دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 5. Capacitor-Inductor

Afarghadan@aut.ac.ir

عظیم فرقدان

مهر ۱۴۰۳

Objectives of the Lecture

- Capacitor and inductor
- Voltage and current characteristics
- Energy
- Series and parallel connection
- duality

The Capacitor

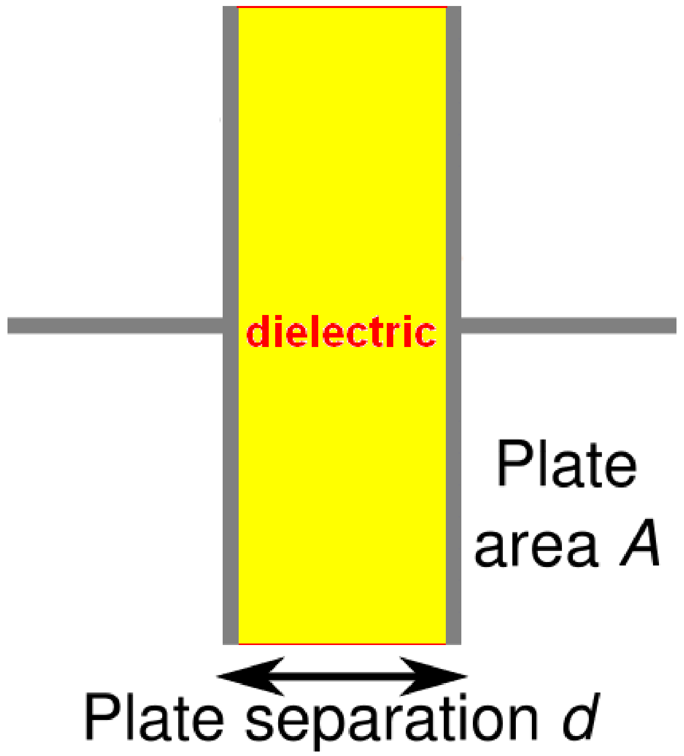
- Composed of two conductive plates separated by an insulator (or dielectric).
  - Commonly illustrated as two parallel metal plates separated by a distance, d.

$C = \varepsilon A/d$

where $\varepsilon = \varepsilon_r \varepsilon_o$

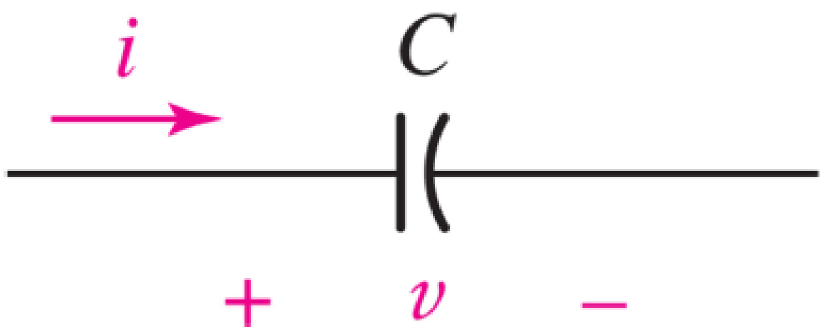
ε_r is the relative dielectric constant

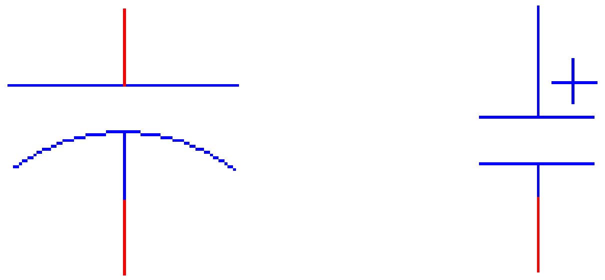
ε_o is the vacuum permittivity



The Capacitor

- The ideal capacitor is a passive element with circuit symbol:





- We define capacitance C by the voltage-current relationship:

$$i_C = \frac{dq}{dt} \qquad q = Cv_C \qquad \boxed{i = C\frac{dv}{dt}} \qquad \boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)}$$

- The unit of capacitance is Farad (F).

- Ceramic capacitors (typically 1 picofarad to 1 microfarad)
- Electrolytic capacitors (typically 1 microfarad to 10 millifarads)
- Supercapacitor (1 to 100 Farads)

Power and Energy

Capacitor power:

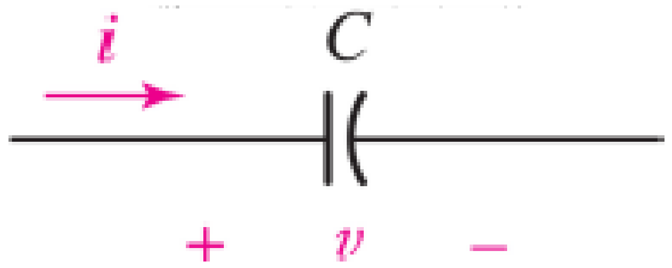
$$\boldsymbol{p(t) = i(t)v(t) = \left(C\frac{dv}{dt}\right)v}$$

Energy stored in capacitor with voltage v:

$$\boldsymbol{E = \int p\,dt = \int Cv\,dv = \frac{1}{2}Cv^2}$$

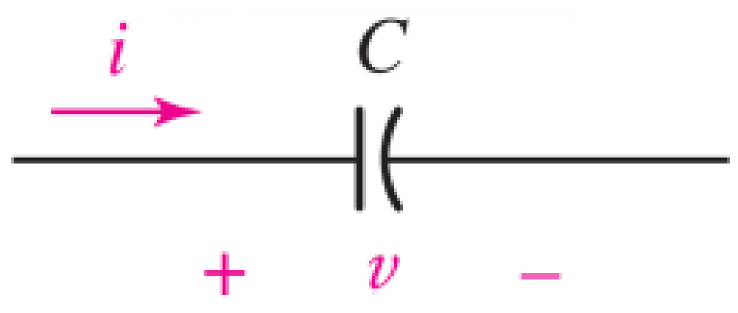
Capacitor behavior

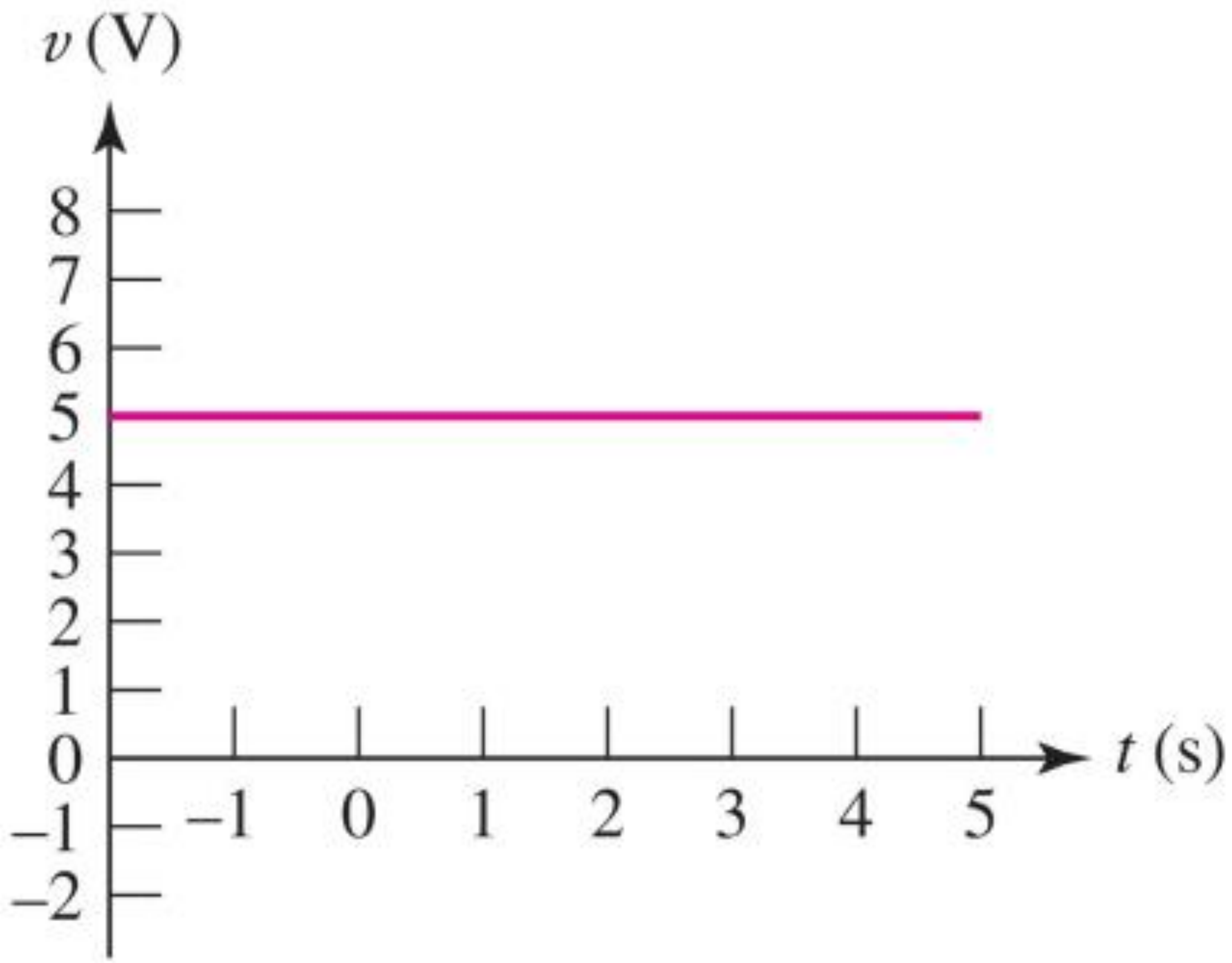
- ✓ Capacitors are open circuits to dc voltages.Why?
- ✓ the voltage on a capacitor *cannot* jump. Why?
- ✓ If we take the direction of current and voltage according to the convention below, if P is positive, the capacitor is storing energy and if it is negative, it is delivering energy.

