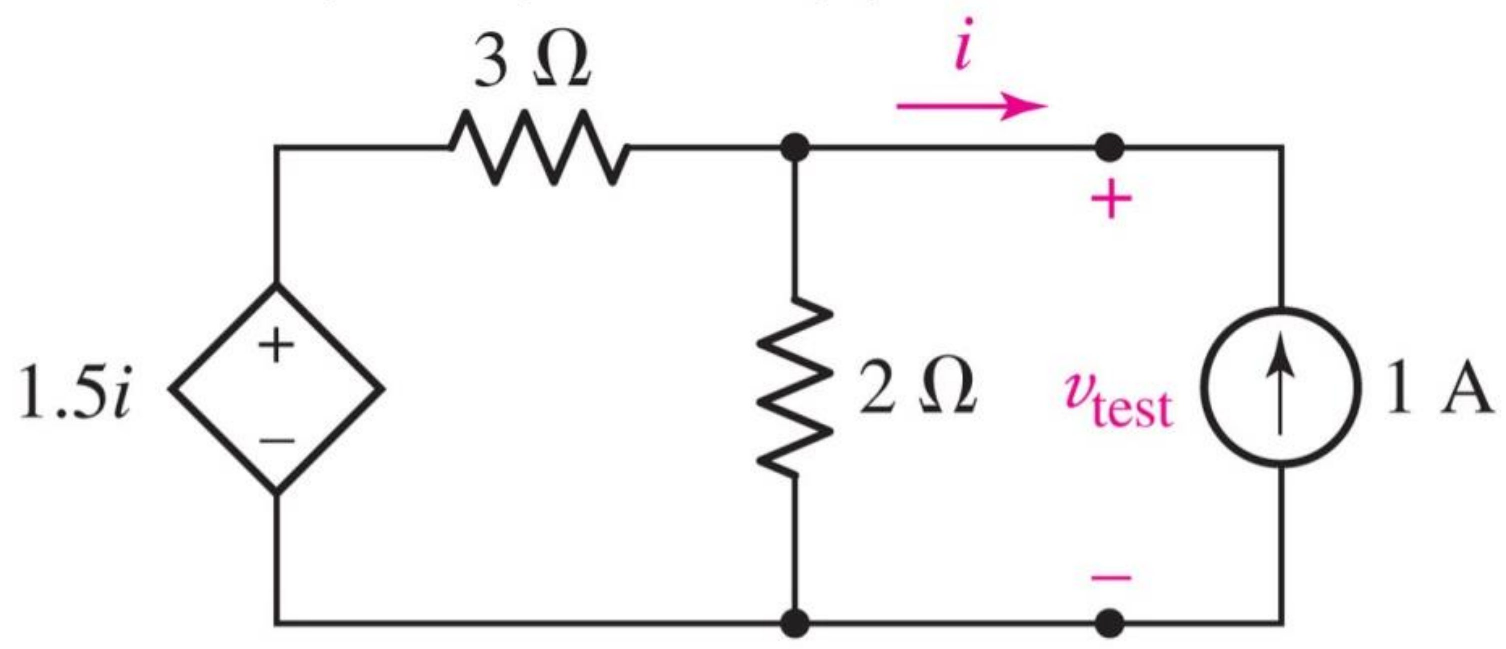


Finding the ratio V_{TH} / I_N fails when both quantities are zero!

Solution: apply a test source.



