دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 2 :Voltage and Current Laws

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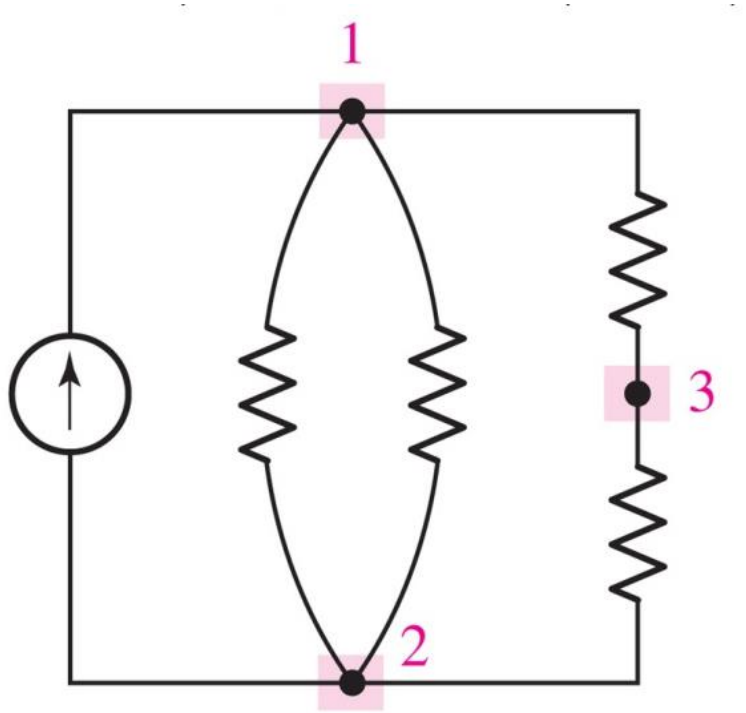
Objectives of the Lecture

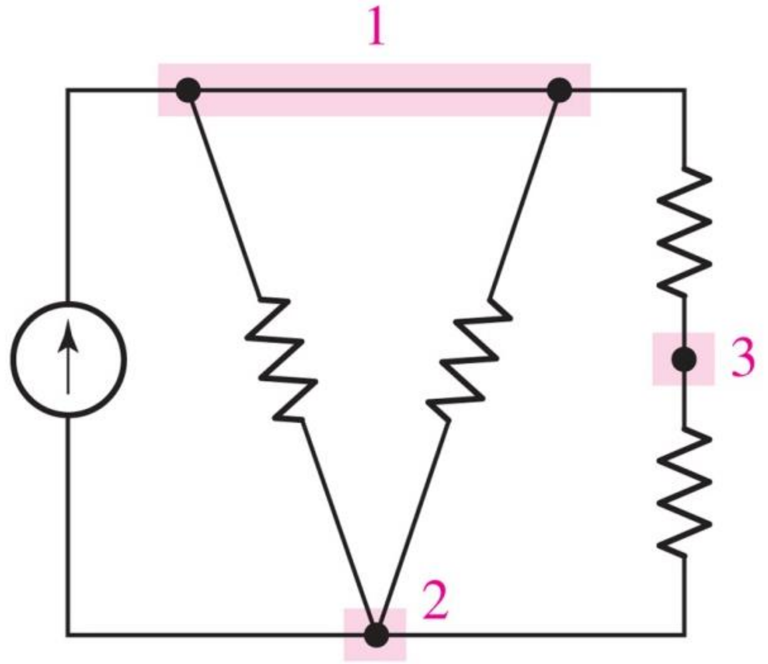
- Present Kirchhoff's Current and Voltage Laws.
- Demonstrate how these laws can be used to find currents and voltages in a circuit.
- Explain how these laws can be used in conjunction with Ohm's Law.

Nodes, Paths, Loops, Branches

- these two networks are equivalent
- there are **three *nodes*** and **five *branches***

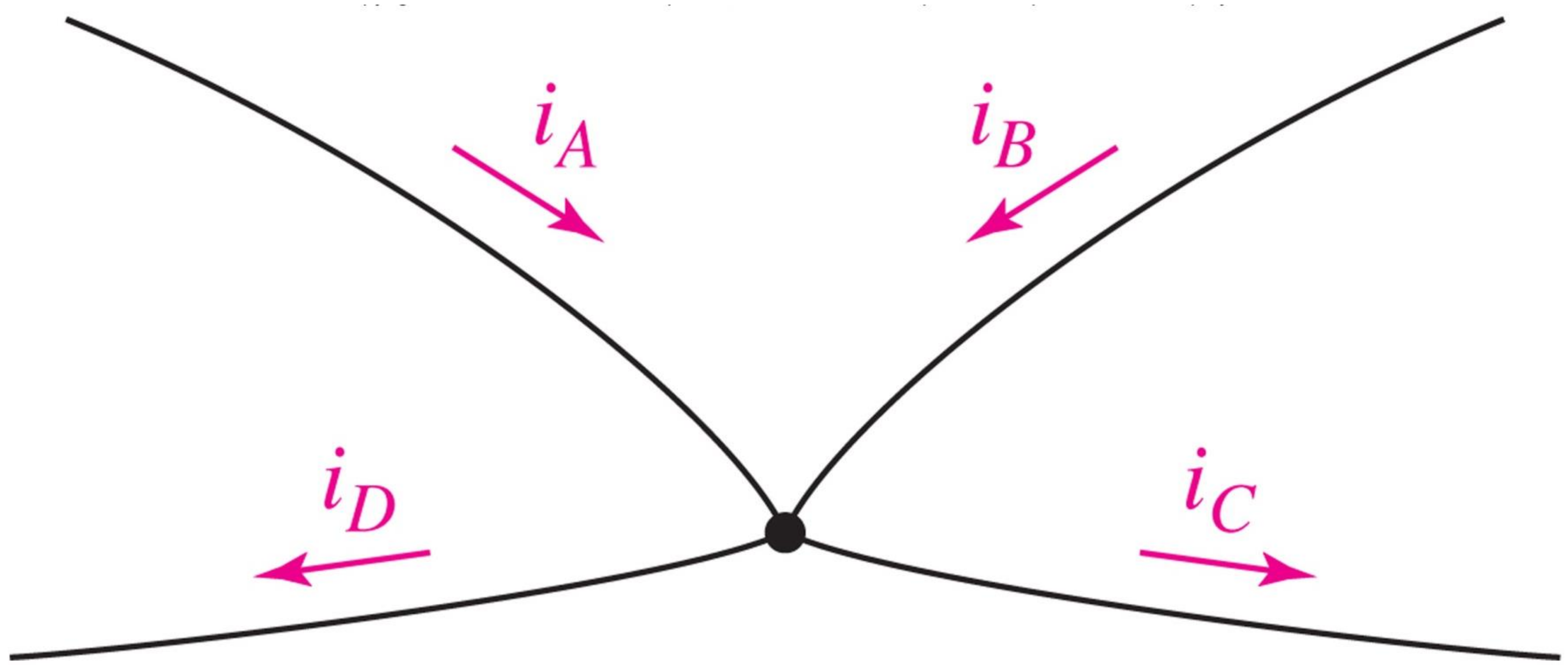


- **Node**
  - point at which 2+ elements have a common connection
    - e.g., node 1, node 2, node 3
- **Branch**
  - a single path in a network; contains one element and the nodes at the 2 ends
    - e.g., 1→2, 1→3, 3→2



- **Path**
  - a route through a network, through nodes that never repeat
    - e.g., 1→3→2, 1→2→3
- **Loop**
  - a path that starts & ends on the same node
    - e.g., 3→1→2→3

KCL: The algebraic sum of the currents entering any node is zero.



$$i_A + i_B + (-i_C) + (-i_D) = 0$$

KCL: Alternative Forms

- Current *IN* is zero:

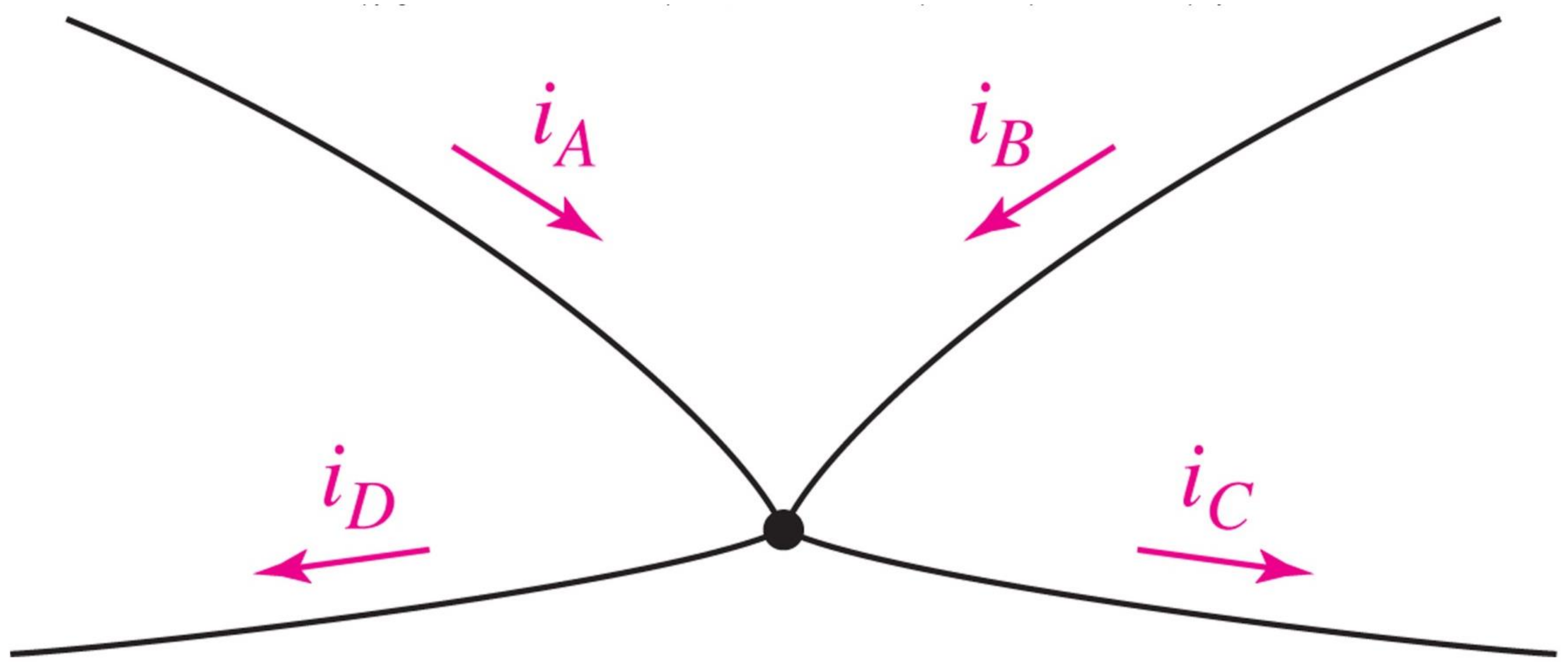
  $$i_A + i_B + (-i_C) + (-i_D) = 0$$

- Current *OUT* is zero:

  $$(-i_A) + (-i_B) + i_C + i_D = 0$$

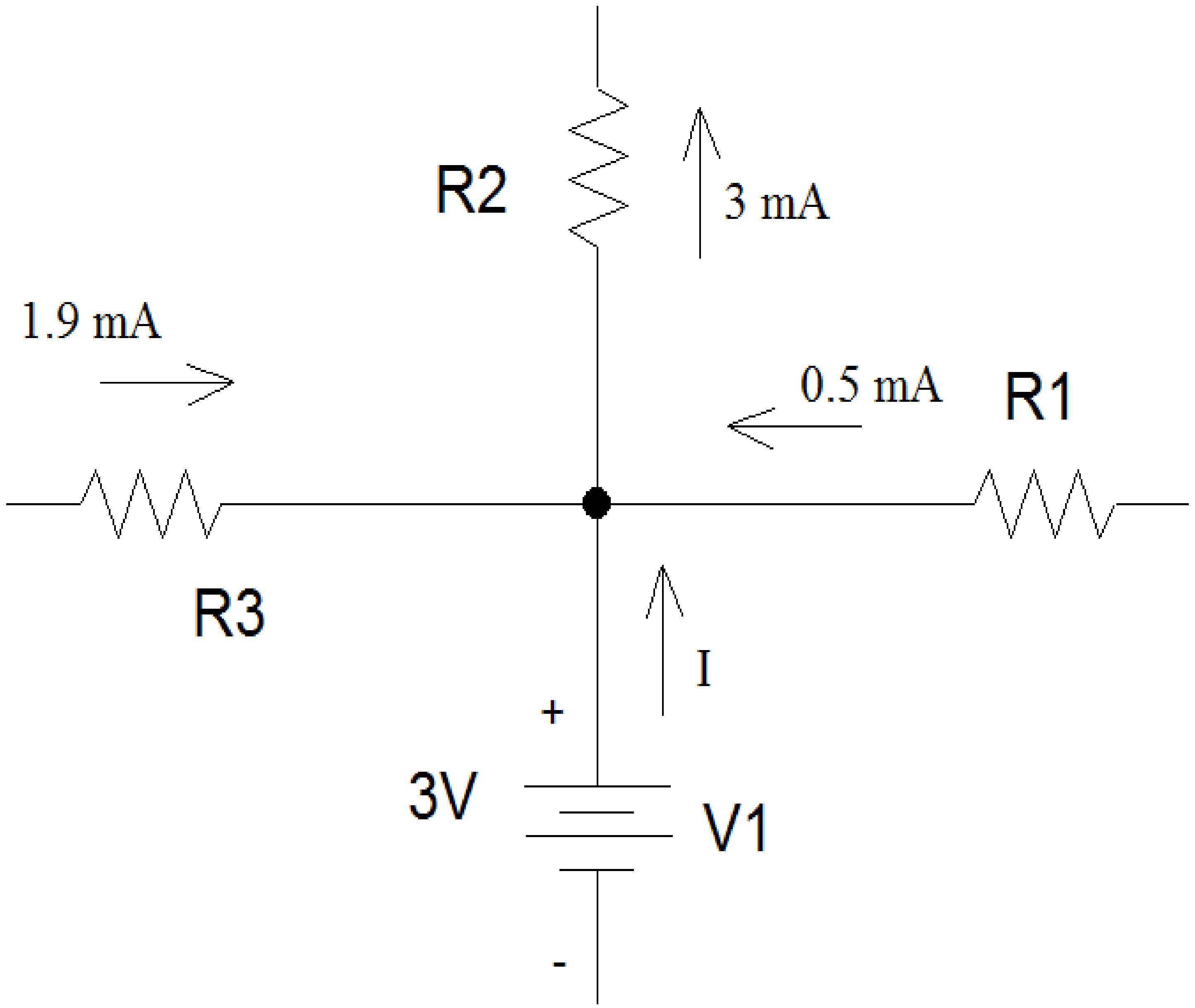
- Current *IN=OUT*:

  $$i_A + i_B = i_C + i_D$$



Example

- Determine I, the current flowing out of the voltage source.



Use KCL

1.9 mA + 0.5 mA + I are entering the node.