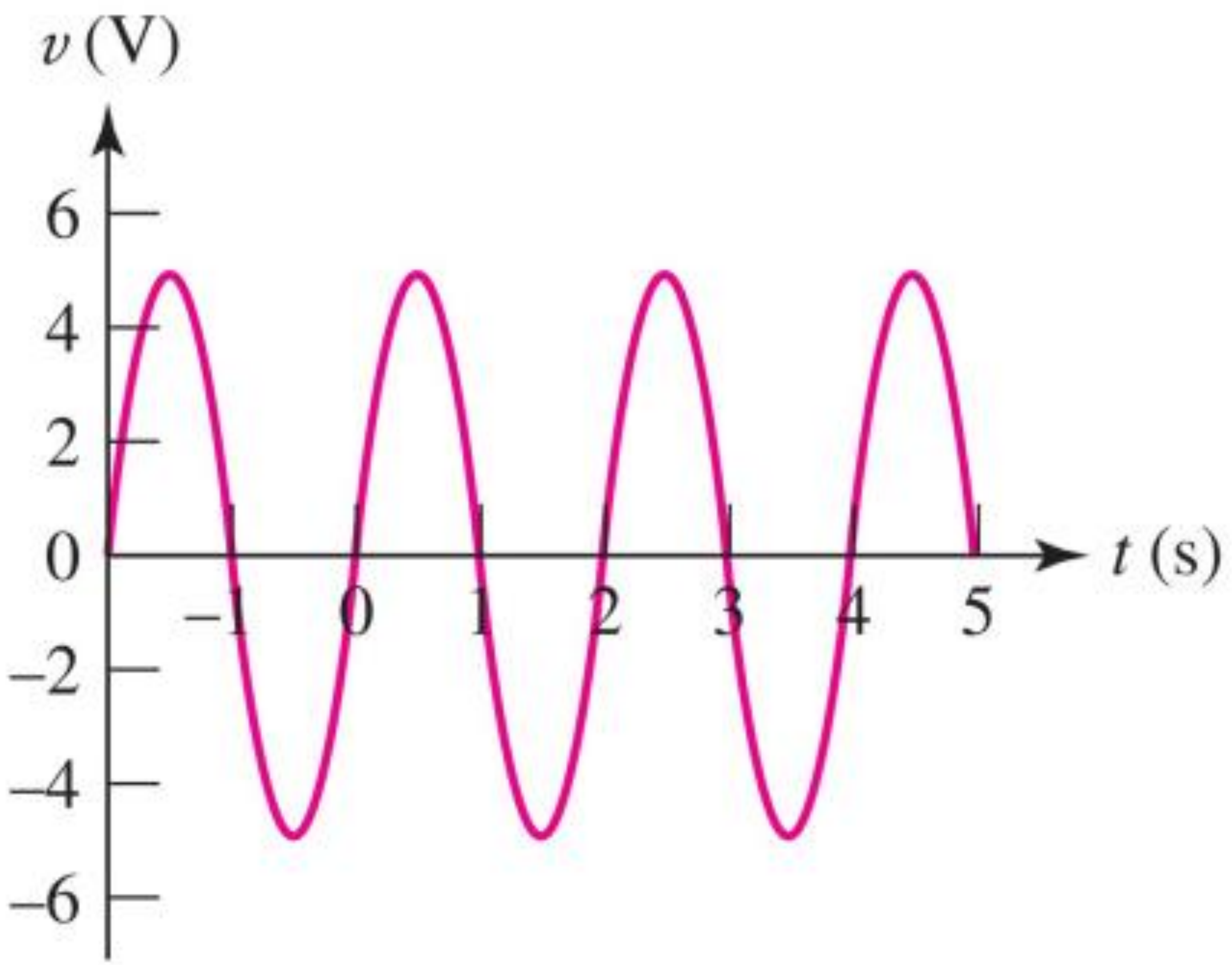


$$i = C\frac{dv}{dt}$$

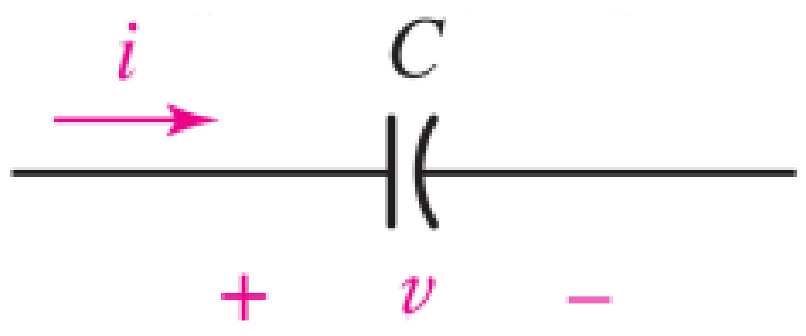
Find *i(t)* for the voltages shown, if C=2 F.

