KAZAKH KBTU BRITISH
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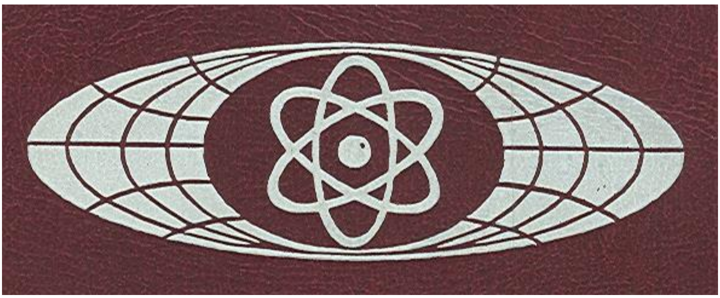
PHYSICS I LAB MANUAL

Mechanics, Molecular Physics and Thermodynamics, Electricity and Magnetism



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KAZAKH KBTU BRITISH
TECHNICAL
UNIVERSITY

FACULTY OF POWER AND OİL & GAS INDUSTRY

PHYSICAL ENGINEERING DEPARTMENT

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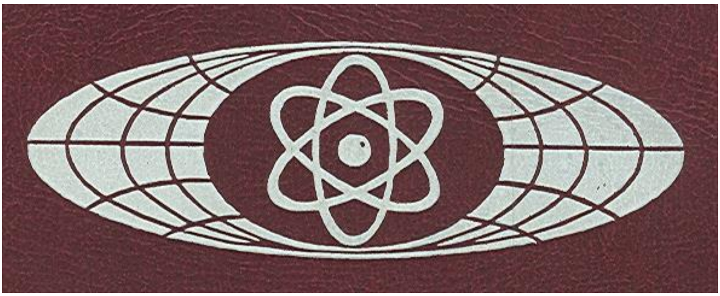

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In this Lab Manual there are descriptions of seventeen laboratory experiments in Mechanics, Molecular Physics & Thermodynamics, Electricity & Magnetism, recommended for Physics Laboratory courses for technical Universities (typical curriculum approved by the Ministry of Education and Science of RK). They include the purpose of the laboratory experiment, a brief theory of studied phenomenon, an experimental set-up, scheme of installation, the order of procedures, tables for entering results of measurements and methods for data processing and analysis.

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Introduction

Laboratory workshop on physics is aimed at training students to work with modern measuring equipment, development of skills of a performing experimental research, processing and analysis of experimental results and extracting concrete physical content in applied problems of their future speciality.

This book contains descriptions of 17 laboratory works, including 5 works on mechanics, 5 works on molecular physics, 7 works on electricity and magnetism, recommended by typical educational program of Department of Education of Republic of Kazakhstan for laboratory workshop for physics course for technical universities. Each description contains objective of work, short theoretical description of studied questions, experimental method, equipment, procedure, tables for experimental results. Least-squares method and Student's method are used for processing experimental results and estimation of errors. A Least Squares Fitting method is usually used for data analysis and making graphs. Graphs can be made by computer or by hand (students can use MathCAD for data proceedings).

Laboratory works on electricity and magnetism are performed on modern educational laboratory complex, made by "Rosuchpribor" of Department of Education of the Russian Federation. This complex includes measuring bench and computer, which enable a student to perform measurements using real and virtual gauges.

When student prepares for laboratory work, he must get acquainted with theory and experimental procedure using these methodical descriptions. The result of preparation of a student to a laboratory work must be reflected in a short laboratory work conspectus. The conspectus must contain the name and the objectives of a laboratory work, worksheets for experimental data, calculation results and errors of measurements. Summary concludes the performed work.

The Lab Report will either be done in the lab or afterwards and then handed in by a deadline stipulated by a lab instructor. The Lab Report must include a brief statement describing the goals of the lab and methods/procedure used, all handwritten data with units and uncertainties, sample printouts of computer generated (or made by hand) graphs and tables, calculations, the major result of calculations (the average value of investigated quantity and uncertainties including all types of error), a short summary considering what could have caused errors in measurement and suggestion how it could be improved and finally, a conclusion including comparison of getting results to accepted values and discussion what the data shows, why, and what was learned. Students must keep all their graded lab reports until they receive the final grade for the course.

The authors would like to thank Drs. Vladimir V. Voronkov and Saadat S. Mamedova for assistance.

1. UNCERTAINTIES OF THE MEASUREMENTS AND ERROR PROPAGATION

1.1. Objectives: familiarization with uncertainty of physical measurements and error propagation. Measurement of length, area and volume of bodies.

1.2. Theory

Measurement of some physical quantity means its comparison with the *unit* of appropriate physical quantity. Two kinds of measurements exist: direct and indirect measurements. In the process of *direct measurement* we simply measure a physical quantity of our interest. When we find it from known relationship with other directly measured physical quantities we deal with *indirect measurement of this quantity*. No measurement made can be ever exact. The *accuracy* (correctness) and *precision* (number of significant figures) of a measurement are always limited by the degree of refinement of the apparatus used, by the skill of the observer, and by the basic physics in the experiment. In doing experiments we are trying to establish the best values for certain quantities, or trying to validate a theory. We must also give a *range of possible true values* based on our limited number of measurements. Why should repeated measurements of a single quantity give different values? Mistakes on the part of the experimenter are possible, but we do not include these in our discussion. We use the synonymous terms *uncertainty, error, or deviation* to represent the variation in measured data. Two types of errors are possible. *Systematic error* is the result of a mis-calibrated device, or a measuring technique which always makes the measured value larger (or smaller) than the "true" value. An example would be using a steel ruler at liquid nitrogen temperature to measure the length of a rod. The ruler will contract at low temperatures and therefore overestimate the true length. Careful design of an experiment will allow us to eliminate or to correct systematic errors. The *least count* is the smallest division that is marked on the instrument. Thus a meter stick will have a least count of 1.0 mm, a digital stop watch might have a least count of 0.01 sec. The *instrument limit of error, ILE* for short, is the precision to which a measuring device can be read, and is always equal to or smaller than the least count.

The Instrument Limit of Error is generally taken to be the least count or the half (1, 1/2) of the least count. If the scale divisions are closer together, you may only be able to estimate to the nearest 1/2 of the least count, and if the scale divisions are very close you may only be able to estimate to the least count. For some devices the ILE is given as a tolerance or a percentage. Resistors may be specified as having a tolerance of 5%, meaning that the ILE is 5% of the resistor's value.

Even when systematic errors are eliminated there will remain a second type of variation in measured values of a single quantity. These remaining deviations will be classified as *random errors,* and can be dealt with in a statistical manner.

Random errors are these that lead to measured values being inconsistent when repeated measures of a constant attribute or quantity are taken. The word random indicates that they are inherently unpredictable and uncontrollable, they are scattered about the true value if a measurement is repeated several times with the same instrument. Random error is caused by unpredictable fluctuations in the readings of a measurement apparatus, or in the experimenter's interpretation of the instrumental reading; these fluctuations may be in part due to interference of the environment with the measurement process.The concept of random error is closely related to the concept of precision. The higher the precision of a measurement instrument, the smaller the variability (standard deviation) of the fluctuations in its readings.

There is one more type of errors called *blunder* which is a big mistake made as a result of lack of care. One may easily avoid a blunder being careful in the process of measurements. The statistical method for finding a value with its uncertainty is to repeat the measurement several times, find the average, and find either the average deviation or the standard deviation. Suppose we repeat a measurement several times and record the different values. We can then find the *average* value, here denoted by a symbol between angle brackets, $< \quad >$, and use it as our best estimate of the reading.

Let the readings obtained in the process of measurement of some physical quantity $\mathbf{X}$ be $\mathbf{X_1, X_2,\ldots,X_i,\ldots X_n}$, where. $\mathbf{X_i}$ is the result of the i-th measurements of X. The *mean value* of X obtained in n measurements is:

$$\langle \mathbf{X} \rangle = (\mathbf{X_1+X_2+X_3+\ldots+X_n})/\mathbf{n} \qquad (1.1)$$

if n approaches to infinity, $\langle \mathbf{X} \rangle$ approaches to the true value of X_0.

The difference between $\langle \mathbf{X} \rangle$ and $\mathbf{X_i}$ is called *absolute error of the i-th measurement* and may be positive or negative:

$$\Delta \mathbf{X_i} = \mathbf{<X> - X_i}, \qquad (1.2)$$

The *standard deviation value* is given by

$$\sigma_n = \sqrt{\frac{\sum (\prec X \succ - X_i)^2}{n-1}}, \quad (1.3),$$

The standard deviation value represents the average distance of a set of scores from the mean. Knowing the standard deviation helps create a more accurate picture of the distribution along the normal curve. A smaller standard deviation represents a data set where scores are very close in value to the mean; a smaller range. A data set with a larger standard deviation has scores with more variance; a larger range. It is convenient to measure the spread of a set of n observations X_I, around their mean value **<X>** . When n approaches infinity standard deviation approaches to a constant limit - σ. The value of $\boldsymbol{\sigma^2}$ is called ***dispersion of the results of the measurements***.

We are more interested in standard deviation of the mean. Mathematicians have shown that the uncertainty of the mean value of a normal distribution is smaller - generally much smaller - than the standard deviation of the entire distribution. They have proven that there is about a 2/3 probability that the "true value" will lie within

$\pm\frac{\sigma}{\sqrt{n}}$ (where σ is a standard deviation of the data, and n is the number of data values) of the mean value, and about 95% probability that the "true value" will lie within twice this distance from the mean value. This number ($\pm\frac{\sigma}{\sqrt{n}}$) is called the "standard deviation of the mean", and whenever we say "standard deviation" in reference to a mean value, we mean "standard deviation of the mean". We will use the standard deviation of the mean (SDEM) as a measure of the precision of our measurements:

$$\frac{\sigma_n}{\sqrt{n}} = \sqrt{\frac{(\prec X \succ - X_i)^2}{n(n-1)}}, (1.4)$$

An uncertainty interval (or error interval) is the "±" part of a measurement.

Confidence probability ,α, is probability that the true value X_0 is within uncertainty interval, between $\langle X\rangle - \Delta X$ and $\langle X\rangle + \Delta X$. The range $2\Delta X$ is called ***confidence interval.*** If n is large enough the confidence probability $\alpha = 0{,}68$ corresponds to the confidence interval:

$\langle X\rangle \pm \sigma_{\langle X\rangle}$, where $\sigma_{\langle X\rangle} = \sigma / (n)^{1/2}$,

$\alpha=0.95$ corresponds to the confidence interval $\langle X\rangle \pm 2\sigma_{\langle X\rangle}$,

$\alpha=0.997$ corresponds to the confidence interval $\langle X\rangle \pm 3\sigma_{\langle X\rangle}$.

Confidence interval (or absolute error for n measurements) is found by:

$$\Delta X = t_s \; S \qquad (1.5)$$

The final result is written in the form

$$X = \langle X\rangle \pm \Delta X \qquad (1.6)$$

for a specified confidence probability α.

Relative error is ratio of the absolute error to the mean value:

$\varepsilon = \Delta X / \langle X\rangle$

Now let's consider the results *of indirect measurements* or so called error propagation. Error propagation is a way of combining two or more *random* errors together to get the third. It is only used in situations where we don't have the ability to measure the same thing several times and thereby estimate the random error on final result directly. These equations assume that the errors are Gaussian (or normal) in nature. Error propagation can also be used to combine several independent sources of random error on the same measurement. For example, you could have a known random error associated with your equipment and find that thing you're studying is physically fluctuating to some degree. If a physical quantity Y is a function of other quantities $X_1, X_2, X_3, \ldots$ and the function $Y=f(X_1, X_2, X_3, \ldots)$ is given it is possible to calculate uncertainty in determination of Y. The most probable value of Y is obtained using mean values of $X_1, X_2, X_3, \ldots$.

Several situations are possible in calculation of errors of indirect measurements.

1) If Y is a function of one variable $Y=f(X)$, then

$$\Delta Y = f'(X)\, \Delta X , \qquad (1.7)$$

where ΔX is the confidence interval for a specified confidence probability α.

2) If Y is a function of several variables $X_1, X_2, X_3, \ldots$ with known values of $\sigma_1, \sigma_2, \sigma_3, \ldots$, then

$$\Delta Y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \nabla x_i\right)^2}, \qquad (1.8)$$

all ΔX_i are found with equal confidence probability α.

3) If function Y= f ($X_1, X_2, X_3, \ldots$) has the form of a product, ratio or power function, it is more convenient to find first relative errors and then confidence interval. Table 1.2 contains errors for several functions of two variables. If three results of measured value are the same you can stop farther measurement.

Table 1.1

#	Function Y	Absolute error
1	$Y=A+B$	$\Delta Y=(\Delta A+\Delta B)$
2	$Y=A-B$	$\Delta Y=(\Delta A+\Delta B)$
3	$Y=A\cdot B$	$\Delta Y=(B\Delta A+A\Delta B)$
4	$Y=A/B$	$\Delta Y=(B\Delta A+A\Delta B)/B^2$
5	$Y=A^n$	$\Delta Y=nA^{n-1}\Delta A$
6	$Y=(A)^{1/n}$	$\Delta Y=(1/n)A\ \Delta A$
7	$Y=\sin A$	$\Delta Y=\cos(A)\Delta A$
8	$Y=\ln A$	$\Delta Y=\Delta A/A$
9	$Y=e^A$	$\Delta Y=e^A\ \Delta A$

1.3. Measuring instruments

Calliper. A calliper or vernier calliper (or just vernier) is a common tool used in laboratories and industries to accurately determine the fraction part of the least count division. The vernier is a convenient tool to use when measuring the length of an object, the outer diameter of a round or cylindrical object, the inner diameter of a pipe, and the depth of a hole (fig.1.1).

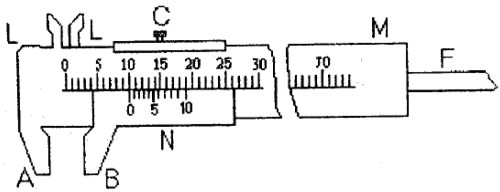

Fig 1.1

The vernier consists of a main scale M engraved on a fixed ruler, rigidly attached to the leg LA and an auxiliary vernier scale engraved on a movable jaw. The movable auxiliary scale is free to slide along the length of the fixed ruler. This vernier's main scale is calibrated in centimetres with the smallest division in millimeters. The auxiliary scale has 10 divisions that cover the same distance as 9 divisions on the main scale. The movable part of the calliper has a stop screw C. When the vernier is closed and properly zeroed the first mark (zero) on the main scale is aligned with the first mark on the auxiliary scale. When measuring external size of an object, legs A and B are moved to touch the object. Then the movable leg is fixed by the stop screw C and a reading is taken. Once the vernier is positioned to make a reading, make a note of where the first mark on the auxiliary scale falls on the main scale. The whole number of millimeters is taken from the main scale up to the zero division of the vernier and fractions of millimeter are taken from the vernier scale (fig 1.2). M divisions of the vernier scale correspond to (m-1) divisions of the main scale.