3 mA is leaving the node.

$$1.9mA + 0.5mA + I = 3mA$$

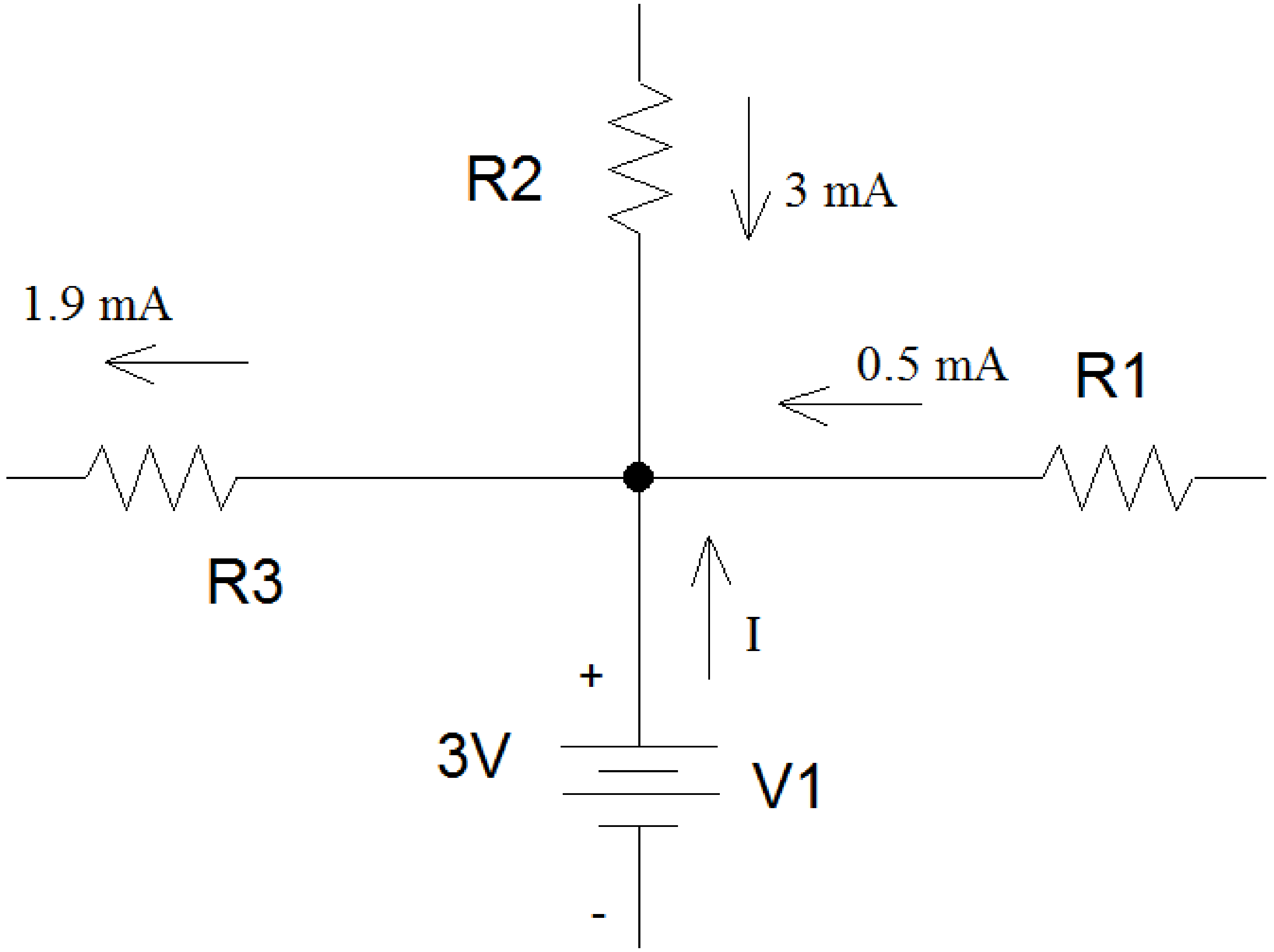
$$I = 3mA - (1.9mA + 0.5mA)$$

$$I = 0.6mA$$

V1 is suppling power.

Example

- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.



Use KCL

3 mA + 0.5 mA + I are entering the node.

1.9 mA is leaving the node.

$$3mA + 0.5mA + I = 1.9mA$$

$$I = 1.9mA - (3mA + 0.5mA)$$

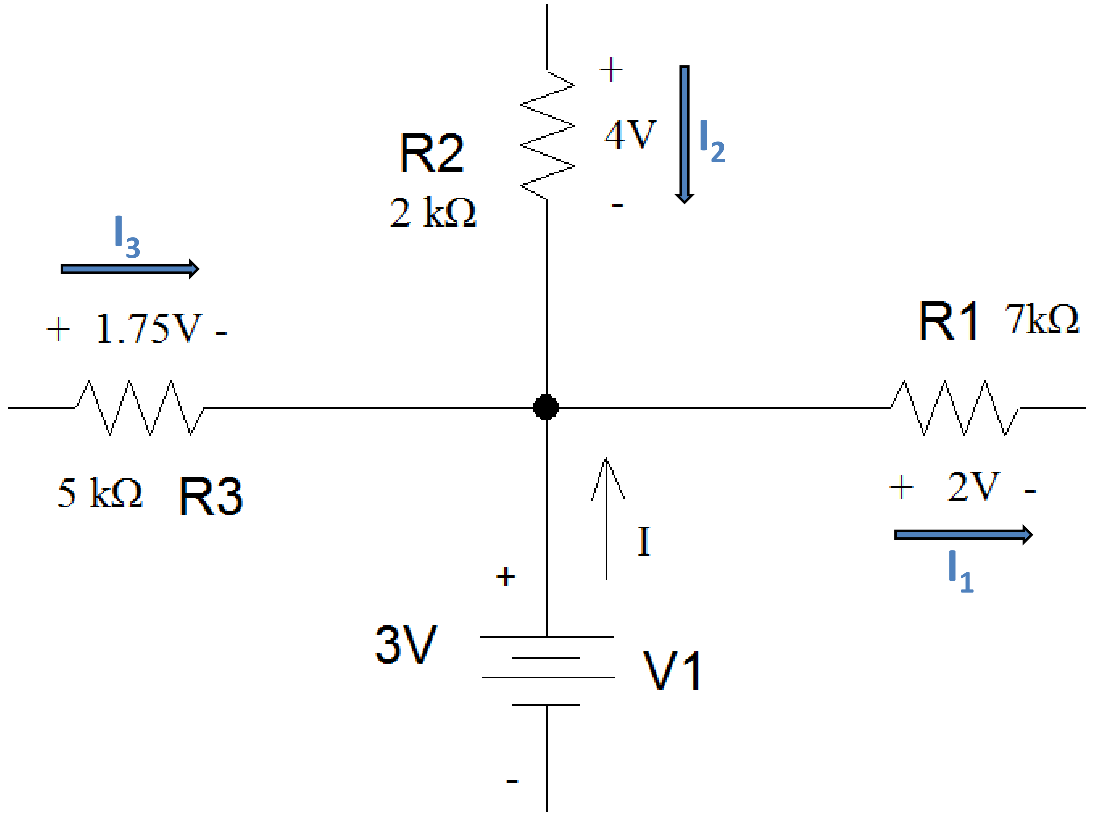
$$I = -1.6mA$$

V1 is absorbing power.

Example

- If voltage drops are given instead of currents,

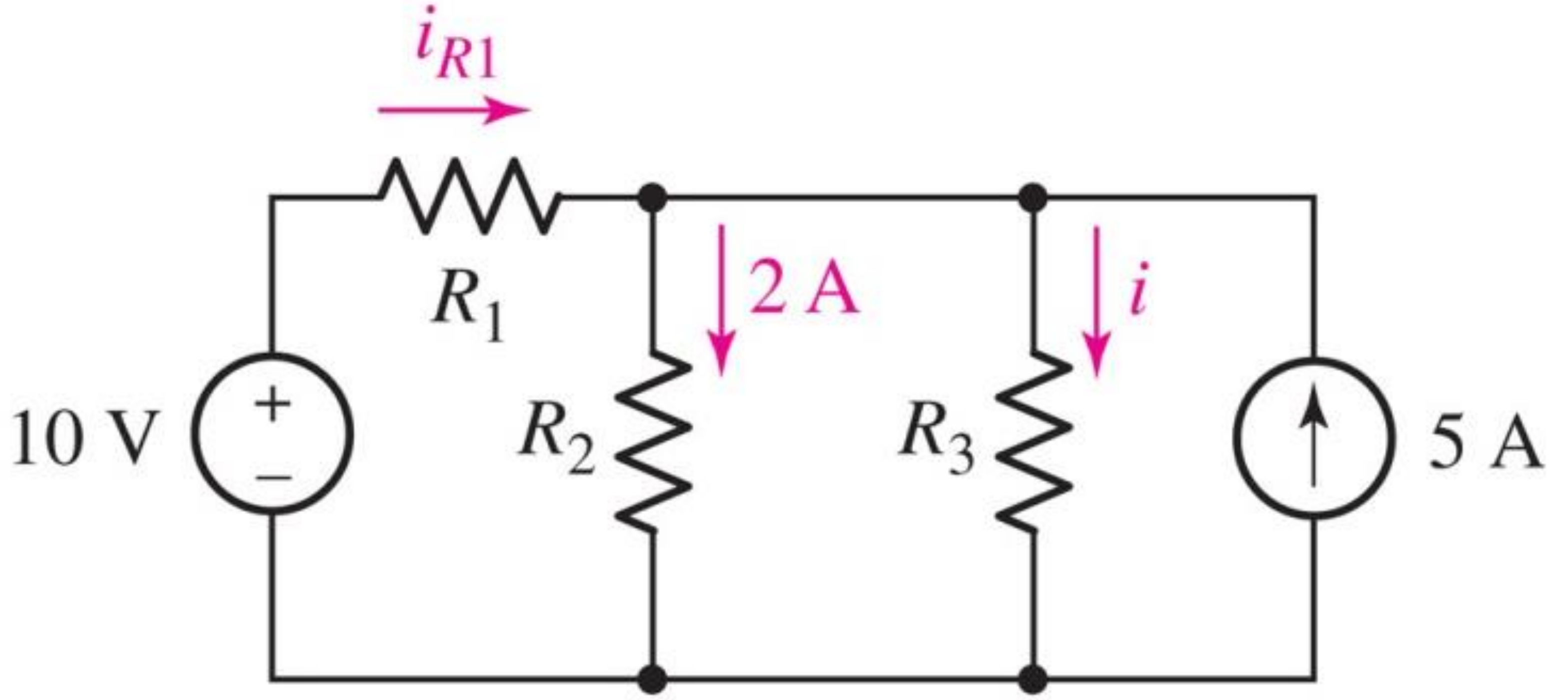
you need to apply **Ohm's Law** to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.



$$I_1 = 2V / 7k\Omega = 0.286\,mA$$
$$I_2 = 4V / 2k\Omega = 2mA$$
$$I_3 = 1.75V / 5k\Omega = 0.35mA$$

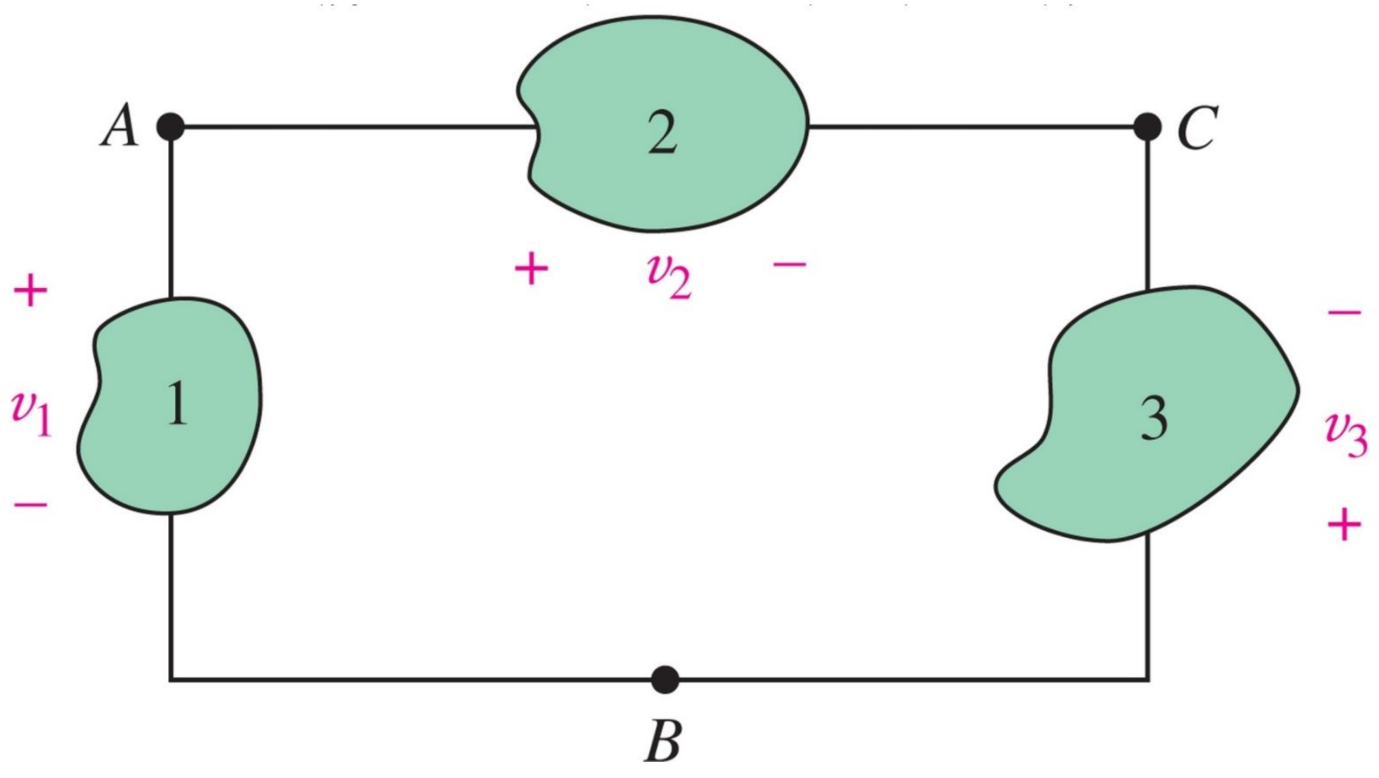
$$I_2 + I_3 + I = I_1$$
$$2mA + 0.35mA + I = 0.286\,mA$$
$$I = 0.286\,mA - 2.35mA = -2.06mA$$

- Find the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.



Answer: i =6 A

KVL: The algebraic sum of the voltages around any closed path is zero.



$$(-v_1) + v_2 + (-v_3) = 0$$

KVL: Alternative Forms

- Sum of ***DROPS*** is zero (clockwise from B):

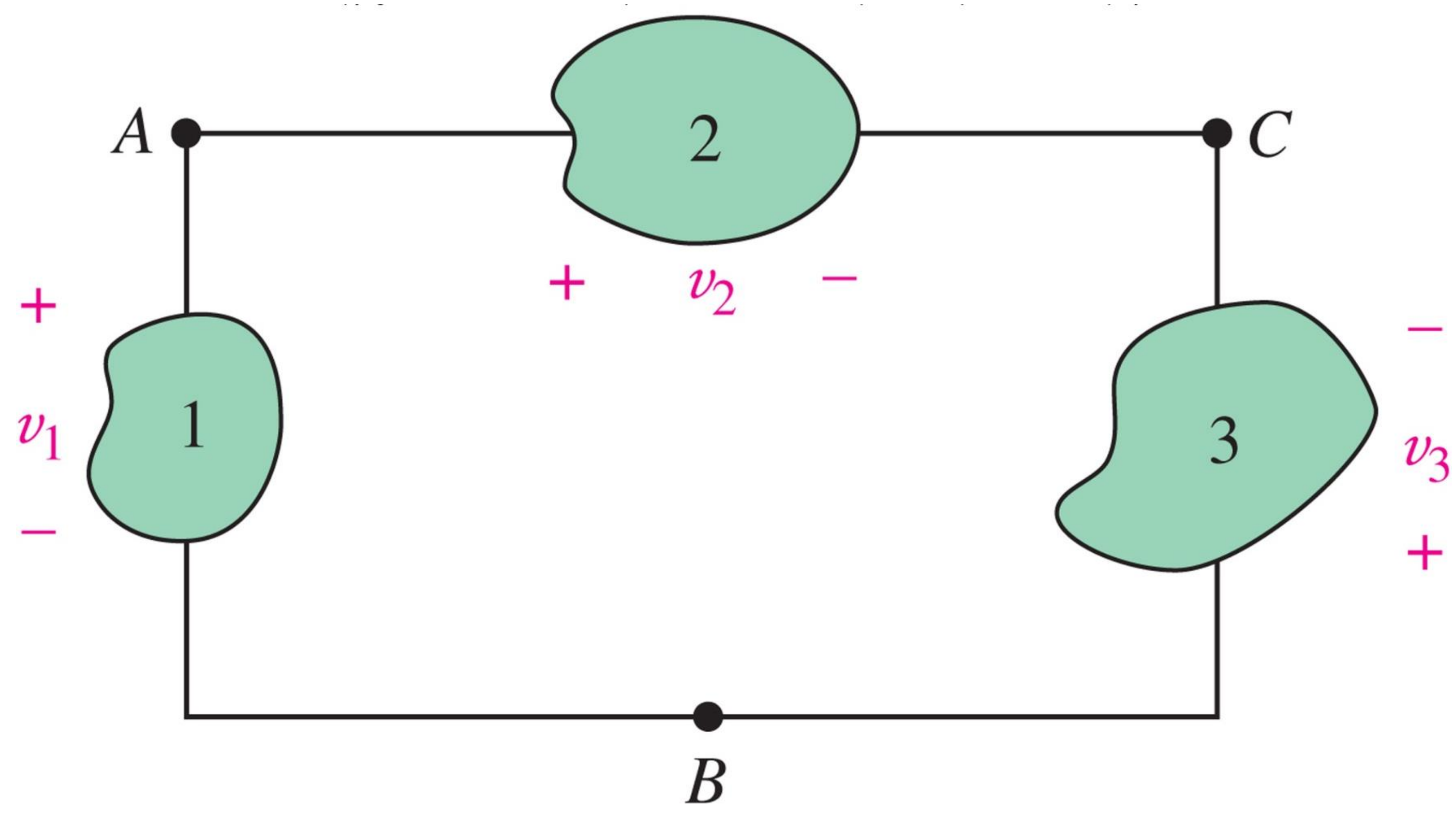
$$(-v_1) + v_2 + (-v_3) = 0$$

- Sum of ***RISES*** is zero (clockwise from B):