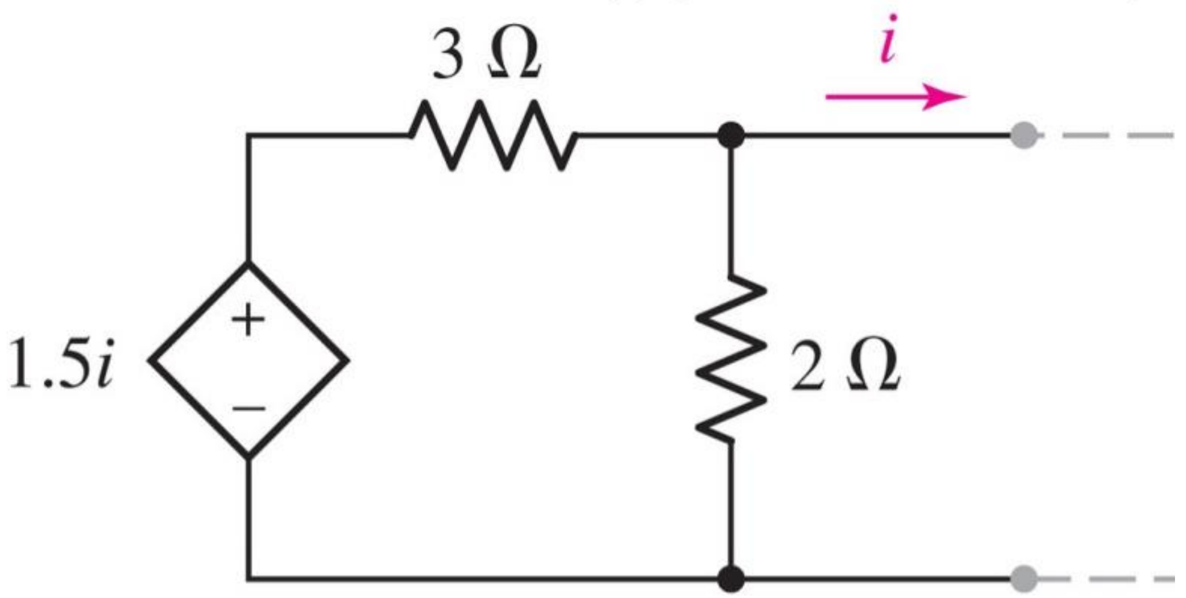
Thévenin Example: Handling Dependent Sources