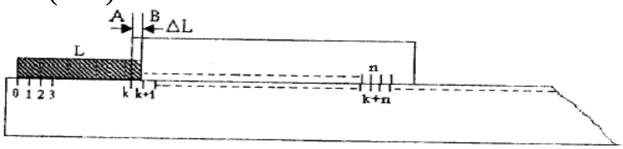

Fig.1.2

When measuring the length of an object L , the n-th division of the vernier scale coincides with a division on the main scale, then measured length is:

$$L= k \cdot a + a\, n /m , \qquad (1.9)$$

where **k** is a whole number of divisions of the main scale, **a** is the main scale division value (usually a is 1mm), **n** is the number of vernier scale division coinciding with a division on the main scale, a/m is the vernier scale division value.

Micrometer. A **micrometer**, sometimes known as a **micrometer screw gauge,** is a device widely used in mechanical engineering and machining for precisely measuring (fig 1.3). A micrometer is composed of *frame, anvil, sleeve (barrel), screw inside the barrel, lock nut, spindle and thimble*. *Frame* is a C-shaped body that holds the anvil and barrel in constant relation to each other. It is thick because it needs to minimize flexion, expansion, and contraction, which would distort the measurement. The frame is heavy and consequently has a high thermal mass, to prevent substantial heating up by the holding hand/fingers. For micrometers the typical accuracy range is 1/100 mm. *Anvil* is a shiny part that the spindle moves toward, and that the sample rests against. *Sleeve / barrel / stock* is a stationary round part with the linear scale on it and sometimes vernier markings. *Lock nut / lock-ring / thimble lock* is a knurled part (or lever) that one can tighten to hold the spindle stationary, when momentarily holding a measurement. *Screw* is the heart of the micrometer, its "measuring screw". One turn of the screw moves the end of the screw to one pitch. *Spindle* is a shiny cylindrical part that the thimble causes to move toward the anvil. *Thimble* is a part that turned by one's thumb.

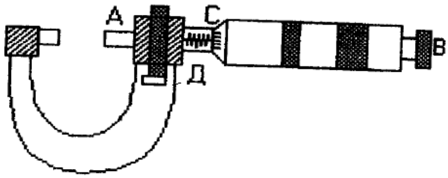

Fig.1.3

The screw has a hollow barrel which passes over collar scale and is graduated in a suitable number of divisions. The horizontal scale with the division value of 0.5 mm contains marks made on both sides of the longitudinal line. Upper divisions are shifted with respect to lower divisions by half division. If the number of divisions in the vertical scale is n =50 and the pitch of the screw is h =0.5mm, then one turn of the screw moves the end of the screw by 0.5 mm and the vertical division value is a=h/n =0.5/50=0/01 mm. An object to be measured must be clamped between unmovable rod and the movable end of the screw A. First you have to read number of divisions from the lower horizontal scale (mm scale), then from the upper horizontal scale made with accuracy 0.5 mm and then to add the reading of vertical scale. The reading of the vertical scale gives hundredth of millimeter. The number of the hundredth corresponds to the mark of vertical scale opposite to the longitudinal line of the horizontal scale.

1.4. Experimental procedure

1.4.1. Familiarization with the structure of a caliper and micrometer. Find the value of division of the main scale and the vernier scale of the caliper and vertical scale for the micrometer.

1.4.2 Choose a measuring instrument for measuring the dimensions of the given object.

1.4.3. Make at least 5 readings for each size of the object and record the readings into table 1.2.

Table 1. 2The linear sizes of the measured object.

№	a_i,mm	b_i,mm	c_i,mm
1			
2			
3			
4			
5			

1.4.4. Calculate the mean values for each parameter and write them down below the table in the form

$<a>=$ $<b>=$ $<c>=$.

1.4.5. Calculate absolute errors of each parameter, square it and write down into table 1.3.

Table 1.3.Data for calculation of errors

№	Δa_1, mm	Δa_1^2, mm^2	Δb_1, Mm	Δb_1^2,mm^2	Δc_1, mm	Δc_1^2, mm^2
1						
2						
3						
4						
5						

1.4.6. Using formula (1.4) calculate standard deviation of the mean value.

1.4.7. Calculate the confidence interval for your number of measurements and confidence probability α=0.95.

1.4.8. Write down the final result of each linear size in the form $X= <X>\pm \Delta X$, where X is either a, b or c and : $<X>$ and ΔX are your numerical results.

1.4.9. Find relative errors for each linear size.

1.4.10. Calculate volumes of given objects, find absolute and relative uncertainties for the volume using most appropriate formula from the table 1.2.

1.4.11 Write down the final result for the volume in the following form: V= <V>± ΔV, where <V> and ΔV are your numerical results.

1.4.12. Make conclusions about your results of measurement and error calculations.

Control questions:

1. Uncertainties of measurements and their kinds.
2. In what way does one find the value of the vernier scale division?
3. What is systematic uncertainty of a calliper and a micrometer?
4. Give examples of direct and indirect measurements.
5. How to get relative or absolute error of indirect measurements?
6. What is an absolute error of a reading if a random error is zero?
7. What are absolute and relative errors?
8. What are mean value, confidence interval, confidence probability?

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Lab 2. DETERMINATION OF THE ACCELERATION OF GRAVITY USING THE ATWOOD MACHINE

2.1. Objectives: The objectives of this lab are to study an application of Newton's Second Law of motion and to measure the constant acceleration of gravity.

2.2 Theory: *Translational motion* is a motion of a rigid body in such a way that any line which is imagined rigidly attached to the body remains parallel to its original direction, all points of the body have the same velocity and have the same acceleration, their paths coincide being superimposed. Hence description of motion of any point of the body is enough to characterize translational motion of whole body totally. It is possible to describe the translational motion of an extended body by following the motion of its centre of mass. This motion is equivalent to that of a point particle, which mass equals that of the body, which is subject to the same external forces as those that act on the body. Newton's second Law relates the resultant force $\boldsymbol{F_{net}}$ acting on such object to its inertia (mass $\boldsymbol{m}$) and its acceleration ($\boldsymbol{a}$), namely:

$$\boldsymbol{F_{net}} = m\boldsymbol{a} \tag{2.1}$$

Where $\boldsymbol{F_{net}}$ and $\boldsymbol{a}$ are vector quantities. We will use a simple machine called the Atwood machine to study this relationship. The Atwood machine is simply two objects suspended by a string over a pulley (see fig. 2.1). This device provides a particularly easy way to study Newton's second law, because the inertia of the system is the sum of the masses, and the resultant force on the system is the difference in their weights (mass times gravitational acceleration). The arrangement of the weights minimizes the effects of random errors so this is a fairly accurate method for measuring **g**. Since this experiment can be done without the use of electronic timers, it was one of the original ways to measure g accurately. (It is doubtful that accurate timers have been available. So, this method is mainly of historic interest). Ideally, the pulley would be firmly supported and frictionless. The cord connecting the two masses would be unstretchable and less mass. In this laboratory, we will perform free fall experiments to measure the acceleration of masses in the Atwood machine to confirm the validity of the Second Law. Consider fig. 2.1 showing the Atwood machine and a free body diagram for each mass. Suppose we apply Newton's second law to each mass.

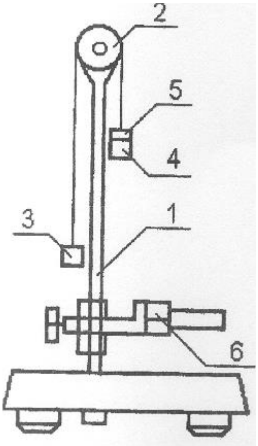

Figure 2.1 Experimental set-up

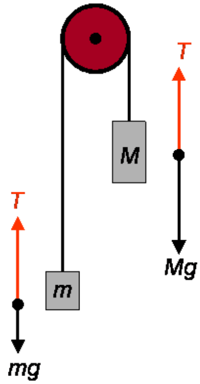

Figure 2.2 A free body diagram for each mass.

The equation of motion of the right block of the mass $M = (m+m_1)$ and the left block of the mass m over massless and frictionless pulley by Newton's second law of motion is:

$$(m+\Delta m)\cdot g - T = (m+\Delta m)\cdot a \qquad (2.2)$$

$$m\,g - T = -\,m\,a \qquad (2.3)$$

where **a** is the acceleration of the system, **T** is the tension of the cord, **g** is the gravitational acceleration, **m** is the mass of blocks 3 and 4, and **Δm** is the additional mass. Solution of these equations gives us the tension of the cord and the magnitude of the acceleration.

$$T = \frac{2m + 2\Delta m}{2m + \Delta m} \cdot mg = \frac{1 + \Delta m / m}{1 + \Delta m / 2m} \cdot mg, \qquad (2.4)$$

$$a = \frac{\Delta m}{2m + \Delta m} \cdot g, \qquad (2.5)$$

$$g = \left(1 + \frac{2m}{\Delta m}\right) \cdot a, \qquad (2.6)$$

More accurate solution requires taking into account mass of the pulley M. In this case cord tensions on each side of the pulley are not the same as in the case of the massless pulley and corresponding equations are:

$$(m+\Delta m) \cdot a = (m+\Delta m)\cdot g \quad - \quad T_2,$$
$$-m\, a = m\, g - T_1, \qquad (2.5)$$
$$I\, \alpha = CmR^2\, \alpha = (T_2 - T_1)\, R$$

where $I = CmR^2$ is rotational inertia of the pulley, R is a the radius of the pulley, α is its angular acceleration and C is a coefficient which depends on distribution of mass of the pulley. (C=1/2 for uniform disk or solid cylinder). Let's express T_1 and T_2 from the first and second equations and substitute into the third equation. The final equation for acceleration is:

$$a = \frac{\Delta m}{2m + \Delta m + M/2} \cdot g, \qquad (2.7)$$

$$g = \frac{2m + \Delta m + M/2}{\Delta m} \cdot a, \qquad (2.8)$$

2.3. Equipment

The experimental setup LKM-4 consists of the pulley and a light cord with two blocks suspended from it, which passes over the pulley. There is also a control board with buttons and toggle switches at the base of the setup. To turn on the setup use the right switch "Set". For measurement of the angular acceleration of the pulley you can choose either the single mode or a double mode. At the switch position "1" the device measures the time needed for the pulley to make one full revolution or the angular displacement 2π. In the position "2" it measures the time needed for the pulley to make two full revolutions or the angular displacement 4π. The switch «Gotov» resets device to zero and starts new reading. Counter and indicator are set to zero by the signal 'Ust.0'. Time measurement begins only after pushing the button "Pusk". Electronic block permits to make direct measurement of period of oscillations.

2.4. Experimental procedures:

1. Put the cord with two blocks 3 and 4 over the pulley. Make sure that the system is in the position of the neutral equilibrium.

2. Pull down the left block and hold it pressed to the base of the panel. Turning the pulley set the slit in the photo detector's gap (The indicator at control panel ISM must be lightening.) Make sure that the switch '0,1/MC/0,01' is set in the position "MC", the switch '1/2'is set in the position "2".

3. Put on the right block an additional weight – first 10 g. Push the button "Pusk". After releasing the system the right block moves down and the timer begins to count a time. Record the reading of the timer and write it into the table 2.1.

4. The angular acceleration of the pulley can be calculated from the measurement of the time needed for the system to make corresponding angular displacement: $\alpha = 2\varphi/t^2$.

If the switch "1/2" is in the position "1" $\varphi = 2\pi$, if it is in the position "2" then $\varphi=4\pi$. The linear acceleration of the blocks can be calculated as:

$$a = \alpha \cdot R = 2\varphi R/t^2 \qquad (2.9)$$

Here **R**=25mm is a radius of the pulley and ***t*** is a reading of the timer.

5. Repeat time measurements 5 times.

6. Repeat the same measurement with additional masses: 20 g, 30g, 40g and 50g. Record all data into table 2.1.

Table 2.1

#	Δm, kg	t, ms	a, m/s^2	g, m/s^2
1				
2				
3				
4				
5				

7. Calculate acceleration of gravity by the following procedure: find average time for each additional mass, calculate for this time linear accelerations a using equation 2.9 and calculate $\boldsymbol{g}$ by formulae (2.8) ($\boldsymbol{\varphi}$=4π, if the switch is in the position "2", **R**=25mm). Take mass of the block **m**=50.5 gram and mass of the pulley **M**= 40g.

8. Find appropriate errors and confident interval for calculated values of the gravitational acceleration using Student's method and compare obtained result with accepted value of the gravitational acceleration for the latitude of Almaty.

(g = 9,804 m/s^2)

9. Analyze obtained results and make conclusion about the precision of your experiment.

Control questions

1. What parameter from your point of view makes more contribution into the final uncertainty of the calculated quantity?
2. How can we take into account the mass of the pulley?
3. Can we test the Newton's second law using this experimental setup?
4. Can you offer alternative method for determination of the block's acceleration?

1.6. References:

1 Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed, Pearson Prentice Hall, New Jersey, USA, 2005.

2. Зайдель А.Н. Ошибки измерений физических величин. - Л.: Наука, 1974.

LAB. 3. DETERMINATION THE SPEED OF A BULLET SHOT FROM A GUN USING BALLISTIC METHOD

3.1. **Objectives:** determination the speed of the bullet by ballistic method based on the conservation of energy and conservation of angular momentum laws.

3.2. **Method:** The "ballistic pendulum" carries this name because it provides a simple method of determining the speed of a bullet shot from a gun. To determine the speed of the bullet a relatively large, massive frame with additional masses is suspended as a pendulum. The bullet is shot into the special block without penetrating clear through it. This is a type of "sticky" collision where the two masses (bullet and block) "stick" to one another and move together after the collision. By noting the angle to which the block and bullet swing after the collision, the initial speed can be determined by using conservation of angular momentum. The angle indicator can be used to measure the maximum angle reached by the pendulum as it swings after the collision. Note: The angle indicator should read zero when the pendulum is hanging in the vertical position. If the reading is measurably different from zero, then take the difference in the angle readings (max angle reading minus vertical angle reading). A" bullet" strikes a "target" after a shot and sticks to it. The time interval of the collision of the bullet and the target is so short that torques due to force of gravity and tension of the thread about vertical axis are zero. So the torque about this axis due to external forces is zero and total angular momentum of the frame and the bullet is conserved in the collision.

The angular momentum of the bullet before collision is $L_i=mvl$, where m is the mass of the bullet, v is its speed, and l is the distance between the axis and the line along which the bullet initially moves. After collision the frame with weights sets in rotation with angular speed ω and its angular momentum is

$$L_f=(I_p+2Ml_1^2)\omega \qquad (3.1)$$

where $\mathbf{I_p}$ is the angular momentum of the frame without weights, **M** is the mass of each additional weight, $\mathbf{l_1}$ is their distance from the axis of rotation. We will ignore rotational inertia of the stacked bullet, because its mass is small compared to the mass of the frame and additional weights (3.30 g).

