دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 3. Node and Mesh Analysis

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Objectives of the Lecture

- Provide step-by-step instructions for nodal analysis, which is a method to calculate node voltages and currents that flow through components in a circuit.

- Provide step-by-step instructions for mesh analysis, which is a method to calculate voltage drops and mesh currents that flow around loops in a circuit.

Circuit Analysis

- as circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's

- *nodal* analysis assigns *voltages* to each node, and then we apply *KCL*

- *mesh* analysis assigns *currents* to each mesh, and then we apply *KVL*

The Nodal Analysis Method

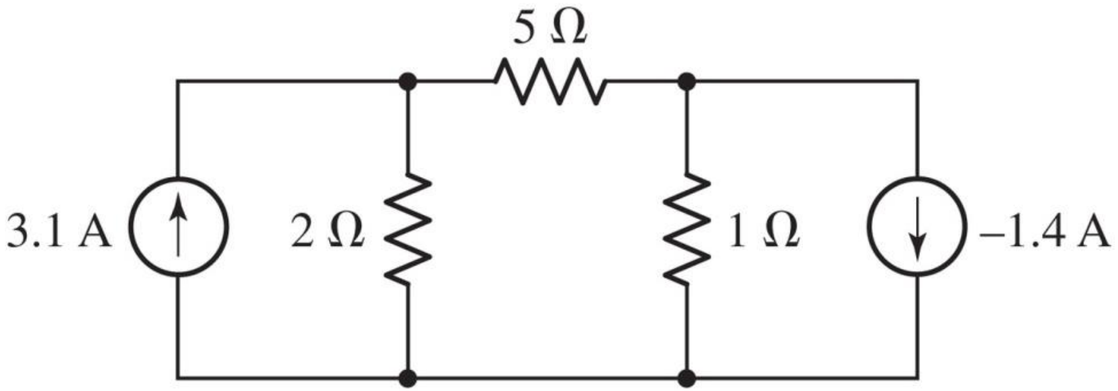
Technique to find currents at a node using Ohm’s Law and the potential differences between nodes.

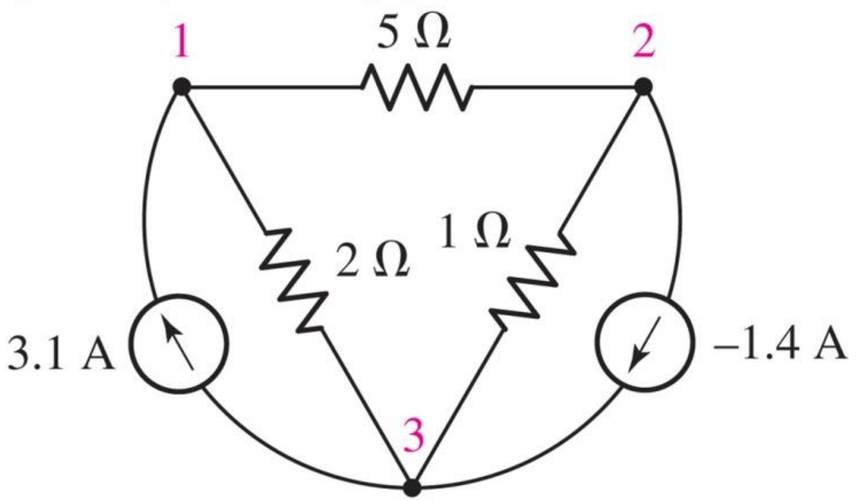
- First result from nodal analysis is the determination of node voltages (voltage at nodes referenced to ground).

- Second result is the calculation of the currents.

The Nodal Analysis Method

- Assign voltages to every node relative to a reference node

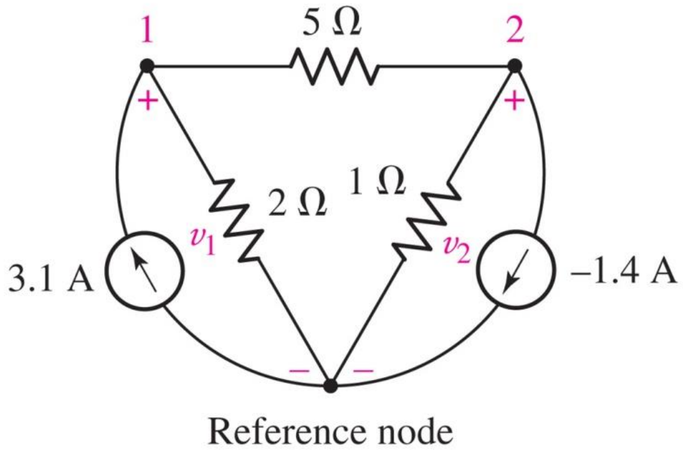




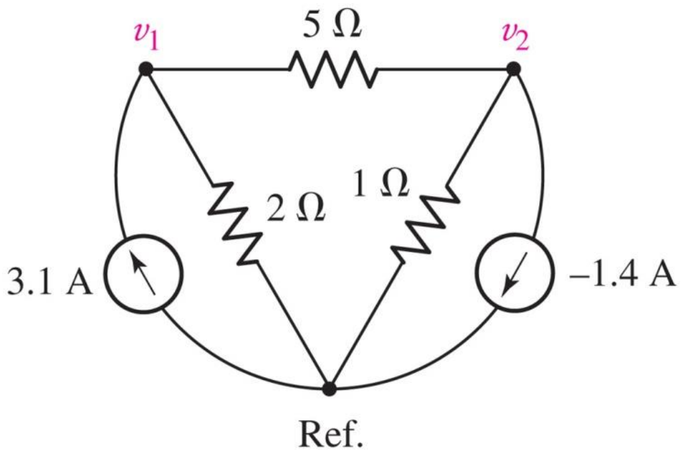
- In this example, there are three nodes

Choosing the Reference Node

- as the bottom node, or
- as the ground connection, if there is one, or
- a node with many connections