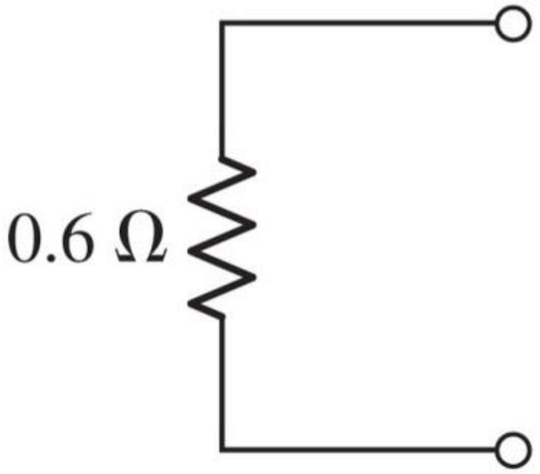


$$\frac{v_{test}}{2} + \frac{v_{test} - (1.5i)}{3} = 1$$

$$i = -1$$

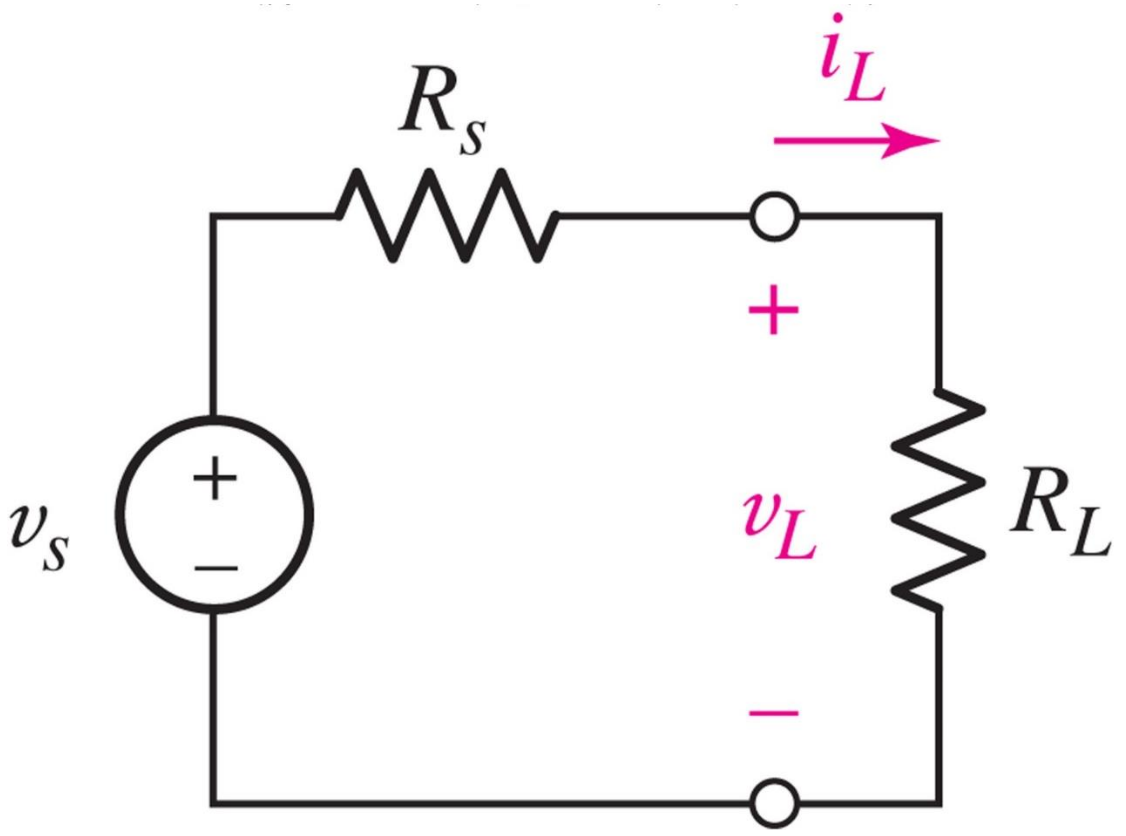
Solve: v_{test} =0.6 V, and so R_{TH} = 0.6 Ω





Maximum Power Transfer

What load resistor will allow the practical source to deliver the maximum power to the load?

