Graded exercise on Epidemics

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$$\frac{dx}{dt} = f(x,y)$$

$$\frac{dy}{dt} = g(x,y)$$

(a) SIR model is given by

$$\frac{dS}{dt} = -\frac{\beta}{N} \cdot SI$$

$$\frac{dI}{dt} = \frac{\beta}{N} SI - \gamma I$$

Let $\frac{S}{N} = x, \quad \frac{I}{N} = y, \quad \frac{\beta}{\gamma} = R_0$

rewriting, $\boxed{\frac{dx}{dt} = -\beta x y}$

$$\frac{dy}{dt} = \beta x y - \frac{\beta y}{R_0}$$

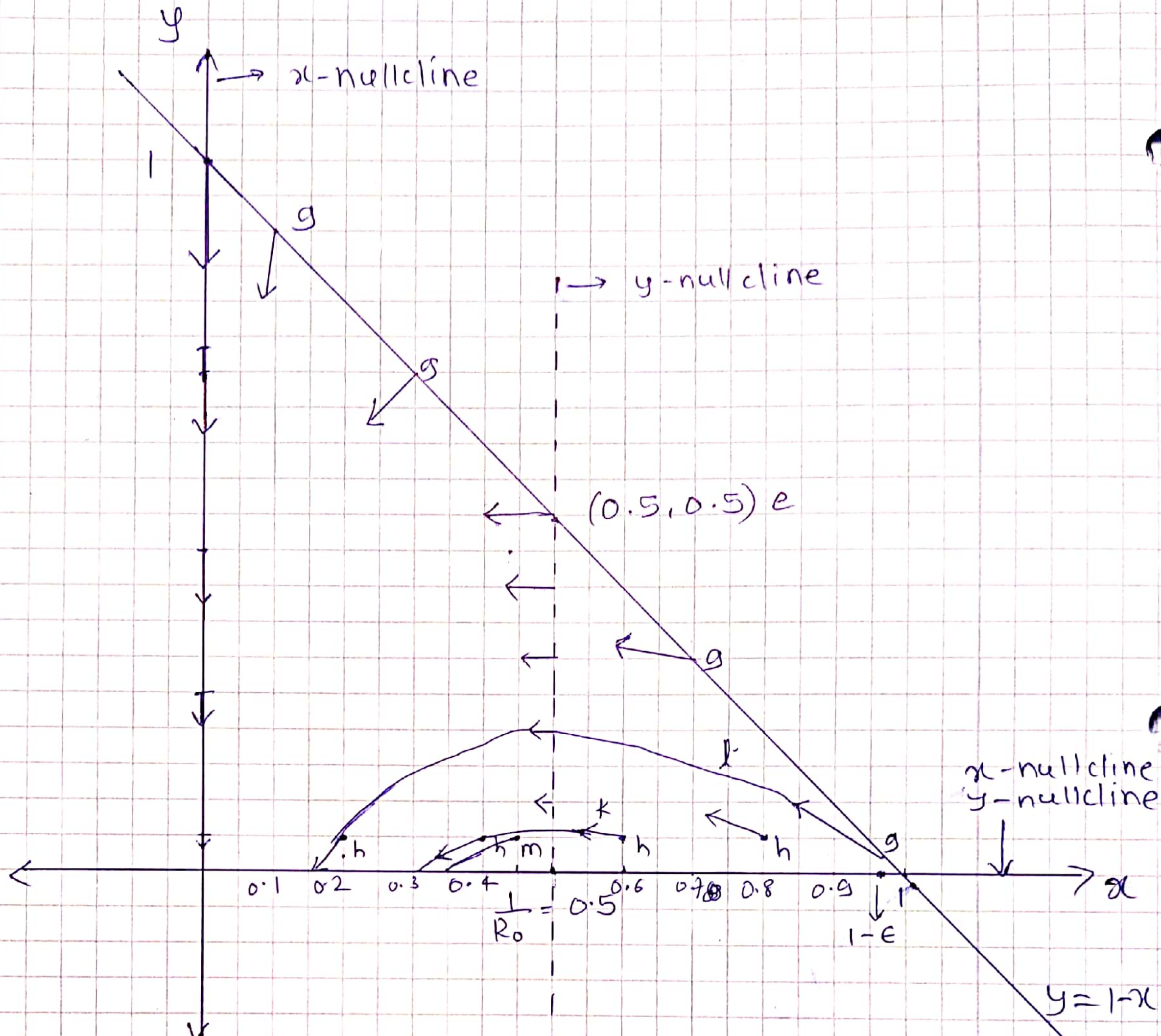
$$\boxed{\frac{dy}{dt} = \beta y\left(x - \frac{1}{R_0}\right)}$$

(b) $R_0 = 2.0$,

Hence, $\frac{dx}{dt} = 0 \Rightarrow \left.\begin{matrix} x = 0 \\ y = 0 \end{matrix}\right\}$ x-nullcline

$\frac{dy}{dt} = 0 \Rightarrow \left.\begin{matrix} y = 0 \\ x = \frac{1}{R_0} \end{matrix}\right\}$ y-nullcline

(c)
(d)
(e)
(j)



(e) $\Delta y = \Delta t \times \frac{dy}{dt} = \Delta t \times \beta x (x - \frac{1}{R_0}) = 0$ $\Delta x = \Delta t \times \frac{dx}{dt} = \Delta t \beta x y = \frac{\Delta t \beta}{4} = 0.1$ for $\Delta t = \frac{0.4}{\beta}$

(f) $$\text{dir}(x) = \frac{\Delta y}{\Delta x} = \frac{g(x,y)}{f(x,y)} = \frac{x - \frac{1}{R_0}}{-x}$$

$$= \frac{1}{R_0 x} - 1 = \frac{0.5}{x} - 1$$

(g) Four points are shown in above figure with label 'g'.