According to the conservation of angular momentum law: $L_i=L_f$ and hence

$$V = (I_p+2Ml_1^2)\ \omega\ /ml \qquad (3.2)$$

Next, we have to calculate the angular speed of the frame ω and the rotational inertia of the frame with weights $(I_p+2Ml_1^2)$ The angular speed can be found by the maximum angle reached by the frame after the collision (or angular displacement) - φ_m . According to the conservation of energy law the rotational kinetic energy of the frame after the collision will be transformed into potential energy of the twisted thread:

$$(I_p+2Ml_1^2)\ \omega^2/2=D\ \varphi_m^2/\ , \qquad (3.3)$$

where **D** is the torsion modulus of the thread. Torsion modulus is a coefficient of proportionality between a torque due to restoring forces M_{el} and a twisting angle – φ:

$$M_{el} = -D\ \varphi \qquad (3.4)$$

Sign minus in the formula shows that the direction of the torque due to elastic forces is opposite to the twisting angle.
So the angular speed can be calculated as:

$$\omega = \varphi\ [D/(I_p + 2Ml_1^2)]^{1/2} \qquad (3.5)$$

Then we have to find a way how to calculate the rotational inertia of the frame with weights and torsion modulus D. The values of these two quantities- D and $(I_p+2Ml_1^2)$ specify the period of the frame oscillations. They can be calculated from the measurement of the period of oscillations of the pure frame without weights and with additional weights. In order to understand the relationship between the period and these quantities let's consider rotation of a frame suspended from the elastic thread under the torque due to the restoring force:

$$Ip\ddot{\varphi} = -D\varphi\ , \qquad (3.6)$$

where I_p is the rotational inertia of the frame and the second derivative from the angular displacement with respect to time is an angular acceleration. This equation can be reduced to

$$\ddot{\varphi} = -\omega_0^2\varphi, \qquad (3.7)$$

where $\omega_0 = \sqrt{\dfrac{D}{I}}$ .The equation describes simple harmonic motion with frequency ω_0. Period of the oscillations can be found by formula:

$$T_0 = 2\pi/\omega_0 = 2\pi\sqrt{I/D}. \tag{3.8}$$

If T_1 is the period of oscillations of the frame without weights and T is that for the frame with weights, then

$$T_1 = 2\pi/\omega_0 = 2\pi\sqrt{I_p/D}, \tag{3.9}$$

$$T = 2\pi/\omega_0 = 2\pi\sqrt{(I_p + 2Ml_1^2)/D}\,.$$

Using this formula we can obtain following expression for angular speed:

$$\omega = 2\pi\varphi_m /T. \tag{3.10}$$

Eliminating torsion modulus from the equations we can find the rotational inertia for the frame with weights:

$$I_p+2Ml_1^2= 2Ml_1^2T^2/(T^2-T_1^2). \tag{3.11}$$

Substitution (3.11) into (3.2) gives the *final formula for the speed:*

$$v = \frac{4\pi M l_1^{\,2} T \overline{\varphi}_m}{ml(T^2 - T_1^{\,2})} \tag{3.12}$$

3.3. Experimental set up: The main element of the experimental setup is a torsion pendulum which is a metal frame 1 suspended from a steel string 2. The string is fastened vertically on the post 3 with the base 4. The frame oscillates about vertical axis passing through its center of symmetry. Two additional masses can be attached to the frame symmetrically from the both sides.

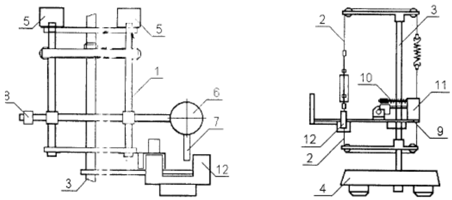

Fig. 3.1 Scheme of the experimental set-up

The disk 'target" 6 is covered with the thin layer of plasticine; the signal flag 7 and counterweight also are attached to the frame. Thin metal ring is used as a bullet. The 'pistol', consisting of a pilot spin with the spring 10 and release device 11, is attached with the holder 9 to the post. The photo detector 12 (lamp + photo receiver) connected with electronic module detects the time and number of oscillations.

3.4. Experimental procedures:

1. Regulate the position of the base using adjusting bolts; the suspension thread should be vertical.

2. Place weights on the frame.

3. Mount "the target" on the frame. Make sure that 'the target' is on the firing line and perpendicular to it and oscillating signal flag crosses optical axis of photo detector.

4. Place "the bullet" on the pilot spin of the "pistol", stretch the spring and fire a shot. Find by sight the angle of maximal displacement of the frame φ_m using the scale of angular displacement and signal flag fixed on the frame. Repeat the shot and angle reading not less than three times.

5. Measure the distance from the frame rotation axis to the center of the mark made by "the bullet" in the target by calliper.

6. Displace the frame with weights through the angle of 40^0 and fix it using electromagnet. Press the key "Sbros" of electronic unit, at that time counters of time and number of oscillation are set to zero. Press the key "Pusk", at that time electromagnet is turned off and torsion oscillations begin. Find time *t* of ***N*** oscillations of the frame. Press the key "Stop"after (*N-1*) total oscillations to register time. The device stops time reading

at the moment of completing of **N** oscillations. Choose **N**=10÷15.

7. Using calliper measure distance $\mathbf{l_1}$ from the rotational axis to the center of weight.

8. For the frame without weights repeat item 6 and find time $\boldsymbol{t_1}$ of $\boldsymbol{N}$ oscillations. N=10÷15.

9. Record mass of weights **M** and "the bullet" **m**.

10. Calculate the average value of the angle of maximum displacement $\overline{\varphi}_m = \frac{\sum_{i=1}^{n} \varphi_m}{n}$, where n is the number of measurements of the angle of maximum displacement.

11. Find the periods of oscillation of the frame with weights $\boldsymbol{T}=t/N$ and without weights $\boldsymbol{T_1}=t_1/N$.

12. Calculate the speed of "the bullet" v using average value of the angle of maximum displacement $\overline{\varphi}_m$.

Control questions

1. What kinds of conservation laws were used in a given experiment? Formulate them for isolated systems.

2. Give definition of elastic and inelastic interactions of two objects. Give examples.

3. Write the laws of conservation for elastic and inelastic collisions of two bodies.

4. Why is it impossible to find the bullet speed by equalization of kinetic energy of the bullet to the potential energy of elastic deformation of the thread at maximal displacement angle of the frame?

5. What simplifying conditions were used in a given work?

6. Obtain and characterize differential equation of torsion oscillations.

3.6. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

2. Савельев И.В. Курс общей физики. т.1, М.: Астрель-АСТ, 2002.- 336с.

3. Трофимова Т.И. Курс физики. М.: Высш. шк., 2002. – 542 с.

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4. THE PHYSICAL PENDULUM

4.1. **Objectives:** study of the law of vibration of a physical pendulum through small angles. Determination of gravitational force using physical pendulum.

4.2. Theory

Physical pendulum is a rigid body of arbitrary shape oscillating under gravitational force about horizontal axis that doesn't pass through its center of mass. In the state of stable equilibrium the line passing through the point of suspension and the center of mass is horizontal one (fig.4.1.)

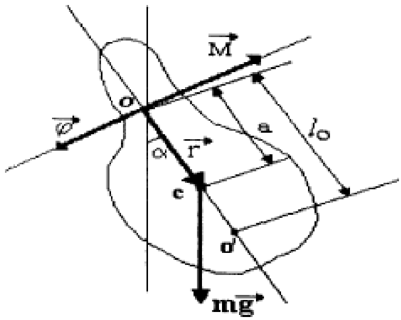

Fig. 4.1

If the physical pendulum is given *a small angular displacement α* and then is released it oscillates under the torque provided by gravitational force about fixed axis. Projection of the torque on the axis is

$$M = mga \cdot \sin\alpha \quad , \tag{4.1}$$

where ***m*** – is the pendulum mass, **g** – free fall acceleration, ***a*** – distance between O and C points. The equation of rotational motion of a physical pendulum, written on the basis of equation of torques in projection to the axis of rotation is:

$$I \cdot \frac{d^2\alpha}{dt^2} = -mga \cdot \sin\alpha, \tag{4.2}$$

where **I** is the rotational inertia about axis of rotation.

If we assume that α is small ($<5^0$), then approximation sinα~α is valid and using

$mga/I=\omega_0^2$ we can reduce *equation of motion* (4.2) to

$$\frac{d^2a}{dt^2}+\omega_0^2\alpha=0, \qquad (4.3)$$

Solution of this linear differential equation of the second order is

$$\alpha=\alpha_0\cdot\sin(\omega_0 t+\varphi), \qquad (4.4)$$

where $\boldsymbol{\alpha_0}$ is the angular amplitude of oscillations measured in radians, φ_0 is the initial phase and $\varphi=(\omega_0 t+\varphi_0)$ is the phase at moment t.

Period of oscillation is the time taken for one complete oscillation $T=\omega_0/2\pi=1/\nu$.

Here $\omega_0=2\pi\nu$ is the angular frequency measured in rad/s or s^{-1}, **f** is the frequency of oscillations measured in Hz (1Hz=1/s). So the period of the physical pendulum is

$$T=2\pi\sqrt{\frac{I}{mag}}, \qquad (4.5)$$

If we compare (4.5) with the period of a simple pendulum

$$T=2\pi\sqrt{\frac{l}{g}}, \qquad (4.6)$$

we can introduce the concept of *effective length* $l_0=\dfrac{I}{ma}$ *of physical pendulum*. $\boldsymbol{l_0}$ is the length of a simple pendulum with the same period $\boldsymbol{T}$. The point on the line passing through the point of suspension O and the center of mass C of the physical pendulum at the distance $\boldsymbol{l_0}$ from the point O is called the *center of oscillations* of the physical pendulum for a given suspension point.

It can be shown that the center of oscillation lies below the center of mass C. By *parallel axes theorem* the rotational inertia of a rigid body about arbitrary axis is

$$I=I_0+ma^2, \qquad (4.7)$$

where $\boldsymbol{I_0}$ is the rotational inertia about parallel axis passing through the center of mass. Then

$$l_0 = \frac{I_0 + ma^2}{ma} = a + \frac{I_0}{ma}, \qquad (4.8)$$

that is $l_0 > a$.

If one uses the center of oscillation as the suspension point of the physical pendulum the period of oscillation stays the same. It means that the center of oscillation and the suspension point of the same physical pendulum are interchangeable.

The dependence of the period T on distance a can be obtained by substitution of (4.7) to (4.5).

$$T = 2\pi\sqrt{\frac{a}{g} + \frac{I_0}{mga}}, \qquad (4.9)$$

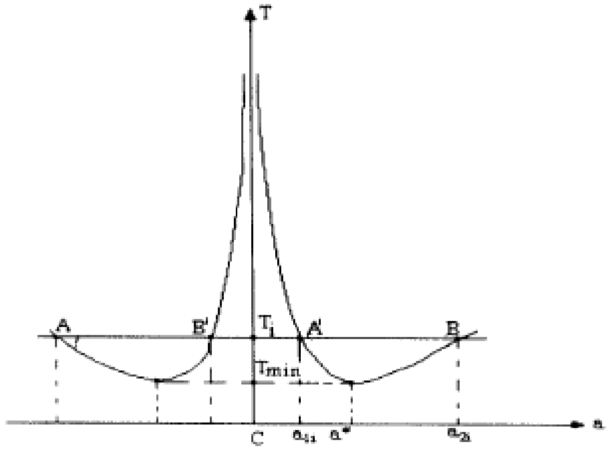

Fig. 4.2

Figure 4.2 illustrates such dependence. When ***a*** approaches zero, the suspension point approaches to the center of mass and the period of oscillation approaches infinity that corresponds to the state of neutral equilibrium. When the suspension point moves away from the centre of mass, ***T*** first decreases then increases approaching to infinity for further rise of ***a***.

If $a \to \infty$ the second term under the root in (4.9) can be neglected and we obtained the expression coinciding with the period of a simple pendulum.

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Abovementioned limit $a\to\infty$ corresponds to neglect the pendulum size compared with the distance a between the suspension point and the center of mass of physical pendulum. That is the definition of a simple pendulum.

One can find minimal value of the period $\boldsymbol{T_{min}}$ which corresponds to some value a* using condition of extremum $dT/da=0$ and (4.9).

$$a^* = \sqrt{\frac{I_0}{m}}, \qquad T_{\min} = 2\pi\sqrt{\frac{2I_0}{mg^2}} \tag{4.10}$$

4.2. Method

Proposed method of determination of the free-fall acceleration is based on the formula (4.6) for the period of oscillation of the simple pendulum. One can obtain expression for **g** using this formula

$$g=4\pi^2 l_0/T \tag{4.11}$$

Effective length of physical pendulum l_0 which period is the same as the period of simple pendulum of the length $\boldsymbol{l=l_0}$ is determined by graphical method using experimental dependence of the period of physical pendulum $\boldsymbol{T}$ on the distance $\boldsymbol{a}$ between suspension point and center of mass (fig.4.2).

4.3. Experimental equipment

Experimental setup LKM-4 consists of the post which supports physical pendulum of the rod shape (fig.2.1). Control board with buttons and toggle switches is in the base of the setup. To power up the setup use right switch “Set”. The switch of choice of the number of measurements is set in the mode of a single measurement in a given experiment. The switch “1/2” in the position “1” permits to measure value of each time interval and in the position “2” to measure total value of two successive time intervals (for example, time taken for two turns of the table).The switch «Gotov» resets device to zero and starts new reading, counter and indicator are set to zero by the signal

‘Ust.0’.Time reading begins at the first signal “Pusk”. Electronics permits to take direct readings of periods of oscillations.

4.4. Experimental procedure

1. Examine ground connection of a measuring unit, power up the setup pressing the switch “Set”. At the same time lamps of photo detector and zeros should shine.

2.Set the switch “Odnokr/Tsikl” in position “Odnokr”, the switch “0,1/MC/0,1” set in position “MC”, the switch “1/2” set in the position “2”.

3. Hang the rod pendulum on the upper hole; give it a small displacement and release. The rod should oscillate freely crossing the light beam of photo detector. Initial displacement should be small but sufficient for 10-20 oscillations needed for measurements.

4. Press and release button “Gotov”. Then the result of measurement will appear at the indicator board. Repeat the experiment at different initial displacement angles $\boldsymbol{\alpha_0}$. Find the initial displacement angle that further decreases of the initial displacement angle does not change the period of pendulum oscillation and use it in further measurements.

5. Measure 3 times the period of oscillations of the pendulum for the first hole. Change the position of the pivot towards the center of mass and continue measurements. Write down your readings in table 4.1.

Table 4.1.Dependence of oscillation period $\boldsymbol{T}$ on distance $\boldsymbol{a}$

a_1,cm	Period T_1,c			$\langle T_1 \rangle$,c
	1	2	3	
2				
4				

4.5 Data handling

1. Find the mean value of oscillation period and record it into table 4.1.

2. Plot the graph $\boldsymbol{T}$ versus $\boldsymbol{a}$. Use all experimental results.
3. Plot separately on a larger scale the region of the dependence T=f(a), where two values of $\boldsymbol{a}$ correspond to the same period $\boldsymbol{T}$.

4. Draw 3-5 horizontal lines and find values $\boldsymbol{a_{1i}}$ and $\boldsymbol{a_{2i}}$ for the points of intersection of these lines and the curve T=f(a).Determine effective lengths $l_{0i}=a_{1i}+a_{2i}$ for each $\boldsymbol{T_i}$.