$$v_1 + (-v_2) + v_3 = 0$$

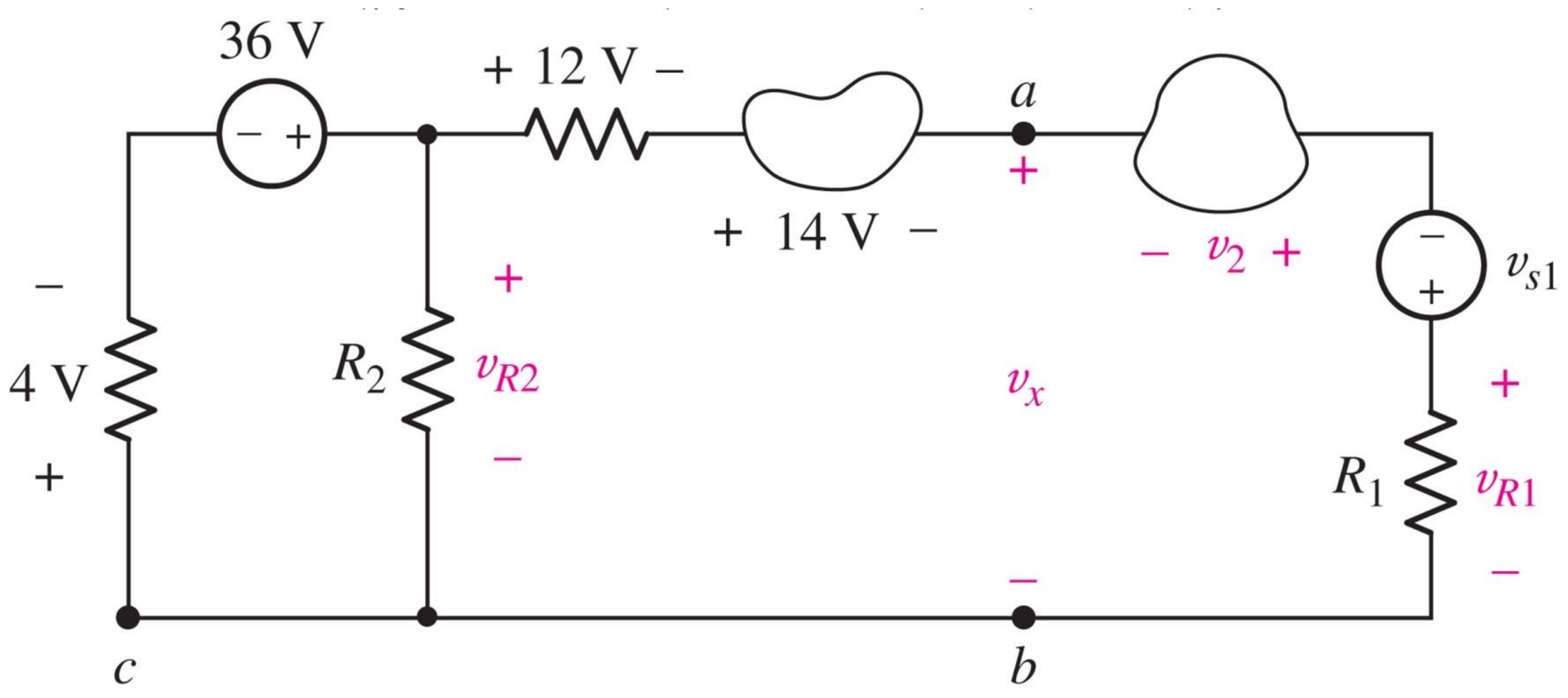
- Two paths, same voltage (A to B):

$$v_1 = v_2 + (-v_3)$$



Example: Applying KVL

- Find v_{R2} (the voltage across R_2) and the voltage v_x.



$$4 - 36 + v_{R2} = 0$$

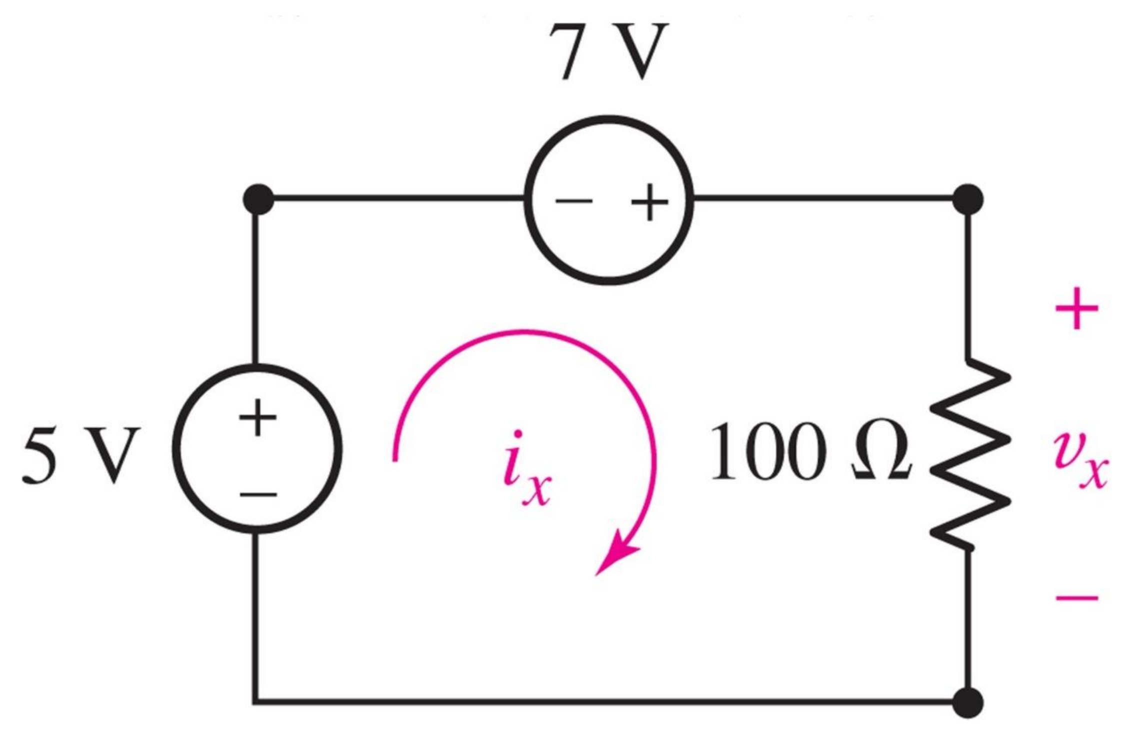
Answer: $v_{R2} = 32$

$$+4 - 36 + 12 + 14 + v_x = 0$$

Answer: $v_x = 6$ V.

$$-32 + 12 + 14 + v_x = 0$$

Example: find the current i_x and the voltage v_x



Answer: $v_x = 12$ *V and* $i_x = 120$ *mA*

3.1 Count the number of branches and nodes in the circuit in Fig. 3.4. If $i_x = 3$ A and the 18 V source delivers 8 A of current, what is the value of R_A? (*Hint:* You need Ohm's law as well as KCL.)

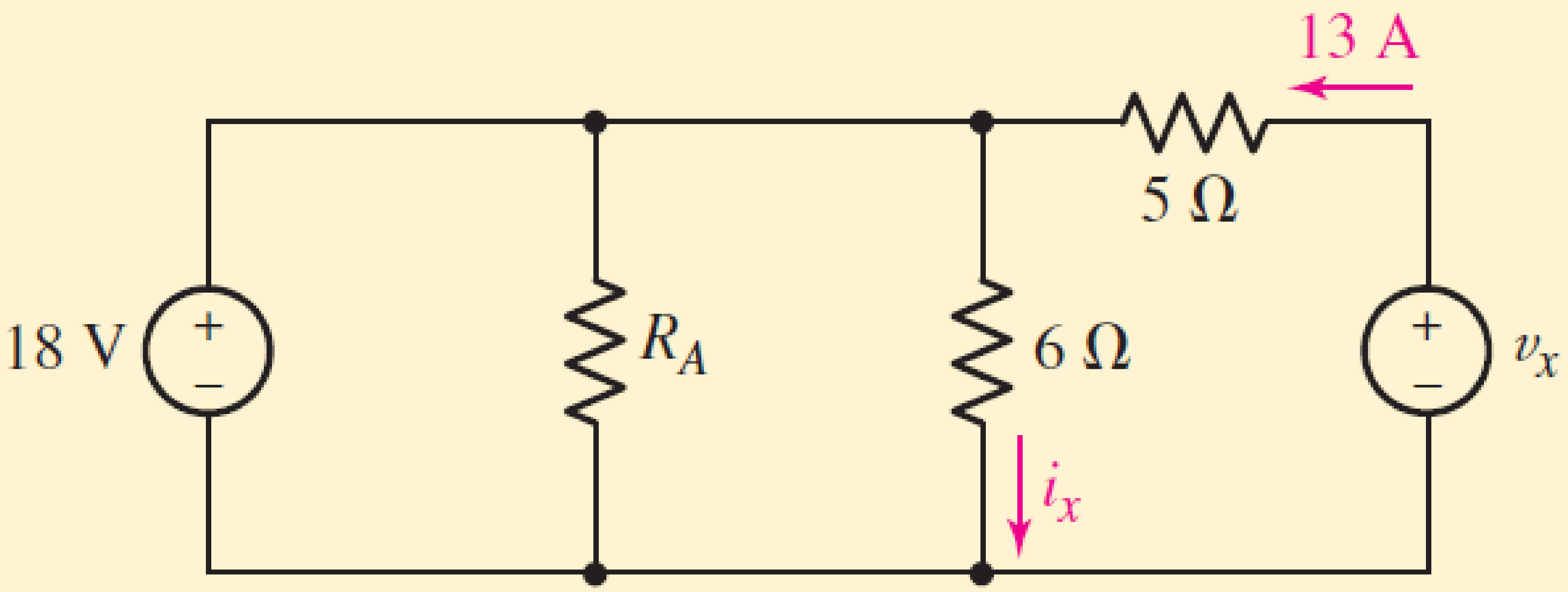
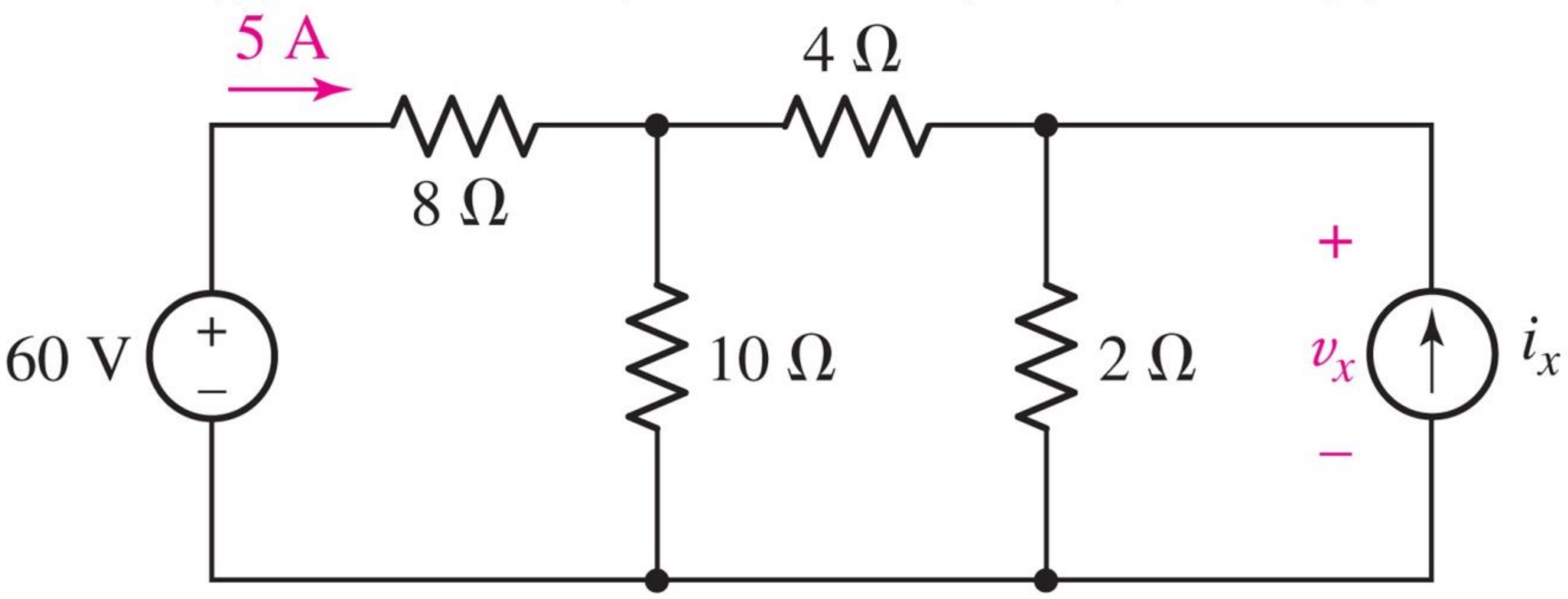
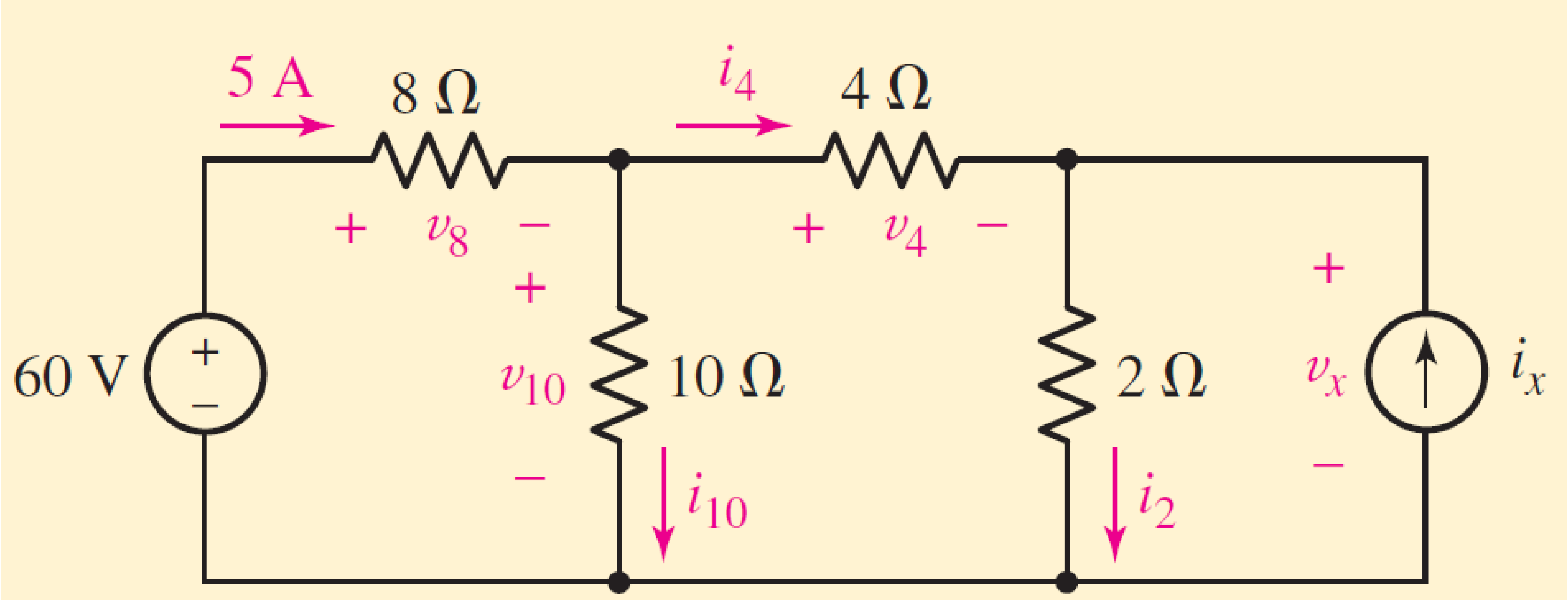


FIGURE 3.4

Ans: 5 branches, 3 nodes, 1Ω.