Reference node

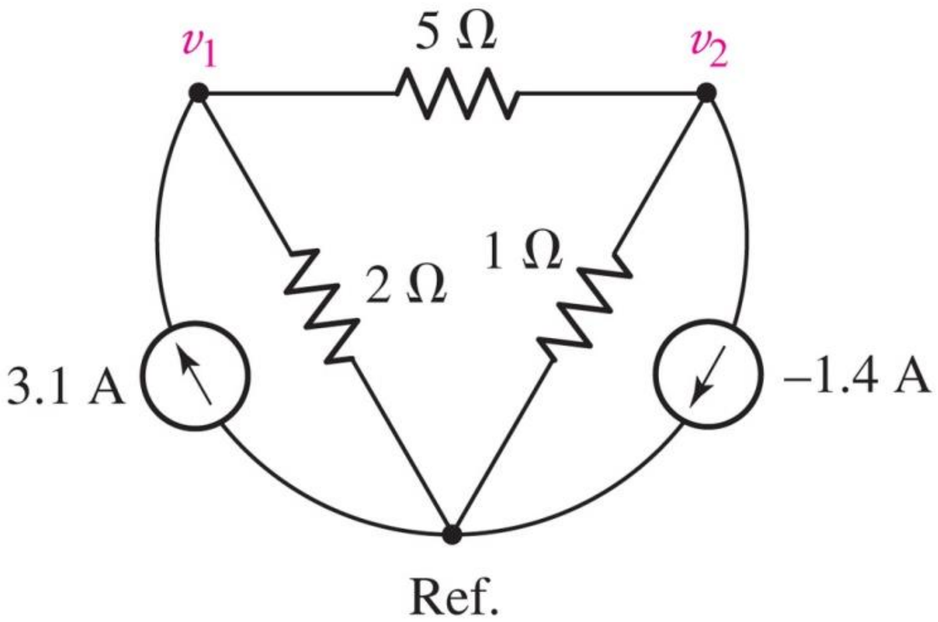


Ref.

- assign voltages relative to reference

Apply KCL to Find Voltages

- Apply KCL to node 1 (Σ out = Σ in) and Ohm's law to each resistor:



Note: the current flowing out of node 1 through the 5 Ω resistor is

$$v_1 - v_2 = i5$$

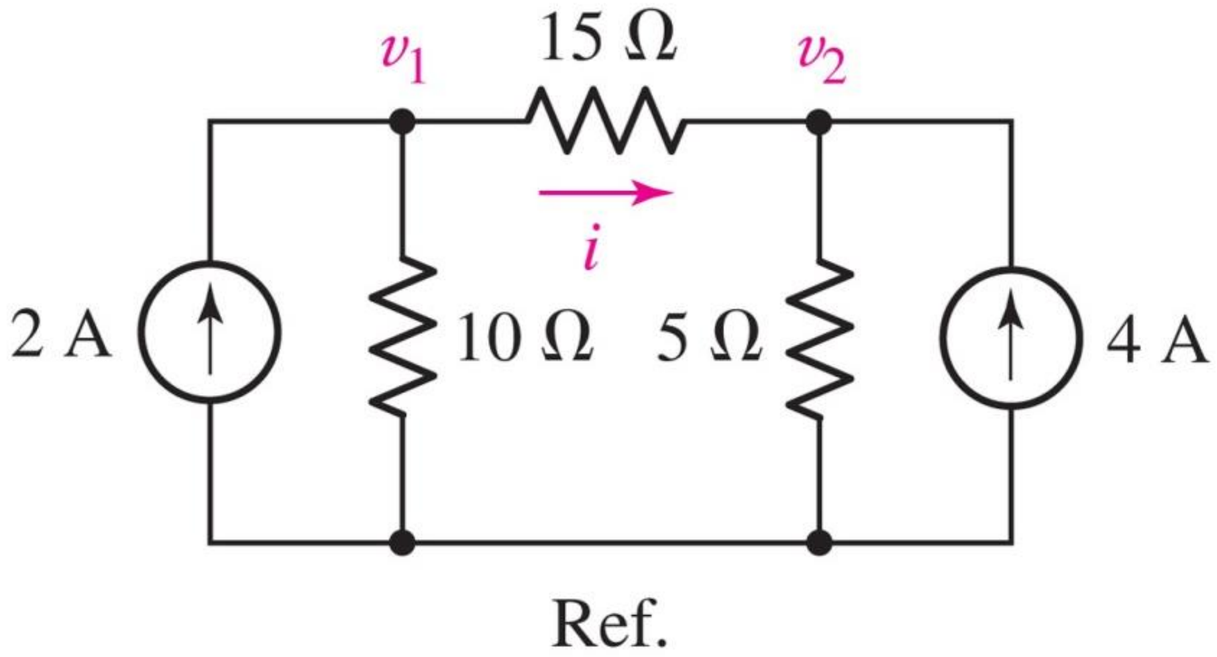
Steps in Node Analysis

- Steps in Nodal Analysis

1. Pick one node as a reference node
2. Label the voltage at the other nodes
3. Label the currents flowing through each of the components in the circuit
4. Use Kirchoff’s Current Law
5. Use Ohm’s Law to relate the voltages at each node to the currents flowing in and out of them.
6. Solve for the node voltage
7. Once the node voltages are known, calculate the currents.

Example: Nodal Analysis

Find the current *i* in the circuit.