5. Find experimental values of the free-fall acceleration using obtained results for l_{0i} and T_i and formula (4.11).

6. Find the average value of free-fall acceleration <**g**> and confidence interval **Δg** by the method of direct measurements for a given confidence probability.

7. Compare obtained results with g=9.804 v/s^2 for Almaty.

8. Find theoretical values of T_{min}, a^* using formula (4.10) and taking into account the rotational inertia of a thin rod about the axis perpendicular to the rod and passing through its center of mass $I_0=ml^2/12$.Compare these theoretical values with experimental results obtained using the graph.

Control questions

1. Give definitions of simple and physical pendulum. Write formulae for their periods.

2. What can be said about a rotational inertia of the physical pendulum about axes passing through different suspension points? Does it stay the same or change? Why does it change? What is physical meaning of effective length of physical pendulum?

3. What curves represent dependence T=f(a) for physical and simple pendulums? Explain them.

4. In what way mass and shape of support of pendulum can influence its motion?

5. Explain why the curve T=f(a) in fig.4.2 is symmetrical to the center of mass although physical pendulum itself may be not symmetrical?

4.7. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

2. Савельев И.В. Курс общей физики. т.1, М: Астрель-АСТ, 2002.-336 с.

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5. DETERMINATION OF ROTATIONAL INERTIA OF RIGID BODYS

5.1. Objectives: experimental verification of the dynamical equation of rotational motion using Oberbeck's pendulum.

5.2. Method

In rotational motion all particles of a body move in circles about straight line called axis of rotation.

An angular acceleration of rotating rigid body ε is proportional to a torque exerted by external forces on a body **M**.

$$\varepsilon = M/I \tag{5.1}$$

Here **I** is the rotational inertia (or moment of inertia) of the body about axis of rotation which characterizes its inertia in rotation. A particle of mass **m** moving in a circle of the radius **r** has rotational inertia

$$I = mr^2 \tag{5.2}$$

To verify experimentally the laws of rotational motion we use in a given experiment Oberbeck's pendulum, which scheme is shown in figure 5.1.

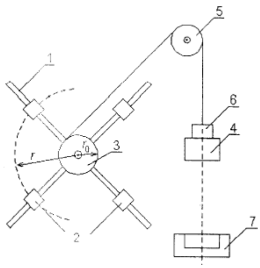

Fig.5.1

Investigated body 1 consists of four rods fixed at the wheel. Blocks 2 are placed on the rods, displacement of blocks along the rods results in change of the rotational inertia of the body. A pulley 3 of the radius $\mathbf{r_0}$ is on the same axis as investigated body. A weight 4 is suspended from a cord passing over a pulley 5.The cord is wrapped about the pulley 3. The weight sets the body in motion. One can add from one to four additional weights 6 to the main weight of the mass $\mathbf{m_0}$. Rotation of the pendulum occurs due to the torques exerted by the force of tension **M** and opposite in direction friction force $\mathbf{M_{fr}}$.

So in accordance with equation (5.1) equation of pendulum motion is

$$I\,\varepsilon = M - M_{fr} \tag{5.3}$$

$$\text{or} \quad M = M_{fr} + I\,\varepsilon. \tag{5.4}$$

One can see from (5.4) that if friction force does not depend on speed and is a constant value, dependence of ***M*** on **ε** is a linear function of a type $y=y_0+kx$, where I plays a role of coefficient ***k***. By the Newton's second law of motion equation of motion of the weight is

$$m \cdot a = m \cdot g - F\ , \qquad (5.5)$$

where **a** is the acceleration of the weight , which can be found using the time of its motion ***t*** and the distance traveled ***h***. For uniformly accelerated motion

$$a = 2h/t^2. \qquad (5.6)$$

From (5.5) and (5.6) one can obtain expression for the torque due to force of gravity

$$M = F \cdot r = m \cdot r_0 \cdot (g - 2h/t^2). \qquad (5.7)$$

Taking into account relationship between linear and angular accelerations $a=\varepsilon \cdot r_0$, one can find from (5.6)

$$\varepsilon = 2h/r_0 t^2 \qquad (5.8)$$

So formulae (5.7) and (5.8) permit to find torque ***M*** exerted by the force of tension and angular acceleration ε using experimental data. One can carry out experiments using weights of different mass and study dependence **M** on ε by plotting corresponding graph. In such a way determination of rotational inertia of the pendulum results in definition of the coefficient of experimentally found function $M=f(\varepsilon)$.

5.3. Experimental procedure

1. When preparing the setup for measurements one should put the post in such a way that it does not touch photo detectors in the process of moving down. Place the blocks on the rods at the distance from axis of rotation **r**=14 cm. Fix the cord at one of the pulleys 3, suspend the weight 4 from another end of the cord and put the cord over the pulley 5. Using additional blocks 6 put mass greater than the minimal mass needed for rotation of the pendulum.

By rotation of the pendulum place the weight in the extreme upper position in such a way that lower plane of the weight coincides with one of the marks of the scale of vertical post.

Fix the weight in such a position. To do that press the key "Set" and electromagnet will operate.

Put the holder of a photo detector at the lower part of the scale of the vertical post and place the photo detector in such a way that the weight with additional blocks will pass through the center of the window of the photo detector when moving downward. The mark on the scale corresponding to the hairline mark on the body of the photo detector and which looks as extension of optical axis of the photo detector crossed by moving weight is taken as the lower position of the weight.

2. Press the key "Pusk", electromagnet is turned off, the weight begins to move down and timer of the unit begins to work. When the weight crosses optical axis of the photo detector, timer stops. Record a reading of the timer ***t***.

Find the distance ***h*** traveled by the weight. It is the distance between the lower plane of the weight in its upper position and the optical axis of the photo detector.

Write down ***h,r,m,t*** and then press the key "Sbros". To improve the accuracy of measurements repeat experiment 5-6 times.

3. Repeat item 2 for the same values ***h*** and ***r***, increasing mass of the weight by additional blocks .Carry out measurements for 4-5 values of ***m***.

4 Repeat items 2 and 3 for new positions of blocks for **r**=11cm and **r**=8cm.

5. For chosen value **r** calculate values ***M*** and ***ε*** for different ***m*** .Plot the graph of the function $M=f(\varepsilon)$ and approximate experimental results $M=f(\varepsilon)$ by linear function using least square method.

6. Find rotational inertia of the pendulum for a given position of blocks.

7. Repeat items 5 and 6 for new positions of blocks. To verify relation (5.2) plot the graph of dependence of rotational inertia on $\mathbf{r}^2$.

8. Calculate theoretical value of rotational inertia of the pendulum for a given position of blocks (item 6) using following formula

$$I_r=I_0+4mr^2+ 4ml^2/12+4mR_r/4, \qquad (5.9)$$

where **m** is the mass of the block, **r** is the distance from the center of mass of blocks m to the axis of rotation, $\mathbf{R_r}$=0.015m is the radius of the block and **l**=0.02 m. $\boldsymbol{I_0}$ is the rotational inertia of the system without blocks

$$I_0=2m_1l_1^2/12, \qquad (5.10)$$

where l_1=0.15 m is the length of one rod of the pendulum, m_l=0.023 kg is the mass of the rod.

Find the relative error η by comparison of theoretical and experimental values of rotational inertia of the pendulum by formula

$$\eta= 100(I_{ex}-I_{th})/I_{th} \qquad (5.11)$$

and compare it with the error of measurements (item 8)

Control questions

1. Formulate and write down the dynamical equation of rotational motion.
2. Make a table of analogy of kinematical characteristics of translational and rotational motion.
3. Make a table of analogy of dynamical characteristics of translational and rotational motion.
4. Write down expressions for rotational inertia of a particle and a rigid body about axis.
5. Derive theoretical expression for one of the following bodies: rod, disk, cylinder and so on.
6. Formulate of parallel axes theorem and its application.
7. Describe the way of determination of the torque exerted by force of friction and rotational inertia of the body using the graph of dependence $M=f(\varepsilon)$.

5.5. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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6. DETERMINATION OF THE LIQUID VISCOSITY BY STOKES' LAW

6.1. **Purpose:** This laboratory investigation involves determining the viscosity of an unknown fluid using Stokes' Law. On completion of this laboratory investigation students will:

- Appreciate the engineering science of 'fluid mechanics.
- Understand the concept of fluid 'viscosity.

- Understand the concept of dimensionless parameters, and most specifically the determination of Reynold's number.
- Be able to predict the settling time of spheres in a quiescent fluid.
- Be able to calculate the viscosity of an unknown fluid using Stokes' Law and the terminal velocity of a sphere in this fluid.
- Be able to correct for the diameter effects of fluid container on the determination of fluid viscosity using a 'falling ball' viscometer.

6.2. Theory: Viscosity is a fluid property that provides an indication of the resistance to shear within a fluid. When an object moves relative to a fluid, the fluid exerts a friction like retarding force on the object. This force, which is referred to as a *drag force,* is due to viscosity of the fluid and also, at high speeds, to turbulence behind the object. Elements of fluid stick to the surface of the object and move, so neighboring molecules are set in motion by cohesive forces. The greater the distance of a molecule from the object the smaller its velocity because of friction between fluid layers. Frictional force between adjacent layers of fluid as they move past one another at different velocities is found on the basis of Newton's law for viscous fluid:

$$F_d = (dv/dy)S, \qquad (6.1)$$

where **S** is the area of contact of fluid layers (fig.6.1), **dv/dy** is called velocity gradient, indicating the rate of change of velocity in direction y, perpendicular to the direction of motion, **η** is coefficient of dynamic viscosity

Specifically, you will be using a fluid column as a viscometer. To obtain the viscometer readings you will use a stopwatch to determine the rate of drop of various spheres within the fluid.

George Gabriel Stokes, an Irish-born mathematician, worked most of his professional life describing fluid properties. Perhaps his most significant accomplishment was the work describing the motion of a sphere in a viscous fluid. This work leads to the development of Stokes' Law, a mathematical description of the special force-drag force, required to move a sphere through a quiescent, viscous fluid at specific velocity. This law will form the basis of this laboratory investigation. This drag force is equal:

$$F_d = 6\pi\mu \, v \, r \qquad (6.1)$$

where F_d is the drag force of the fluid on a sphere, μ is the fluid dynamic viscosity, v is the velocity of the sphere relative to the fluid, and r is a radius of the sphere. ***Viscosity is*** a fluid property that relates the shear stress in a fluid to the angular rate of deformation. Using this equation, along with other well-known principle of physics, we can write an expression that describes the rate at which the sphere falls through a quiescent, viscous fluid. First we must draw a free body diagram (FBD) of the sphere, that means to sketch the sphere and all of the internal and external forces acting on the sphere as it is dropping into the fluid. Figure 1 shows a sketch of the entire system (sphere dropping through a column of liquid).

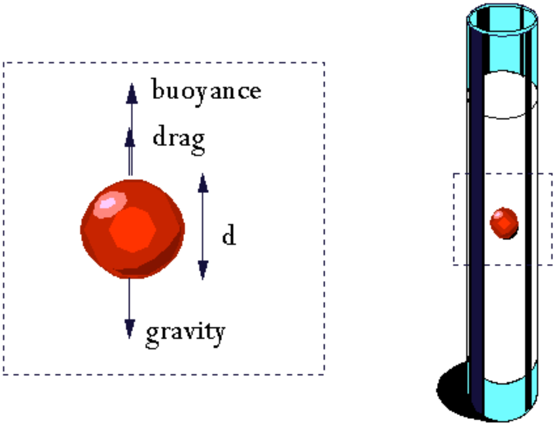

Figure 6.1.*Free-body diagram of a sphere in a quiescent fluid.*

There are three forces acting on the sphere; ***Fb***, ***Fd***, and ***mg***. The first two forces arise from the buoyancy effect of displacing the fluid in question and from the viscous drag of the fluid on the sphere, respectively. Both forces act upwards -- buoyancy tending to 'float' the sphere (***Fb***) and the drag force (***Fd***) resisting the acceleration of gravity. The only force acting downwards is the body force resulting from gravitational attraction (***mg***). By summing forces in the vertical direction we can write the following equation:

$$F b + \mathrm{F}d = mg \qquad (6.2)$$

The buoyancy force is simply the weight of displaced fluid. As you know the volume of a sphere (V_{sphere}) is written as: $V_{sphere} = 4/3\pi r^3$

Combining this volume with the mass density of the fluid, we can now write the buoyancy force as the product:

$$F = 4/3\ \pi\ r^3\ g\ \rho_{fluid} \qquad (6.3)$$

where ***g*** is the gravitational acceleration and ***r*** is the radius of the sphere. Combining all of the previous relationships that describe the forces acting on the sphere in a fluid we can write the following expression,

$$4/3\ \pi\ r^3\ g\ \rho_{fluid} + 6\pi\mu\ v\ d = mg \qquad (6.4)$$

Rearranging and regrouping the terms from the above equation we arrive at the following relationship:

$$V_t = 2r^2 g\,(\rho_{sphere} - \rho_{fluid})/9\mu \qquad (6.5)$$

While Stokes' Law is straight forward, it is subject to some limitations. Specifically, this relationship is valid only for 'laminar' flow. Laminar flow is defined as a condition where fluid particles move along in smooth paths in lamina (fluid layers gliding over one another). The alternate flow condition is termed 'turbulent' flow. This latter condition is characterized by fluid particles that move in random in irregular paths causing an exchange of momentum between particles. Engineers utilize a dimensionless parameter known as the Reynold's number to distinguish between these two flow conditions. This number is a ratio between the inertial and viscous forces within the fluid:

$$Re = \rho V d / \mu \qquad (6.6)$$

where **Re** is Reynold's number, ρ is the mass density of the fluid, ***V*** is the velocity of the fluids relative to the sphere, and ***d*** is the diameter of the sphere. The application of the Reynold's number to fluids problems is to determine the nature of the fluid flow conditions – laminar or turbulent. For the case where we have a viscous and incompressible fluid flowing around a sphere, Stokes' Law is valid providing the Reynold's number that has a value less than 1.0. When utilizing Stokes' Law, it is appropriate to verify the application of this law also.

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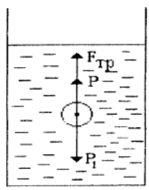

Fig. 6.2.

The coefficient of dynamic viscosity **η** is related to the coefficient of kinematic viscosity **ν** by relation ν=ηρ, where **ρ** is the fluid density.

Viscosity depends on nature and properties of fluid.

Consider forces on a sphere falling in a liquid (Fig.6.2).Three forces act on the falling sphere: $\mathbf{P_1}$ is the gravitational force, **P** is the buoyant force, $\mathbf{F_{fr}}$ is the drag force. If **r** is the radius of the sphere, **v** is the velocity of motion; **ρ** is the density of the liquid **η**, **g** is the free-fall acceleration. The force of gravity on the sphere is

$$P_1{}^{=}(4/3)\pi r^3\rho_1 g\,, \qquad (6.2)$$

where $\boldsymbol{\rho_1}$ is the density of the sphere material, **r** is the radius of the sphere.