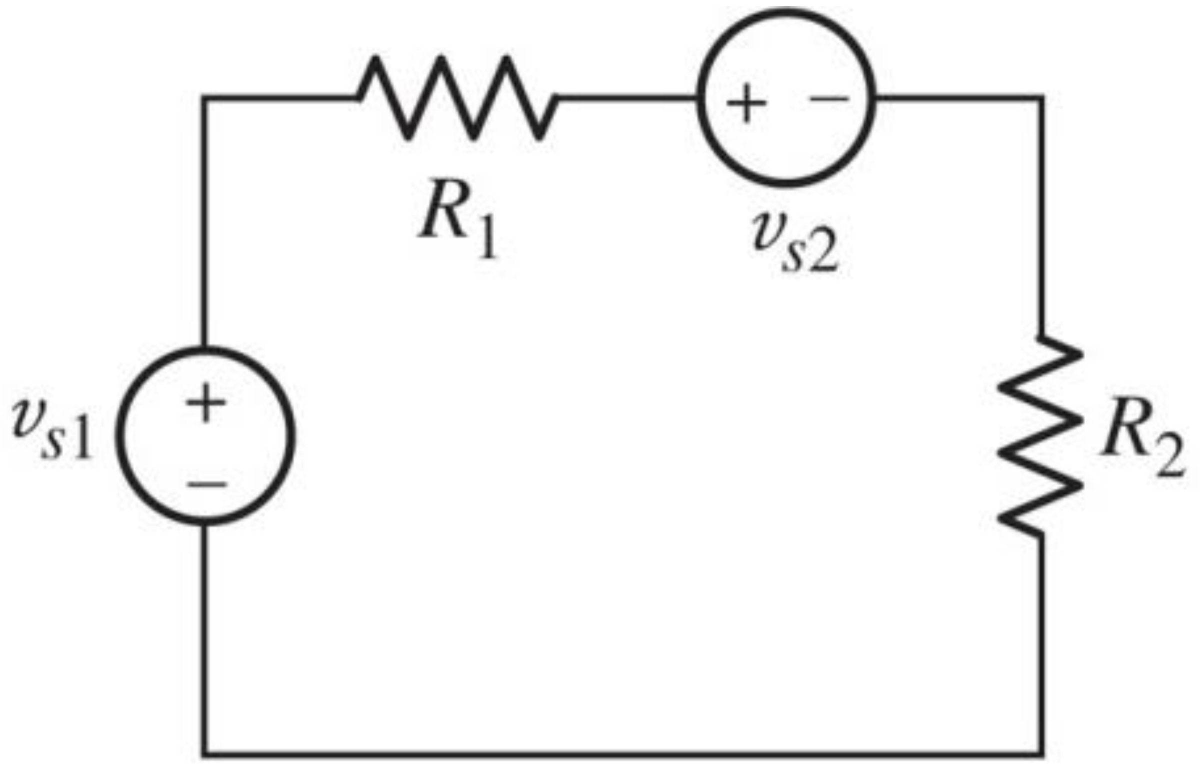
Solve for the voltage v_x and and the current i_x

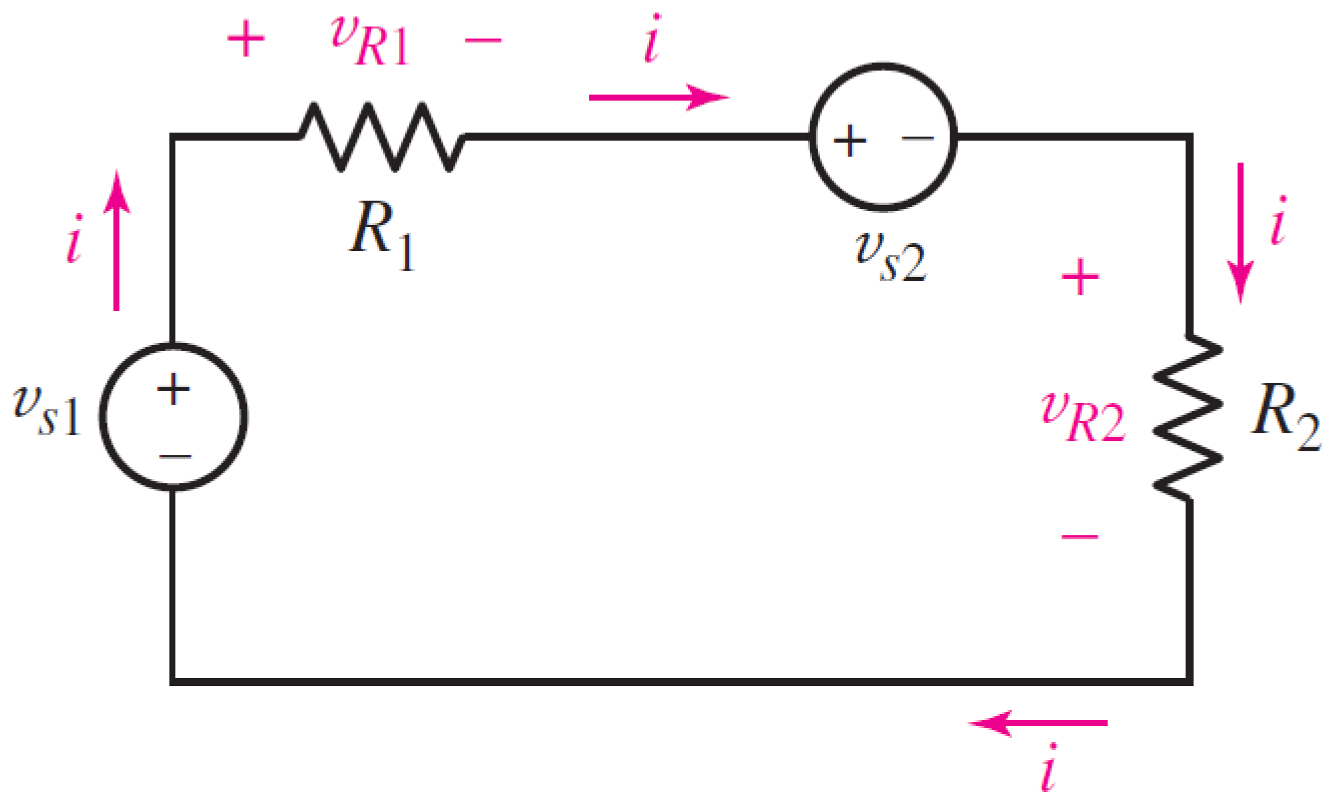




Series Connections

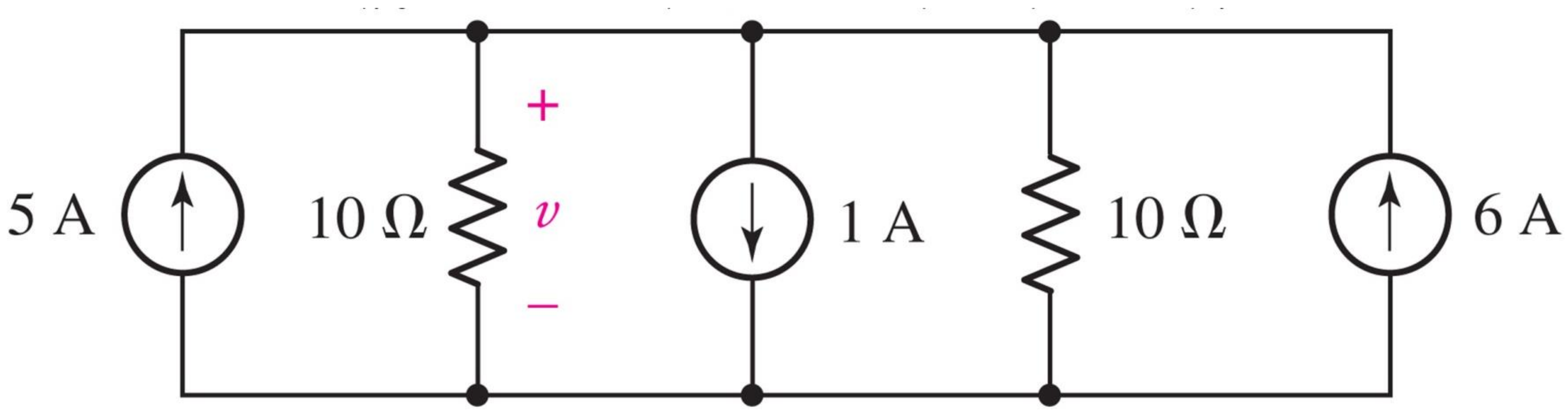
❖ All of the elements in a circuit that carry **the same current** are said to be connected in **series**.





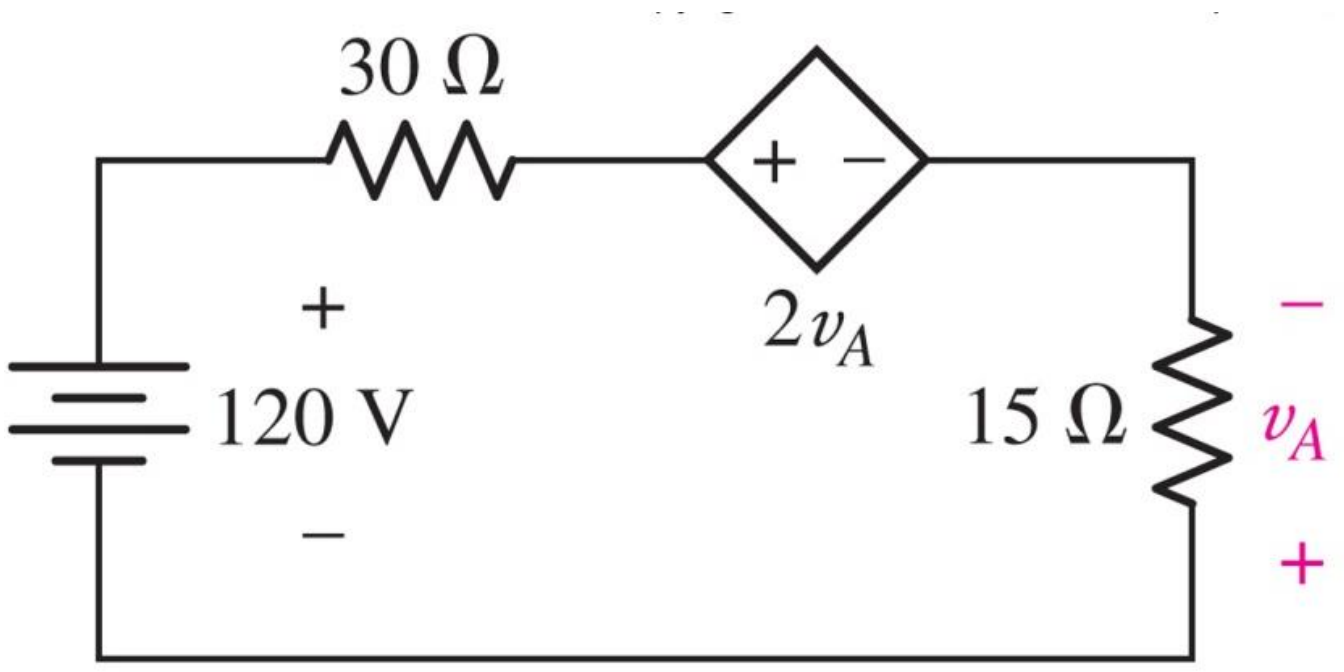
Parallel Connections

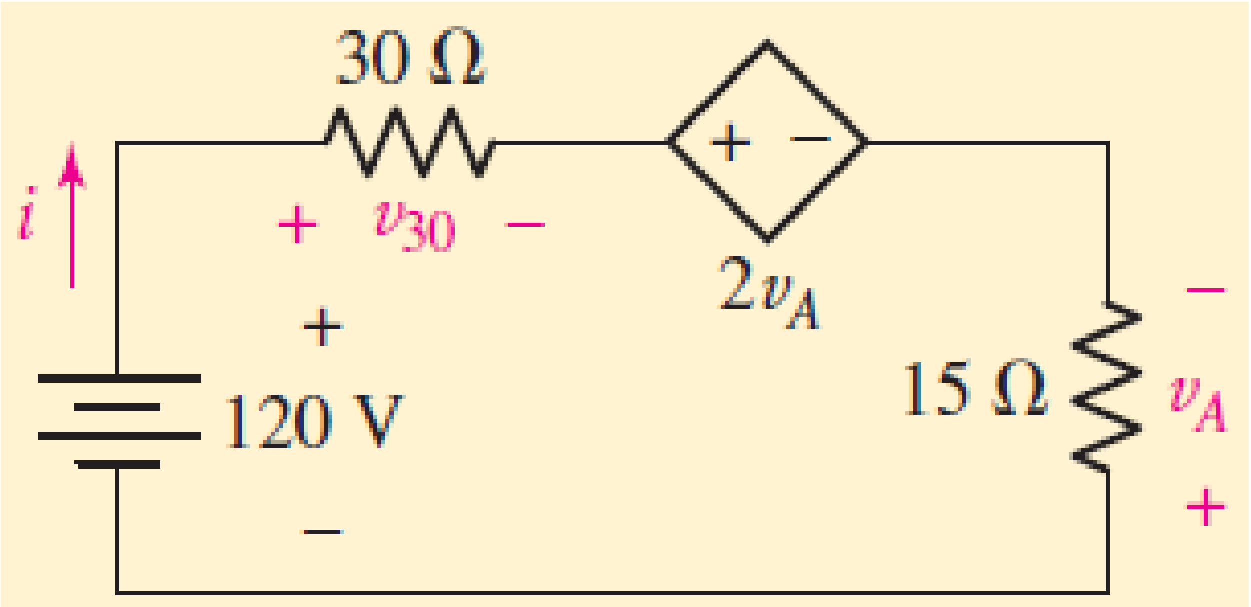
❖Elements in a circuit having a common voltage across them are said to be connected in *parallel*.



Example: Single Loop Circuit

- Calculate the power absorbed by each circuit element.





$$-120 + v_{30} + 2v_A - v_A = 0$$

$$v_{30} = 30i \quad \text{and} \quad v_A = -15i$$

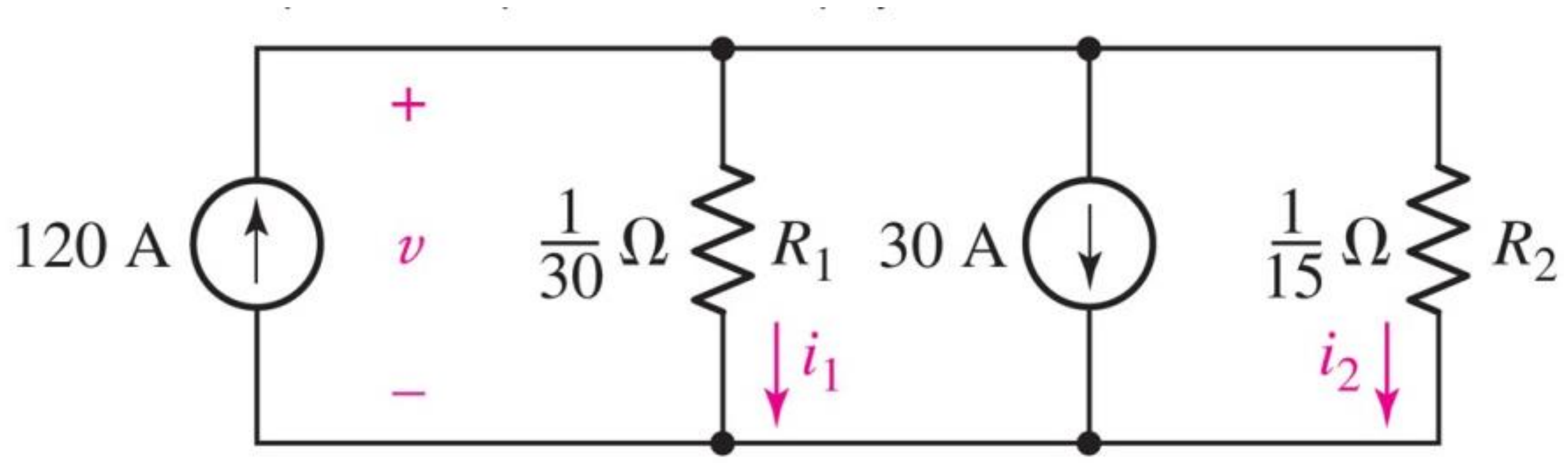
$$-120 + 30i - 30i + 15i = 0$$

$$i = 8 \text{ A}$$

$$\sum p_{\text{absorbed}} = \sum p_{\text{supplied}}$$

Example: Single Node-Pair Circuit

Find the voltage v and the currents i_1 and i_2.



