اعمال ریاضی با اعداد

اعمال ریاضی باینری: جمع

- قوانین: مانند جمع دسیمال
- با این تفاوت که 10 = 1+1 ← تولید نقلی

```
  +1 +1    +1
   1 0 1 1 0 1
+  0 1 1 0 0 1
-------------
 1 0 0 0 1 1 0
```

- 0+0 = 0(c0) (sum 0 with carry 0)
- 0+1 = 1+0 = 1(c0)
- 1+1 = 0(c1)
- 1+1+1 = 1(c1)

Carry	1	1	1	1	1	0
Augend	0	0	1	0	0	1
Addend	0	1	1	1	1	1
Result	1	0	1	0	0	0

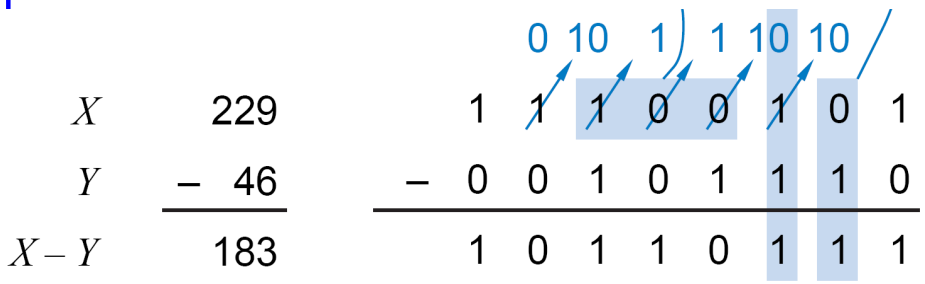
سرریز (Overflow)

- اگر تعداد بیت ها = n و حاصل جمع n+1 بیت نیاز داشته باشد
  - ← سرریز

اعمال ریاضی باینری: تفریق

- قوانین:
  - 0-0 = 1-1 = 0 (b0) (result 0 with borrow 0)
  - 1-0 = 1 (b0)
  - 0-1 = 1 (b1)
  - …

Borrow	1	1	0	0	
Minuend	1	1	0	1	1
Subtrahend	0	1	1	0	1
Result	0	1	1	1	0



روش انجام محاسبات

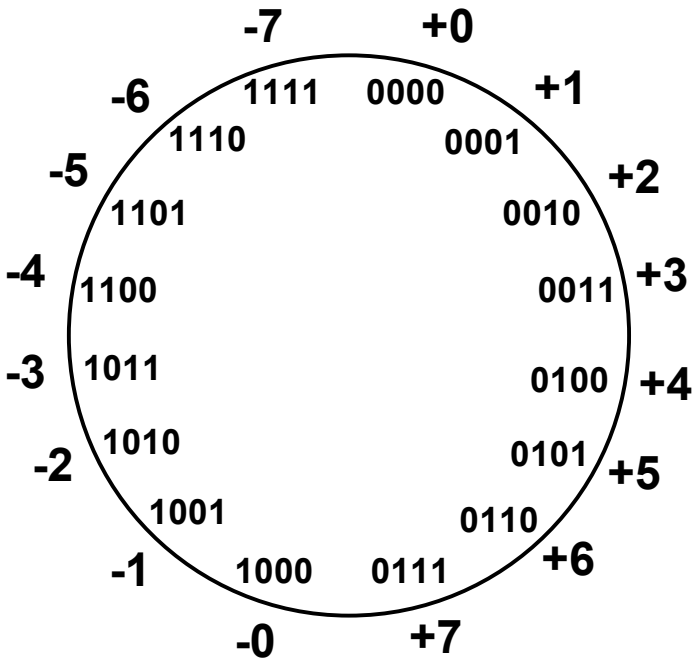
- الگوریتم های اعمال ریاضی مبنای 10 را به خاطر آورید.
- آنها را برای مبنای مورد نظر تعمیم دهید.
- قانون مبنای مورد نظر را به کار برید.
  - برای باینری: 1+1=10

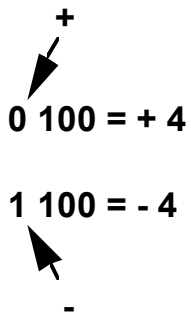
نمایش اعداد

- **نمایش اعداد مثبت:**
  - در بیشتر سیستم ها یکسان است.
- **نمایش اعداد منفی:**
  - اندازه-علامت (Sign magnitude)
  - مکمل 1 (Ones complement)
  - مکمل 2 (Twos complement)
    - در بیشتر سیستم ها: مکمل 2
- **فرض:**
  - ماشین با کلمه های 4 بیتی:
    - ← 16 مقدار مختلف قابل نمایش.
    - تقریباً نیمی مثبت، نیمی منفی.