The buoyant force is defined by Archimedes' principle

$$P_1{}^{=}(4/3)\pi r^3\rho g\,, \qquad (6.3)$$

where **ρ** is the density of the liquid.

If the **v** is the velocity of the sphere (vortex-free motion) in the liquid unrestricted by walls, the drag force by the Stokes's law is

$$Fd=6\pi\eta rv. \qquad (6.4)$$

Using the second Newton's law of motion one can write the equation of motion of the sphere as

$$m(dv/dt)=P_1\text{-}P\text{-}F_d. \qquad (6.4)$$

If one substitutes all three forces and the sphere mass as $m^{=}(4/3)\pi r^3\rho_1 g$, separates the variables and integrate (6.4), one obtains [1] the relaxation period of sphere motion **τ** after which the sphere moves with constant maximal velocity, called *terminal velocity,* equal

$$v_0=(2/9)(\ \rho_1\text{-}\ \rho_)gr^2/\eta, \qquad (6.6)$$

When radius of the vessel R is compared with r, drag force differs from (6.4) and equals to

$$F_c=6rv\pi\eta[1+2.4(r/R)]. \qquad (6.7)$$

Then

$$v_0=(2/9)gr^2(\ \rho_1\text{-}\ \rho_)gr^2/\eta[1+2.4(r/R)], \qquad (6.8)$$

Substitution of all forces into equation (6.5) for the uniform motion of the sphere at velocity $\mathbf{v_0}$, found by the formula $v_0=S/t$, gives the formula for the liquid viscosity

$$\eta=2/9)gr^2(\ \rho_1\text{-}\ \rho_)gr^2/v_0[1+2.4(r/R)], \qquad (6.9)$$

The Stokes's law is valid for Reynolds number Re <0.5, where

$$Re=2v_0r\rho/\eta \qquad (6.10)$$

6.3. Equipment

Experimental setup consists of two transparent cylindrical vessels filled with castor oil and glycerin. Marks separated by 10 cm are made on the walls of the vessels. These marks are used to find the distance traveled by the sphere in the liquid. Steel spheres of small diameters are used for the measurements.

6.4. Experimental procedure

1. Measure the diameter of each sphere 3 times at different positions using micrometer.

2. Find the depth of the liquid at which the uniform motion of the sphere begins. To do it, find the motion of the sphere at the first 10cm interval, then at the second 10 cm interval. If they are equal then the reference line was correctly chosen.

3. Measure using stop watch the time required for the sphere to travel the distance of 60 cm marked at the vessel. Record your readings in table 6.1.

Table 6.1

Diameter of the sphere,mm	D_1,mm	D_2,mm	D_3,mm	D_4,mm	D_5,mm
T,s					

4. Repeat 2 and 3 for the second liquid (by order of the instructor).

5. To find the effect of vessel walls (by order of the instructor) you can work out the viscosity using two values of terminal velocity (two formulae (6.6) and (6.8))and compare obtained results.

6. Calculate uncertainty of obtained result.

7. Using formula (6.10) work out Reynold's number

6.5. Control questions

1. Specify viscosity.

2. Relationship of dynamic and kinematic viscosities, their SI units.

3**.** On what physical quantities does viscosity of liquid depend?

4. What forces act on a sphere falling in a liquid?

5. Why after some moment does motion of a sphere in a liquid become uniform?

6. In what way do the diameters of a sphere and of a vessel influence the motion of a sphere?

6.6. References:

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LAB.7. BINOMIAL PROBABILITY DISTRIBUTION

7.1. Objectives: experimental verification of binomial probability law.

7.2. Theory

We deal with two types of laws of nature: deterministic and statistical ones. They differ in manifestation of casual links.

For example, the position of the planets in solar system may be exactly predicted at any moment of time using laws of mechanics. It is an example of deterministic laws.

The event of disintegration of a given atom of radium is an example of event which may or may not occur. In any given time interval atom may or may not disintegrate. When we toss a coin we can not predict using laws of mechanics if we will get head or tail. We may predict only probability of the event. Laws of this kind we call statistical.

We know that the probability of getting head and tail is the same and it is equal to 1/2 or 50%, that is, for 10 coin-tossings we get 5 heads and 5 tails.

In kinetic theory binomial probability law is used to calculate probability of macrostate of a system made up of enormous number of particles and for determination relation between equilibrium states and its the most probable states. One may get Maxwell's distribution of molecular speeds on the basis of binomial distribution.

Consider main concepts of probability.

An *event* is called *certain* if its occurrence is *inevitable* .An event is called *impossible* if there is certainty in its nonoccurrence. Random event may or may not occur. Measure of probability of its realization of such an event is probability of the event.

The probability ***p*** of the event **A** is the ratio of the number of successes for **A** and the number of trials.

For example,

1) In tossing of a coin the probability getting head is equal to the probability of getting tail and equal to ½.

2) In throw of dice each face of which has one of the following numbers 1,2,3, 4,5 ,6 the probability to get any number is the same and equal to 1/6.

3) The *binomial distribution* describes process in which each identical trial has one of two outcomes. For example coin tossing (heads or tails), quality control (pass or fail) and drug testing (kill or cure).

We are interested in the probability of a certain number of successes after n trials. Let us define the probability of success in one trial to be p, the probability of failure is then (1-p)=q. Probability of k successes out of n trials is a product of two factors;

1) Since k successes have probability p^k and n- k failures have a probability q^{n-k} ,the combined result has a probability obtained by multiplying these together:

$p^k q^{n-k}$.

2) Since the order does not matter, the number of ways of selecting k successes from n trials is

$$C_n^k = \frac{n!}{k!(n-k)!}$$

It is a number of k-element combinations of n objects.

The product of both factors is the *binomial probability distribution*

$$W(k/n,p) = C_n^k p^k q^{n-k} = \frac{n!}{k!(n-k)!} p^k q^{n-k}, \quad (7.1)$$

The binomial distribution holds when following conditions are met:

1) The number of trial n is fixed.

2) Outcome of each trial does not depend on the outcomes of the other trials.(independent elementary events).

3) Probability p of the event A does not depend on the number of the trial, that is p=const.

4) Probability that event A does not occur is q=1-p.

The set of conditions is called mathematical model of binomial experiment.

Example: The probability of 6 heads after 10 tosses is

$W(6/10,1/2)=(1/2)^6(1/2)^4 10!/(6!4!)=210/1024=0,205$

In binomial distribution (7.1) k is a *random* value, because it is unknown when this event occurs. Random value may be *discrete* and *continuous* ones.

Discrete random value is one that takes on final number of values.

Continuous random value takes on continuous range of values.

For example random value k in equation (7.1) is a discrete value, and readings taken in the process of measurements some physical quantities a (say, acceleration or size) is of continuous random value, because in the process of measurement one can obtain any value from a certain interval.

The expected value of discrete random variables is called *mathematical expectation* or the *mean*. It is arithmetic mean of a random value found taking into account its probability of occurrence. It is determined as

$$m_k = \sum_{k=0}^{n} kw(k/n, p) . \qquad (7.2)$$

It can be shown that in the case of the binomial distribution

$$m_k = np. \qquad (7.3)$$

If p=q, the mathematical expectation coincides with the most probable value. Then the maximal probability in the case of coin tossing corresponds to uniform distribution (both states in equal parts).

Dispersion is average of squares of deviations of a random value from its mathematical expectation taking into account its probability

$$\sigma_k^2 = \sum_{k=0}^{n} (k - m_k)^2 w(k/n, p). \qquad (7.4)$$

Dispersion characterizes scattering of value about its mean.

For binomial distribution

$$\sigma^2 = npq \qquad (7.5)$$

7.3. Experimental technique

Tossing of coins is used to test the binomial distribution experimentally. Closed container contains 12 coins. To find experimentally probability of getting 0,1,2 ,…k , ..12 heads from

12, one should toss 12 coins simultaneously as many times as possible and register in every trial number of heads. Let the number of trials be n. After determination the number of trials for which we got 0,1,2, ..,k,..12 heads, that is a frequency of occurring of random value, we can find relative frequency N(k)/N,where k=0,1,2,3,…,12.

7.4. Experimental procedure

1. Make from 100 to 150 trials and for each trial calculate the number of heads (or tails, it is an experimenter's option).Put chosen mark (x,*,0,) over the range corresponding to value of k and at the end of the experiment you will obtain ready histogram.

2.Find from obtained histogram frequency of occurring of the random value k, N(k) and its experimental probability W_{exp}=N(k)/N. Write down obtained results in Table 7.1. Calculate C_{12}^{k} and W and write them down in Table 7.2

Table 7.1

k	N(k)	W_{exp}=N(k)/N
1		
2		
…		
12		
∑		

Table 7.2

K	$C^k_{12}=12!/(k!(12-k)!)$	$W_{BD}= C^k_{12}(1/2)^k(1/2)^{12-k}= C^k_{12}/4096$
1		
2		
…		
12		

3. Plot the experimental dependence W_{exp}=N(k)/N=f(k) and theoretical dependence W_{BP}=f(k) on the same graph using the results of Tables 7.1 and 7.2.

4. Analyze obtained results and make conclusion of the reasons of discrepancy of data.

Control questions

1. Give definitions and examples of deterministic and statistic laws.

2. What types of events were described in the text? Characterize them. What is probability of event? Give examples.

3. What is the meaning of discrete random value and continuous random value?

4. What is the meaning of mathematical expectation and dispersion?

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1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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8. DETERMINATION OF AIR VISCOSITY

8.1. Objectives: Experimental determination of air viscosity.

8.2. Theory

Consider a layer of liquid between two plates at distance y_1 from each other. The lower plate 1 is at rest, and the upper plate 2 of area S is being moved under the influence of an external force. Plate 2 moves towards the direction of the axes *x* at constant velocity V_0. Then the friction force **f**, applied to the upper plate 2, is equal in magnitude and opposite in direction to the external force **F**.

If $y_1 << \sqrt{S}$ then distribution of velocity of liquid between the plates is linear and grows from 0 to v_0 when *y* changes from 0 to y_1. Newton found the following regularity from measurement of velocity v_0 and force **F**:

$$f = \eta \frac{v_0}{y_1} \cdot S \tag{8.1}$$

where η is a coefficient of internal friction or viscosity of liquid (gas), filling the space between the plates. The relation of friction force to the surface area of the plate

$$\sigma_{xy} = \frac{f}{S} = \eta \frac{v_0}{y_1} \tag{8.2}$$

is called the shearing stress of friction. Here indices ***x*** and ***y*** show the direction of motion (*x*) and the direction of change of speed (*y*).

Now, imagine that the liquid consists of thin parallel layers. Every layer moves uniformly, the upper layer drags the next lower layer with force ***f*** forward, the lower layer drags the upper one with force –***f*** backwards. So drag force ***f*** passes from one layer of liquid to another one. Every layer is under the influence of two equal and opposite forces, therefore its motion is uniform. The aforesaid is true to friction stress **σ**, as it is equal to the friction force over surface area unit. The SI units of viscosity are kg/(m · s).

Viscosity of liquids decreases when the temperature increases; and vice versa for viscosity of gases. This fact shows that different mechanisms of internal friction act in these media.

Along with the dynamical viscosity η of liquid or gas, the kinematical viscosity ν of liquid (or gas) is used also. The kinematical viscosity is defined as:

$$\nu = \frac{\eta}{\rho}, \tag{8.3}$$

where ρ – is the density of liquid.

In general, when speed of liquid in the vicinity of a body, arbitrary changes in the direction perpendicular to the considered surface patch, the shearing friction stress is defined by the formula:

$$\sigma_{xy} = \eta \frac{dv}{dy} \tag{8.4}$$

Where y shows normal direction to the surface patch.

8.3. Flow of a viscous liquid in a pipe

Consider a part of a cylindrical pipe, far from its beginning. Suppose v and σ are independent of x, depend only on r, i.e. the flow is steady-state. Here the cylindrical coordinate system is chosen so that x axis is directed along the pipe axis; r axis is perpendicular to x and goes along the pipe radius.

Chose two cross sections at distance l between them. The forces applied to the volume of the liquid between the sections 1 and 2 and cylindrical side surface with radius r are in equilibrium. A force acts in the direction of flow along the axis. This force is due to the difference of pressure in sections 1 and 2: $(P_1-P_2)\pi r^2$. Friction force equal to $\sigma \cdot 2\pi rl$ acts on the side surface of this volume of liquid. These forces are equal in magnitude:

$$(P_1-P_2)\cdot\pi r^2 = \sigma \cdot 2\pi rl \quad . \tag{8.5}$$

From this follows:

$$\sigma = \frac{P_1 - P_2}{l} \cdot \frac{r}{2} \quad . \tag{8.6}$$

From this expression it follows that in flow of liquid, shearing stress changes from zero value at *r=0* to its maximum value

$$\sigma_{max} = \frac{P_1 - P_2}{l} \cdot \frac{R}{2} \tag{8.7}$$

on the pipe wall at *r=R*.

If flow in the pipe is laminar, then according to the Newton's law the shearing friction stress is defined by the formula (8.4).

Formula (8.4) can be written in the form:

$$\sigma = -\eta \frac{dv}{dr} \tag{8.8}$$

Sign minus is used because σ is a positive value, but it must be negative in the selected coordinate system. Substituting (8.8) to (8.6):

$$\frac{dv}{dr} = -\frac{P_1 - P_2}{l} \cdot \frac{r}{2\eta}$$

Integrating this equation by *r:*

$$v = -\frac{P_1 - P_2}{l} \cdot \frac{r^2}{4\eta} + C \tag{8.9}$$

The integration constant *C* can be found from the condition that the speed of liquid is zero on the pipe walls, i.e. $v = 0$ at $r=R$ (*R* is the pipe radius), then:

$$C = \frac{P_1 - P_2}{l} \cdot \frac{R^2}{4\eta} \tag{8.10}$$

By substituting (8.10) to (8.9):

$$v = \frac{P_1 - P_2}{l} \cdot \frac{1}{4\eta}\left(R^2 - r^2\right) \tag{8.11}$$

This expression shows that speed is distributed along the cross section of the pipe by the parabolic law. It is equal to zero on the pipe wall at *r=R* and is maximal at *r=0:*

$$v_{max} = \frac{P_1 - P_2}{l} \cdot \frac{R^2}{4\eta} . \qquad (8.12)$$

When speed distribution is known, then volumetric rate of flow ***q*** through cross section of the pipe can be calculated. For that aim take thin ring of radius ***r*** and width ***dr***. Then the surface area of the ring is $2\pi r dr$. Then the volumetric rate of flow through this ring is

$$dq = v \cdot 2\pi r dr.$$

Substituting here speed from formula (8.11) and integrating by the whole pipe cross section, we find the volumetric flow rate:

$$q = 2\pi \int_0^R v r dr = \frac{P_1 - P_2}{l} \cdot \frac{\pi R^4}{8\eta} , \qquad (8.13)$$

that is equal to

$$P_1 - P_2 = \frac{8\eta \cdot q \cdot l}{\pi \cdot R^4} \qquad (8.14)$$

Equality (8.14) is the Poiseuille law for laminar flow: the difference of pressures necessary for volumetric flow rate ***q*** is proportional to the pipe length ***l***, the liquid viscosity and inversely proportional to the forth power of the pipe radius ***R***.