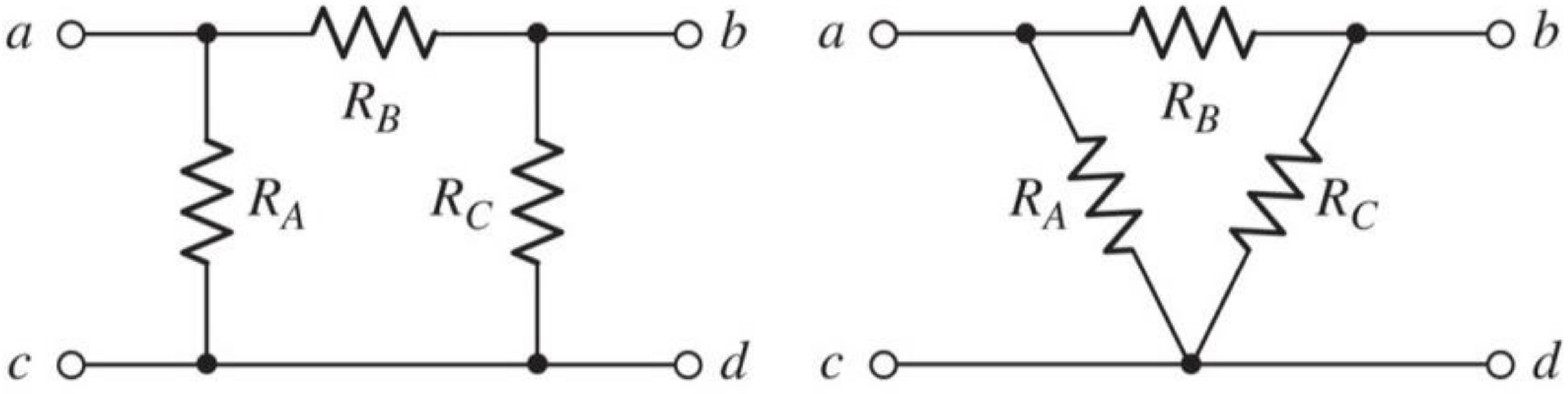


Answer: $R_L = R_s$

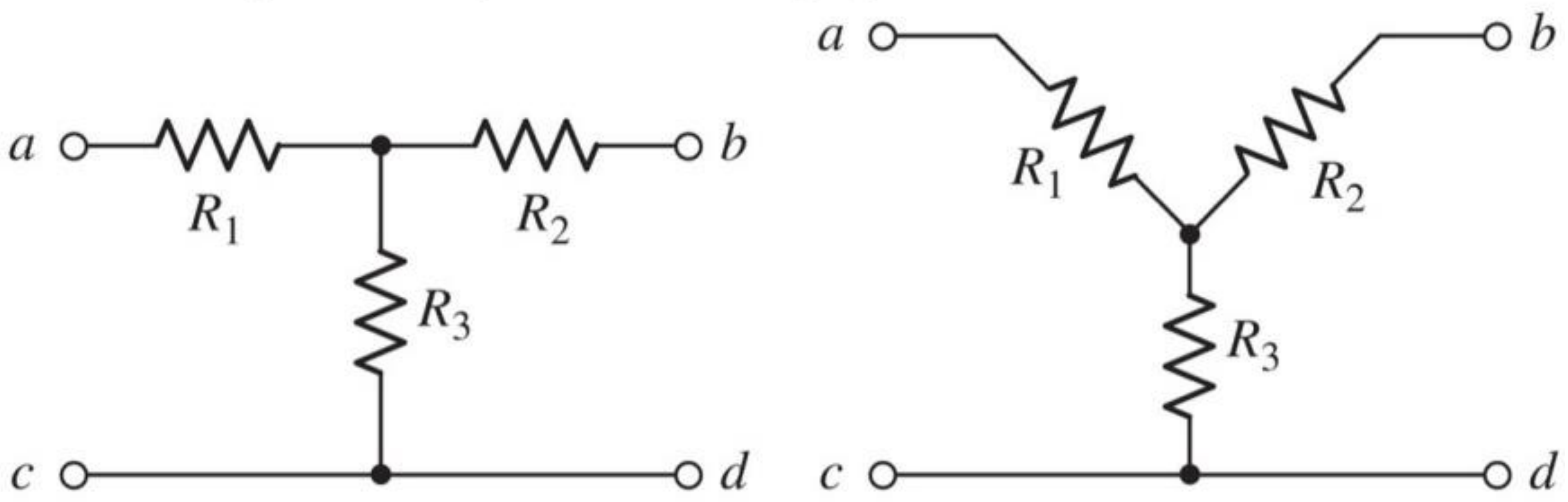
[Solve $dp_L/dR_L = 0$.]

[Or: $p_L = i(v_s - iR_s)$, set $dp_L/di = 0$ to find $i_{max} = v_s/2R_s$. Hence $R_L = R_s$]

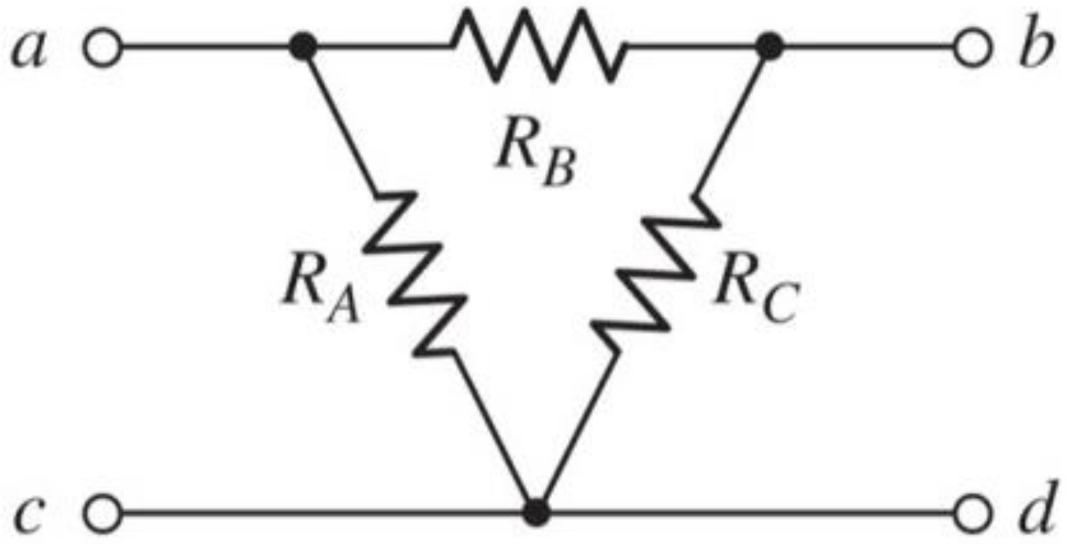
- The following resistors form a Δ:



- The following resistors form a Y:



Δ-Y (delta-wye) Conversion

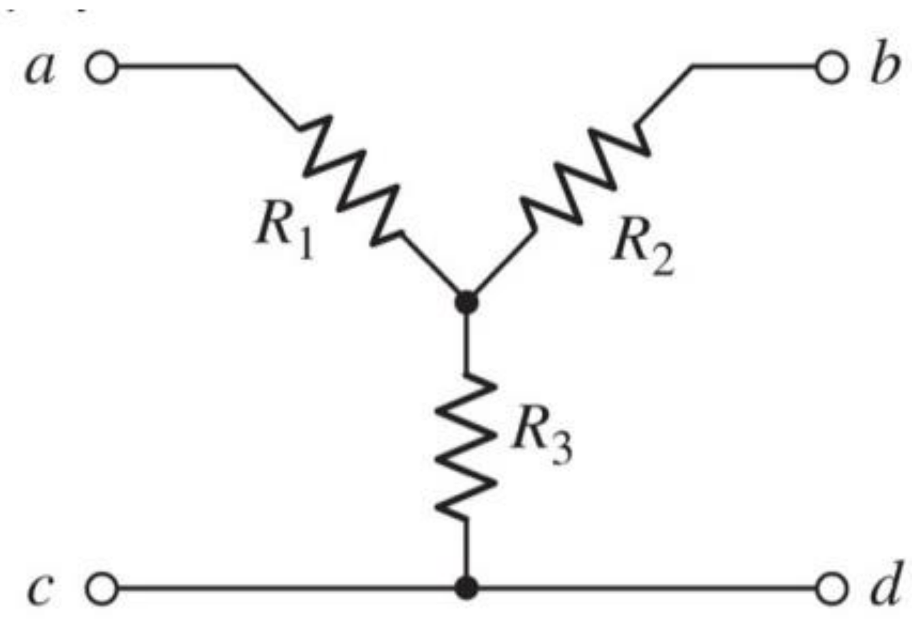


this Δ is equivalent to the Y if

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$



this Y is equivalent to the Δ if

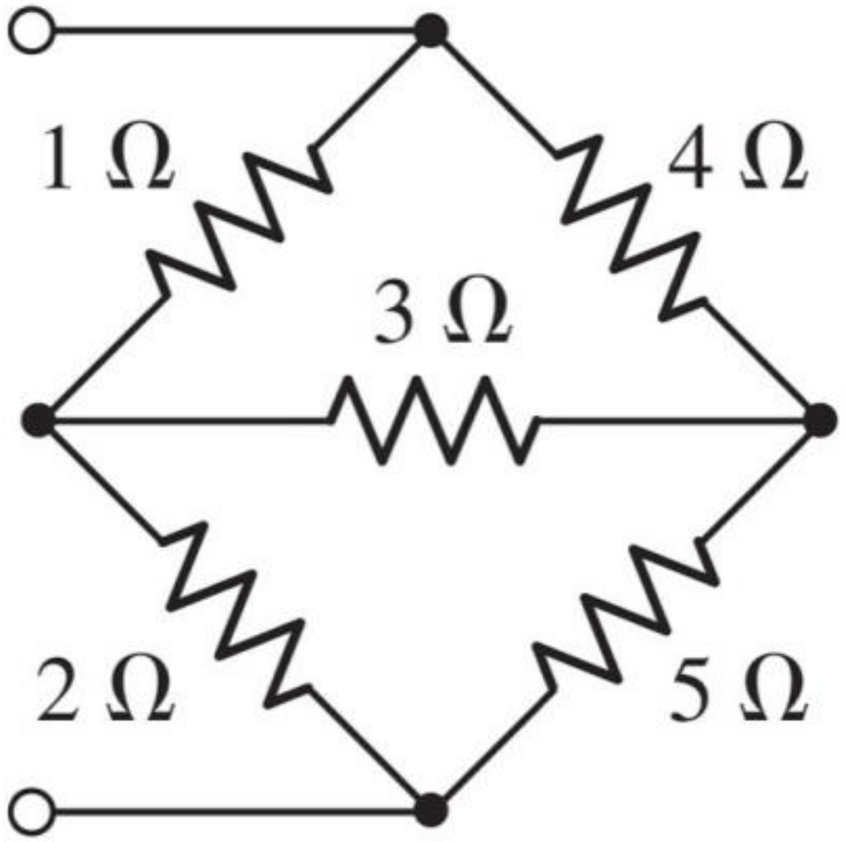
$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