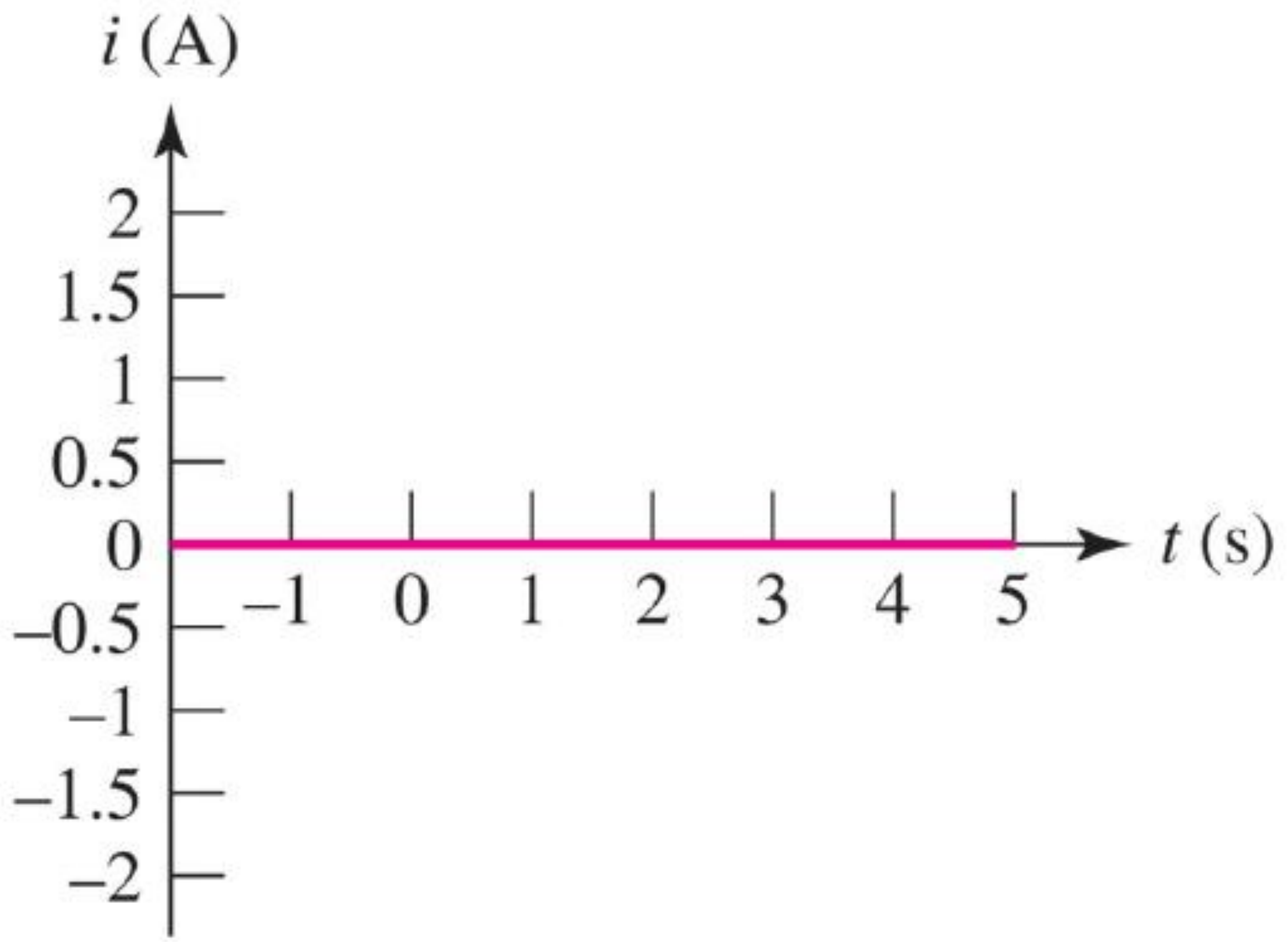


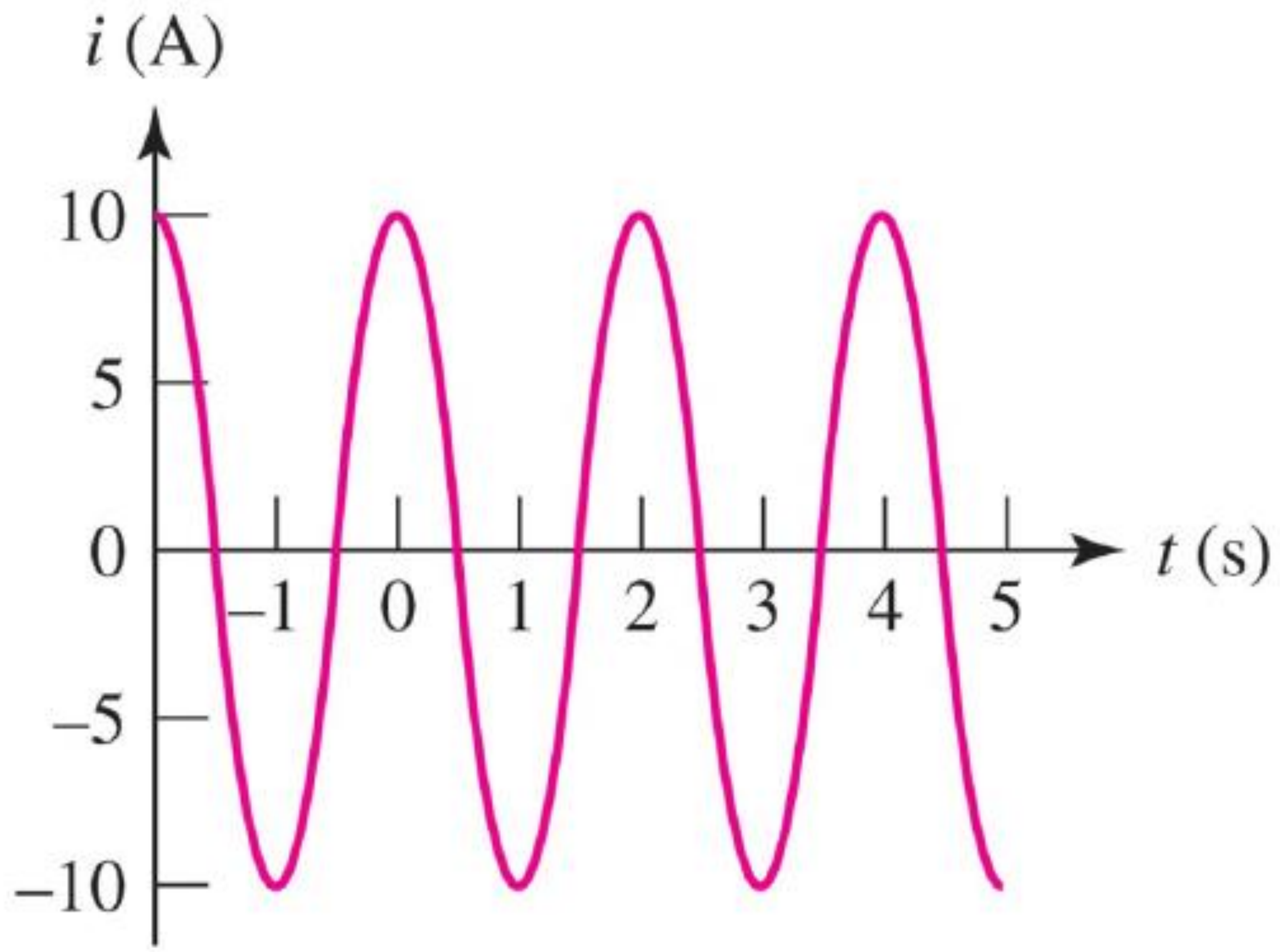




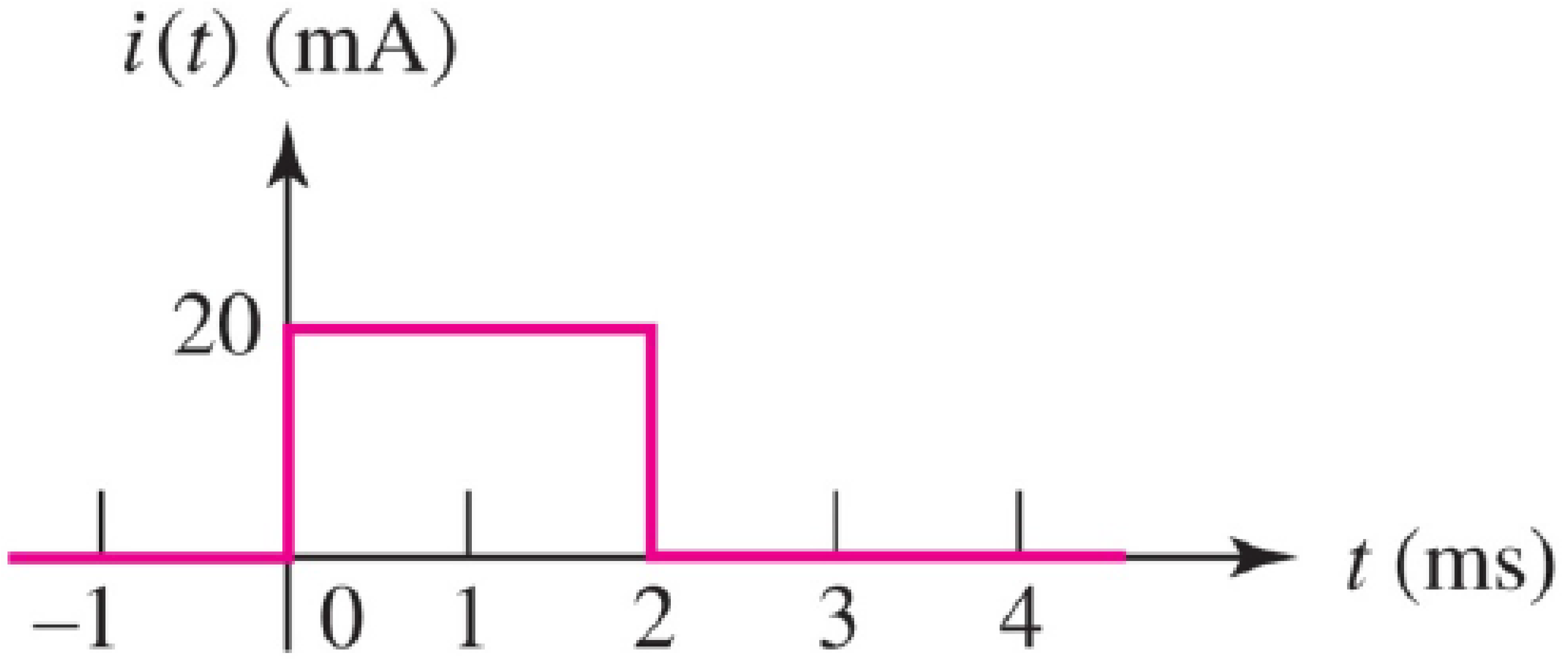
Solution: apply *i(t)=2dv/dt* and graph:

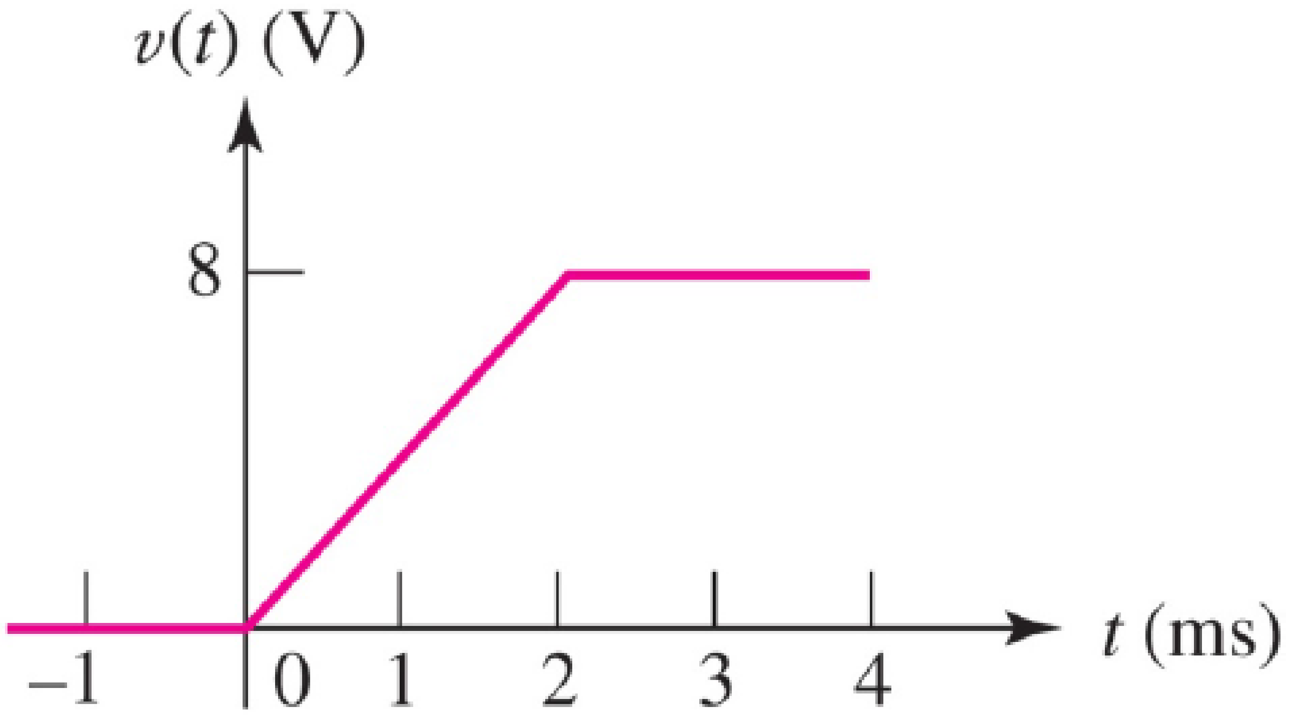






Show that the following graphs are matching voltage and current graphs for a capacitor of C=5μF.