The Poiseuille law is used for experimental determination of viscosity **η** by measured values of volumetric flow rate ***q***, radius ***R*** and pressure difference P_1-P_2 for a pipe of length ***l*** by the formula:

$$\eta = \frac{\pi R^4 (P_1 - P_2)}{8 \cdot q \cdot l} \qquad (8.15)$$

If the volumetric flow rate q is known, then average velocity of flow can be found:

$$v_{cp} = \frac{q}{\pi R^2} = \frac{P_1 - P_2}{l} \cdot \frac{R^2}{8\eta} \quad (8.16)$$

Comparison of formulae (8.12) and (8.15) shows that the maximal speed of flow on the axis of pipe is two times higher the average velocity of flow.

$$\mathbf{v_m} = 2 \cdot \mathbf{v_{cp}}$$

8.4. Experimental method

A gas is pumped into a container of volume V_0. Then the gas is bled out through the capillary or radius R and length l. If the pressure difference is small enough $(P-P_0) << P_0$, where P is the pressure in the container, P_0 is the atmospheric pressure, the gas discharge is defined by its viscosity η:

$$q = \frac{P - P_0}{l} \cdot \frac{\pi R^4}{8\eta}$$

Pressure decrease in the container is described by the isothermal process with decreasing gas mass:

$$dP/dt = -(P/V_0) \cdot q.$$

When pressure decrease is small, P can be changed by average $\langle P \rangle$ during the observation time. In the result we get:

$$dP/dt = -(P - P_0)/\tau \text{, где}$$

$$\tau = 8l\eta V_0 / \pi R^4 \langle P \rangle,$$

whence it follows that the pressure difference $\Delta P = (P - P_0)$ exponentially fades, τ is a constant:

$$\Delta P = \Delta P_{нач} \cdot \exp(-t/\tau)$$

The angular coefficient of $\ln(\Delta P)$ on time gives τ, using which the viscosity can be found:

$$\eta = \tau \cdot \pi r^4 \langle P \rangle / 8lV_0. \quad (8.18)$$

If temperature $\boldsymbol{T}$ of the gas in the capillary is not equal to the temperature $\boldsymbol{T_0}$ of the gas in the container, then the volume of the gas passed through the capillary is $V = V_0\, T/T_0$.

8.5. Experimental procedure

Connect the container to the manometer with the aid of two connecting pipes (connecting pipe Step1), and to the pumping bulb. Close valve K1. Connect throttle capillary to connecting pipe Step2 through hosepipe. Pump air to the container up to pressure of 220-260 millimeters of mercury, after that clamp down the hosepipe of the pumping bulb. Wait for 1-2 minutes. If it is necessary, correct the pressure (add air by the pumping bulb or discharge air through connecting valve Step2, cautiously opening valve K1).

Open valve K1 and measure dependence of pressure in the container on time:

- turn on mode ‘Stopwatch’ by pressing the left button of the three buttons, after that the corresponding note blinks in the upper part of the display;

- when the pressure approaches the initial pressure ΔP_{init}, switch on the stopwatch by the middle button;

- when the chosen meanings of the pressure ΔP are reached, fix the measurement by the right button, read and write the registration, then press the right button again.

Draw a picture of dependence of $\ln\Delta P$ against time, determine linear intervals, determine the time constant $\tau = -d(ln\Delta P)/dt$ by the slope angle of the straight line, corresponding to the laminar flow (for small ΔP). Calculate the viscosity coefficient, using the following parameters: $r=0.2\ mm$ – the capillary radius, $l=40\ mm$ – the capillary length. Put down the experimental data into the table 8.1.

Table 8.1

ΔP, mm merc.	$ln\Delta P$	t, min-s	t, s
200	5.30		
160	5.08		
140	4.94		

120	4.79		
100	4.61		
80	4.38		
70	4.25		
60	4.09		
50	3.91		
40	3.69		
30	3.40		
20	3.00		

Calculate $\boldsymbol{\tau}$ in [s], air viscosity $\boldsymbol{\eta}$ in [кPa*s]

8.6. Control questions

1. Explain, how air (gas) viscosity manifests itself?

2. What physical quantities does air viscosity depend on? Explain dependence of gas viscosity on temperature, using the molecular-kinetic theory of gases.

3. What quantity is transferred from one layer to another at internal friction? What determines such transference?

4. Formulate the Poiseuille law. What are its practical applications?

5. Derive the formula for viscosity in this laboratory work.

8.7. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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9. DETERMINATION OF THE RATIO OF HEAT

CAPACITIES USING STANDING WAVE

9.1. Objectives. Determination of the ratio of the heat capacities for gases using the standing wave method.

9.2. Theory

To characterize thermal properties of any body including gas physical quantity called thermal capacity is used.

Heat capacity of any body is amount of heat needed to change its temperature by 1K.

$$C = \frac{dQ}{dT}, \tag{9.1}$$

where dQ is the amount of heat needed to change temperature of a body by dT. Dimension of C is J/K. Specific heat capacity is a heat capacity of the unit mass

$$c = \frac{C}{m} = \frac{1}{m} \cdot \frac{dQ}{dt}. \tag{9.2}$$

Specific heat has dimension J/(kgK). Molar heat capacity is a heat capacity per one mole of a substance. Molar heat capacity as well as specific heat capacity refers not to an object but the material of which this object is made up.

In determining and using the heat capacities of any substance, we need to know the conditions under which heat transfer occurs. Heat capacities at constant volume ($C_{v)}$ and heat capacity at constant pressure (C_p) are the most important characteristics. To increase temperature of the given mass of a substance at constant pressure by 1K we need more heat than for similar operation at constant pressure.

$$C_v = \left(\frac{dQ}{dT}\right)_v \neq C_p = \left(\frac{dQ}{dT}\right)_p. \tag{9.3}$$

$\mathbf{C_P}$ for ideal gas is greater than **Cv** because the heat added to the gas at constant volume is used to change only its internal energy while in the case of constant pressure part of the added heat is used to perform work for gas expansion.

By equipartition theory we may express the ratio of $\mathbf{C_P}$ and $\mathbf{C_V}$ as

$$\gamma = \frac{C_p}{C_v} = \frac{i+2}{i}, \tag{9.4}$$

where $\boldsymbol{i}$ is degree of freedom. *Degree of freedom of mechanical system is a number of independent variables needed to y specify uniquely its position and configuration in space.* For monatomic gases i=3, for diatomic gases i=5, for polyatomic (three and more atoms in molecule) i=6.

In accordance with (9.4) γ does not depend on temperature but experimentally found that the ratio C_p/C_v is temperature dependent. Such disagreement between theory and experiment shows us that theoretical representation of molecules as hard spheres which motion obeys laws of classical mechanics is inconsistent with reality. Molecules consist of interacting atoms and their motion is controlled by quantum mechanics laws. When we deal with heat capacity of monatomic gases that is not affected by interaction of atoms within a molecule and by energy of such interaction the classical theory of heat capacity is in a good agreement with experiment. For complex molecules internal processes play important role and classical theory gives approximate results. Quantum theory totally explains all experimental data on heat capacity.

The ratio C_p/C_v plays important role in ideal gas theory .This ratio is used in the equation of adiabatic process. If we know γ, we can calculate $\mathbf{C_v}$ from measured values of $\mathbf{C_p}$ and $\boldsymbol{\gamma}$ and there is no need to measure C_v. There are several ways to measure C_p/C_v.. The most convenient is one based on the measurement of sound speed in gases. In acoustics speed of sound in gases is determined by the formula

$$V = \sqrt{\frac{\gamma RT}{\mu}}, \tag{9.5}$$

where **R**=8.31J/(mol ·K) is the universal gas constant, **T** is the gas temperature, **μ** is the molar mass the gas. For the given temperature and molar mass and known sound speed one can determine C_p/C_v from equation (9.5)

$$\gamma=\mu V^2/RT. \tag{9.6}$$

The following equation can be used to find speed of sound

$$V=\nu\lambda \qquad (9.7)$$

where ν is the sound frequency and λ is the sound wavelength.

Method of standing waves is used to find wavelength. A standing wave is produced when a wave reflected from obstacle interferes with an incident wave. The superposition of two identical waves moving simultaneously through a medium in opposite directions results in a standing wave. If both waves have similar phases at the origin and their initial phases are zero, then equations describing both waves are

$$y_{ins}=a_0\cos(\omega t-kx),$$
$$y_{ref}=a_0\cos(\omega t+kx) \qquad (9.7)$$

Superposition of these waves gives

$$y= y_{ins} + y_{ref}=a_0\cos(\omega t-kx)+ a_0\cos(\omega t+kx)=$$
$$[2 a_0\cos(2\pi x/\lambda)]\cos\omega t =A \cos\omega t. \qquad (9.8)$$

Expression (9.8) shows that oscillation of the frequency ω appears at all points of the medium. Such an oscillation is called a standing wave. Multiplier $[2 a_0\cos(2\pi x/\lambda)] =[2 a_0\cos\omega(x/\nu)]$ which is time independent is the amplitude of the resultant oscillation.

So the amplitude of oscillation depends on the coordinate x specifying the position of a point of the medium. Points with the zero amplitude are called *nodes,* those with maximal amplitude are *antinodes*. Maximal amplitude is observed at points with

$$\cos(2\pi x/\lambda)=1. \qquad (9.9)$$

Position of antinodes is defined by condition

$$2\pi x_k/\lambda=k\pi, \qquad (9.10)$$

where k=1,2,3…. So position of antinodes are

$$x_k =k\lambda/2. \qquad (9.11)$$

In a closed at both sides tube (by microphone and telephone) resonant wave is established when the length of the tube **l** equals to the whole number of $\lambda/2$: $l= k\lambda/2$.For such a situation intensity of sound is maximal. Then sound velocity **V** is

$$V= \nu\lambda=2l\nu/k \qquad (9.12)$$

9.3. Experimental procedure

Switch on the setup IST-3 (toggle switches «Set» and «Vkl») and make sure that generator (judging by sound of the loud speaker,"Amplituda" handle) and heater (judging by load current at pressed button 1) run.

Obtain series of successive resonant frequencies gradually increasing the frequency of the signal with the handle "Chastota" judging by amplitude maximum recorded by indicator "Ind".

Repeat the experiment at several values of temperature beginning from the room temperature to 90-110^0.Put the handle "Nagrev" in the position "10B"-"20B" and set required temperature values by regulator "Temperatura".

Green indicator runs if the temperature of the sensor is lower than the temperature set by the regulator. If the sensor's temperature approaches to the set temperature both indicators (green and red) run and the system begins to operate in the termostating regime. At this moment sensor's temperature appears on the screen at the pressed button "T1" (accuracy 0.1 degree).

Values of sound speed **V** and **γ** are found as the average taken over several resonances.

$V_n=2l\nu/n$, n=1,2,3…

l=180 cm is the length of the resonator,

$\gamma=V^2M/RT$, $M=29\cdot10^{-3}$kg/mol.

R=8.31J/(mol·K),T is absolute temperature.

Record all obtained readings into the table 9.1.

Table 9.1

T,^{0}C				
ν,Hz				
N,Hz				
N,Hz				
<V>m/s				
γ				

9.4. Control questions

1.What is degree of freedom? What is **i** for a particle, monatomic, diatomic and polyatomic molecules and for rigid body?

2. What is the relationship between heat capacity of a body, molar and specific heat capacities?

3. How can the disagreement between experimental data and predictions of classical theory for molecules made up of several atoms be explained?

4. Write the equation of standing wave and characterize its main properties.

9.5. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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10. DETERMINATION OF SPECIFIC (LATENT) HEAT OF CRYSTALLIZATION (FUSION) AND ENTROPY CHANGE FOR ROSE'S ALLOY

10.1. Objectives: determination of specific heat and entropy change for crystallization (fusion) of Rose's alloy.

10.2. Theory: A substance often undergoes a change in temperature when energy is transferred between it and its surroundings. There are situations, however, in which the transfer of energy does not result in a change in temperature. This is the case whenever the physical characteristics of the substance change from one state to another; such a change is commonly referred to as a **phase transition**. Two common phase transitions are from solid to liquid (melting) and from liquid to gas (boiling); the other is a change in the crystalline structure of a solid. All such phase changes involve a change in internal energy but no change in temperature. The increase in internal energy in boiling, for example, is represented by the breaking of bonds between molecules in the liquid state; this bond breaking allows the molecules to move farther apart in the gaseous state, with a corresponding increase in intermolecular potential energy.

As you might expect, different substances respond differently to the addition or removal of energy as they change phase because their internal molecular arrangements vary. Also, the amount of energy transferred during a phase change depends on the amount of substance involved. (It takes less energy to melt an ice cube than it does to thaw a frozen lake.) If a quantity $\boldsymbol{Q}$ of energy transfer is required to change the state of a mass $\boldsymbol{m}$ of a substance, the ratio $\lambda = Q/m$ characterizes an important thermal property of that substance. Because this added or removed energy does not result in a temperature change, the quantity is called the **latent heat** (literally, the "hidden" heat) of the substance. The value of c for a substance depends on the nature of the phase change, as well as on the properties of the substance.

When a solid is heated up to a certain temperature it begins to melt and all supplied heat is used to destroy crystal structure and rise of temperature stops at the melting point until the fusion process is complete. So solids have a definite *temperature of fusion.*

If melted substance is gradually cooled crystallization occurs at a certain temperature. The temperature remains constant from the beginning of crystallization till it termination. This temperature is called temperature of crystallization.

Temperature of fusion and temperature of crystallization of chemically pure substances are the same. Amount of heat released at crystallization equals to the amount of heat absorbed by a body at fusion. The crystallization temperature for *solid solutions* is lower then for pure substances and they are important for cooling mixtures in preparation of easily melting mixtures widely used in many safeguard devices.

The amount of heat required to heat a body from $\mathbf{T_1}$ (say, room temperature) to fusion temperature $\mathbf{T_f}$ is:

$$Q = mc(T_f - T_1), \qquad (10.1)$$

The amount of heat required to melt the body is

$$Q = \lambda \cdot m\ , \qquad (10.2)$$

where $\boldsymbol{\lambda}$ is the specific heat of fusion.

In Thermodynamics, *a phase transition* is the transformation of a thermodynamic system from one phase to another. At phase-transition point, physical properties may undergo abrupt change- for instance, volume of the two phases may be vastly different. Phase transitions are divided into two broad categories: The *first-order phase transitions* are those that involve a latent heat. During such transition, a system either absorbs or releases a fixed (and typically large) amount of energy. During this process, the temperature of the system will stay constant as heat is added. Because energy cannot be instantaneously transferred between the system and its environment, first-order transitions are associated with "mixed-phase regimes" in which some parts of the system have completed the transition and others have not. This phenomenon is familiar to anyone who has boiled a pot of water: the water does not instantly turn into gas, but forms a turbulent mixture of water and vapor bubbles.

The second class of phase transitions are the *continuous phase transitions*, also called *second-order phase transitions*. These have no associated latent heat. Examples of second-order phase transitions are the ferromagnetic transition, superconductor and the super fluid transition.