نمایش اعداد

اندازه–علامت:





High order bit is sign: 0 = positive (or zero), 1 = negative

Three low order bits is the magnitude: 0 (000) thru 7 (111)

Number range for n bits = $[-(2^{n-1} - 1), +(2^{n-1} - 1)]$

Representations for 0

Cumbersome addition/subtraction

Must compare magnitudes to determine sign of result

نمایش اعداد

مکمل 1:

N is positive number, then $\overline{N}$ is its negative 1's complement

$$\overline{N} = (2^n - 1) - N$$

Example: 1's complement of 7

$2^4 = 10000$

$-1 = 00001$

1111

$-7 = 0111$

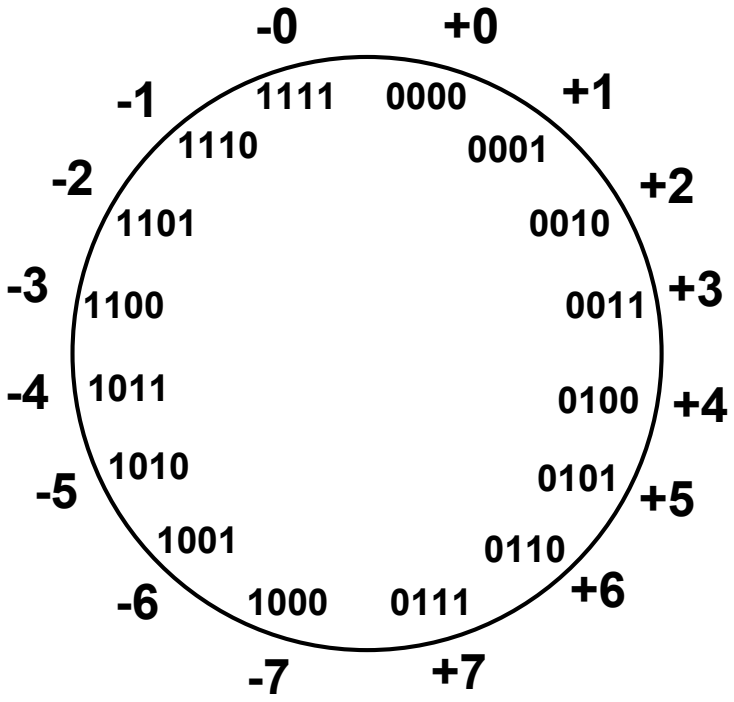
1000 = -7 in 1's comp.

Shortcut method:

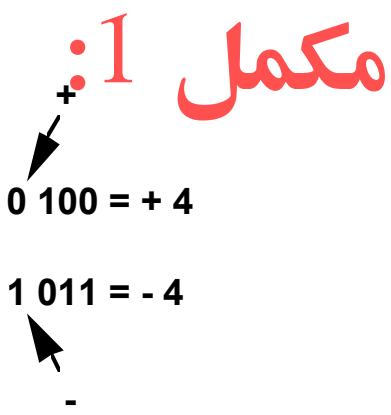
simply compute bitwise complement

0111 -> 1000

نمایش اعداد



مکمل 1:



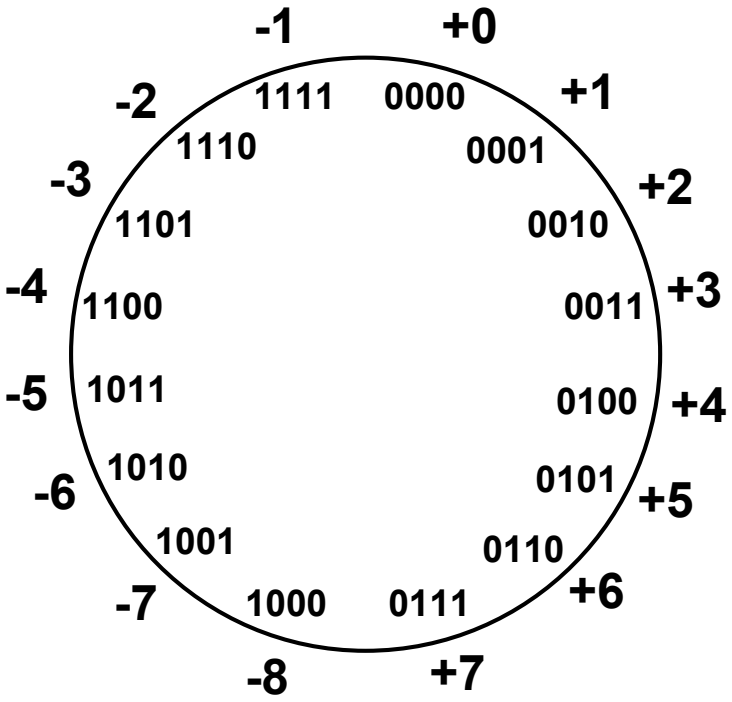
Subtraction implemented by addition & 1's complement

Still two representations of 0! This causes some problems

Some complexities in addition

نمایش اعداد

like 1's comp except shifted one position clockwise



مکمل 2:

+

0 100 = + 4

1 100 = - 4

-

Only one representation for 0

One more negative number than positive number

نمایش اعداد

$N^* = 2^n - N$

مکمل 2:

Example: Twos complement of 7

$2^4 = 10000$

sub 7 = 0111

1001 = repr. of -7

Example: Twos complement of -7

$2^4 = 10000$

sub -7 = 1001

0111 = repr. of 7

Shortcut method:

Twos complement = bitwise complement + 1

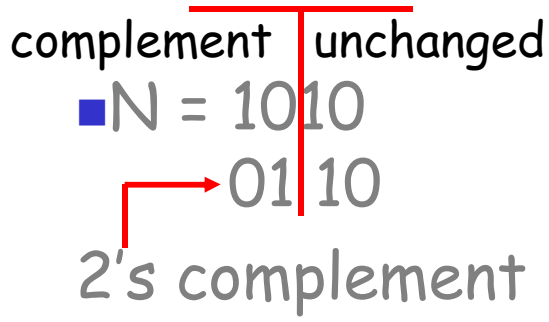
0111 -> 1000 + 1 -> 1001 (representation of -7)

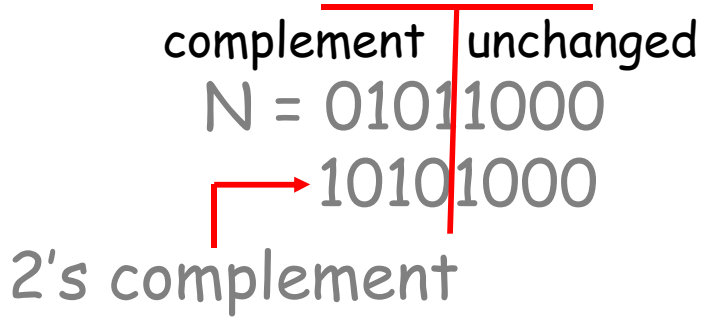
1001 -> 0110 + 1 -> 0111 (representation of 7)

مکمل 2

- **Here's an easier way to compute the 2's complement:**
  1. Leave all least significant 0's and first 1 unchanged.
  2. Replace 0 with 1 and 1 with 0 in all remaining higher significant bits.

- Examples:





جمع و تفریق
مکمل 2

4	0100		-4	1100
+ 3	0011		+ (-3)	1101
7	0111		-7	11001

If
(carry-in to sign = carry-out)
then ignore carry

if
(carry-in ≠ carry-out)
then overflow

4	0100		-4	1100
- 3	1101		+ 3	0011
1	10001		-1	1111

Simpler addition scheme makes twos complement the most common choice for integer number systems

جمع و تفریق
مکمل 2

Why can the carry-out be ignored?