$$-120 + i_1 + 30 + i_2 = 0$$

$$i_1 = 30v \quad \text{and} \quad i_2 = 15v$$

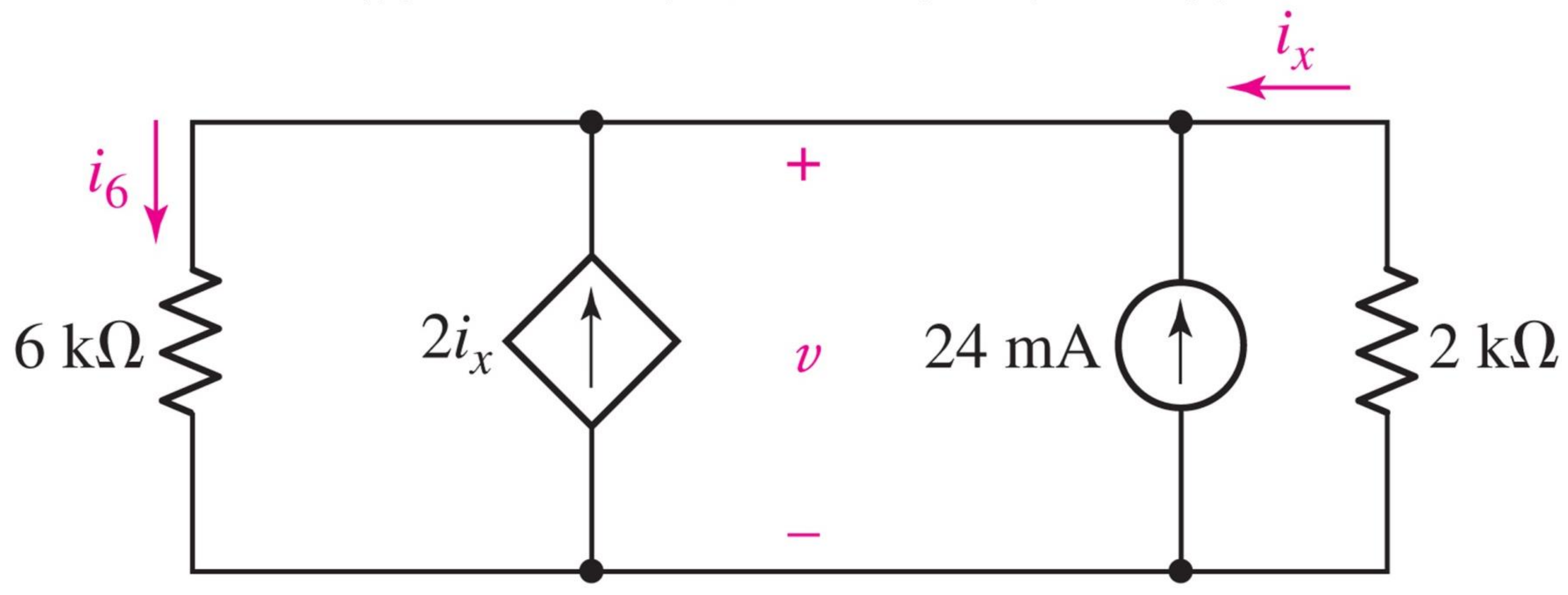
$$-120 + 30v + 30 + 15v = 0$$

$$v = 2\ \text{V}$$

$$i_1 = 60\ \text{A} \quad \text{and} \quad i_2 = 30\ \text{A}$$

Example: Single Node-Pair Circuit

Determine the value of v and the power supplied by the independent current source.



$$i_6 - 2i_x - 0.024 - i_x = 0$$

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

and so $v = (600)(0.024) = 14.4$ V.

$$p_{24} = 14.4(0.024) = 0.3456 \text{ W } (345.6 \text{ mW})$$

Example: Single Node-Pair Circuit

3.8 For the single-node-pair circuit of Fig. 3.18, find i_A, i_B, and i_C.

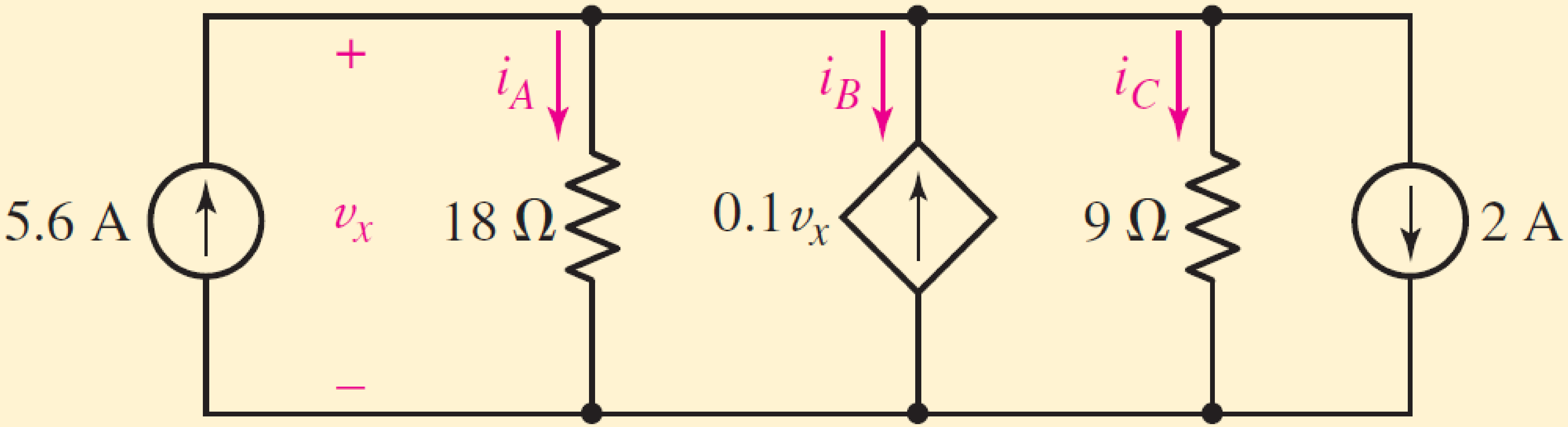
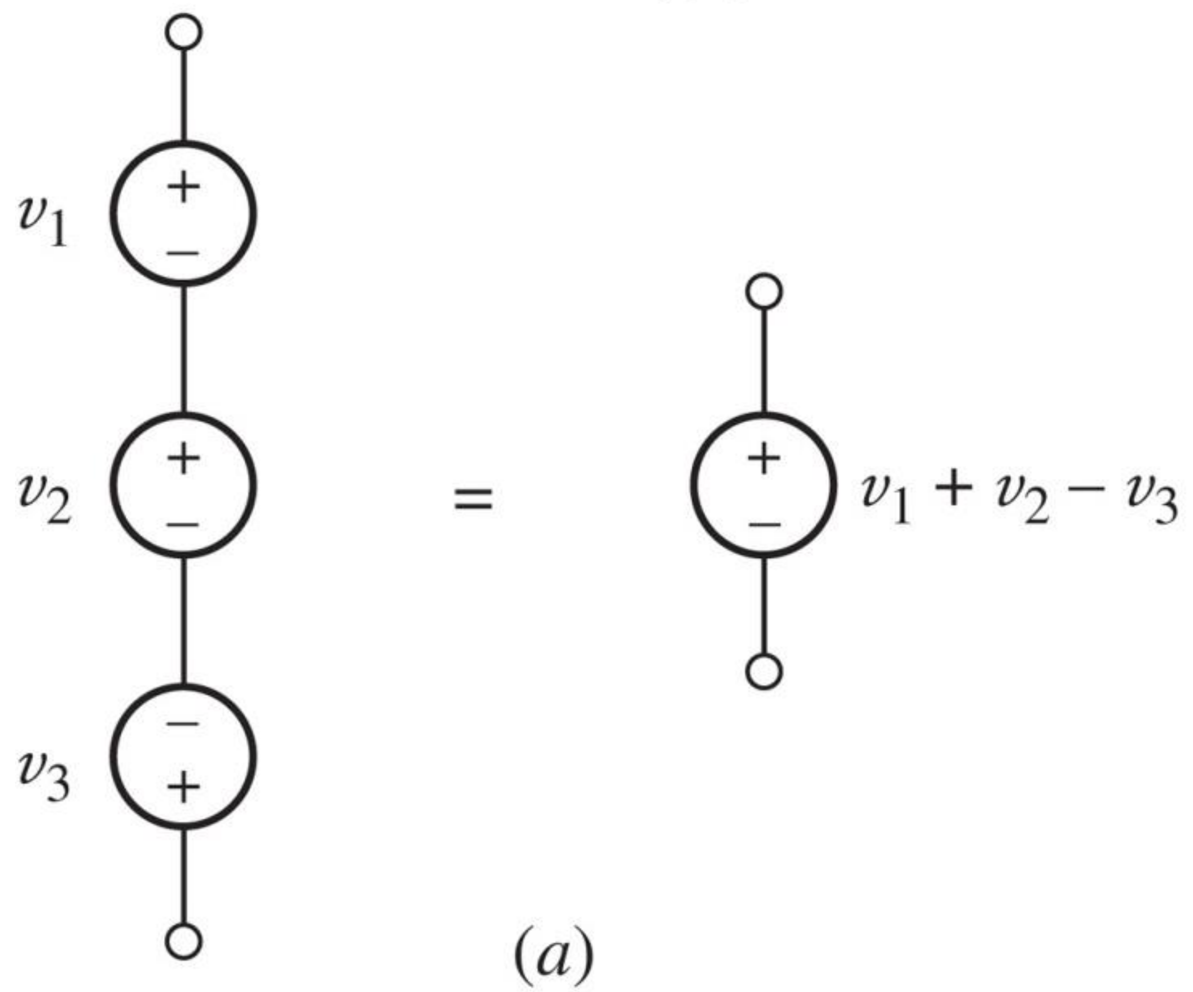


FIGURE 3.18

Ans: 3 A; −5.4 A; 6 A.

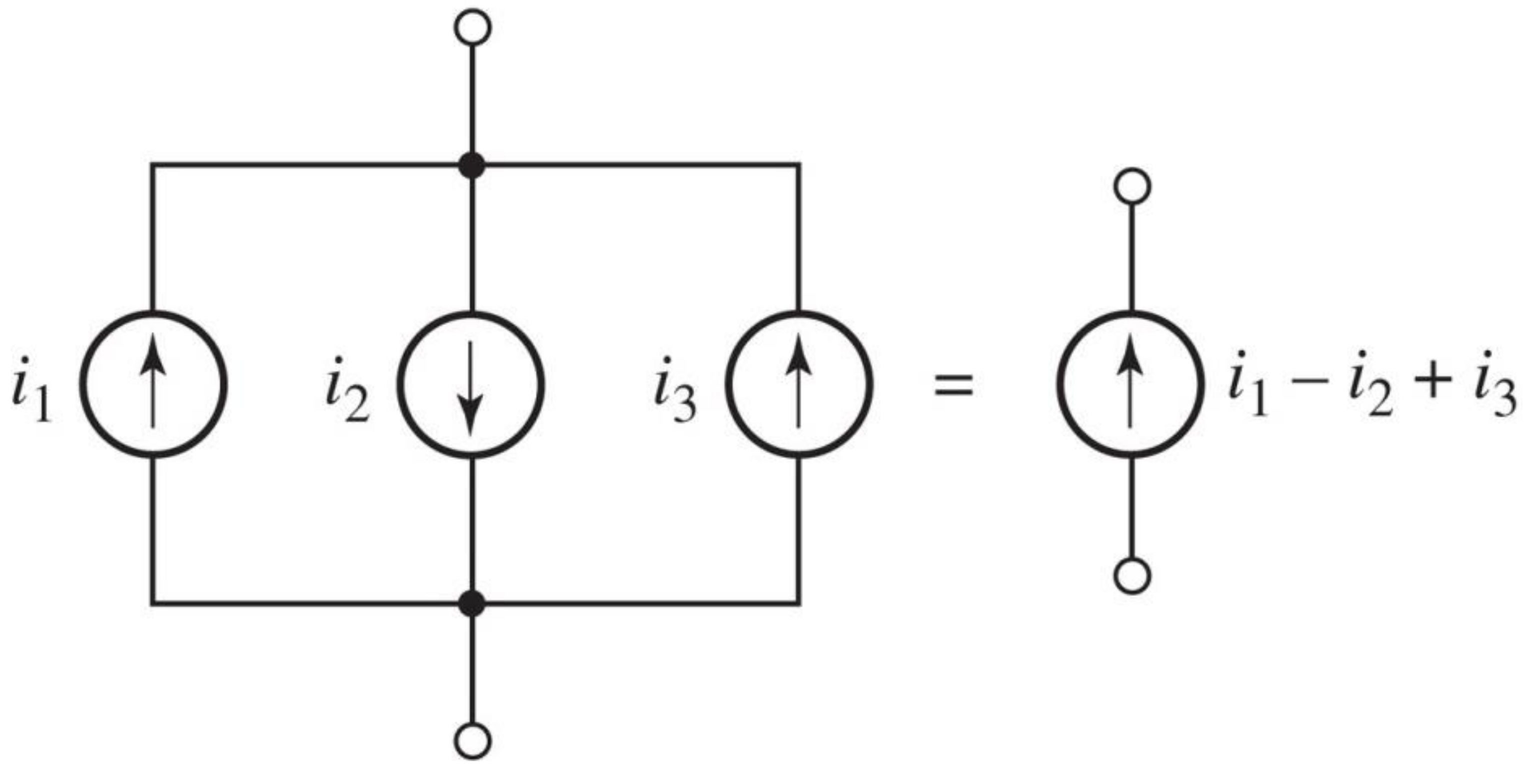
Series and Parallel Sources

Voltage sources connected in series can be combined into an equivalent voltage source:



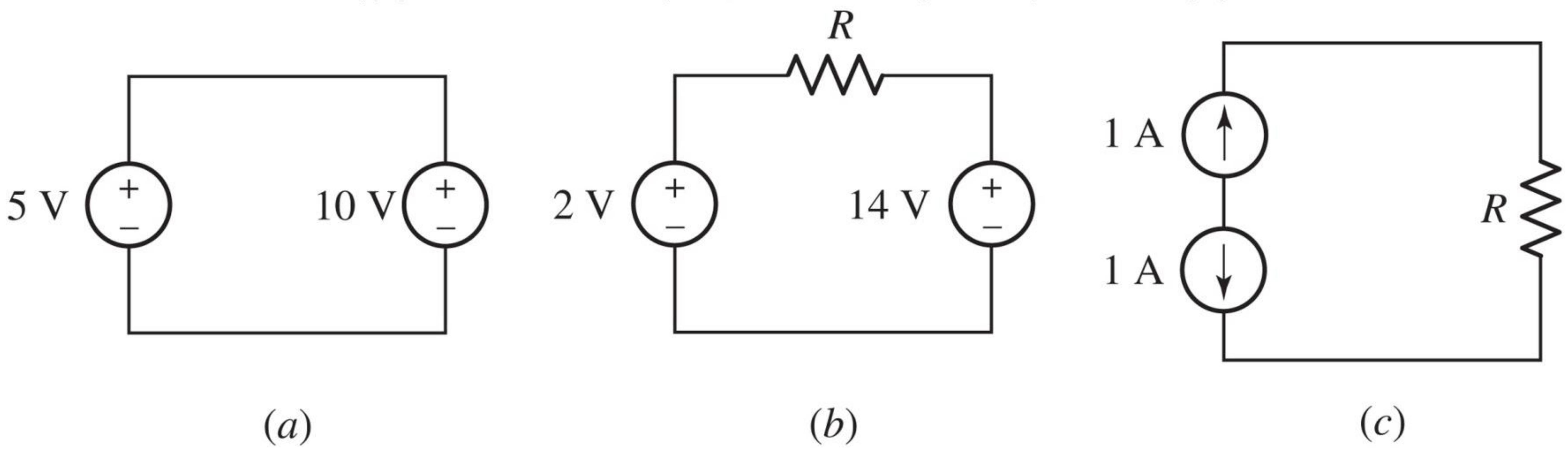
(a)

Current sources connected in parallel can be combined into an equivalent current source:



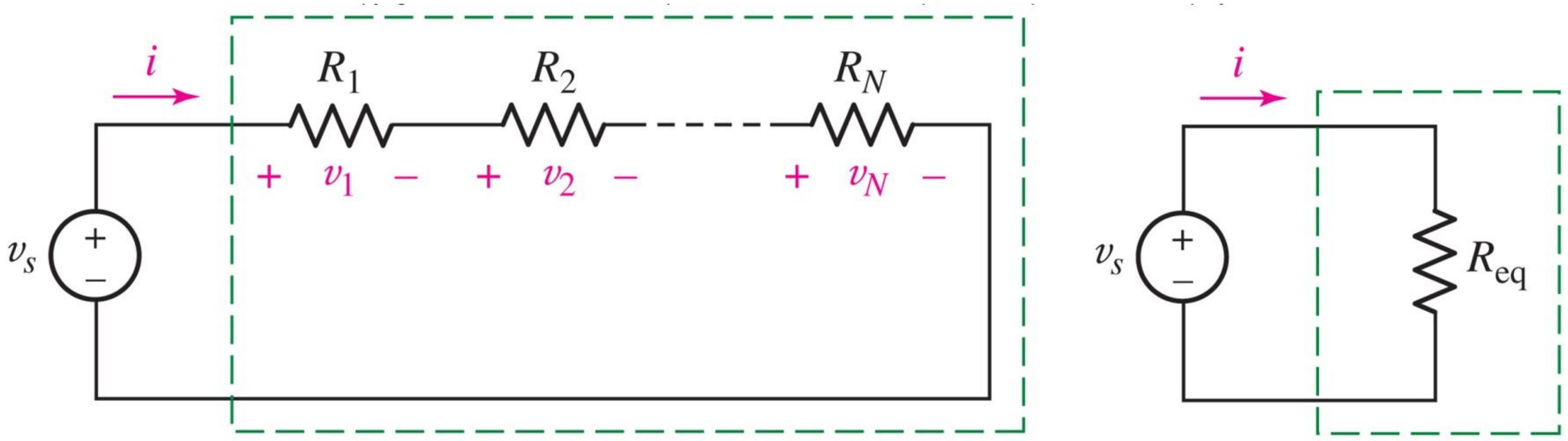
Impossible Circuits

- Our circuit models are idealizations that can lead to apparent physical absurdities:



- V_s in parallel (a) and I_s in series (c) can lead to “impossible circuits”

Resistors in Series

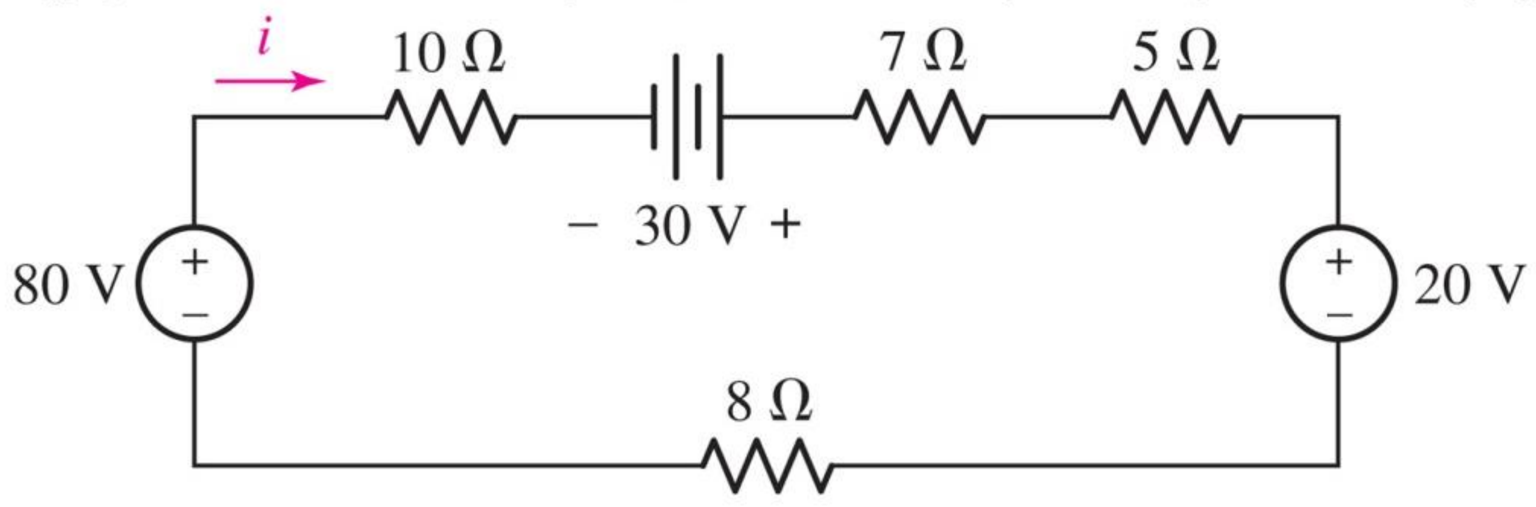


Using KVL shows:

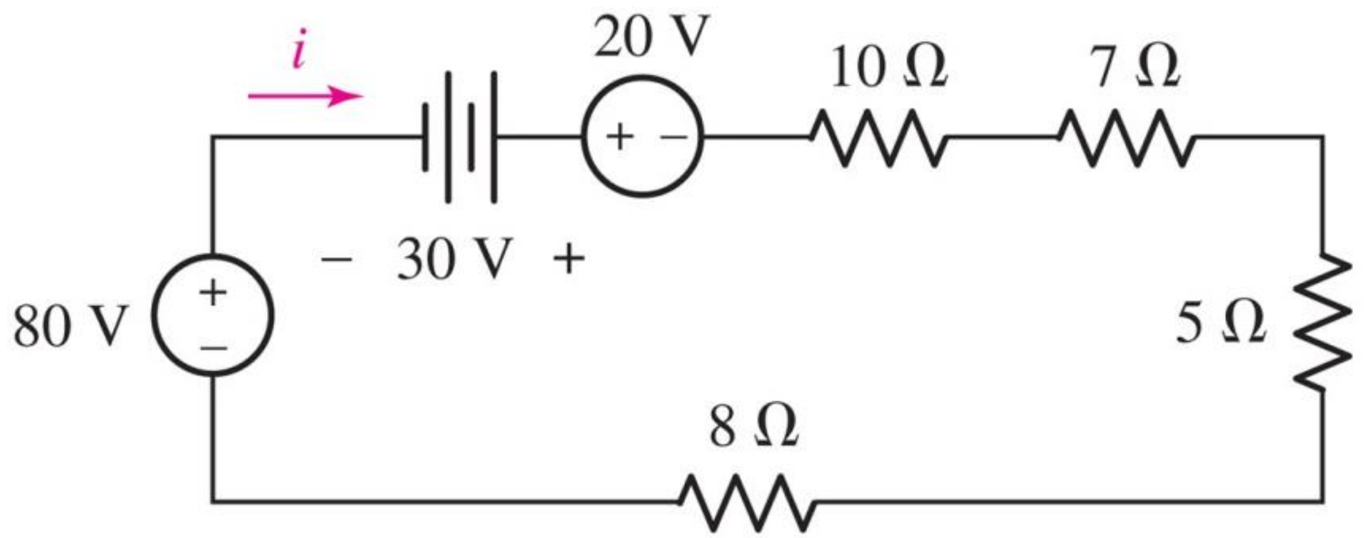
$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

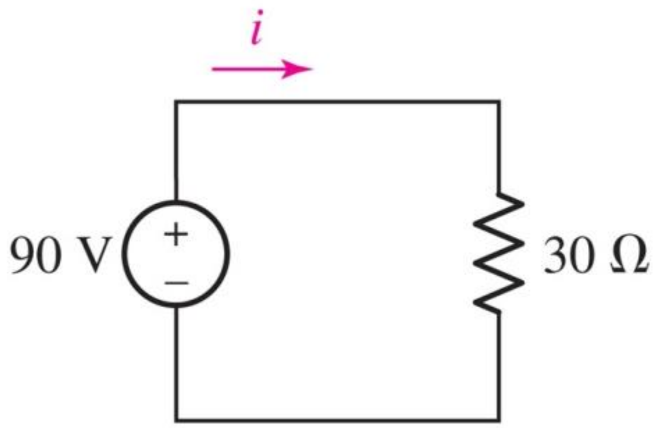
Example: Circuit Simplifying

Find *i* and the power supplied by the 80 V source.

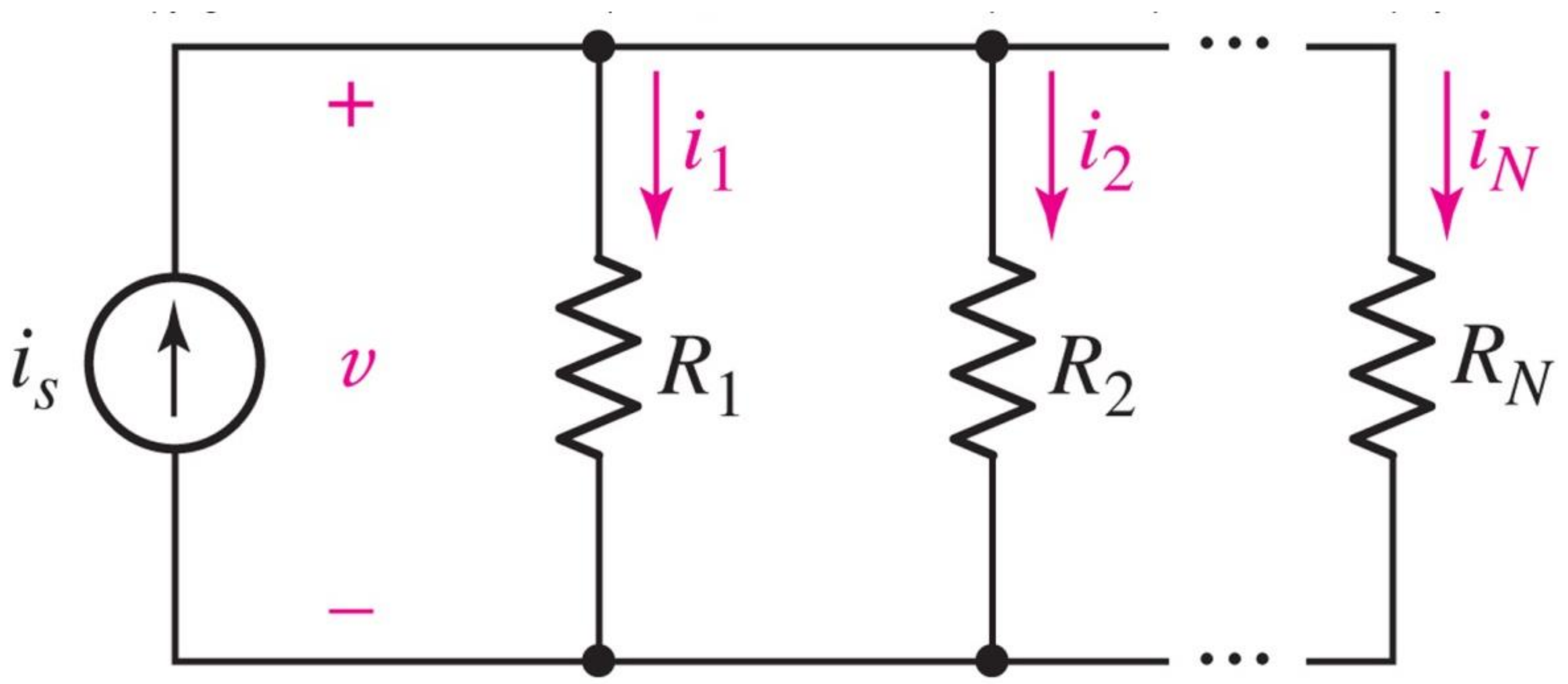


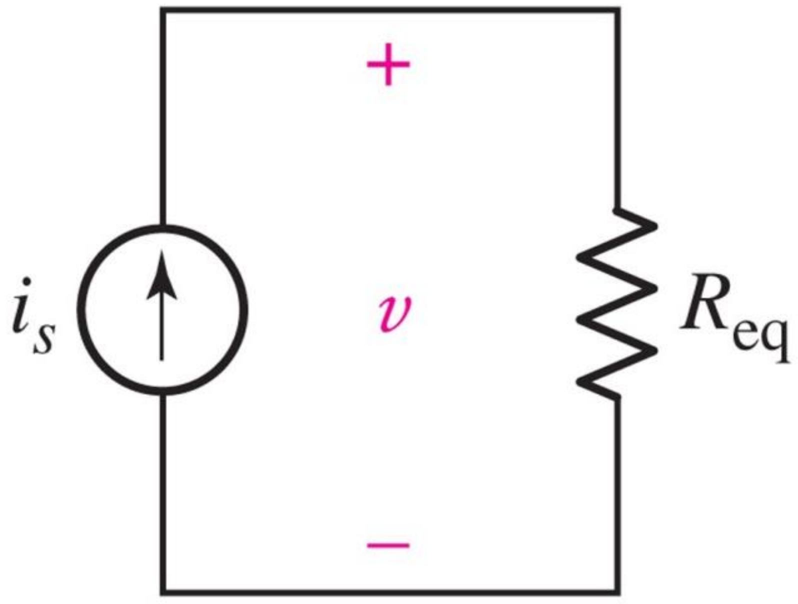
Answer: i = 3 A and p = 240 W supplied





Resistors in Parallel





Using KCL shows:

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Two Resistors in Parallel

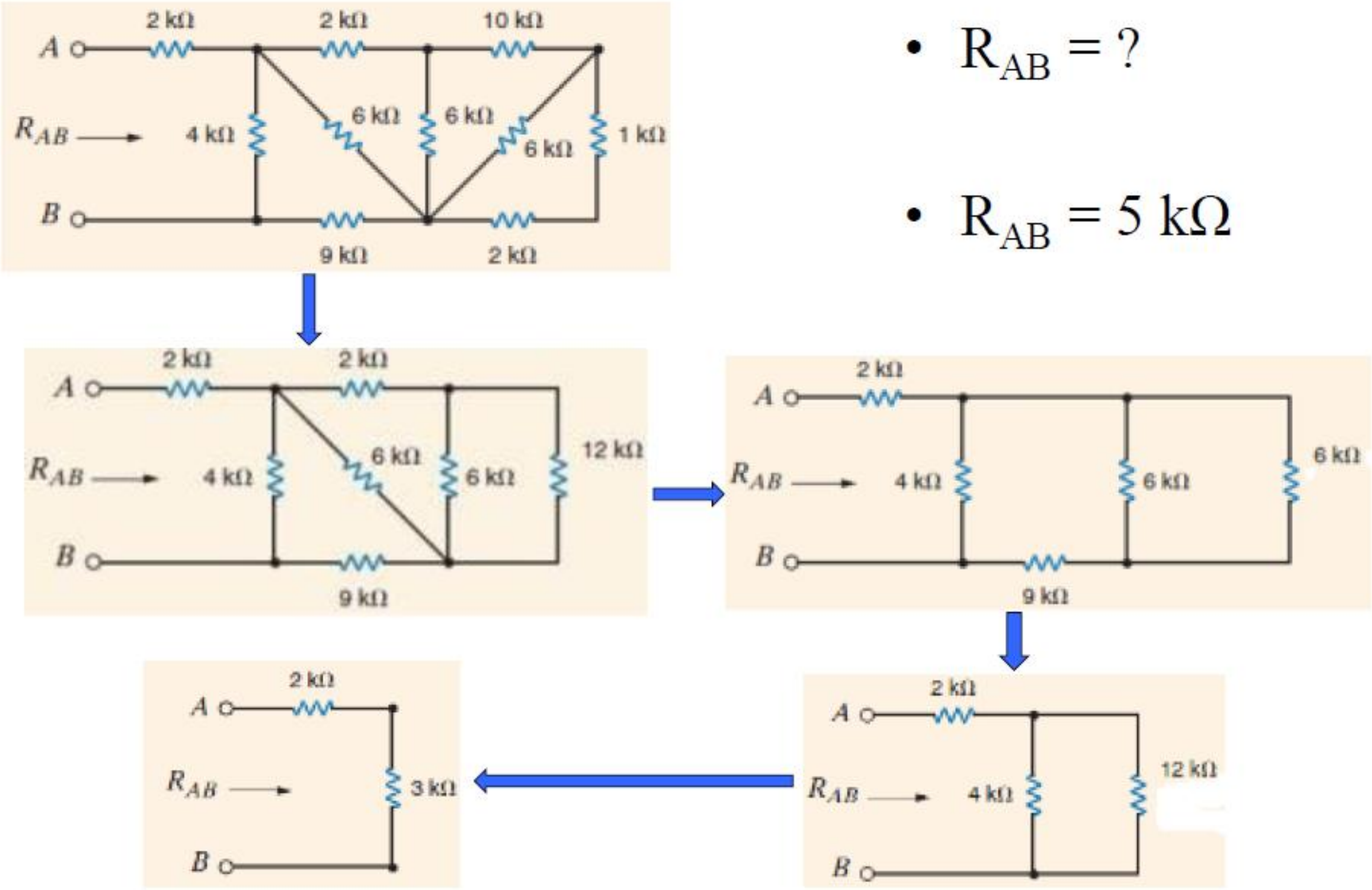
Two resistors in parallel can be combined using the shortcut.