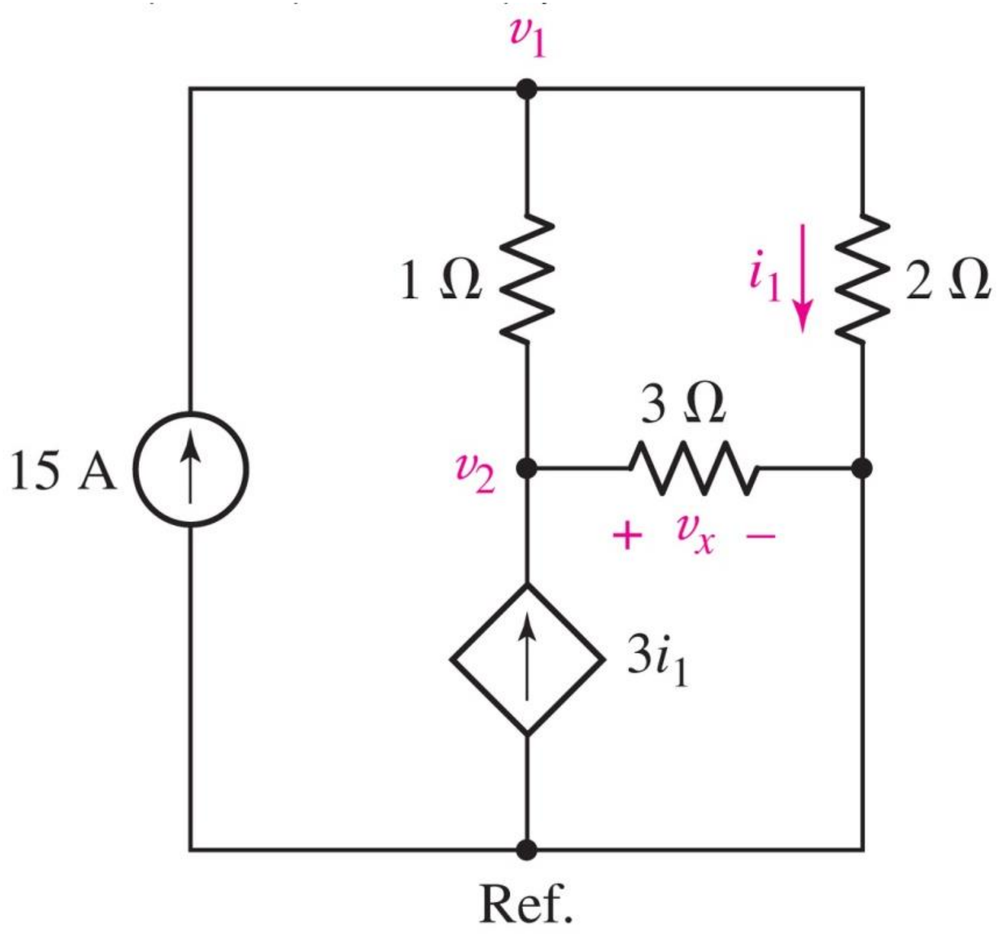
Answer: i = 0 (since v_1=v_2=20 V)

Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source.