-M + N when N > M:

$$M^* + N = (2^n - M) + N = 2^n + (N - M)$$

Ignoring carry-out is just like subtracting 2^n

-M + -N where N + M < or = 2^{n-1}

$$-M + (-N) = M^* + N^* = (2^n - M) + (2^n - N)$$

$$= 2^n - (M + N) + 2^n$$

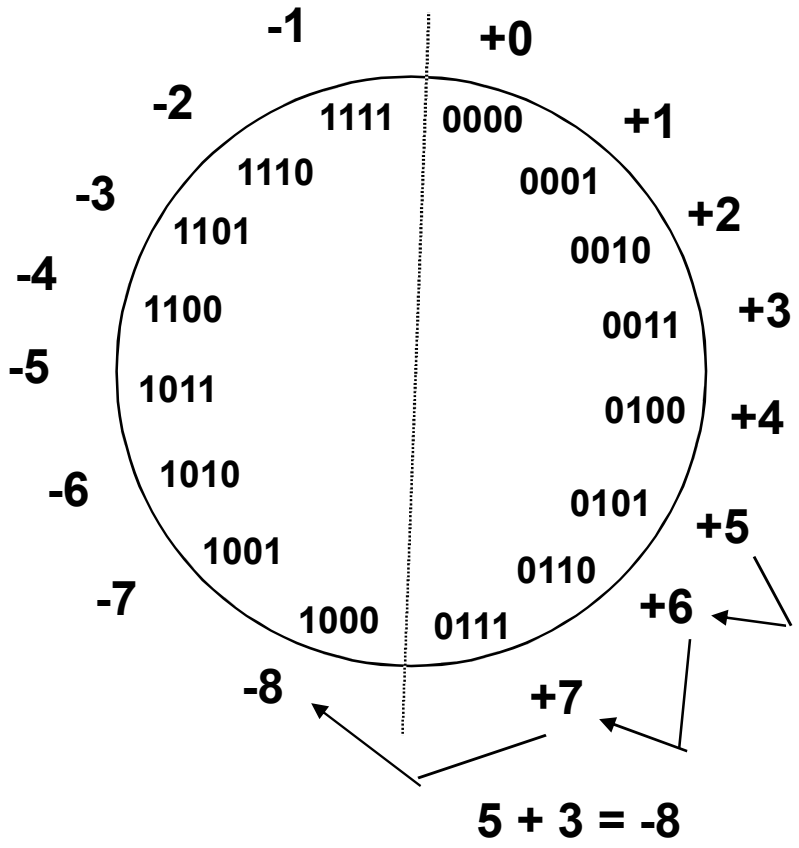
After ignoring the carry, this is just the right twos compl. representation for -(M + N) or (M+N)*