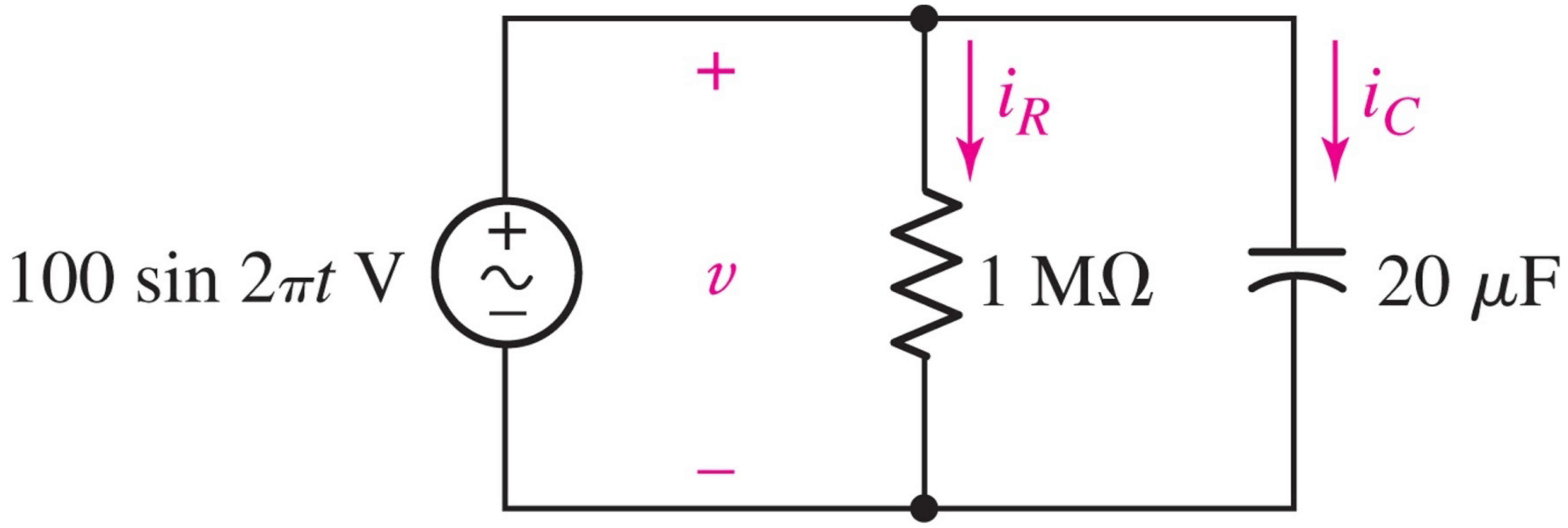




$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$$

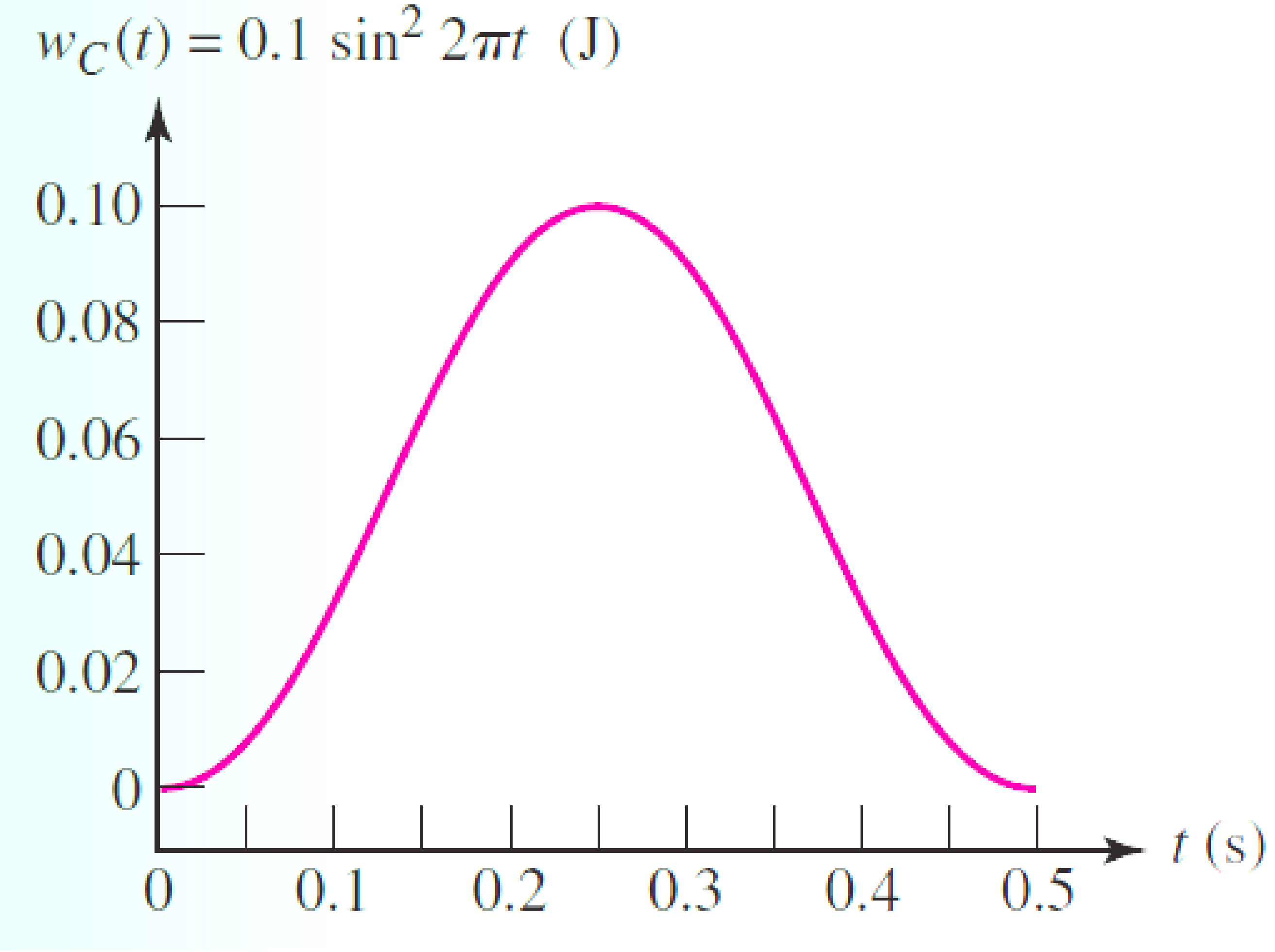
Example: Capacitive energy

Determine the maximum energy stored in the capacitor, and plot *iR* and *iC*.



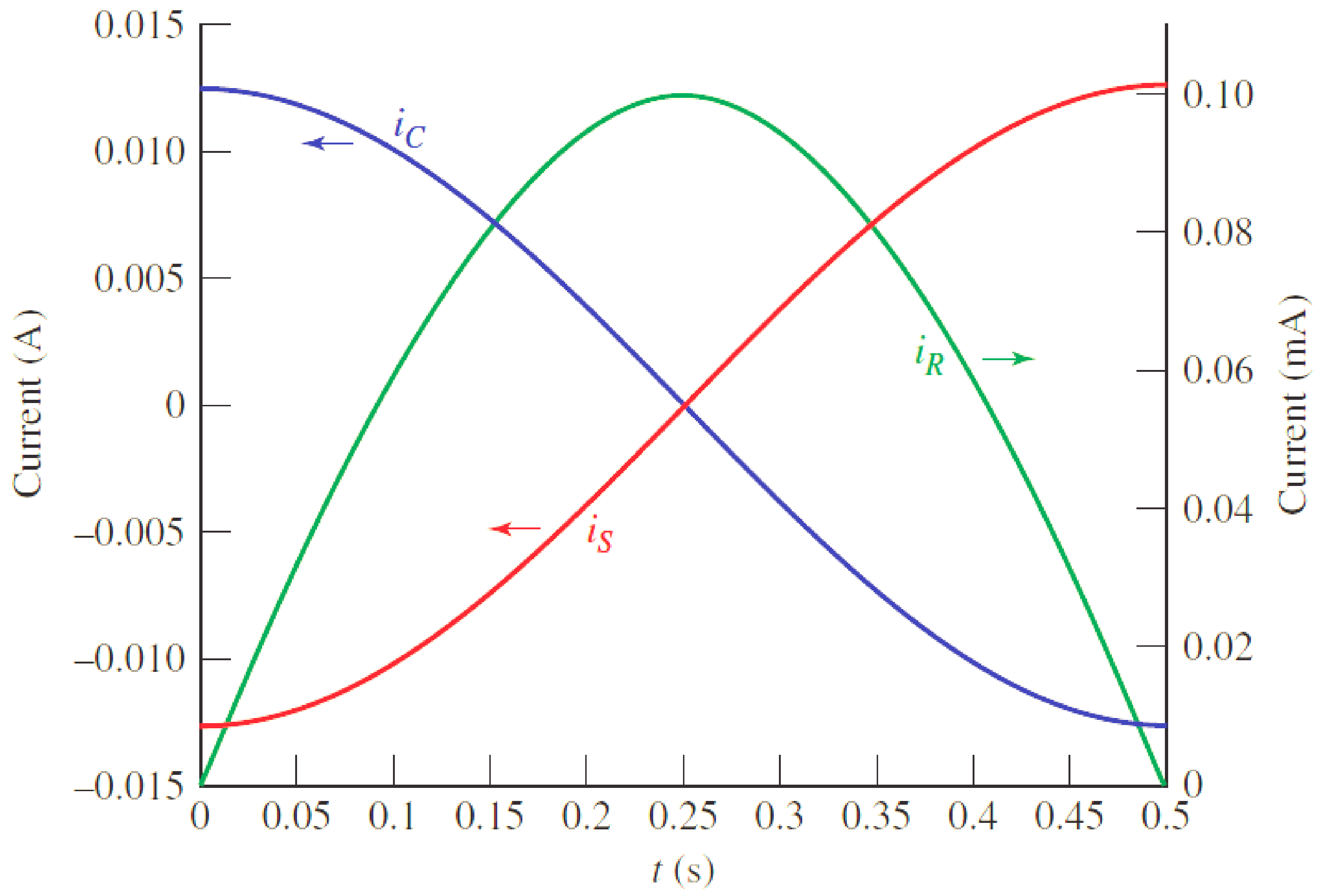
Example: Capacitive energy

Determine the maximum energy stored in the capacitor, and plot *iR* and *iC*.



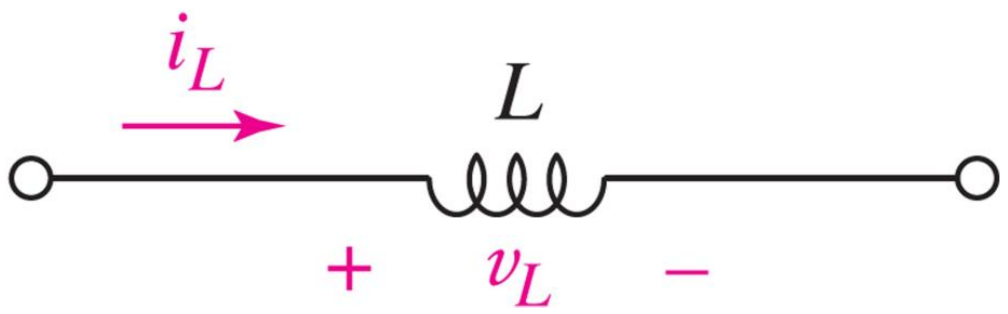
$$i_C = 20 \times 10^{-6} \frac{dv}{dt} = 0.004\pi \cos 2\pi t$$