product / sum

$$R_{eq} = R_1 \| R_2 \qquad = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

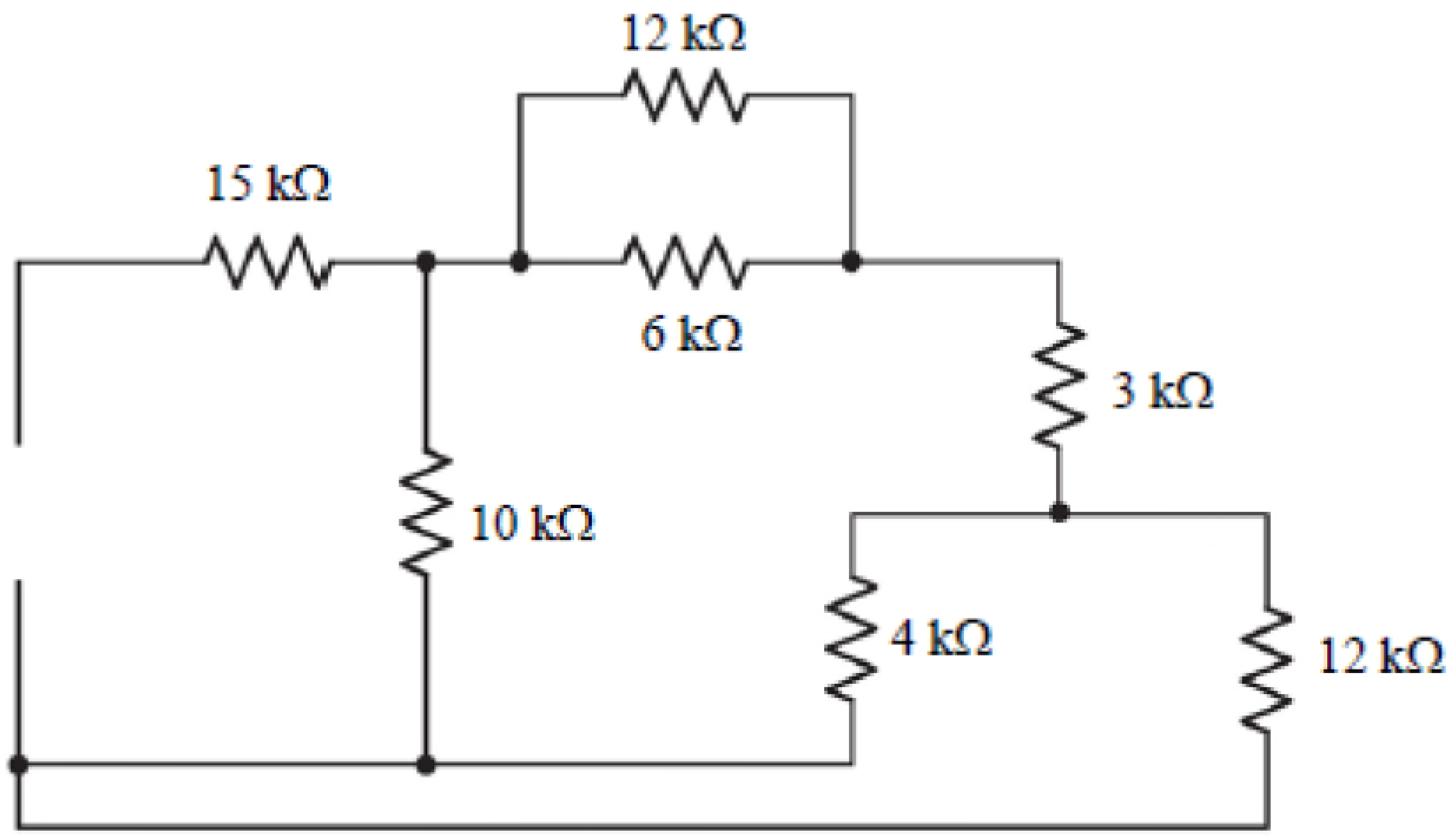
Example



- $R_{AB} = ?$
- $R_{AB} = 5\ k\Omega$

Example

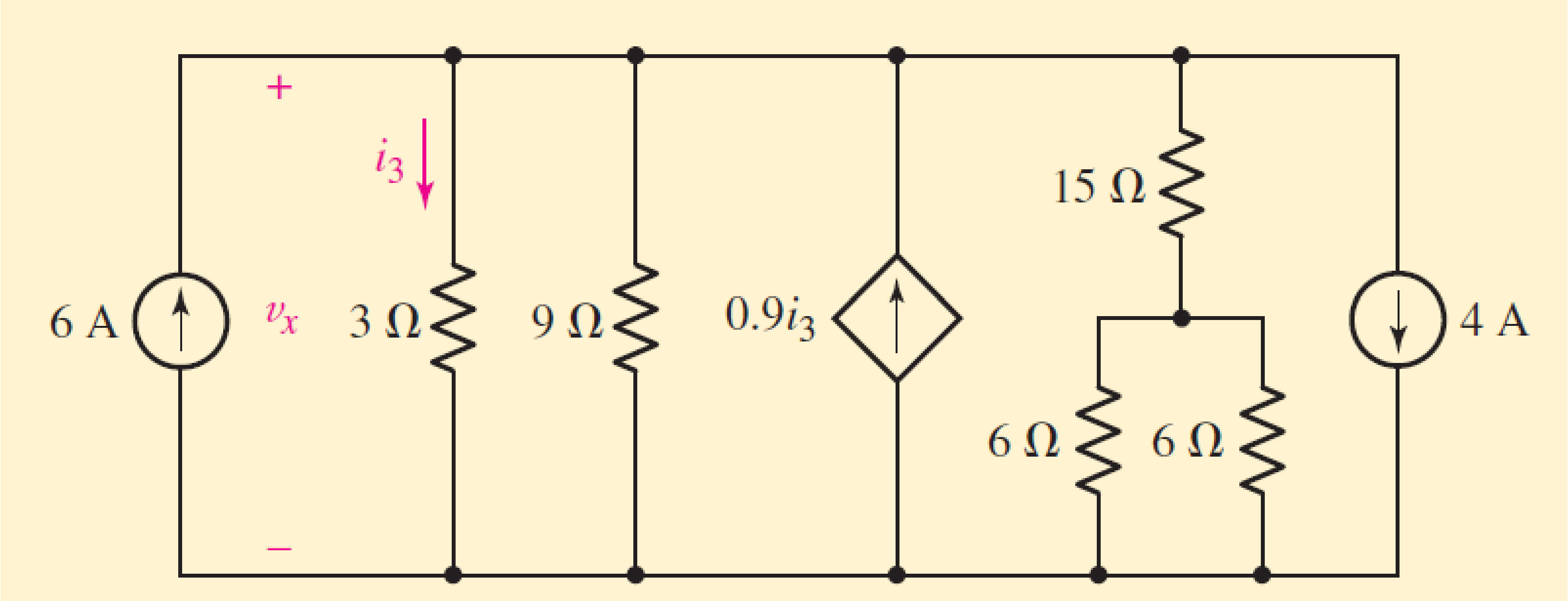
- Determine the equivalent resistance of this network between the open-circuit terminals.

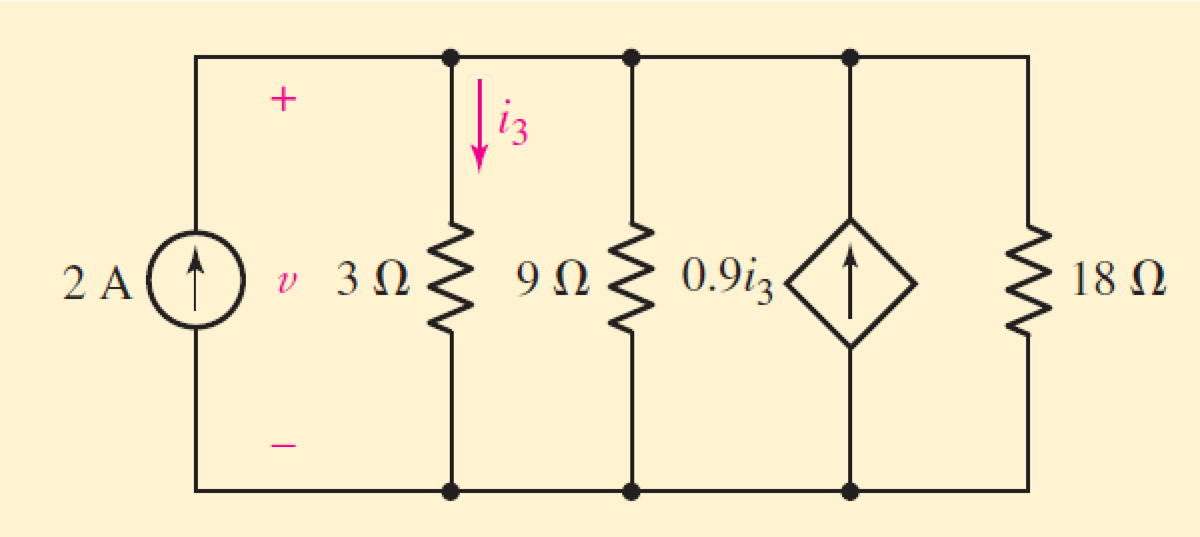


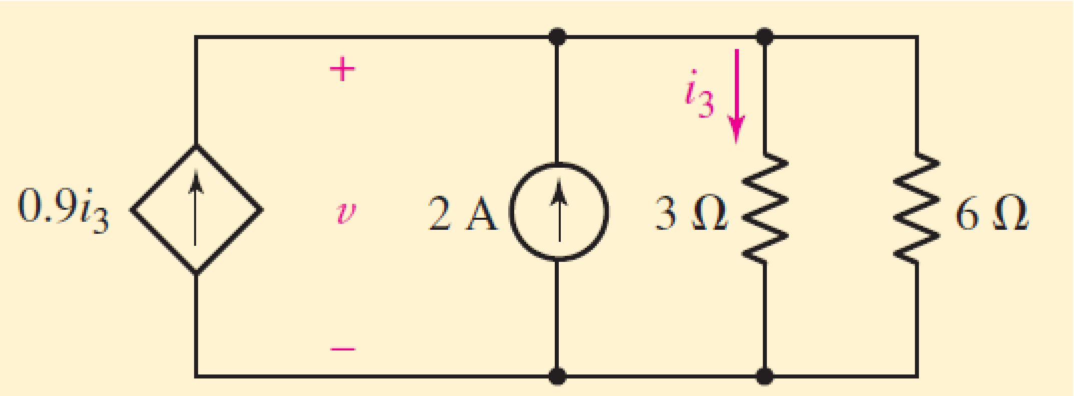
- 20 kΩ

Example

Calculate the power and voltage of the dependent source







$$-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0$$

$$v = 3i_3$$

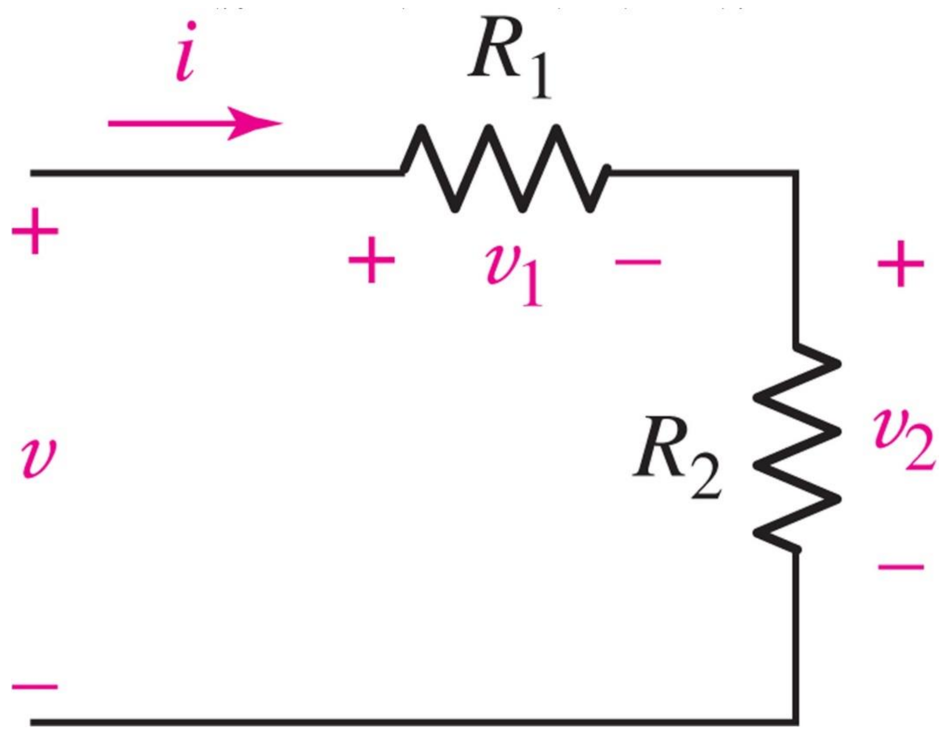
$$i_3 = \frac{10}{3}\ \text{A}$$

$$v = 3i_3 = 10\ \text{V}$$

$$v \times 0.9i_3 = 10(0.9)(10/3) = 30\ \text{W}$$

Voltage Division

Resistors in series “share” the voltage applied to them.



$$v_1 = \frac{R_1}{R_1 + R_2} v$$

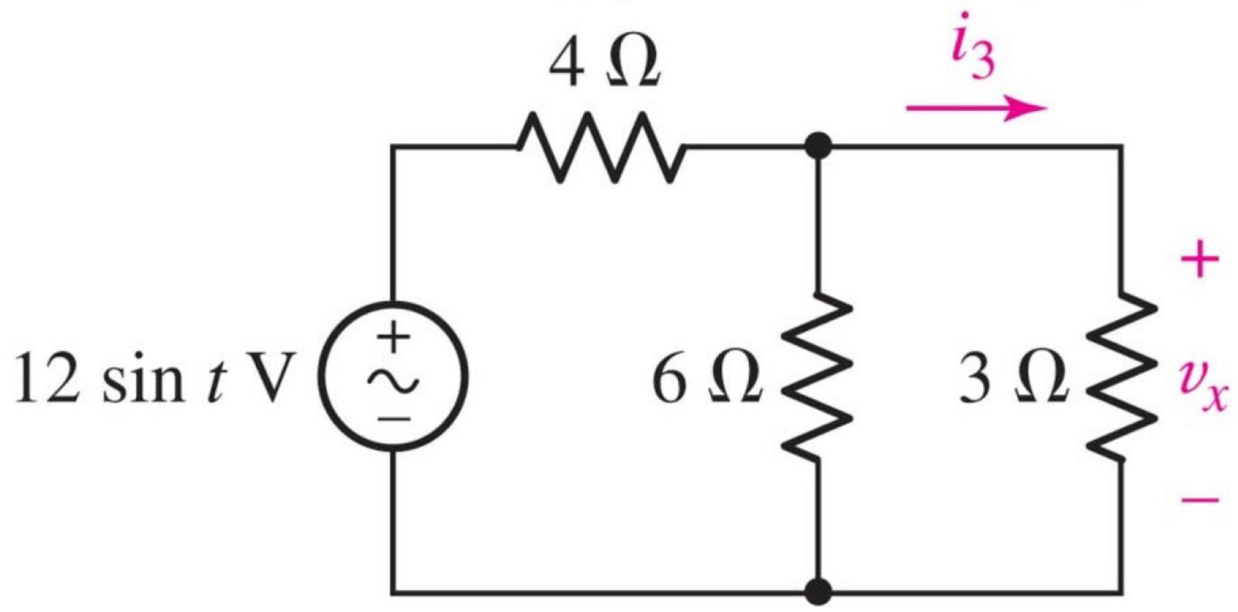
$$v_2 = \frac{R_2}{R_1 + R_2} v$$

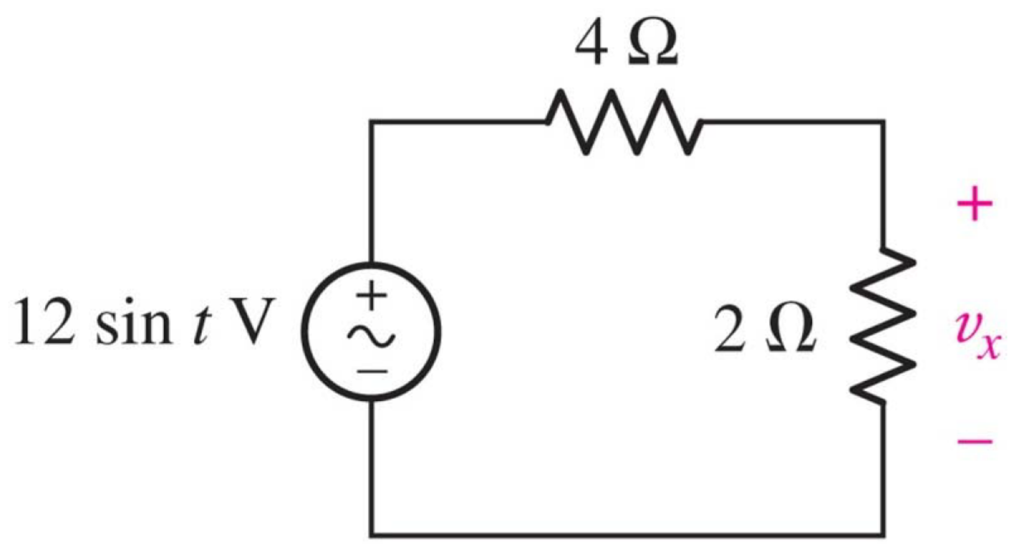
$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v}$$

Example: Voltage Division

Find v_x

We first combine the 6 Ω and 3 Ω resistors, replacing them with $(6)(3)/(6+3) = 2\ \Omega$.