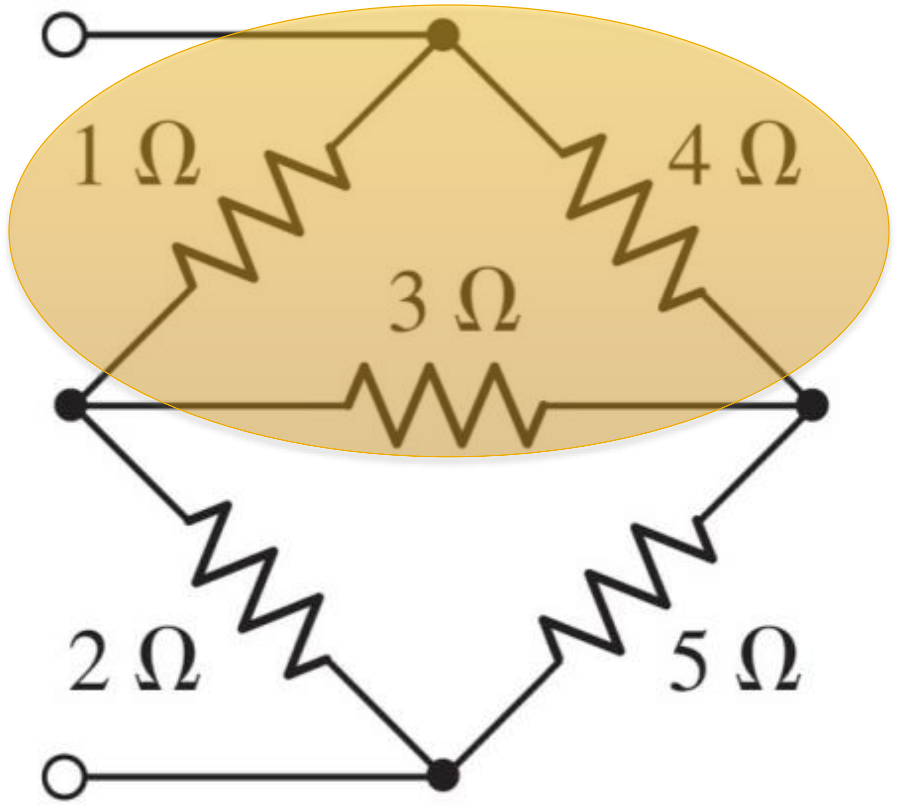
$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