$$i_R = \frac{v}{R} = 10^{-4} \sin 2\pi t$$



The inductor

- The ideal inductor is a passive element with circuit symbol



- The current-voltage relation is

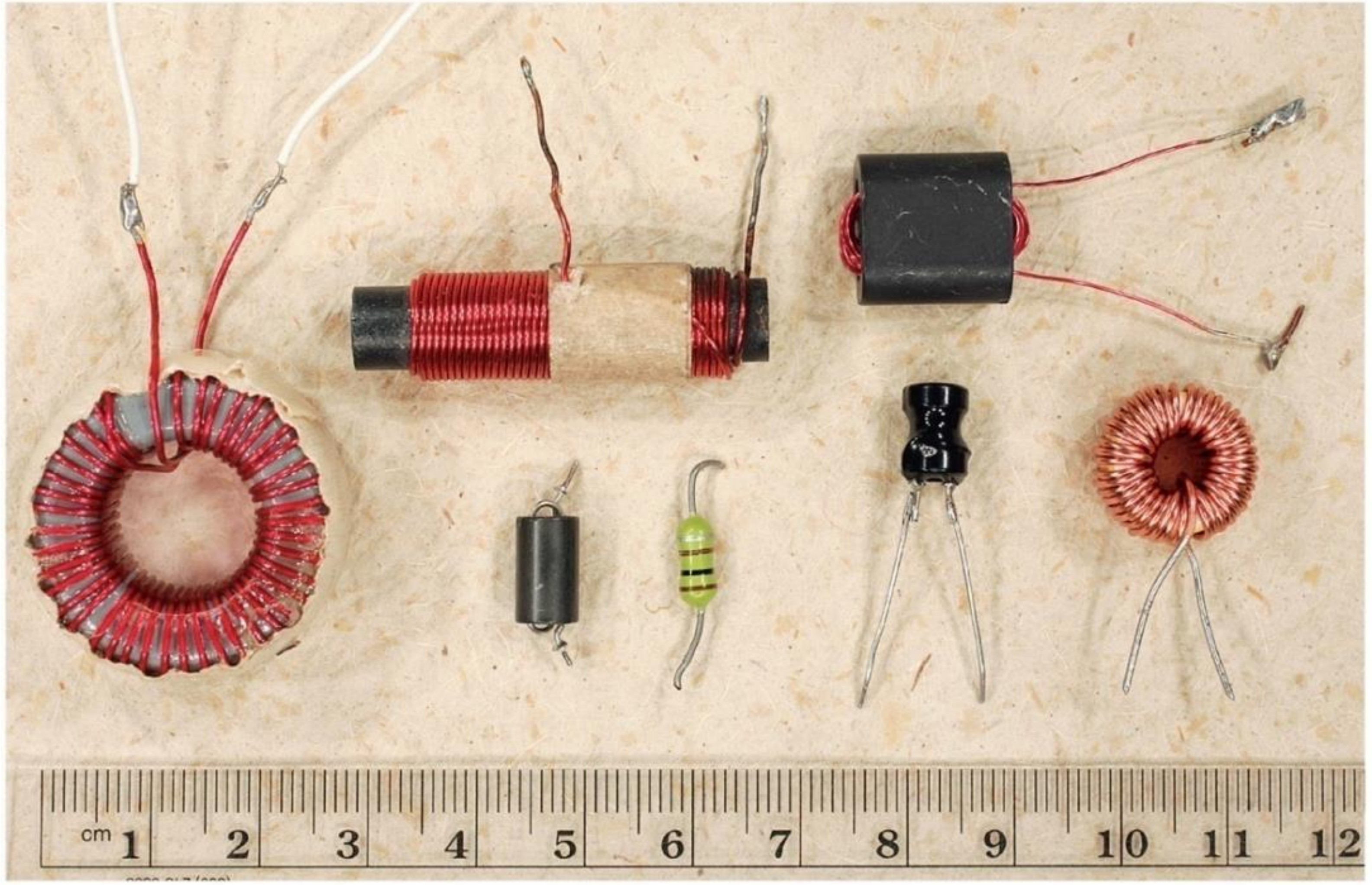
$$\boxed{v = L\frac{di}{dt}}$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)}$$

- The unit of inductance L is henry (H)

Some inductors

Inductors can be bulky and typical values range from μH to H



Energy stored in the inductor

Since:

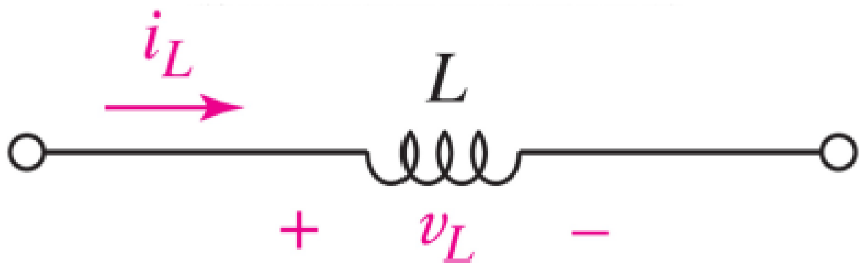
$$p(t) = i(t)v(t) = i\left(L\frac{di}{dt}\right)$$

then the energy stored in a inductor is:

$$E = \int p dt = \int L i di = \frac{1}{2}Li^2$$

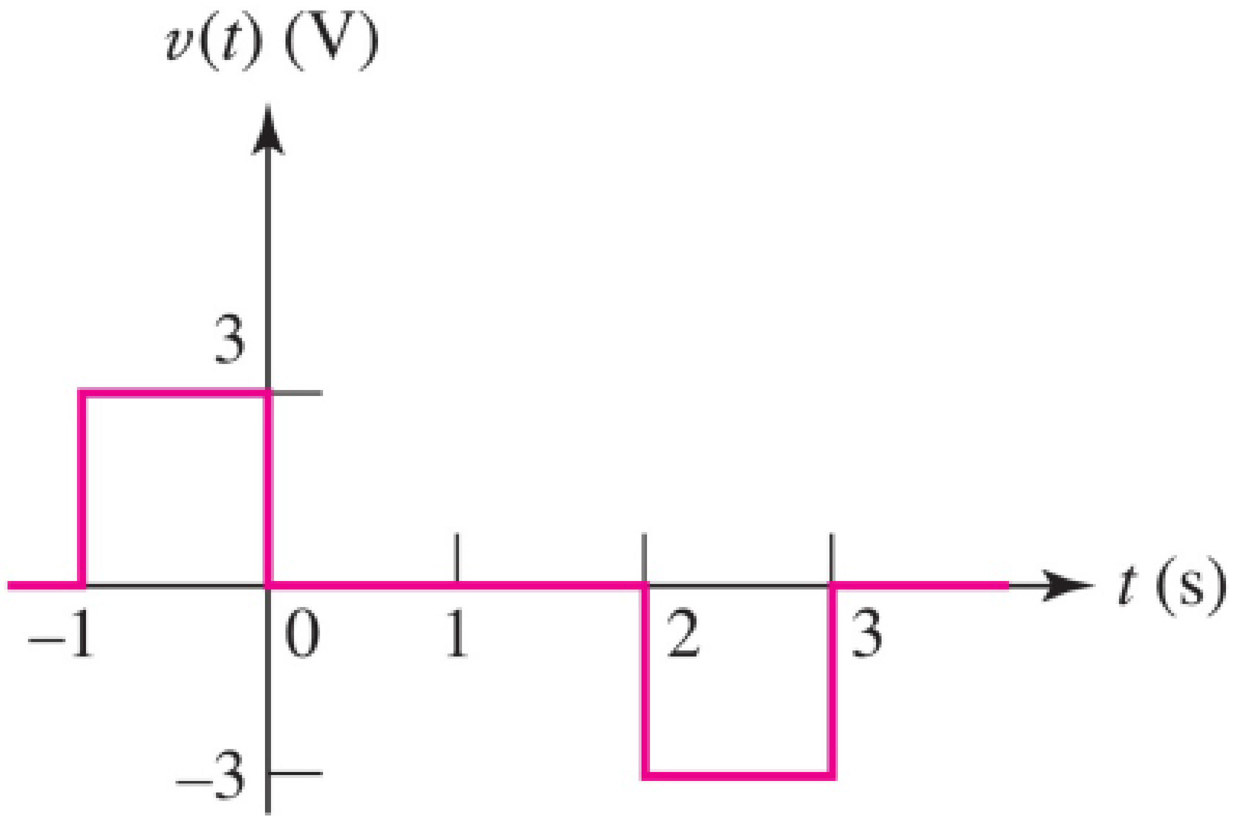
Inductor behavior

- ✓ Inductors are short circuits to dc voltages
- ✓ The current through an inductor *cannot* jump
- ✓ If we take the direction of current and voltage according to the convention below, if P is positive, the inductors is storing energy and if it is negative, it is delivering inductors.