Key step: eliminate i_1 from the equations using $v_1=2i_1$

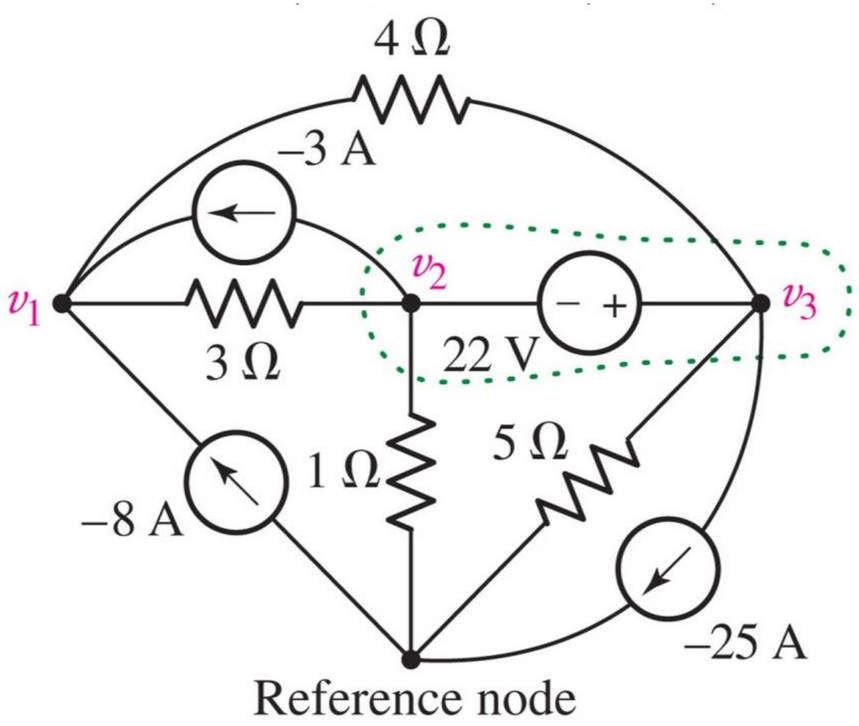
Answer: 4.5 kW

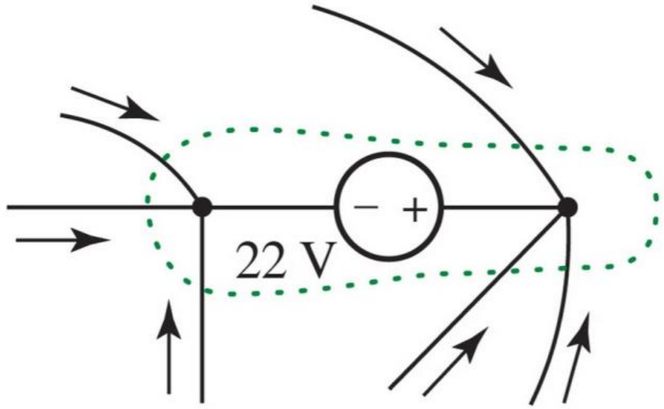


Voltage Sources and the Supernode

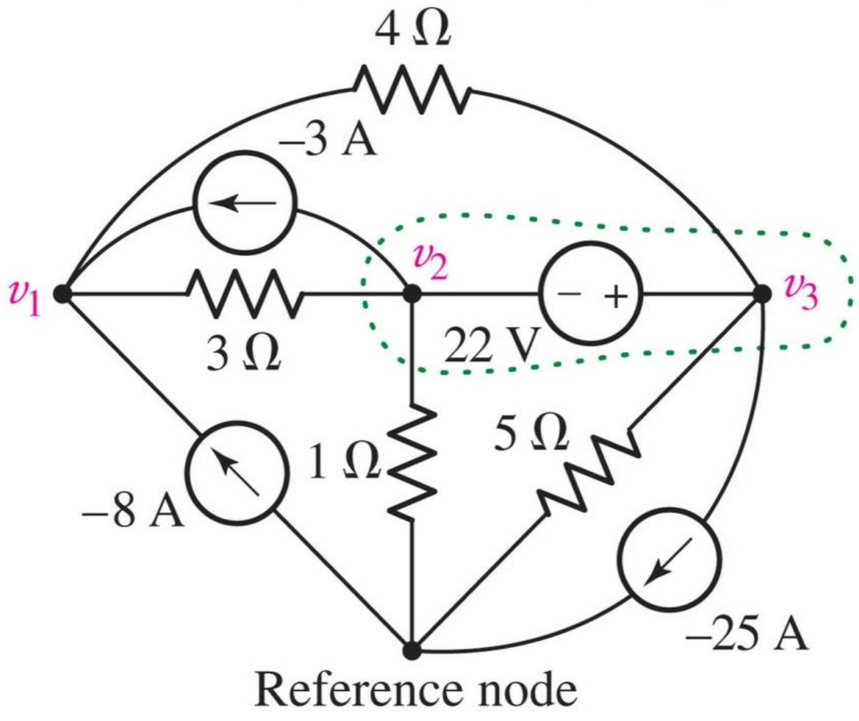
- What is the current through a voltage source connected between nodes?

We can eliminate the need for introducing a current variable by applying KCL to the *supernode*.





The Supernode



- Apply KCL at Node 1.
- Apply KCL at the supernode.
- Add the equation for the voltage source inside the supernode.

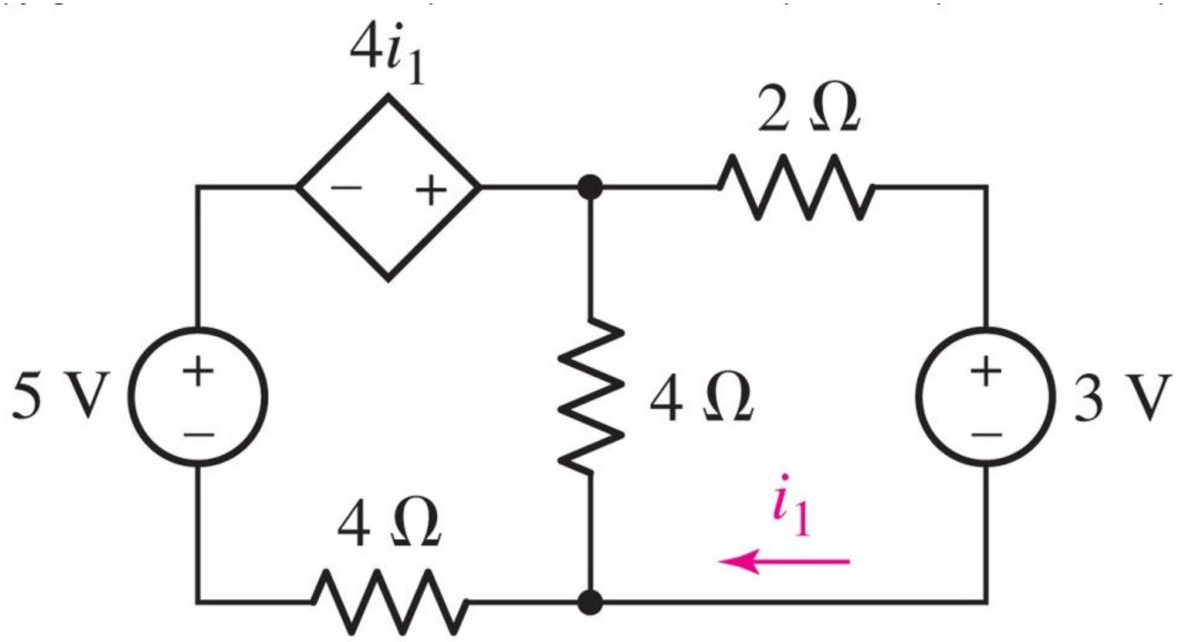
$$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{3} = -3 - 8$$

$$\frac{v_2}{1} + \frac{v_2 - v_1}{3} + \frac{v_3}{5} + \frac{v_3 - v_1}{4} = -(-25) - (-3)$$