If the melting temperature is known we can find change in entropy for this process. For reversible processes increment of entropy equals:

$$dS = \frac{\delta Q}{T} \qquad (10.3)$$

When system goes from state A to state B, the change in entropy during such transformation equals:

$$\Delta S = \int_a^b \frac{dQ}{T}. \qquad (10.4)$$

Change in entropy at heating and melting of a substance in accordance with (10.3) and (10.4) is

$$\Delta S = \int_{T_1}^{T_2} \frac{\delta Q_1}{T} + \int_1^2 \frac{dQ_2}{T} = \int_{T_1}^{T_2} \frac{mcdT}{T} + \frac{\lambda m}{T_f},$$

or finally

$$\Delta S = mc \cdot \ln \frac{T_f}{T_1} + \frac{\lambda m}{T_f}. \qquad (10.5)$$

10.3. Experimental procedure

Rose's metal, also known as “Rose's alloy” is a fusible alloy with a low melting point. Rose's metal consists of 50% Bi (bismuth), 25–28% Pb (lead) and 22–25% Sn (tin); its melting point is 100 °C. Rose's metal is typically used as a solder. Its special property is that it does not contract on cooling. It was used to secure cast iron railings and balusters in pockets in stone bases and steps.

We will use the same experimental set up as used in laboratory work 9.First place the substance under investigation (Rose’s alloy, mass m=120 g) into crucible, then weight it to find the mass of the substance (mass of the empty crucible is M=60.0±0.5,g) and put it on the plate of the thermostat. For a better contact add 2-3 drops of glycerin on the plate.

1. Set the regulator “Temperature” to the maximal position and heat the substance up to 120^0 using constant power supply (30-40 W is recommended).

2. Register temperature dependence on time. In temperature interval 30^0-90^0C detect a time interval required to increase the temperature by 10^0 using stop watch. Starting from 90^0C measure the time required to increase the temperature by 2^0C.
3. When temperature approaches 120^0C turn off the heater and turn on fan to cool the substance under investigation.
4. Register the time dependence of temperature in the process of cooling of the sample. Measure time in minutes.
5. Plot T= f(t) for heating and cooling on the same graph.
6. Determine the fusion temperature of Rose's alloy and calculate the entropy change for the process. When a temperature of the system rises by ΔT the sample absorbs amount of heat:

$$Q=(N-N_1)\Delta t-C_0\Delta T,$$

where **N** is the power of the heater, $\mathbf{N_1}$ is the power loss in the given temperature interval, $\mathbf{C_0}$ is a heat capacity of the crucible oven, $\boldsymbol{\Delta t}$ – time interval.

10.4. Control questions

1. What does the fusion temperature depend on?
2. What is the difference between crystalline and amorphous bodies?
3. Does the increase in pressure always result in the rise of fusion temperature?
4. What type of phase transition is a fusion?
5. Give statistic and thermodynamic definition of entropy and describe its main properties.
6. How do we find change in entropy for isochoric and isobaric processes?

10.5. References:

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.
2. Кикоин И.К., Кикоин А.К., "Молекулярная физика".М.: 1963.
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11. INVESTIGATION OF ELECTROSTATIC FIELDS

11.1. Objectives:

- Understanding of how electric fields are produced and their effect on charged objects.
- Understanding of electric potential and how it is related to electric field.
- Ability to sketch field and potential patterns for simple geometries.
- Ability to experimentally map the positions of equipotential surfaces.
- Acquire a knowledge of the concepts which allows students to predict the electric field and potentials associated with more complex geometries.

11.2. Theory: An electric field surrounds charged particles and represents the force per unit charged felt by other charged particles in that field. If the electric field does not change in time, then the force felt by charged particles in the electric field is given by:

$$\vec{F} = \vec{E}(\vec{r})q \qquad (1)$$

Where $\vec{E}(\vec{r})$ is the electric field vector at a position $\vec{r}$ from the source producing the electric field and q is the charge being affected by the electric field. The quantity q is positive for positive charges, and negative for negative charges. Columbs law describes the force felt by two point charges separated by a distance, in scalar form this law is:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad (2)$$

$$\epsilon_0 = 8.85418782 \times 10^{-12}\frac{A^2 s^4}{m^3 kg}$$

is the permittivity of free space, q_1 and q_2 are the charges of the two particles, r is the scalar distance between the two particles. (2) can be slightly simplified by combining the product of coefficients into one constant called Coloumb's constant.

$$F = k\frac{q_1 q_2}{r^2} \tag{3}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551787 \times 10^9 \frac{Nm}{C^2}$$

Equation 2 can be written in vector form in the following way:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{4}$$

where $\hat{r}$ is a unit vector of the displacement vector between the two charges. Using (4) and (1) we can determine the electric field produced by a point charge.

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r} \tag{5}$$

When there is more than one charge the electric field produced in the surrounding space is the vector sum of all the individual electric fields produced by each point charge (superposition principle):

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i^2}\hat{r}_i \tag{6}$$

Where $\vec{r}_i$ is the displacement between the charge producing the electric field and the point at which the electric field is to be calculated. $\hat{r}_i$ is the unit vector of the displacement vector.

Electric potential is the negative of the work done in moving an object. The electric potential is defined as the negative of the work done *per unit charge* by the electric field in moving a point charge from infinity to a distance r away from the source of the field. Starting from the definition of work:

$$V = -W = -\int_{\infty}^{r} \frac{\vec{F}}{q} \cdot \vec{dr} \tag{9}$$

The force is defined as:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{r} \tag{10}$$

Finally we have:

$$\begin{aligned} V &= \int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}dr \\ V &= \left[-\frac{1}{4\pi\epsilon_0}\frac{Q}{r}\right]_{\infty}^{r} \\ V &= -\frac{1}{4\pi\epsilon_0}\frac{Q}{r} - \left(-\frac{1}{4\pi\epsilon_0}\frac{Q}{\infty}\right) \\ V &= -\frac{1}{4\pi\epsilon_0}\frac{Q}{r} \end{aligned} \tag{14}$$

(14) give the expression for the electric potential at a distance **r** from the source. Note that if the charge producing the electric field is positive then the electric potential is negative. If the charge is negative, then the electric potential is positive.

We can represent electric field with field lines and equipotential lines following rules given below:

1. Field lines start at positive charges and terminate on negative charges and for a single charge continue to infinity.

2. Vector **E** is directed along the tangent to a field line at any its point.

3. Density (number of field lines per unit perpendicular to area) is equal or proportional to the magnitude of **E**.

The relationship between potential and electric field is

$$\vec{E} = -grad\varphi \tag{11.6}$$

$$E_n = \frac{d\varphi}{dn}. \tag{11.7}$$

Equation (11.7) holds for magnitudes of vectors. Strength of electric field is numerically equal to the change of potential over unit length in the direction of maximal change in potential (it is the direction of the normal to an equipotential surface at a given point).Sign minus says that vector **E** points lower potential.

11.3. **Method**

Model experiments are widely used for physical simulation of complex physical phenomena. *When physical modeling* an object under investigation and its model has the same physical nature but geometric sizes of the object and its model are different. Mathematical simulation of electric field of charged bodies used in the laboratory work is based on the fact that electric field of direct steady current in medium with low conductivity is *potential*.

Their similarity is based on following properties:

1.Electrostatic field is a potential one, $\oint_L E_l dl = 0.$

Field in conducting homogeneous medium is potential also. $\oint_L \vec{j} \cdot d\vec{l} = 0$ or

$\oint_L \vec{E} \cdot d\vec{l} = 0$ because by Ohm's law $\vec{j} = \sigma\vec{E},$ where $\vec{j}$ is the current density and σ is the medium conductivity .

2. Boundary conditions are similar as well. Tangential and normal components of electric field vector at interface of two dielectrics satisfy conditions: $E_{\tau 1} = E_{\tau 2,} E_{n1} = E_{n2}$.

Therefore to study electric field of charged bodies one can use field of current in medium with low conductivity. When modeling current line is analogue of electric field line and surface of equal voltage is analogue of equipotential surface. To study potential distribution in stationary fields one uses special probe which is thin metal rod isolated along its length but with bare point.

Electrodes which shape is the same as the shape on natural bodies but made on another scale are used for modeling.

11.4. Experimental procedure.

1.Set up one variant of conducting list configuration (plot board 1,2,3 or 4) and connect up the power supply and multimeter MY62 in voltmeter mode as fig. 11.1 shows (plot board 1 is shown).

2. Prepare the sketch of electrodes with coordinate grid on the sheet of paper.

3. Switch on power supply and with the probe make sure that one electrode has zero potential and the other one has potential equal to the voltage of power supply (15 V) after setting 0-15V by regulator of voltage.

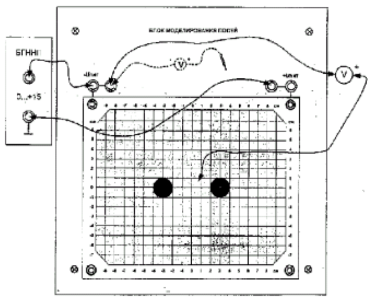

Fig. 11.1

4. Choose the step of change in potential ΔU (for example, 1,2 or 2,5 V) in order to get 8-10 equipotential lines.

5. Moving the probe from the zero potential point to another electrode along the axis of symmetry find points with potentials ΔU, $2\Delta U$, $3\Delta U$…Mark found points at the prepared sketch with the coordinate grid.

6. Moving the probe from the point with the potential ΔU near electrode (moving slightly to or from it) find points of equal potential and mark them on the sketch. Connect points of equal potential by the smooth curve. Construct in similar way other equipotential lines.

7. Map electric field lines using obtained equipotential lines.

8. Taking into account relationship between electric field **E** and potential $\mathbf{E}=|\Delta\varphi/\Delta L|$, where ΔL is the shortest distance between two equipotential lines along field line , find electric field intensity **E** at two or three points and draw a vector **E** at these points at the sketch.

11.5. Control questions

1. What is electric field? Give its main characteristics ($\mathbf{E}$,φ).

2. Speak on properties of electrostatic field: notion of electric flux, circulation around closed path, divergence, curl of **E**.

3. Speak on principle of superposition of electric fields. Gauss' theorem and its application.

4. Rules of construction of electric field lines and equipotentials.

5. Essence of the method of mathematical simulation.

11.6. References

1 Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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12. DETERMINATION OF RESISTANCE AND RESISTIVITY

12.1. Objectives: master the method of determination of resistance and resistivity of conductors. Measurement of the resistance of a resistor with multimeter and virtual meters. Calculation of resistivity.

12.2. Theory. Ohm's law holds for the direct current circuits: the current **I** flowing in metal conductor is proportional to the voltage **U** across a conductor:

$$I=U/R, \tag{12.1}$$

where R is the resistance of the conductor. $U=\varphi_1-\varphi_2$, that is the voltage across the conductor equals potential difference (voltage drop) in the case of the uniform conductor or leg of the circuit. Nonelectric forces (any force except electrical force) act in nonuniform leg .The voltage across such a leg is

$$U=\varphi_1-\varphi_2\pm\varepsilon_{12}, \tag{12.2}$$

where $\boldsymbol{\varepsilon_{12}}$ is the electromotive force (emf) of a source in the leg 1-2. By definition emf is a physical quantity equal to the work done by nonelectric force in moving the unit positive charge from the point 1 to the point 2.

$$\varepsilon=W_{nonel}/q, \tag{12.3}$$

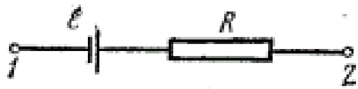

Fig.12.1

Differential form of the Ohm's law is expressed as

$$\vec{j}=\sigma\vec{E}, \tag{12.4}$$

.Current density *j* is the current per unit area of the perpendicular cross-section of the conductor. $\boldsymbol{\sigma}=1/\rho$ is the conductivity; $\boldsymbol{\rho}$ is the resistivity of the conductor which depends on material. Resistance of a uniform cylindrical conductor is

$$R=\rho l/A, \tag{12.5}$$

where **l** is the length of the conductor and **A** is its cross-sectional area.

If conductors are connected in series the equivalent resistance equals the sum of individual resistances

$$R=R_1+R_2+\ldots=\sum R_k. \qquad (12.6)$$

For resistors connected in parallel their equivalent resistance is found from relation of the inverse resistances

$$1/R=\sum 1/R_k. \qquad (12.7)$$

Resistivity of most metals is directly proportional to temperature: ρ~T. At low temperature one observes departure from such a dependence (Fig.12.2)

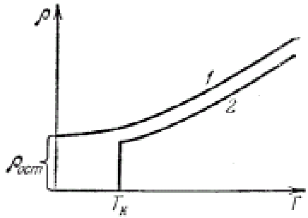

Fig. 12.2.

Usually temperature dependence of resistance follows curve 1. The remaining resistance depends on metal purity and residual mechanical stress in a sample. Curve 2 is observed for superconducting materials (Hg, Pb, Zn and so on).

12.3. Method

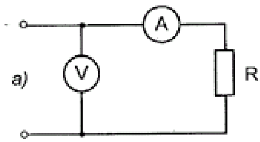

Fig.12.3a

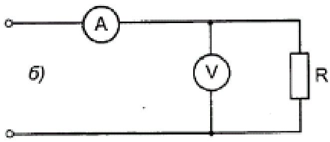

Fig.12.3b

Two ways of resistance determination are shown in figure 12.3. By the voltmeter method (fig.12.3a) resistance is found using following formula

$$R=(U/I)[1-IR_A/U], \qquad (12.8)$$

and by the ammeter method (12.3b):

$$R=(U/I)[1+IR_v/U] \qquad (12.9)$$

One can see that both methods give approximate values when compared with R=U/I. Formulae (12.8) and (12.9) are derived using Ohm's law for the circuit leg.

12.4. Experimental procedure

1. Set the given length of the conductor within the interval from 0.5 v to 2.5m using a screwdriver at the mini-unit "Resistivity"

2. Put the mini-unit at the jack panel and connect it with the right multimeter operating in the mode of resistance measurement (the limit of measurement 200Ω) and record the length of the conductor (m), its diameter (mm), resistance (Ω).

3. Set up the circuit to measure the resistance by the voltmeter method (fig.12.3b), using the right multimeter MY62 as voltmeter and the left multimeter MY 60 as ammeter. Set up corresponding modes and the limits 20V and 200mA.

Solid line shows connection of the voltmeter by the voltmeter method (fig.12.3a) and broken line demonstrates connection by ammeter method (fig.12.3b).

4. Switch on the unit of voltage generator and set up current in the circuit on order of 200mA by regulator. Record in table 12.1 values of current and voltage and find the resistance. Repeat measurements five times for each method with multimeters and with virtual meters.

Tabl.12.1

Scheme of measurements	Multimeters			Virtual meters		
	U,V	I,A	R, Ω	U,V	I, A	R,Ω
Voltmeter method						
Ammeter method						

5. Measure the same parameters with virtual meters on the screen of a computer. To do that replace multimeters with virtual voltmeter VO and virtual ammeter, their clips are on the connector and switch on the computer.