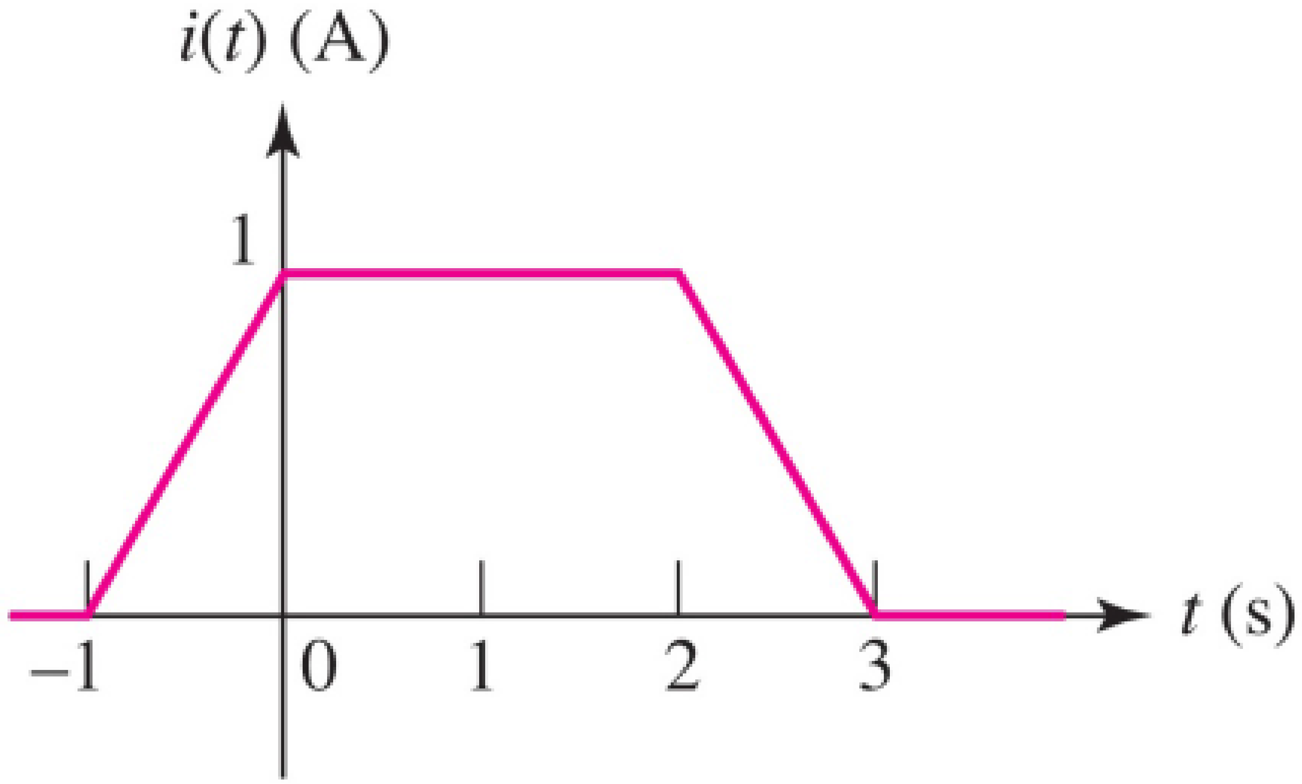


$$v = L\frac{di}{dt}$$

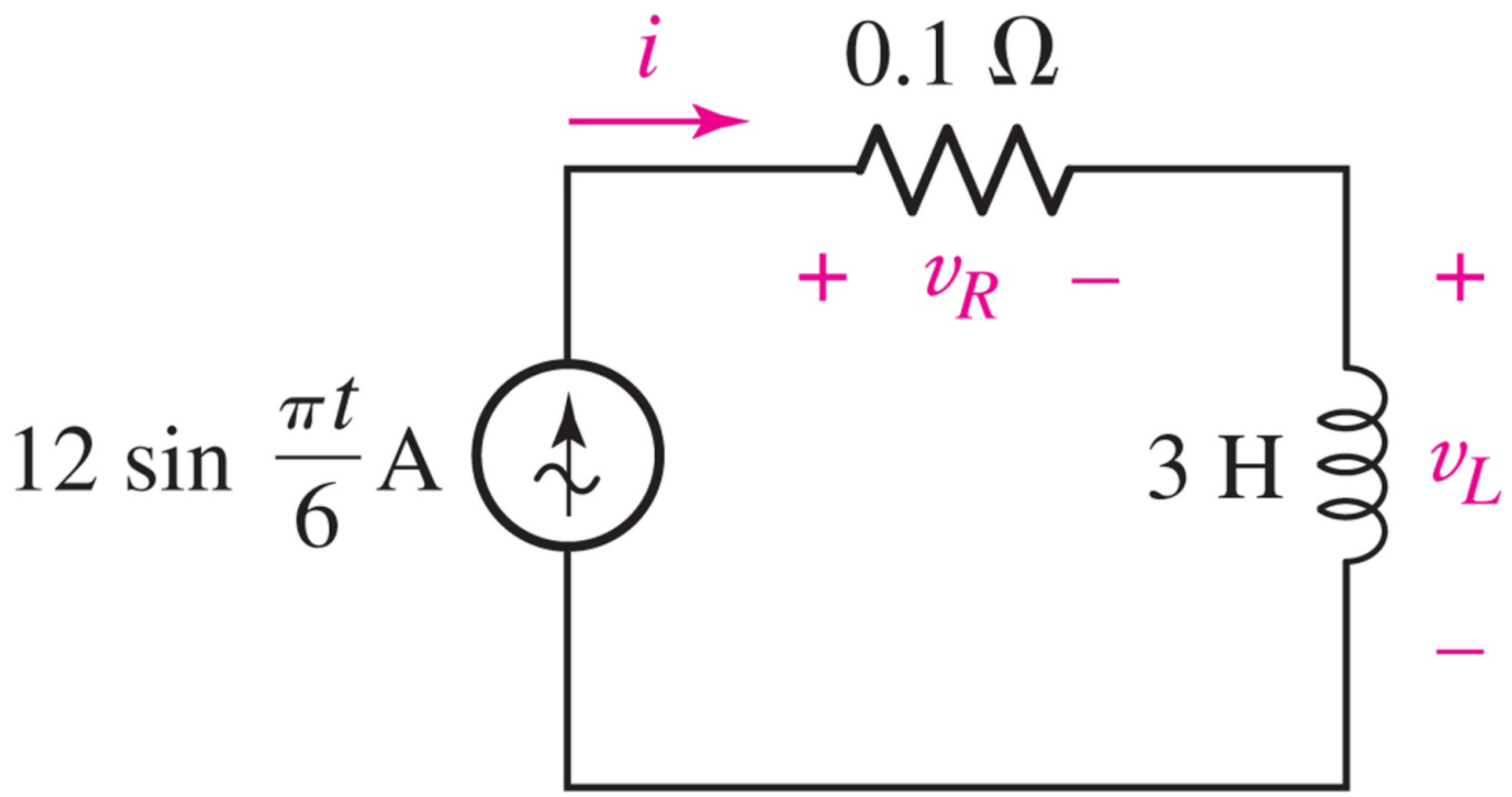
Show that the following graphs are matching voltage and current graphs for an inductor of L=3 H.



$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)$$

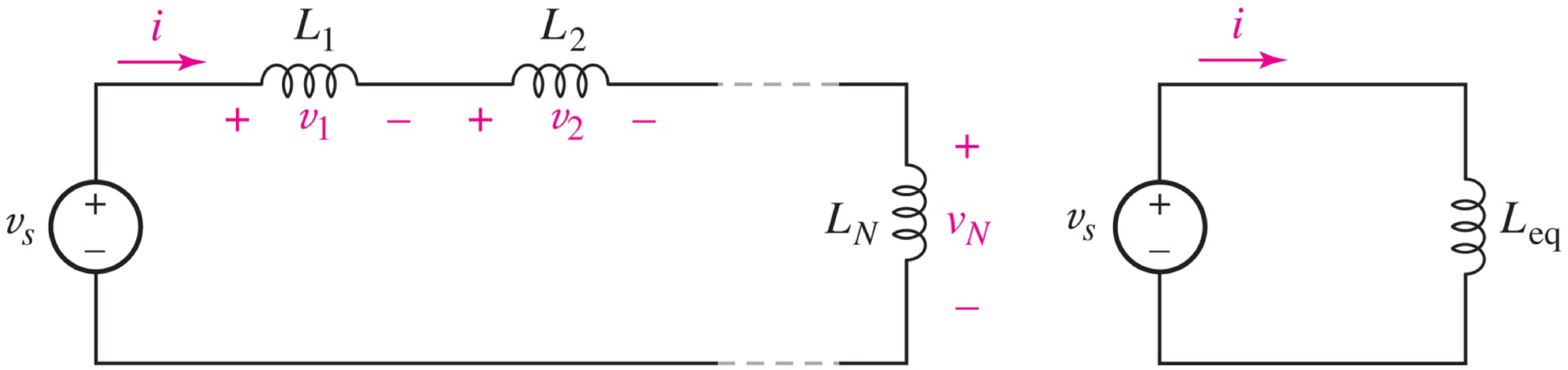


Determine the maximum energy stored in the inductor, and find the energy lost to resistor from t=0 to t=6 s.



Answer: 216 J, 43.2 J

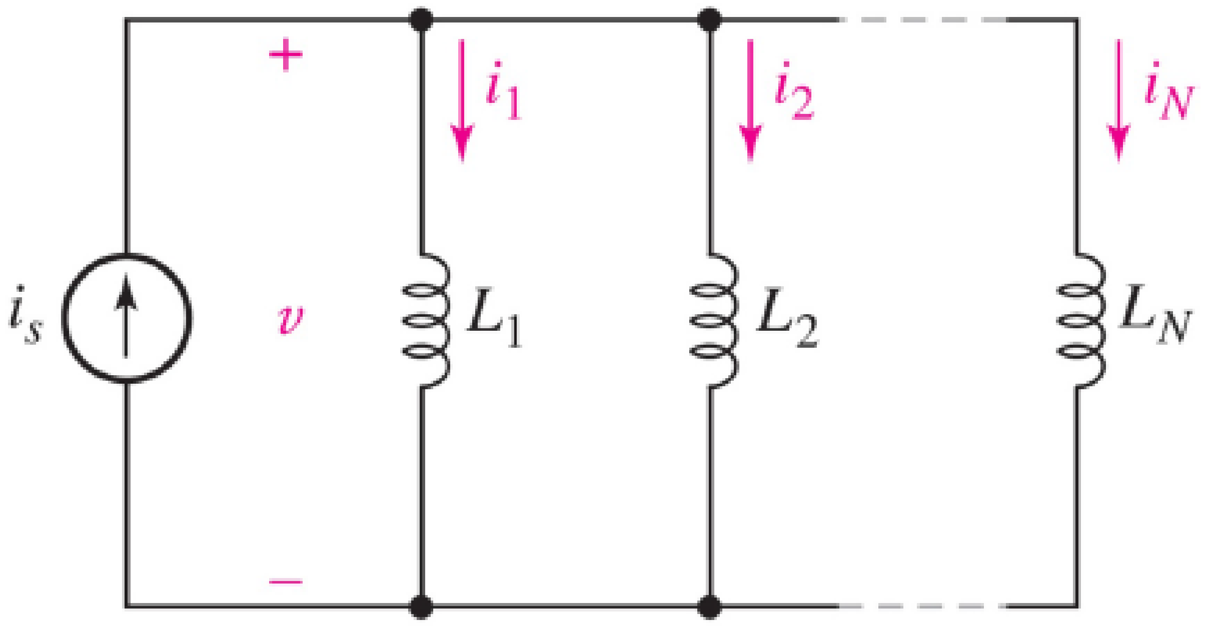
Series connection of inductors

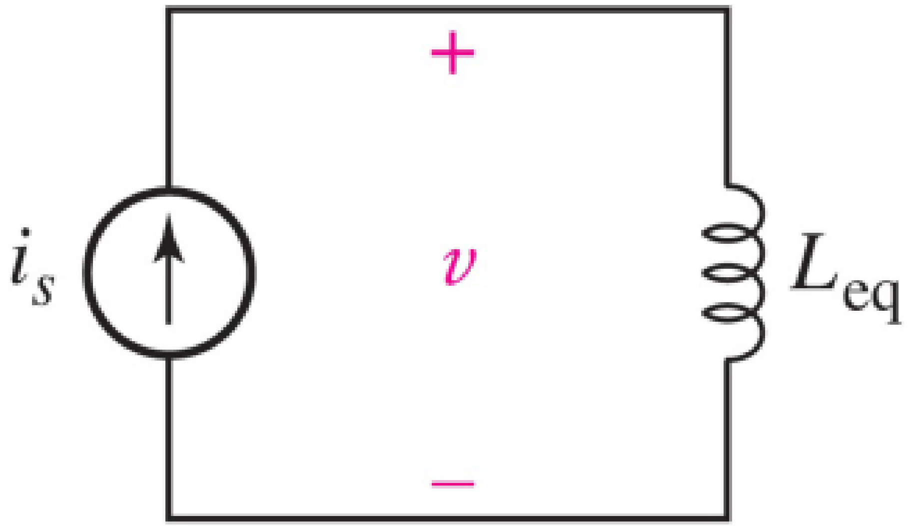


Apply KVL to show:

$$\boxed{L_{eq} = L_1 + L_2 + \cdots + L_N}$$

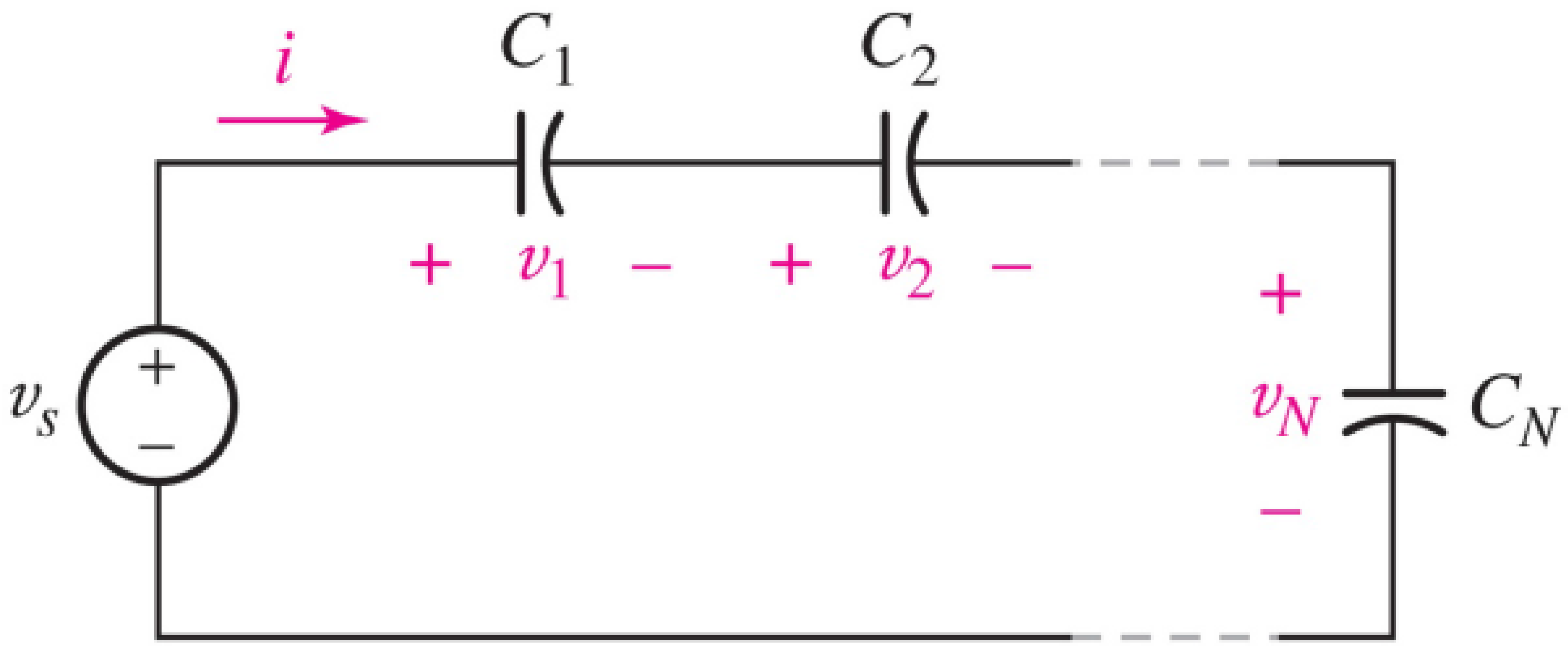
Parallel connection of inductors

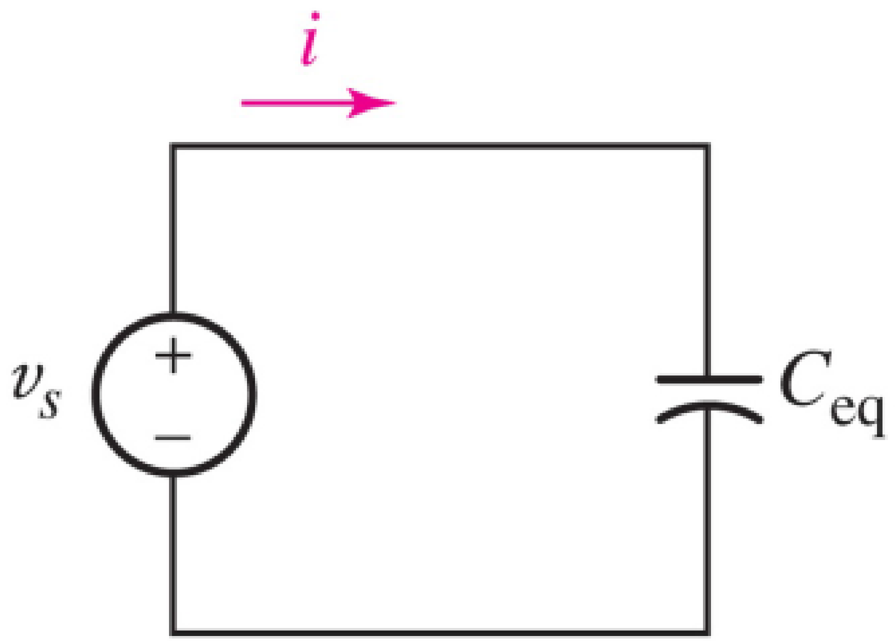




Apply KCL to show

$$\boxed{L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \cdots + 1/L_N}}$$

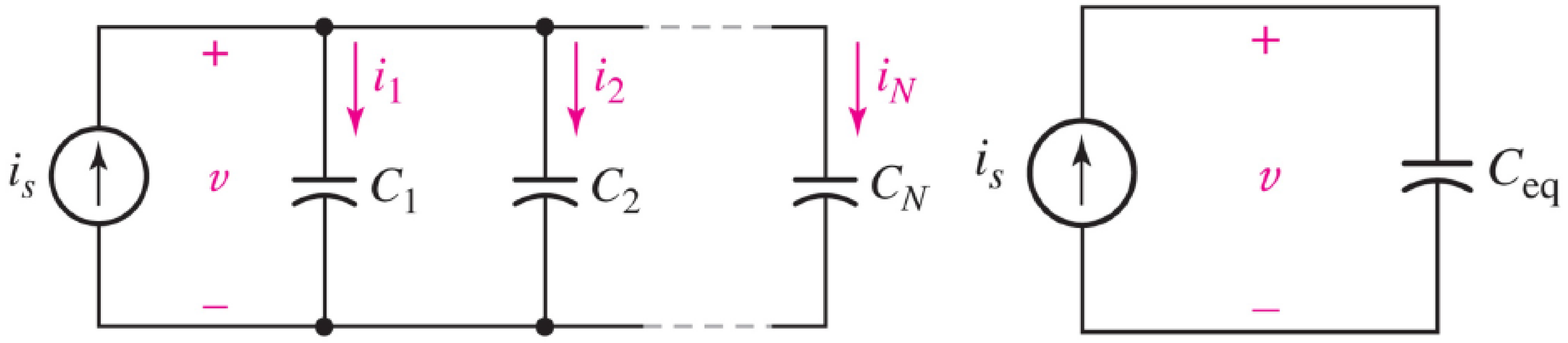




Apply KVL to show:

$$\boxed{C_{eq} = \frac{1}{{}^{1}\!/\!_{C_1} + {}^{1}\!/\!_{C_2} + \cdots + {}^{1}\!/\!_{C_N}}}$$

Parallel connection of capacitors



Apply KCL to show

$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_N}$$

Two-elements equivalent inductor and capacitor

Two capacitors in *series*:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

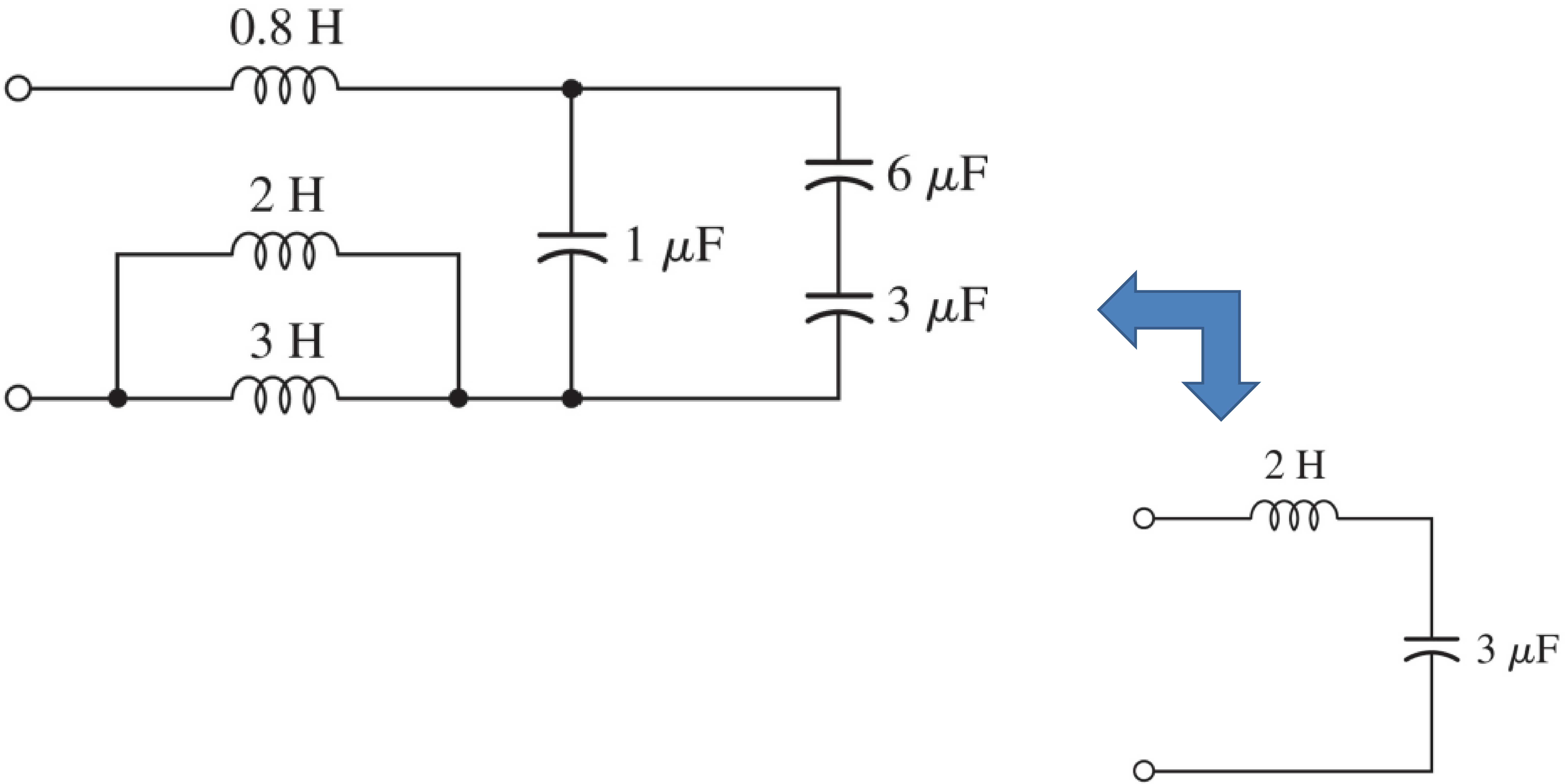
Two inductors in parallel:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Two resistors in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Example: Simplifying LC