For $x > 0.5$, $\text{dir}(x) < 0$

$x < 0.5$, $\text{dir}(x) > 0$

Points are chosen at $x = 0.1, 0.3, 0.7, 0.9$
(arrow lengths are ≈ 0.1 for better visualization)

(h)

$(0.2, \epsilon) \rightarrow (-0.2, -0.3) \times \beta \epsilon \Delta t$

$(0.4, \epsilon) \rightarrow (-0.4, -0.1) \times \beta \epsilon \Delta t$

$(0.6, \epsilon) \rightarrow (-0.6, 0.1) \times \beta \epsilon \Delta t$

$(0.8, \epsilon) \rightarrow (-0.8, 0.3) \times \beta \epsilon \Delta t$

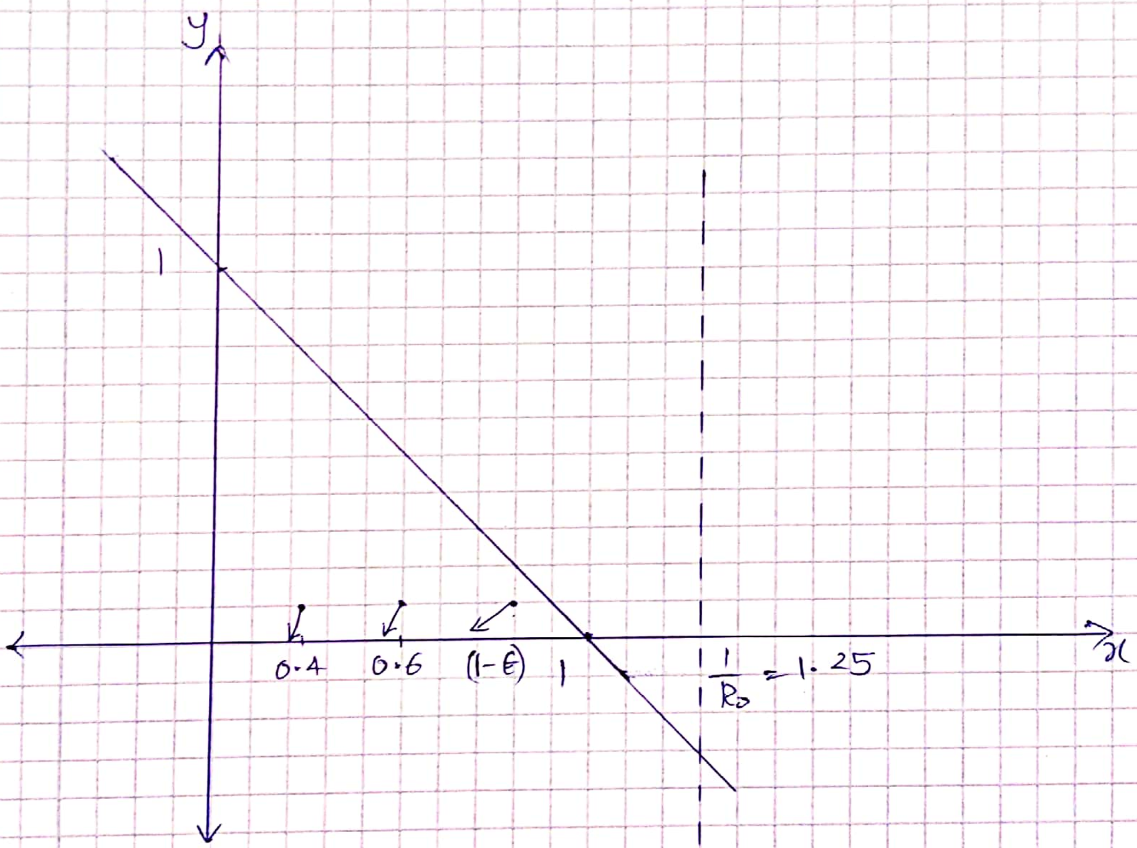
(i) (Arrows are added in the figure with label 'h') (k), (l), (m) $\rightarrow$ labelled in the figure accordingly.

(n) $R_0 = 0.8 \Rightarrow \frac{1}{R_0} = 1.25 > 1$

Thus, $\Delta x = -\beta x y \Delta t \leq 0 \quad \forall \; x, y \in [0,1]$

$\Delta y = \beta y (x - 1.25) \leq 0 \quad \forall x, y \in [0,1]$.

$dir(x) = \frac{1}{R_0 x} - 1 = \frac{1.25}{x} - 1 \rightarrow$ decreases with x



$(0.45, \epsilon) \rightarrow (-0.45, -0.8) \times \beta \epsilon \Delta t$

$(0.6, \epsilon) \rightarrow (-0.6, -0.65) \times \beta \epsilon \Delta t$

$(1-\epsilon, \epsilon) \rightarrow (-1, -0.25) \times \beta \epsilon \Delta t$