UNIVERSITY OF MALAYA

EXAMINATION FOR THE DEGREE OF MASTER OF DATA SCIENCE

ACADEMIC SESSION 2020/2021 : SEMESTER II

WQD7011 : Numerical Optimization

June 2021 Time : 2 hours

INSTRUCTIONS TO CANDIDATES :

Answer **ALL** questions (40 marks).

(This question paper consists of 4 questions on 4 printed pages)

1. Figure 1 shows a contour plot of a function. There is a point marked with a red dot in the diagram, along with 3 search directions.

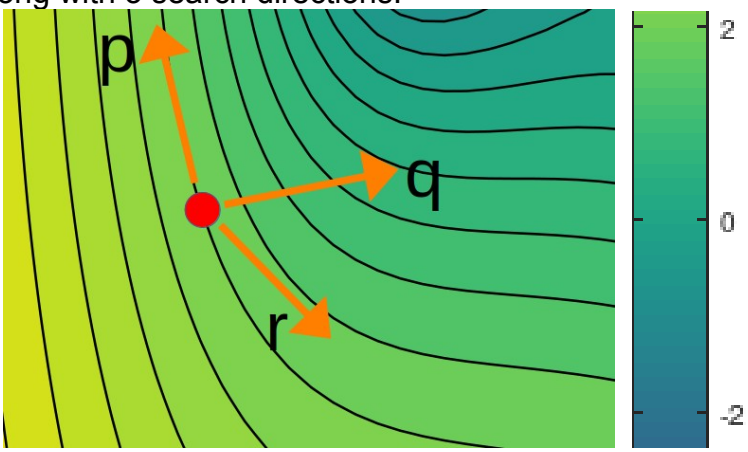

Figure 1

a) Explain why the point is not a local minimizer.

b) At this point, three search directions, *p*, *q* and *r* have been proposed as the next direction for a line search method. Are these directions guaranteed a decrement in the objective function? Why?

c) Which is the one proposed by the Steepest Descent method? Explain your answer.

(5 marks)

2. Answer this question based on the graph in Figure 2:

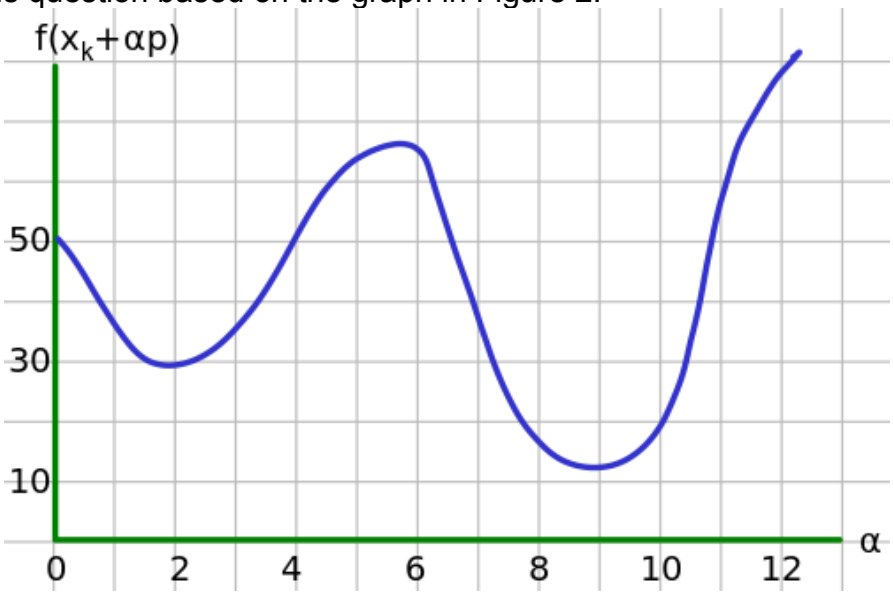

Figure 2

a) Consider a search for a local minimizer starting from point x_k. Given that $\nabla f(x_k)^T p_k = -2000$ and c_1 = 10^{-3}, determine whether each of the following step lengths fulfilled the requirement of Armijo condition and justify your answers:

i) $\alpha = 3$
ii) $\alpha = 4$
iii) $\alpha = 6.5$

(3 marks)

b) Figure 3 shows the graph that given in Figure 2, with additional three points labeled with (i), (ii), and (iii). Discuss whether each of these points is acceptable by the curvature condition.