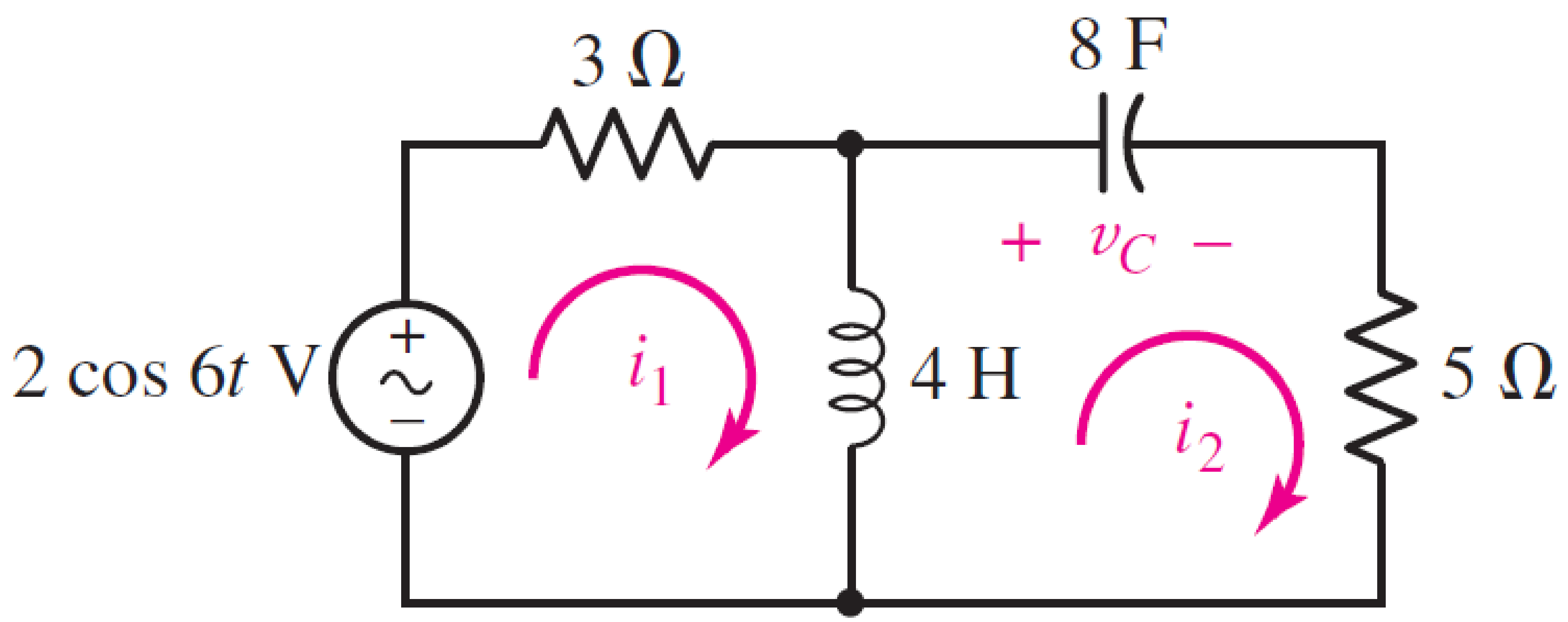


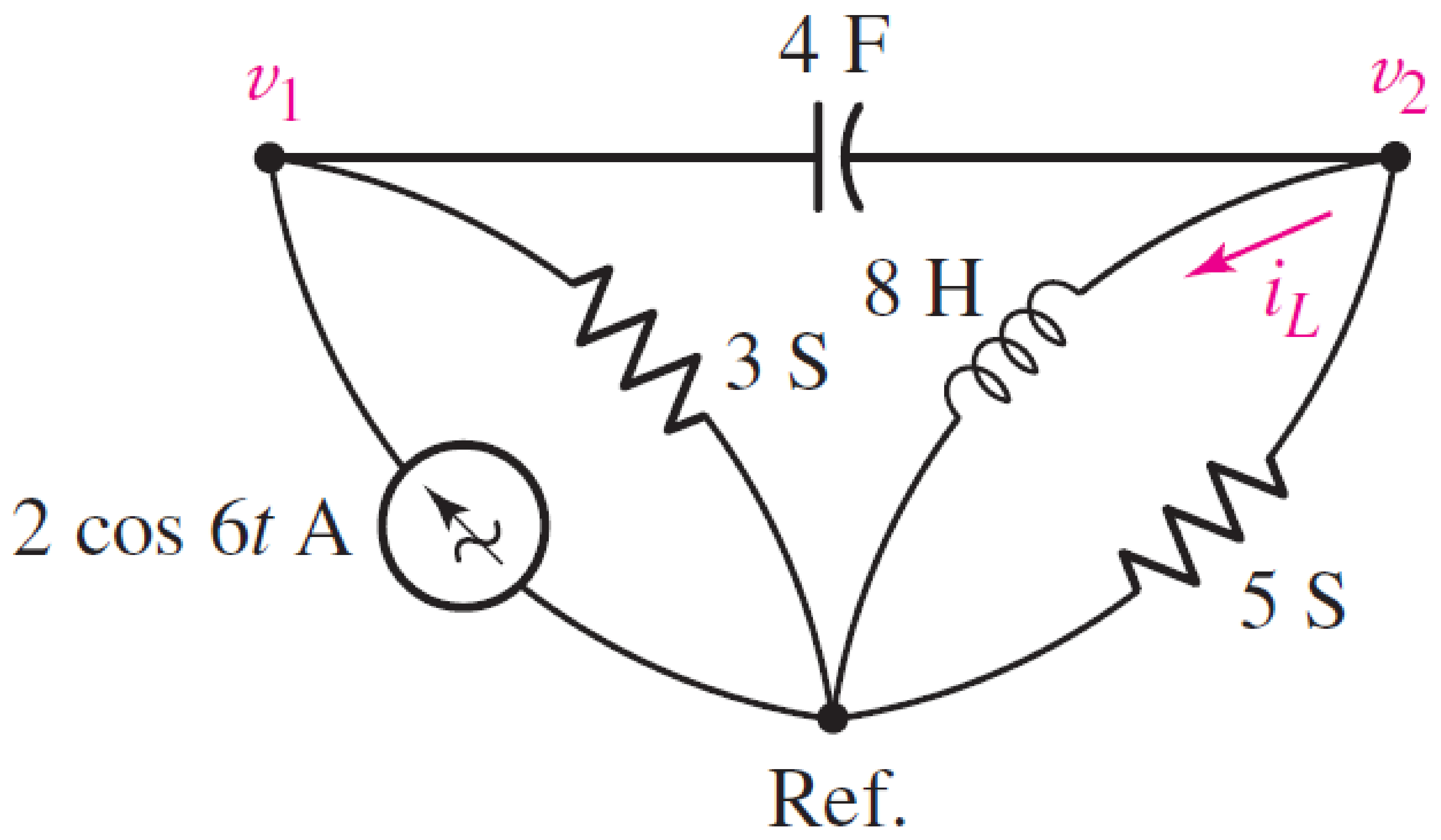
Duality

- Two circuits are dual if:
  - The current equations of the first mesh should be equal to the voltage equations of the second node. (and vice versa)

Duality	
R	G
KVL	KCL
C	L
$v(t)$	$i(t)$
$v_s(t)$	$i_s(t)$
$v_C(0)$	$i_L(0)$

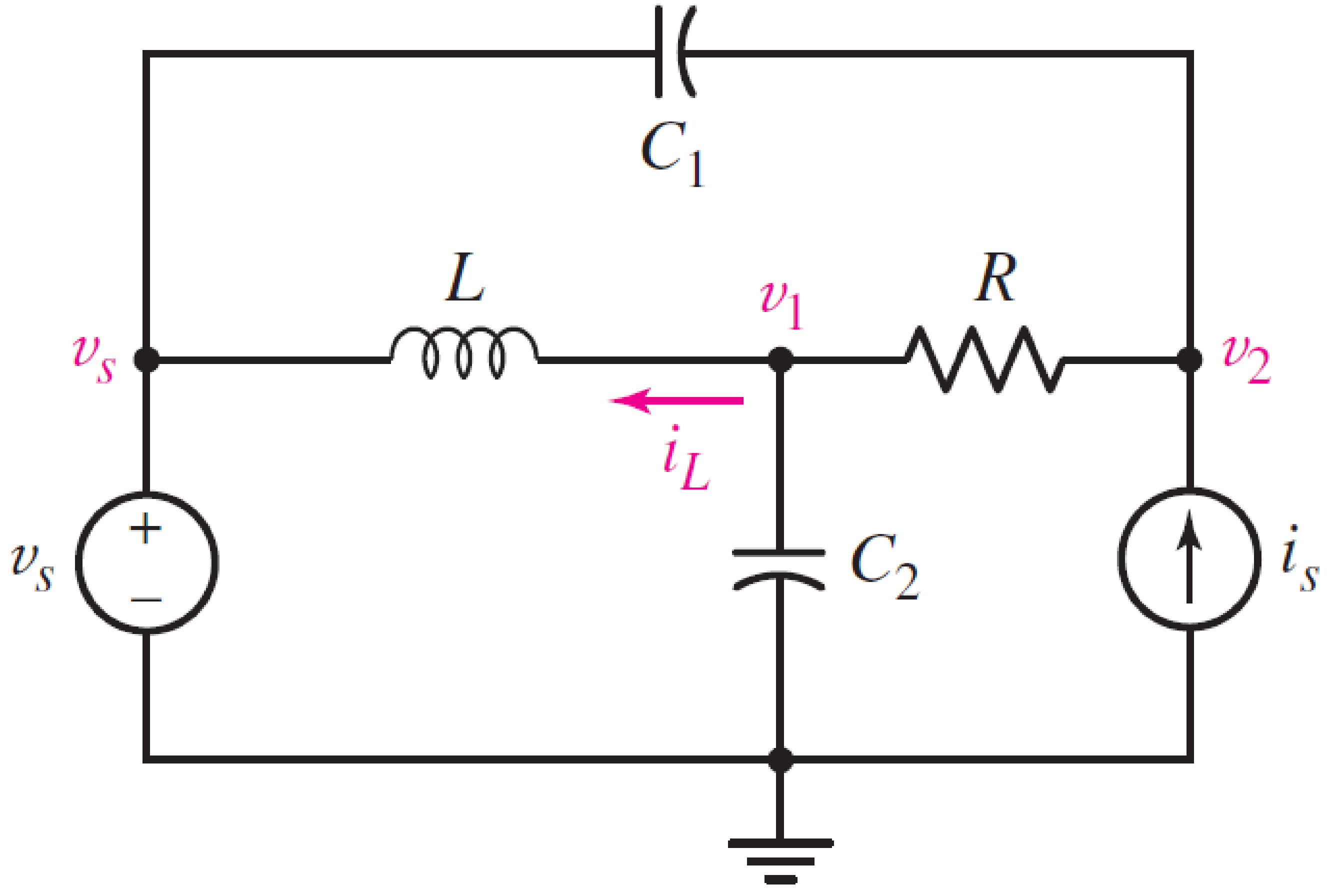
Example of duality





Practice

Write the nodal equations for the nodes v_1 and v_2. Also, get the dual of the circuit and write the current equations of its meshes.



Thanks