$$v_3 - v_2 = 22$$

Dependent Source Example

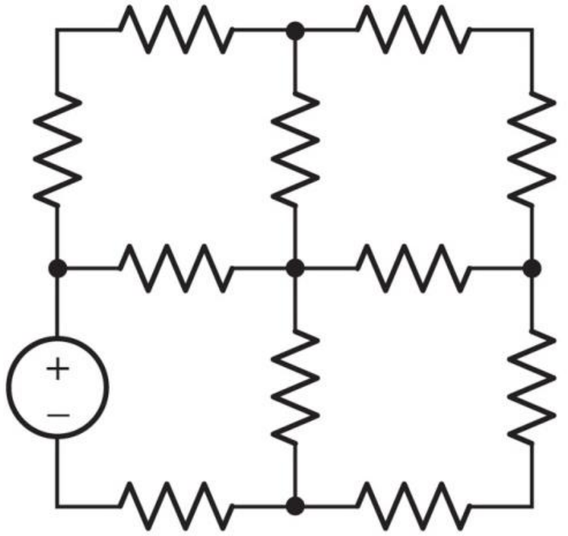
Find i_1



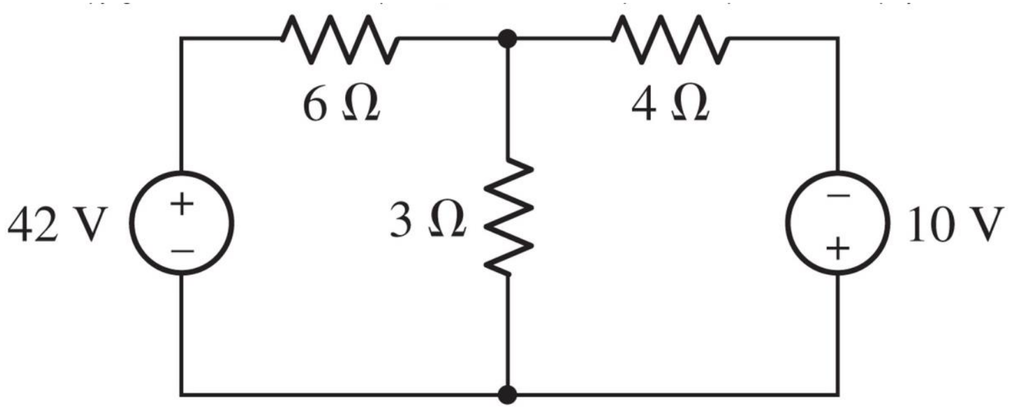
Answer: $i_1 =$ *- 250 mA.*

Mesh Analysis: Nodal Alternative

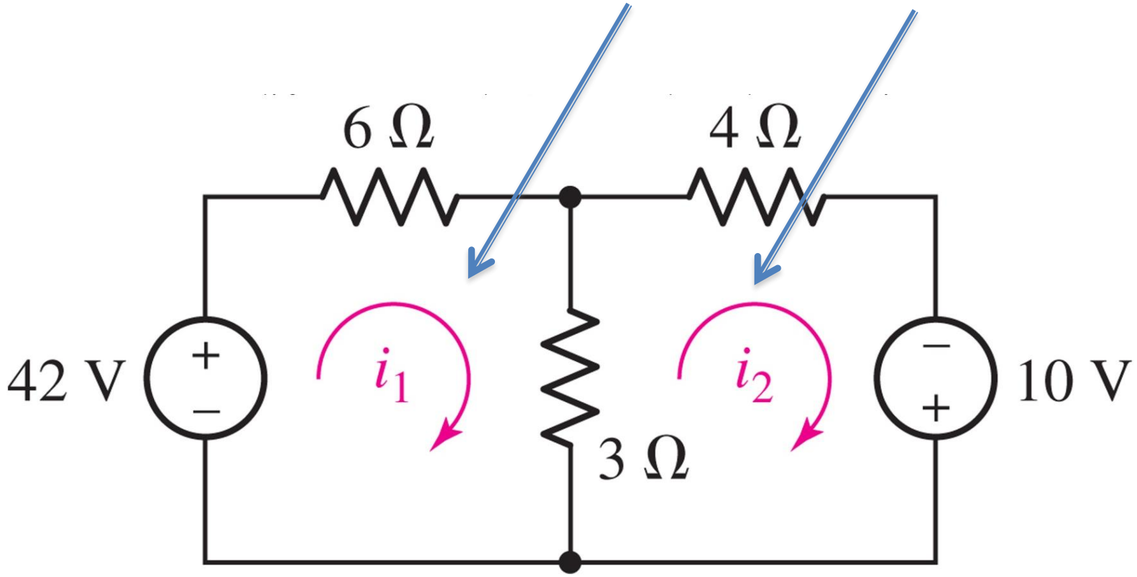
- a mesh is a loop which does not contain any other loops within it
- in mesh analysis, we assign currents and solve using KVL
- assigning mesh currents automatically ensures KCL is followed
- this circuit has four meshes:

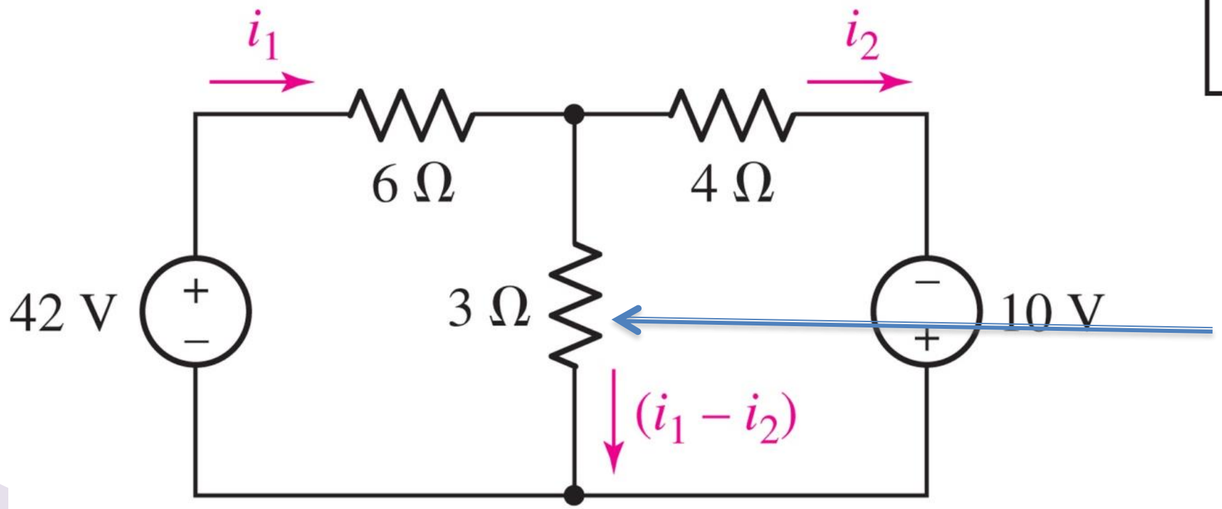


The Mesh Analysis Method



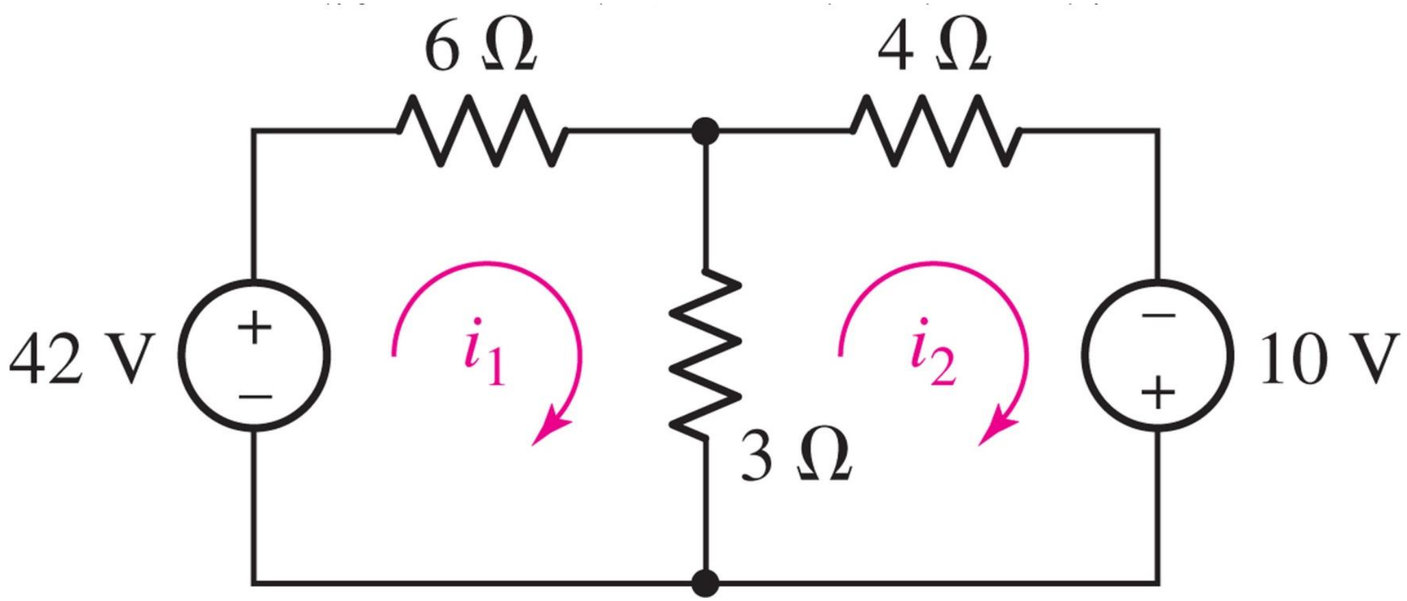
Mesh currents





Branch currents

Mesh: Apply KVL



Apply KVL to mesh 1 (Σ drops=0):

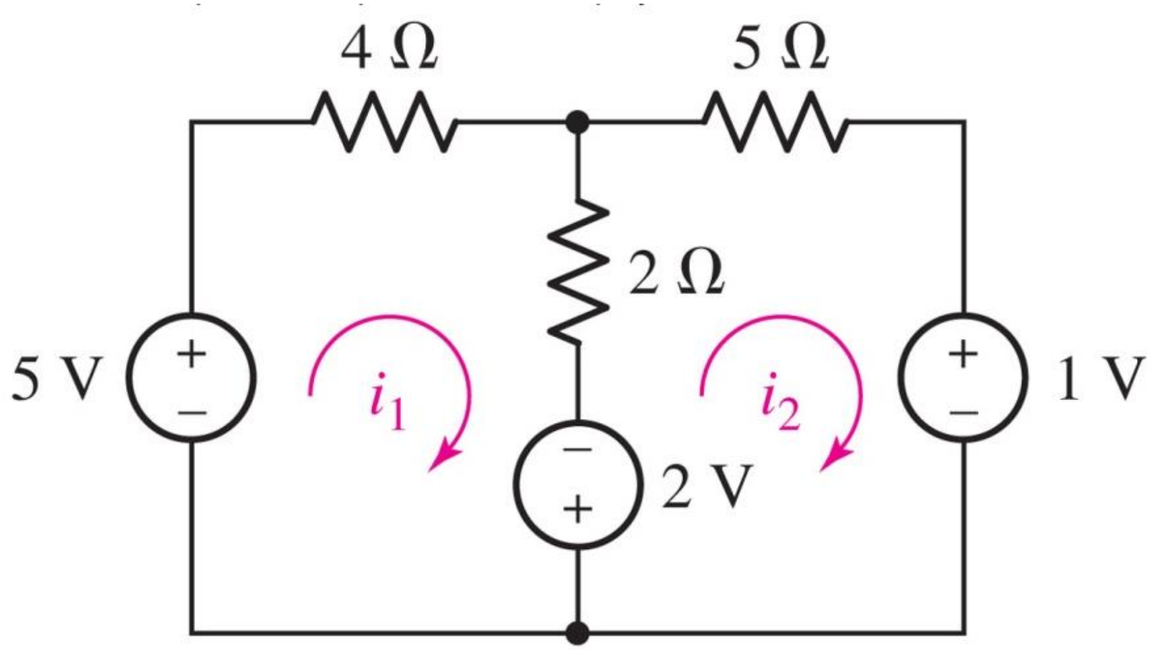
$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

Apply KVL to mesh 2 (Σ drops=0):

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

Example: Mesh Analysis

Determine the power supplied by the 2 V source.



Applying KVL to the meshes:

$$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$$

$$+2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$$

Solve: i_1=1.132 A, $i_2 = -0.1053$ A.