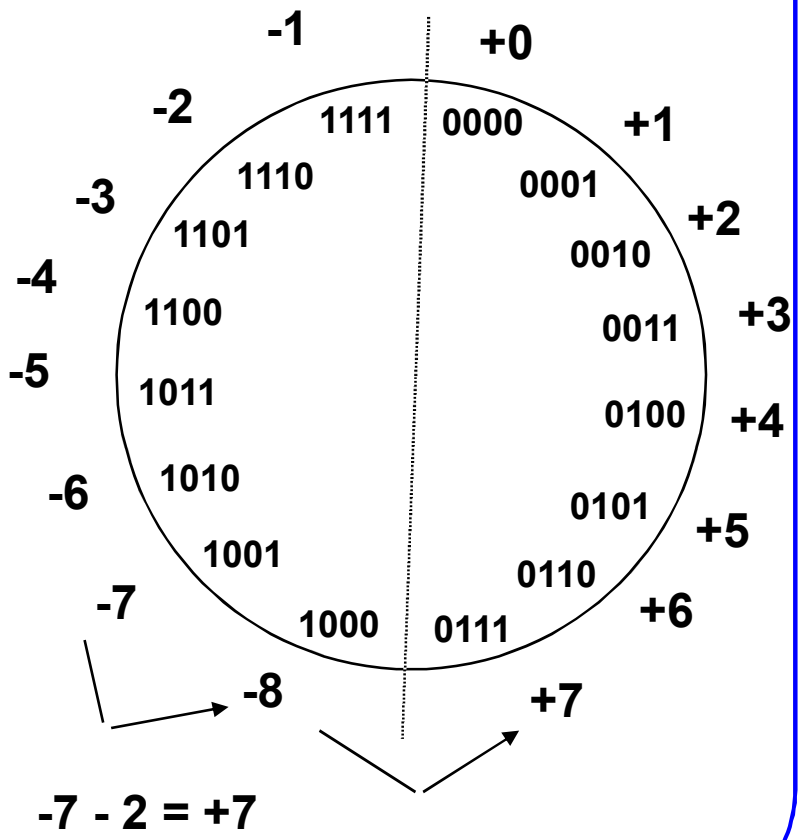
Overflow Conditions

سرریز

Add two positive numbers to get a negative number

or two negative numbers to get a positive number





سرریز

Overflow Conditions

	0 1 1 1
5	0 1 0 1
3	0 0 1 1
-8	1 0 0 0

Overflow

	1 0 0 0
-7	1 0 0 1
-2	1 1 1 0
7	1 0 1 1 1

Overflow

	0 0 0 0
5	0 1 0 1
2	0 0 1 0
7	0 1 1 1

No overflow

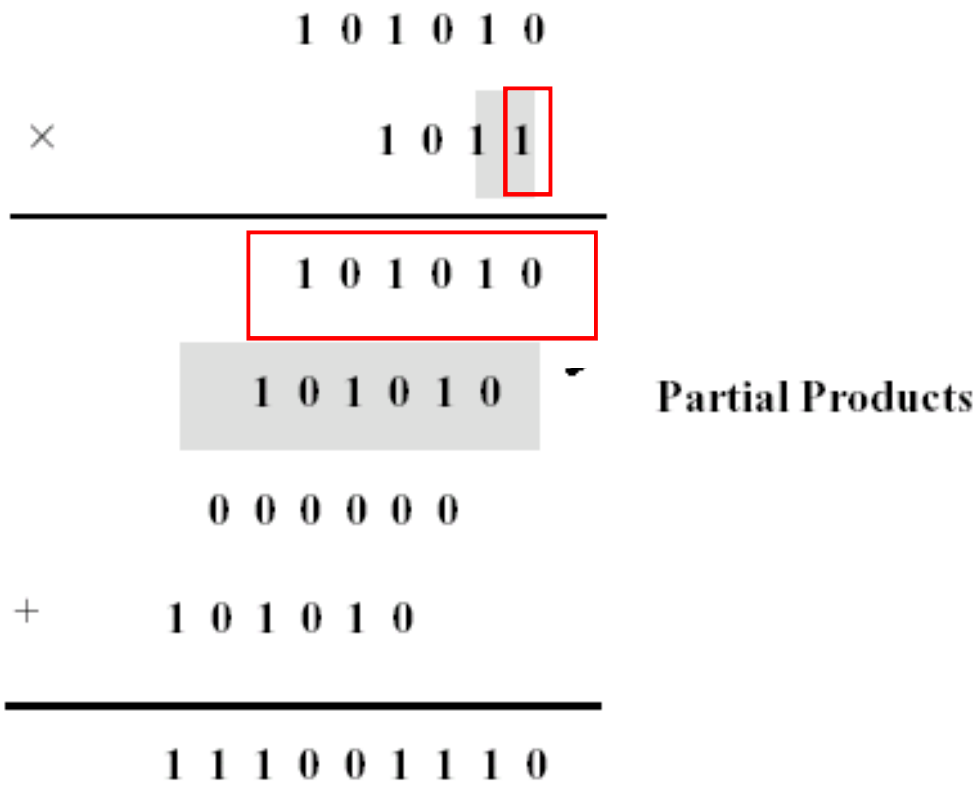
	1 1 1 1
-3	1 1 0 1
-5	1 0 1 1
-8	1 1 0 0 0

No overflow

Method 1: Overflow when carry in to sign ≠ carry out

Method 2: Overflow when sign(A) = sign(B) ≠ sign (result)

The Binary Multiplication



ضرب باینری

- **Shift-and-add algorithm, as in base 10**

M’cand	0	0	0	1	1	0	1
M’plier	0	0	0	0	1	1	0
(1)			0	0	0	0	0
(2)		0	1	1	0	1	
(3)	0	1	1	0	1		
Sum	1	0	0	1	1	1	0

- **Check: 13 * 6 = 78**

Binary-Coded Decimal (BCD)

- A decimal code:
  Decimal numbers (0..9) are coded using 4-bit distinct binary words
- Observe that the codes 1010 .. 1111 (decimal 10..15) are NOT represented (invalid BCD codes)

TABLE 1-3
Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Table 1-3 Binary-Coded Decimal (BCD)