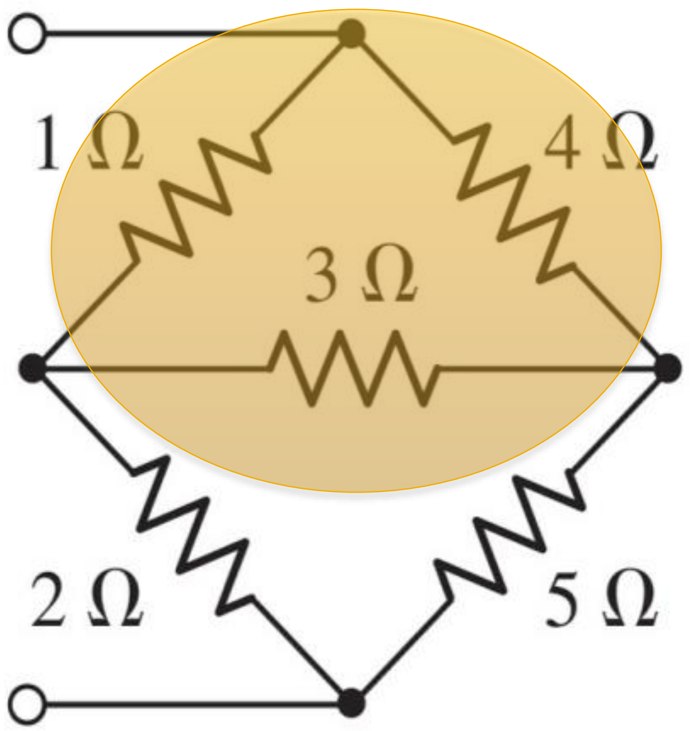
$$R_3 = \frac{R_C R_A}{R_A + R_B + R_C}$$

Example: Δ-Y Conversion

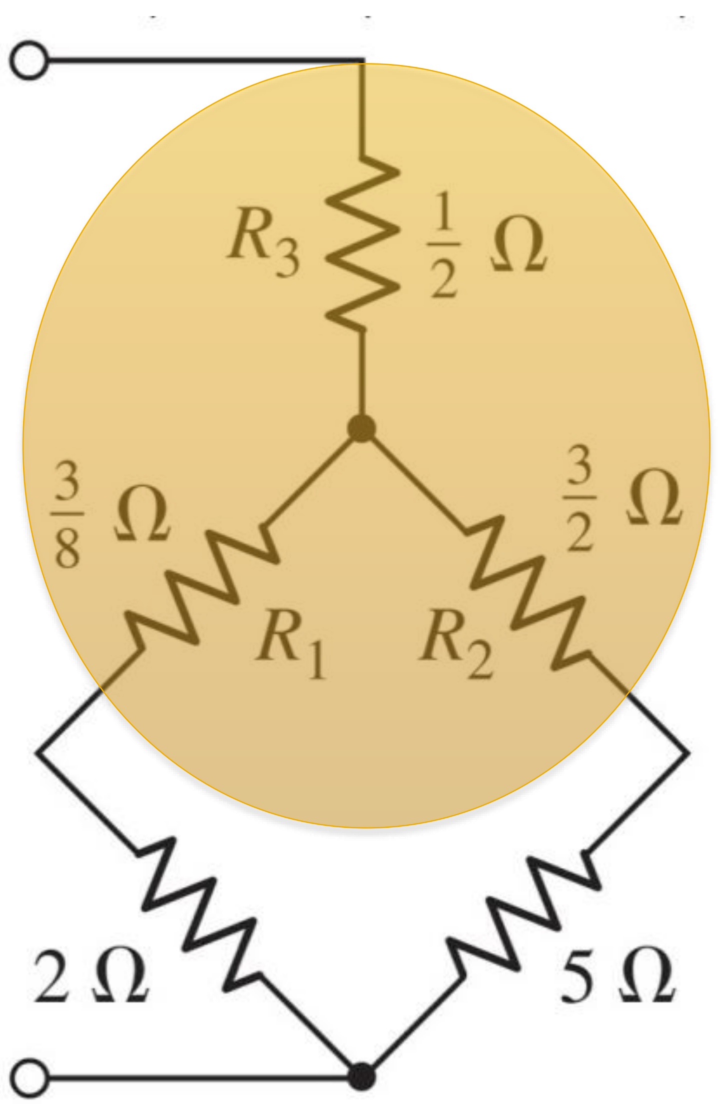
How do we find the equivalent resistance of the following network? Convert a Δ to a Y