Answer: 2.474 W

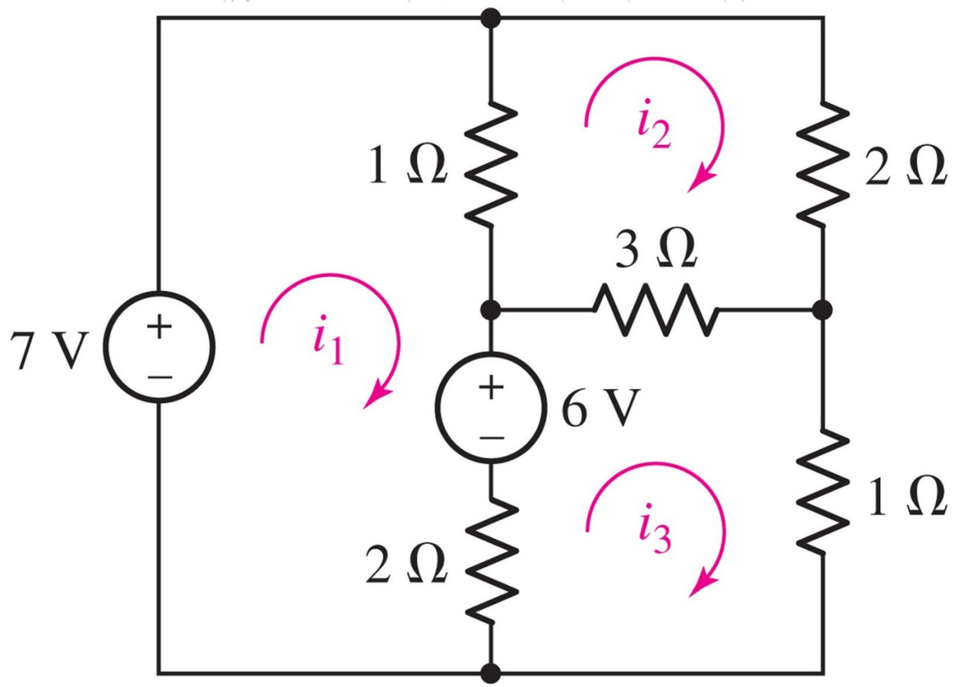
A Three Mesh Example

Follow each mesh clockwise

$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

Simplify

$$3i_1 - i_2 - 2i_3 = 1$$
$$-i_1 + 6i_2 - 3i_3 = 0$$
$$-2i_1 - 3i_2 + 6i_3 = 6$$

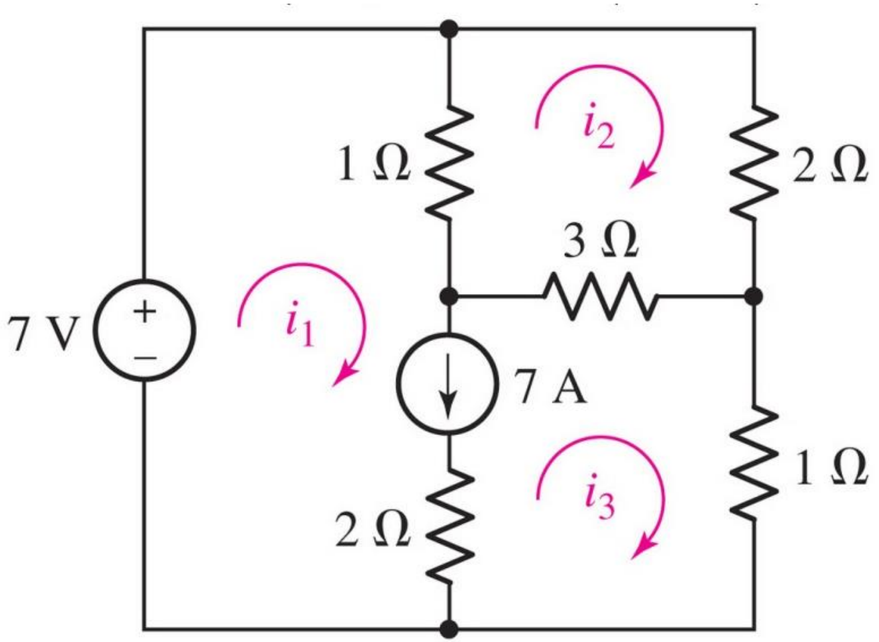


Solve the equations:

$i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

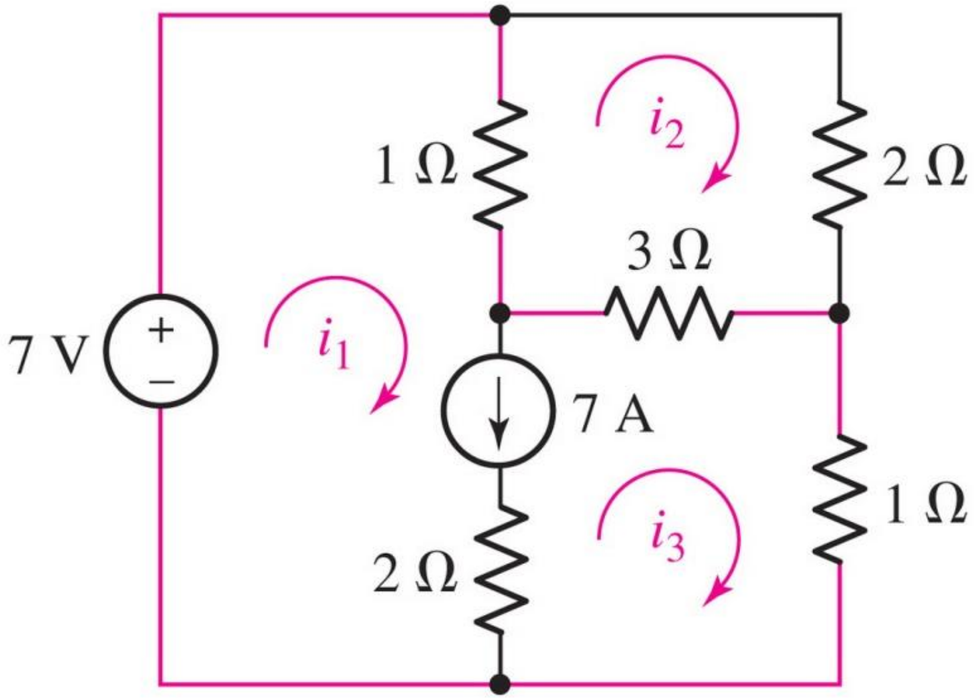
Current Sources and the Supermesh

What is the voltage across a current source in between two meshes?



We can eliminate the need for introducing a voltage variable by applying KVL to the *supermesh* formed by joining mesh 1 and mesh 3.

The Supermesh



Apply KVL to mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Apply KVL supermesh 1/3:

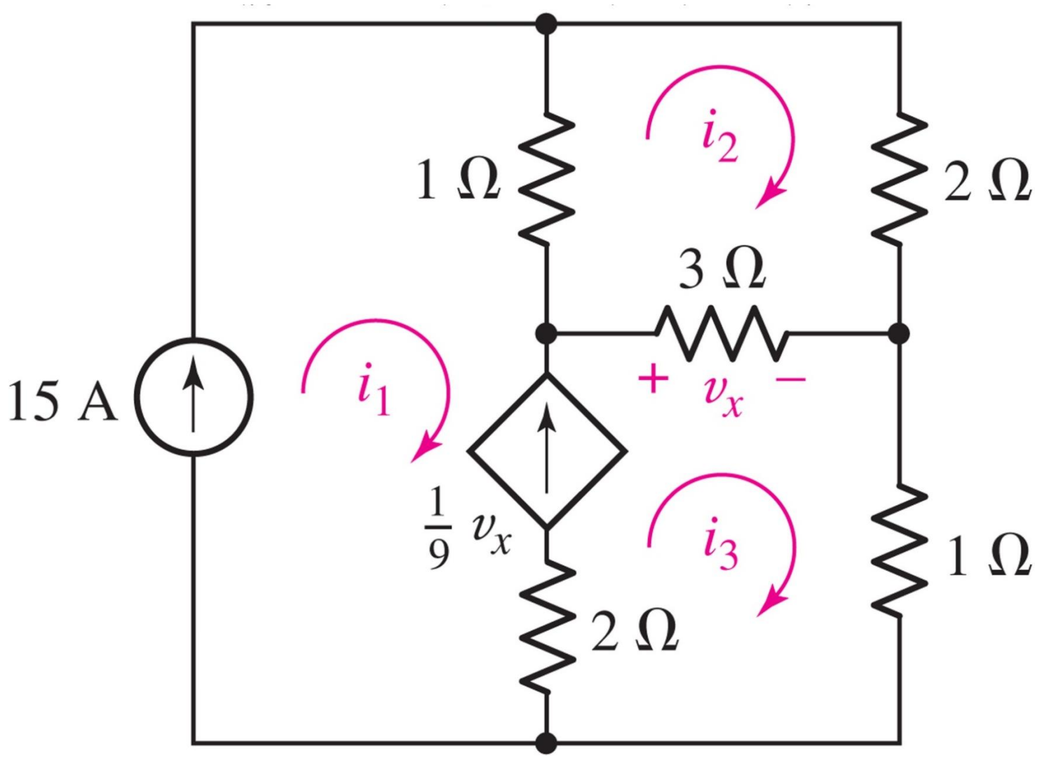
$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Add the current source:

$$7 = i_1 - i_3$$

Dependent Source Example

Find the currents.



Key step:

$$\frac{v_x}{9} = i_3 - i_1$$

Answer: $i_1 = 15\ A$, $i_2 = 11\ A$, *and* $i_3 = 17\ A$

Node or Mesh: How to Choose?

- use the one with fewer equations, or
- use the method you like best, or
- use both (as a check), or

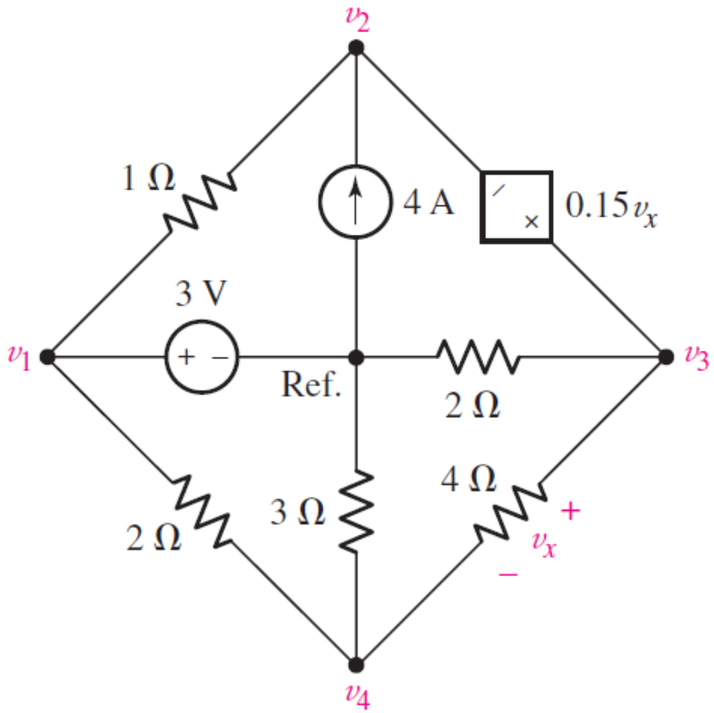
Steps in Mesh Analysis

1. Identify all of the meshes in the circuit
2. Label the currents flowing in each mesh
3. Label the voltage across each component in the circuit
4. Use Kirchoff’s Voltage Law
5. Use Ohm’s Law to relate the voltage drops across each component to the sum of the currents flowing through them.
6. Solve for the mesh currents
7. Once the voltage across all of the components are known, calculate the mesh currents.

Practice 1

Determine v_x with node analysis

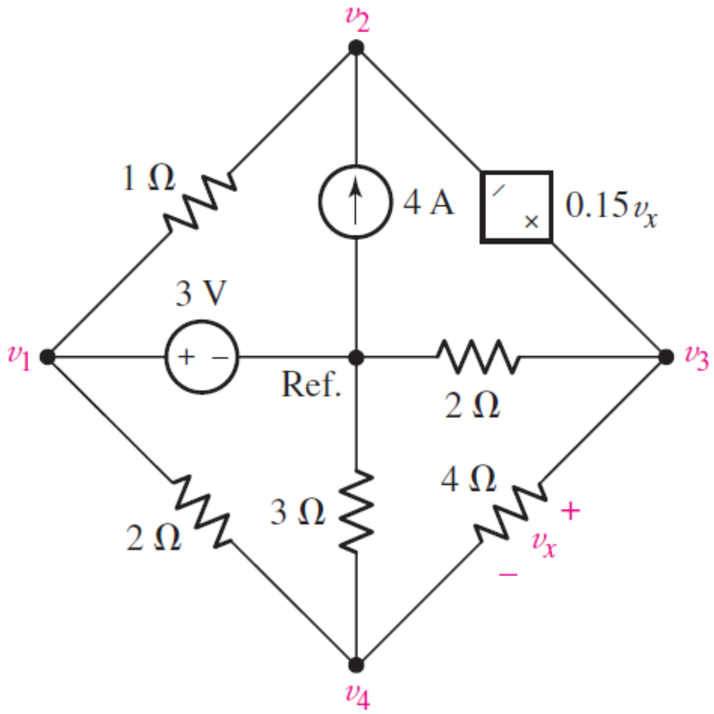
- identify the nodes & supernodes
- write KCL at each node (except the reference)



Practice 2

Determine v_x with mesh analysis

- identify the mesh & supermeshs
- write KVL at each mesh



Thanks