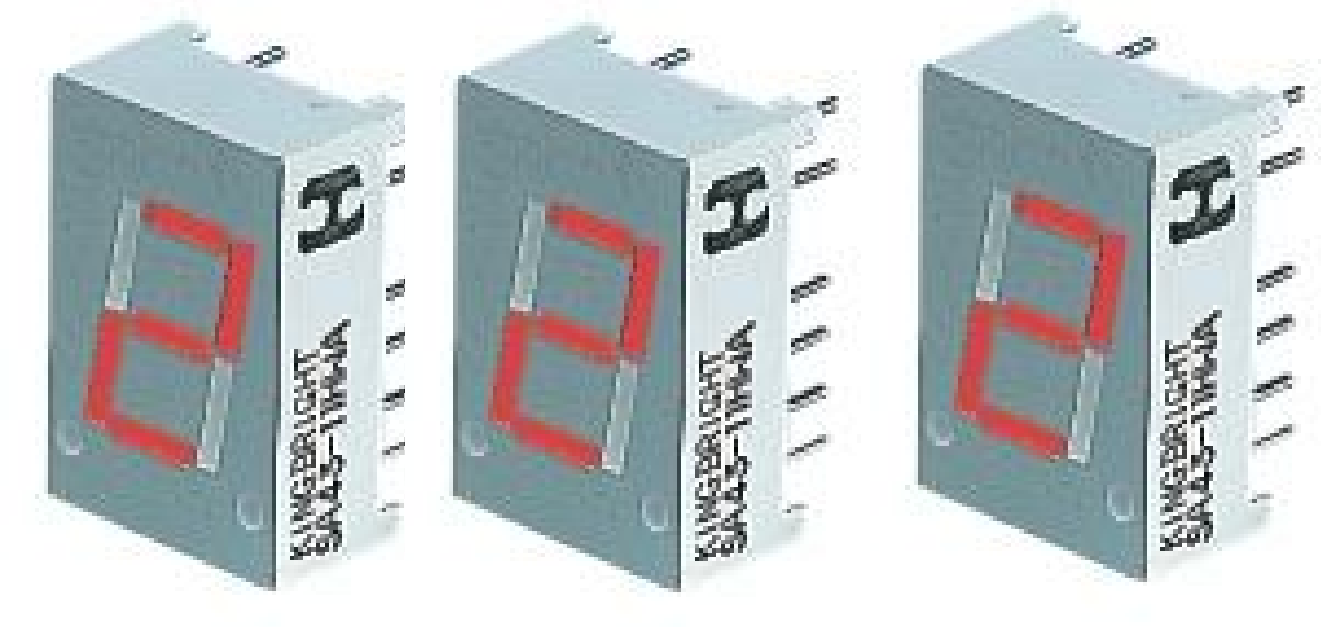
Binary-Coded Decimal

- To code a number with *n* decimal digits, we need *4n* bits in BCD

  e.g. $(365)_{10} = (0011\ 0110\ 0101)_{BCD}$

- This is different from converting to binary, which is $(365)_{10} = (101101101)_2$
- Clearly, BCD requires more bits. BUT, it is easier to understand/interpret

Application



BCD Addition

Case 1:

0001	1
0101	5
(0) 0110	(0) 6

Case 2:

0110	6
0101	5
(0) 1011	(1) 1

WRONG!

Case 3:

1000	8
1001	9
(1) 0001	(1) 7

Note that for cases 2 and 3, adding a factor of 6 (0110) gives us the correct result.

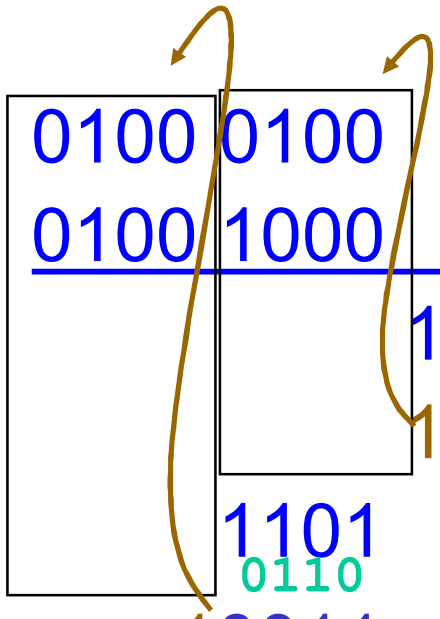
BCD Addition (cont.)

- **BCD addition is therefore performed as follows**
  1) Add the two BCD digits together using normal binary addition
  2) Check if correction is needed
     a) 4-bit sum is in range of 1010 to 1111
     b) carry out of MSB = 1
  3) If correction is required, add 0110 to 4-bit sum to get the correct result;

     → **BCD carry out = 1**

BCD Addition (cont.)