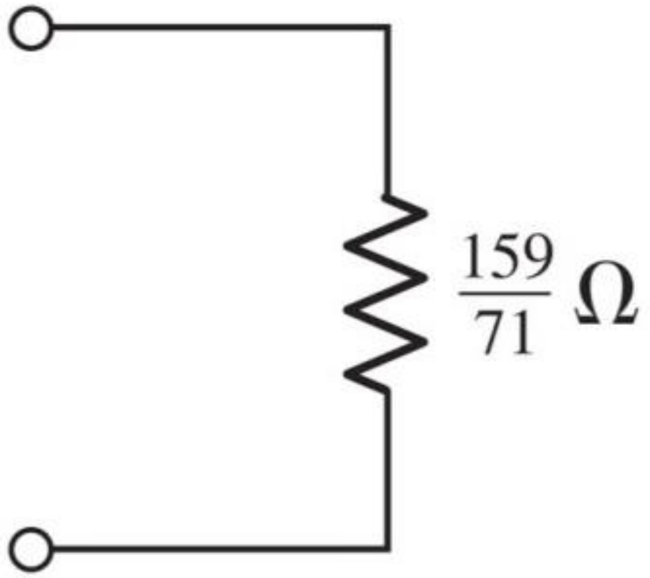






use the Δ to Y equations





use standard serial and parallel combinations

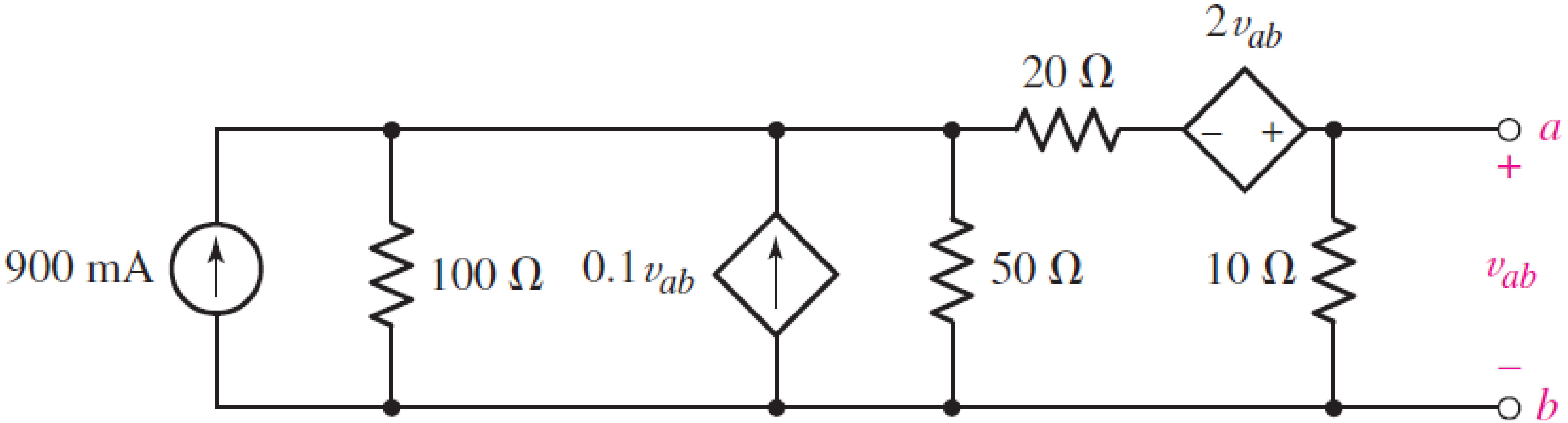
Summary

What you learned in this slide:

- The Superposition Theorem and how to use it
- Practical Sources
- Source Transformation and its application in circuit simplification
- Thevenin and Norton Equivalent Circuits and how to calculate them
- Maximum Power Transfer Theorem
- Star-Delta Transformation and its application in circuit simplification

Practice

If a load R_L is connected across terminals a and b, determine its value for maximum power transfer. Also, calculate the maximum power.



Thanks