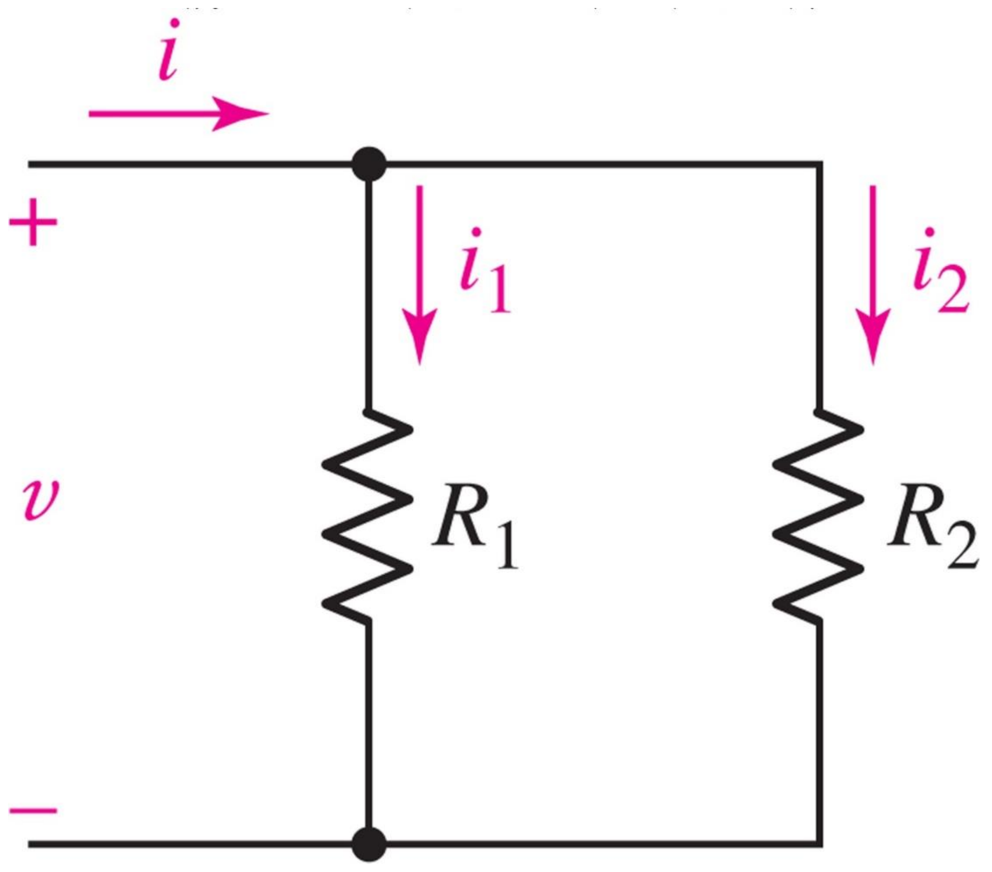




$$v_x = (12 \sin t)\frac{2}{4+2} = 4 \sin t \qquad \text{volts}$$

Current Division

Resistors in parallel “share” the current through them.



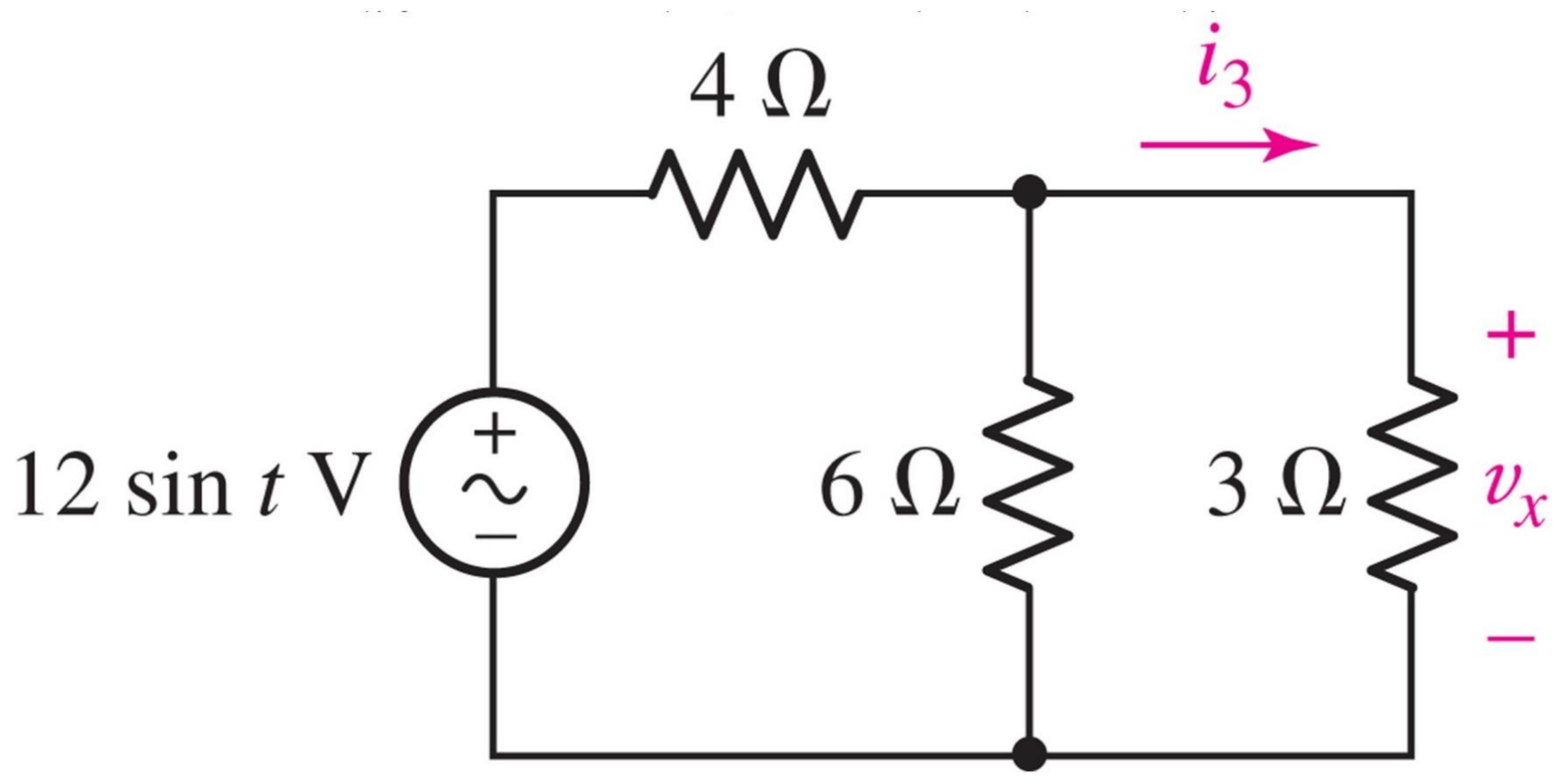
$$i_1 = i\,\frac{R_2}{R_1 + R_2}$$

$$i_2 = i\,\frac{R_1}{R_1 + R_2}$$

$$i_k = i\,\frac{\dfrac{1}{R_k}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}}$$

Example: Current Division

Find $i_3(t)$



$$i(t) = \frac{12 \sin t}{4 + 3\|6} = \frac{12 \sin t}{4+2} = 2 \sin t \quad \text{A}$$

$$i_3(t) = (2 \sin t)\left(\frac{6}{6+3}\right) = \frac{4}{3} \sin t \quad \text{A}$$

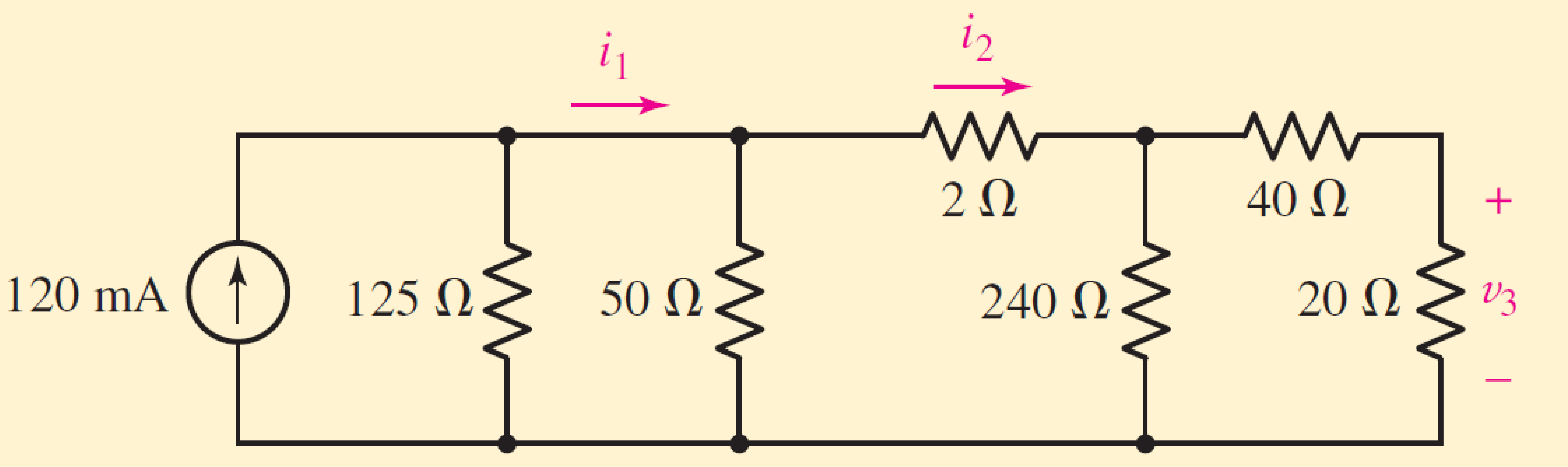
SUMMARY AND REVIEW

- ***KCL***: the algebraic sum of the **currents entering any node is zero.**
- ***KVL***: the algebraic **sum of the voltages** around any closed path in a circuit is zero
- All elements in a circuit **that carry the same current** are said to be connected in ***series.***
- Elements in a circuit having **a common voltage across them** are said to be connected in ***parallel.***
- Voltage sources in series can be replaced by a single source.
- Current sources in parallel can be replaced by a single source.

SUMMARY AND REVIEW

- A series combination of N resistors can be replaced by a single resistor having the value $R_{eq} = R_1 + R_2 + \cdots + R_N$.
- A parallel combination of N resistors can be replaced by a single resistor having the value $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots + 1/R_N$
- **Voltage division** allows us to calculate what fraction of the total voltage across a **series string of resistors** is dropped across any one resistor (or group of resistors).
- **Current division** allows us to calculate what fraction of the total current into a **parallel string of resistors** flows through any one of the resistors.

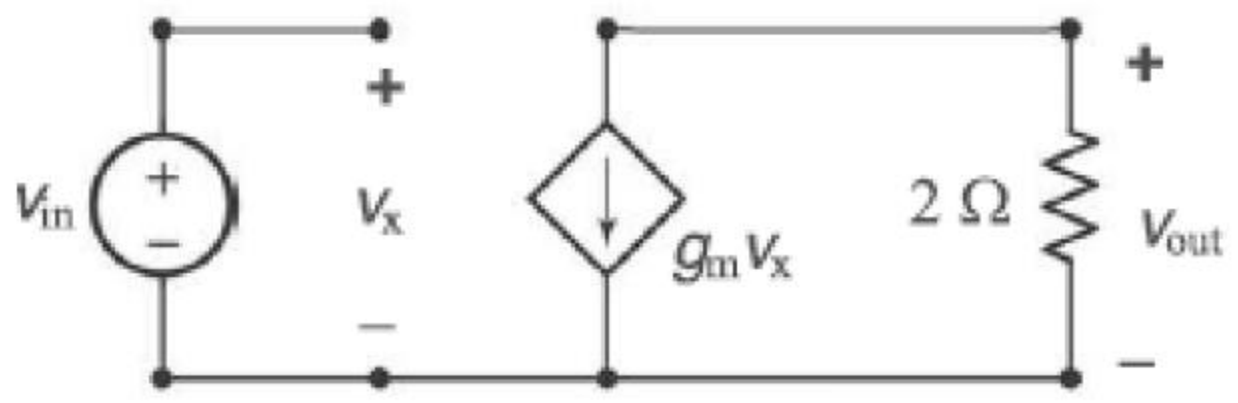
find $i1$, $i2$, and $v3$.



Ans: 100 mA; 50 mA; 0.8 V.

Practice 2

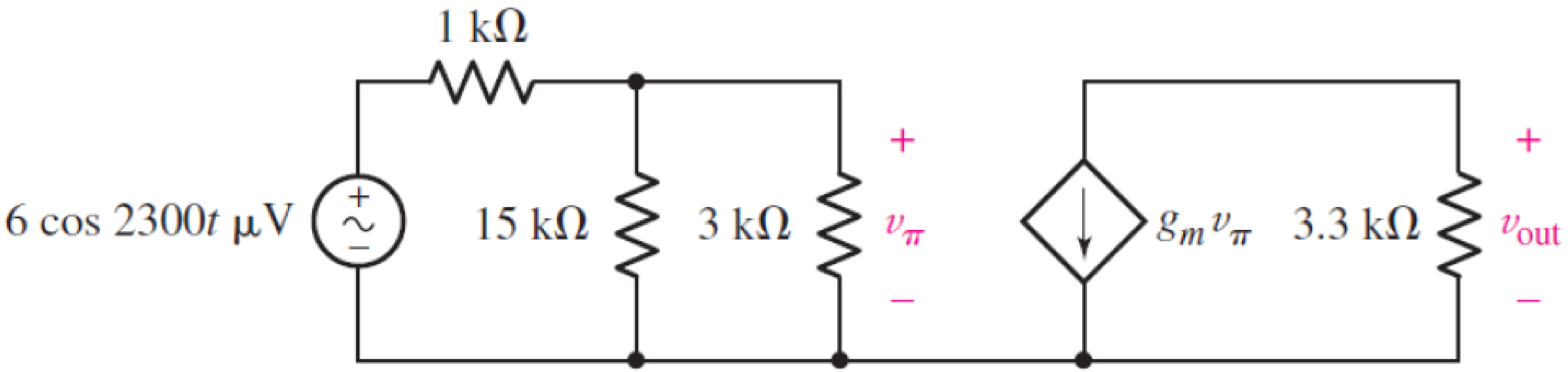
In the circuit below, v_{in}=3sin(ωt) mV and g_m=10 A/V. Determine v_{out}.



$$v_{out} = -g_m v_x \times R$$
$$v_{out} = -10 \times 3\sin(\omega t) \times 2$$
$$v_{out} = -60\sin(\omega t)\ \text{mV}$$

Practice 2

Calculate the amplifier output *v*out if the transconductance *gm* is equal to 322 mS.



Thanks