(3 marks)

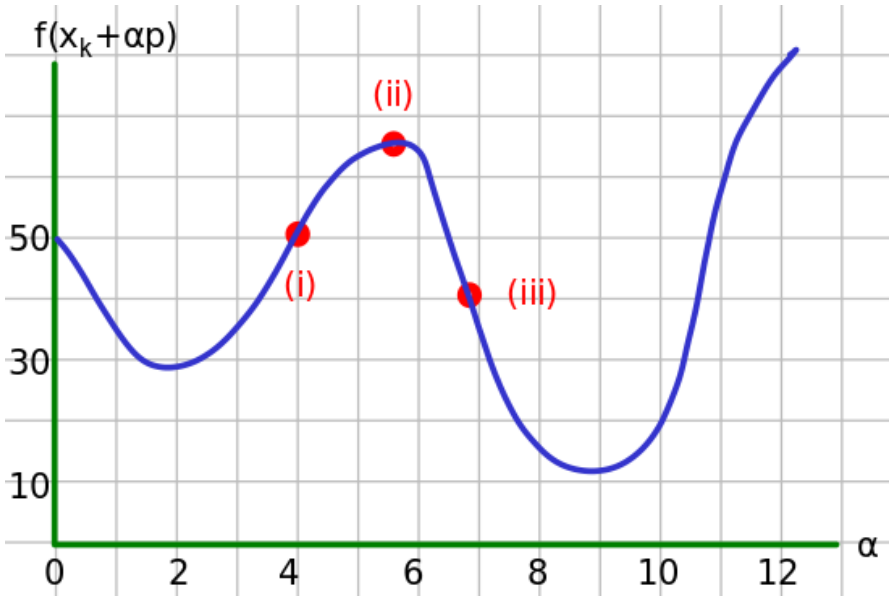

Figure 3

c) Briefly describe the backtracking line search algorithm.

(2 marks)

d) Assume that, for some reason, we start the backtracking line search algorithm with an initial step length $\bar{\alpha}$ = 11. Discuss what is the difference if we have different setting on the contraction factor , ρ as 0.9 and 0.4 respectively.

(2 marks)

3. Consider the following function f(x):$R^3 \rightarrow R$ in a linear programming problem:

$$\min f(x) = 5x_1 - 12x_2 - 10x_3 + 3x_4$$

subject to :

$$-2x_1 + 5x_2 - 3x_3 + x_4 \leq 10$$
$$5x_1 + 2x_3 = 30$$
$$x_1, x_2, x_3, x_4 \geq 0$$

a) Find the optimal solution of the function using simplex method as follow:
   i. Add slack and artificial variables to convert this problem to a standard form. Identify an augmented matrix that representing the problem.

(3 marks)

   ii. From the matrix, identify the initial basic feasible solution for the system, as well as the corresponding value of f(x).

(2 marks)

   iii. Start the pivot process to identify the optimal solution of the problem. State the values of x and f(x).

(5 marks)

b) Instead of finding min f(x), find max f(x) by writing an Octave program.

(5 marks)

4. A function f(x):$R^2 \rightarrow R$ is defined as follow:

$$f(\mathbf{x}) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

a) Find the gradient and Hessian of the function.

(2 marks)

b) A search for a local minimizer starts at the initial position $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the search direction at this initial point if we are using:
i) Steepest descent method
ii) Newton method
Would you recommend to use Newton method without modification at this position? Explain your answer.

(4 marks)

c) Write Octave code to plot the contour of the function for $-8 \leq x_1 \leq 8$ and $-8 \leq x_2 \leq 8$.

(2 marks)

d) Based on the plot in part (c), discuss the function.

(2 marks)

END