- **Example: Add 448 and 489 in BCD.**



0100 0100 1000 (448 in BCD)
0100 1000 1001 (489 in BCD)
10001 (greater than 9, add 6)
0110
10111 (carry 1 into middle digit)
1101 (greater than 9, add 6)
0110
10011 (carry 1 into leftmost digit)
1001 0011 0111 (BCD coding of 937_{10})

BCD Negative Number Representation

- **Similar to binary negative number representation but for r = 10.**
  - BCD 9's complement
    - invert each BCD digit (0→9, 1 → 8, 2 → 7,3 → 6, …7 → 2, 8 → 1, 9 → 0)
  - BCD 10's complement
    - $-N \equiv 10^n - N$; 9's complement + 1

Excess-3

- مانند BCD ولی هر رقم 3+
  - جمع سرراست تر
  - self-comlpement code
    - (مکمل هر رقم = مکمل 9 آن)

Decimal Digit	BCD				Excess-3			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

ASCII character code

- We also need to represent letters and other symbols → alphanumeric codes
- ASCII = American Standard Code for Information Interchange. Also known as Western European
- It contains 128 characters:
  - 94 printable (26 upper case and 26 lower case letters, 10 digits, 32 special symbols)
  - 34 non-printable (for control functions)
- Uses 7-bit binary codes to represent each of the 128 characters

ASCII Table

$b_3b_2b_1b_0$	Row (hex)	$b_6b_5b_4$ (column) 000 0	001 1	010 2	011 3	100 4	101 5	110 6	111 7
0000	0	NUL Null	DLE	SP Space	0	@	P	`	p
0001	1	SOH	DC1	!	1	A	Q	a	q
0010	2	STX	DC2	"	2	B	R	b	r
0011	3	ETX	DC3	#	3	C	S	c	s
0100	4	EOT	DC4	$	4	D	T	d	t
0101	5	ENQ	NAK	%	5	E	U	e	u
0110	6	ACK	SYN	&	6	F	V	f	v
0111	7	BEL Bell	ETB	'	7	G	W	g	w
1000	8	BS BkSpc	CAN	(	8	H	X	h	x
1001	9	HT Tab	EM	)	9	I	Y	i	y
1010	A	LF Line Fd	SUB	*	:	J	Z	j	z
1011	B	VT	ESC Escape	+	;	K	[	k	{
1100	C	FF	FS	,	<	L	\	l	|
1101	D	CR Crg Ret	GS	-	=	M	]	m	}
1110	E	SO	RS	.	>	N	^	n	~
1111	F	SI	US	/	?	O	_	o	DEL

Unicode

- Established standard (16-bit alphanumeric code) for international character sets
- Since it is 16-bit, it has 65,536 codes
- Represented by 4 Hex digits
- ASCII is between 0000_{16} .. $007B_{16}$

Unicode Table

http://www.unicode.org/charts/

Unicode

062B 1579	ث 062C 1580	ج 062D 1581	ح 062E 1582	خ
0633 1587	س 0634 1588	ش 0635 1589	ص0636 1590	ض
063B 1595	ک 063C 1596	پ 063D 1597	ئ 063E 1598	ی
0643 1603	ك 0644 1604	ل 0645 1605	م 0646 1606	ن
064B 1611	◌ً 064C 1612	◌ٌ 064D 1613	◌ِ 064E 1614	◌َ
0653 1619	◌ٓ 0654 1620	◌ٔ 0655 1621	◌ٕ0656 1622	◌ٖ
065B 1627	◌ٰ065C 1628	◌.065D 1629	◌ٗ065E 1630	◌ٙ
0663 1635	٣ 0664 1636	٤ 0665 1637	٥ 0666 1638	٦
066B 1643	, 066C 1644	, 066D 1645	* 066E 1646	ـ
0673 1651	إ 0674 1652	ٕ 0675 1653	ا 0676 1654	ۇ
067B 1659	ٻ 067C 1660	ٽ 067D 1661	ٹ 067E 1662	پ
0683 1667	ڃ 0684 1668	چ 0685 1669	څ 0686 1670	چ
068B 1675	ڋ 068C 1676	ڌ 068D 1677	ډ 068E 1678	ڈ