6. Switch on virtual meters by double click of the mouse at the label "VP Physics".As a result unit 'Pribory 1" will be opened and ammeter, which contains voltmeters and ammeters. Part of them is activated that is limits of measurements are switched on.

7. Activate required virtual meters by clicking on buttons "Откл" in corresponding windows. To switch off a meter, make a click in window of measurement limits.

8. Choose the limits of ammeter and voltmeter measurements pressing corresponding buttons of the connector (fig.3).Chosen limits will be automatically shown in the windows of corresponding devices. In the case when measured signal exceeds allowable for a given canal level the window with the data begins to give intermittent red warning light and warning “Overload! Move to largest limit” appears at the upper part of the panel. It disappears as soon as limit of measurement becomes greater than measured value.

9. To close the window of a virtual device click the key “ Vykl”.

10. Calculate the average value of the resistance for every method of measurement with multimeters and virtual meters, treat the results with Student’s method, compare relative errors and make conclusions.

11. Find the resistivity of conductor material and using reference data identify the material of the conductor.

12.5. Control questions

1. Describe the relationship between the resistance of a conductor and its resistivity. Plot the graph and explain dependence of resistance of metals on temperature.

2. Derive formula for the equivalent resistor connected in series and in parallel.

3. Ohm’s law for uniform and nonuniform legs of direct current circuit, closed circuit and differential form of the law. Derive formulae for resistance found by the voltmeter and ammeter methods.

12.6. References

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

2. Трофимова Т.И. Курс физики. М.: Высшая школа, 2002.

13. STUDY OF HALL EFFECT IN SEMICONDUCTORS

13.1. Objectives: familiarity with electro-physical method of measurement of the number of charge carriers per unit volume in semiconductors.

1. Determination of Hall's EMF dependence on magnetic field.

2. Calculation of the number of carriers per unit volume in semiconductors.

13.2. Theory

Hall Effect is widely used in measuring engineering, for example, in Hall device to measure the number of carriers per unit volume and magnetic field. When a metal or semiconductor plate carrying direct current is placed in a perpendicular magnetic field then a potential difference $U_H=\varphi_1-\varphi_2$ is generated in a direction perpendicular to both the current and the magnetic field (fig.13.1a).The Hall voltage is

$$U_H=RbjB, \tag{13.1}$$

where b is the width of the plate, j is the current density, B is the magnetic field and R is the Hall constant.

The electron theory explains the Hall Effect. In absence of magnetic field electric current is due to electric field $\mathbf{E_0}$ (fig.13.1b). Equipotential surfaces of the field are the set of planes perpendicular

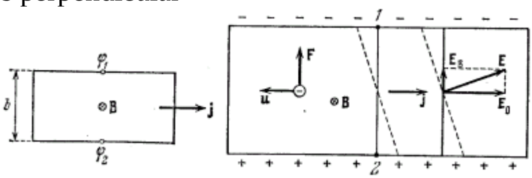

Fig. 13.1

to $\mathbf{E_0}$.Two planes from the set are shown by the solid lines on figure. Potentials at points 1 and 2 are the same because both points lie on the same equipotential plane. Majority charge carriers in n-type semiconductor and charge carriers in metal are electrons. Drift velocity of electrons **u** is opposite to **j**.

In presence of magnetic field magnetic force **F** acts on charge carriers in direction across the conductor. Magnitude of the force is

$$F=euB. \qquad (13.2)$$

As a result electrons deflect to the upper edge of the plate and excess of positive charge is observed at the lower edge. Electric field E_B generated by the charge separation is

$$eE_B=euB;\ E_b=uB.$$

Superposition of $\mathbf{E_B}$ and $\mathbf{E_0}$ produces net electric field **E**. Equipotential surfaces are perpendicular to **E**, that is, they will change their position. Their new position is shown by broken lines in figure 13.1b.Point 1 and 2 will have different potentials.

$$U_H=bE_b=buB.$$

Taking into account that j=enu, where j is the current density and n is the the number of carriers per unit volume, we can write

$$U_H=(1/en)bjB. \qquad (13.3)$$

Comparison of (13.3) and (13.1) shows that R_H equals 1/(en).

For the p-type semiconductor Hall voltage changes its sign. So it is possible by Hall Effect to find the type of a semiconductor.

13.3. Experiment procedure.

1. Set up the electric circuit shown in figure 13.2 after switching on ammeter (left multimeter) to measure current of Hall sensor (as shown by dotted line in figure).

2. Set voltage generator regulator to zero position, switch on the voltage generator and record value of current of Hall device.

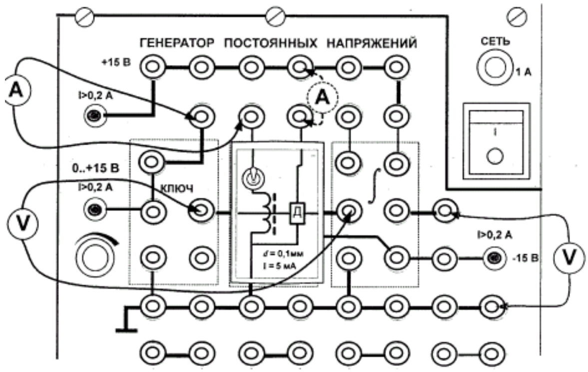

Fig. 13.2

3. Shift the connection of the ammeter to position shown by solid line to measure current in the coil. Connect by a wire the jacks to which the ammeter was connected previously.

4. Adjust the current 20mA in the coil and record the value of Hall EMF U_1 in table 2. Set the opposite current in the coil using the switch on the muni-unit and record the new value of hall EMF U_2.

5. Increase current in the coil in 20 mA steps up to 200mA and record in Table 2 values of U_1 and U_2.

Table 13.1

I,mA	U_1,mV	U_2,mV	U_{av},mV	B mT
20				
40				
…				
200				

6. Calculate the average value of The Hall EMF for forward and back currents and magnetic field B in the core gap: $B=\mu_0 Iw/\delta$, where I is the current in the coil, w=1200 is the number of the coil turns, δ=1,7mm is the width of the gap.

7. Plot the graph of U(V), approximate it with straight line with χ^2 method and find its inclination: $K=\Delta U/\Delta B$.

8. Find Hall coefficient R_H and the number of charge carriers per unit volume for the semiconductor using formulae: $R_H=Kd/I_{\text{Hall device}}$, $n=1/(eR_H)$. Here d=1mm, $E=1.6\times10^{-19}$C.

13.4. Control questions

1. Write the expression for Lorentz force.

2. Derive the expression for $\mathbf{R_H}$, Hall constant. Consider cases of n-type and p-type semiconductors.

3. How can you find the charge carrier concentration in semiconductor? When do you use Hall device, for what purpose?

13.5. References

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

2.Савельев И.В. Курс общей физики.М.: Наука, 1989.

3. Трофимова Т.И. Курс физики. М.: Высш. шк., 2002.

4. Иродов И.Е. Основные законы электромагнетизма. М.: Высш. шк., 1983.

14. STUDY OF MAGNETIC FIELD ON THE AXIS OF A CYLINDRICAL COIL

14.1. Objectives: Familiarization with the method of measurement at various points on the axis of a cylindrical coil. 1. Measurement of dependence of magnet field on distance from the coil axis. 2. Verification of measurement results by theoretical calculation.

14.2. Theory

It's experimentally known that a magnetic field arises in the space around a conductor with current in it. Moving electric charges and alternative electric fields are sources of magnetic field as well. There circulating electric currents are exist in magnets, these currents are the sources of magnetic field as well.

The main characteristics of magnetic field is magnetic field vector ***B***, its unit is tesla (*T*).

There are no magnetic monopoles in nature, that's why magnetic field properties differ from electric field:

A) Unlike potential electrostatic field, magnetic field are eddy or solenoidal. It is manifested mathematically that magnetic field circulation is not zero:

$$\oint \vec{B}d\vec{l} = \mu_0 \Sigma I_k . \qquad (14.1)$$

This equation expresses Ampere's law, it is used for magnetic field calculation, for instance in case of straight conductor with current, magnetic field of solenoid etc.

B) Gauss theorem (in integral (14.2) and differential (14.3) forms) for magnetic field expresses the fact that there are no magnetic monopoles. The Gauss theorem is not used for calculation of magnetic fields:

$$\oint \vec{B}d\vec{S} = \mathbf{0} \qquad (14.2),$$

$$\mathrm{div}\,\vec{B} = 0. \qquad (14.3)$$

The Biot-Savart law is used for calculation of magnetic field, created by constant electrical current:

$$d\mathbf{B} = (\mu_0/4\pi)I(d\mathbf{l} \times \mathbf{r})/r^3. \qquad (14.4)$$

For defining the resultant magnetic field ***B*** in compliance with the field superposition principle, it is necessary to integrate this expression.

In this laboratory work, the magnetic field on the axis of a solenoid (or a coil) with current *I* is equal:

$$B = (\mu_0 nI/2)(\cos \alpha_1 - \cos \alpha_2), \qquad (14.5)$$

where $n=w/l$ is the number of turns of the solenoid per unit of length, α_1 and α_2 are the angles to the solenoid edge from point A_1, where magnetic field is measured (Fig.14.1). The magnetic field at the centre of a very long solenoid, which length $l>>R$, R is its radius, is $B = \mu_0 n I$, and at its edges equal $\mu_0 nI/2$. The last two formulas show that magnetic field decreases from the centre of the solenoid to its edges.

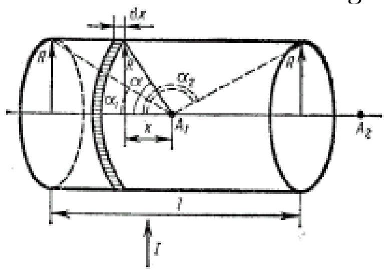

Fig.14.1

14.3. Experimental procedure

1. Fix the studied coil on the jack panel, connect it to the dc power supply through the ammeter (the left gauge of the two multimeters), as it is shown in Figure 14.2.

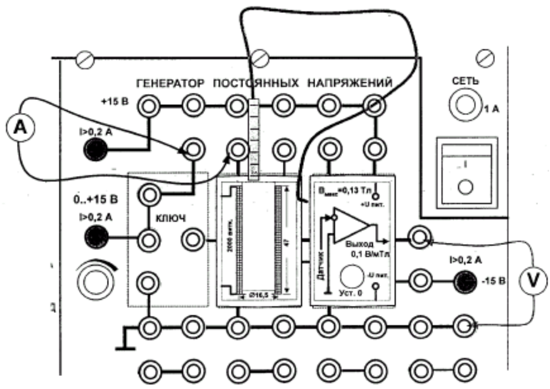

Fig. 14.2

2. Mount gauge “Teslameter” for measuring magnetic field to the jack panel.

3. Switch off dc power supply of the coil by unplugging the pin of the wire of the ammeter. Switch on the generator unit.

4. Check zero reading of the voltmeter at switched off power supply of the coil. It is considered to be correctly adjusted, if the reading of the teslameter is not higher 20 mV.

5. Switch on power supply of the coil, set current 0.2 A by the voltage regulator. At lower meaning of the current the error of measurement of magnetic field increases because the magnetic field is too weak. During experiment observe the current to stay at value 0.2 A.

6. By moving the probe with the Hall sensor along the coil axis at intervals in 5mm, write the voltage of the teslameter $\mathbf{U_T}$ into the table 14.1. The coordinate $x=0$ is the centre of the coil. The magnet field is determined by the formula:

$$B\ [\text{мТл}] = 10\ U_T\ [\text{В}]. \qquad (14.6)$$

7. Make picture of experimental B(x), using suitable scales, make scales on the axes.

8. Calculate the meaning of the magnetic field by formula (14.5). Draw a theoretical curve B(x) in the picture with the experimental B(x). Calculating theoretical B(x) by formula (14.5), use the following data:

$\mathbf{w} = 2000$ (turns); $\mathbf{R} = 8{,}25.10^{-3}$ (m); $\boldsymbol{\ell} = 0{,}047$ (м); $\boldsymbol{\mu_0} = 4\pi.10^{-7}$ H/m. The cosinuses in the formula (14.5) are:

$\cos(\alpha_1) = (x - \ell/2)/\sqrt{(R^2 + (x - \ell/2)^2)}$. and
$\cos(\alpha_2) = (x + \ell/2)/\sqrt{(R^2 + (x + \ell/2)^2)}$.

Table14.1

x, mm	25	20	15	10	5	0	10	15	20	25
U, V										
B, mT										

14.4. Control questions

1. What is the value of magnetic field on the axis of an infinitely long solenoid? Derive this formula from the Ampere's law.

2. Write the Biot-Savart formula in vector and scalar form, make a picture for it.

3. State the main properties of magnetic field and main theorems of magnetostatics.

14.5. References

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

2.Савельев И.В. Курс общей физики.М.: Наука, 1989.

3. Трофимова Т.И. Курс физики. М.: Высш. шк., 2002.

4. Иродов И.Е. Основные законы электромагнетизма. М.: Высш. шк., 1983.

15. STUDY OF THE MUTUAL INDUCTION

15.1. Objective: study of phenomena of mutual induction of two coaxial coils.

15.2. Theory. This work deals with mutual induction coefficient of a long coil 1 (**L_1**) and a short coil 2 (**L_2**), which is put on coil 1, it can move along its axis (fig. 15.1). Power supply of one coil (e.g. coil 1) is performed by acoustic generator PQ, which voltage is

$$U=U_0\cos\omega t, \tag{15.1}$$

it renders through resistor **R**. The magnitude **R** must obey condition $R \succ\succ \sqrt{R_1^2 + L_1^2\omega^2}$. Here L_1 is inductance of coil 1, **R_1** is its active resistance. Then the current in coil 1 is determined by formula:

$$I_1=\frac{U}{R}=\frac{U_0}{R}\cos\omega t = i\cos\omega t\ . \tag{15.2}$$

Alternative current in coil 1 creates alternative EMF of mutual inductance in coil 2:

$$\varepsilon_2 = -M_{21}\frac{dI_1}{dt_2} = M_{21}\frac{U_0}{R}\omega\sin\omega t \tag{15.3}$$

The amplitude of mutual inductance EMF is:

$$\varepsilon_{02} = M_{21}\frac{U_0}{R}\omega = M_{21}\frac{U_0}{R}2\pi f\ , \tag{15.4}$$

where ***f*** is the frequency of acoustic generator. Using formula (15.4) we obtain

$$M_{21}=\frac{\varepsilon_{02}R}{2\pi f U_0}\ . \tag{15.5}$$

If coils 1 and 2 are swapped then

$$M_{12}=\frac{\varepsilon_{01}R}{2\pi f u_0} \tag{15.6}$$

15.3. Experimental equipment

Cartridge FPE-05/06 “Mutual inductance” is supposed for study of mutual inductance phenomenon. The cartridge consists of two coaxial inductance coils 1 (**L_1**) and 2 (**L_2**), and rod with scale (Ш) exposing relative position of coils 1 and 2. The schematic circuit of the cartridge is in fig. 15.1. Switchers P_1 and P_2

are put in opposite position for swapping coils 1 and 2. Schematic line connection is in fig. 15.2.

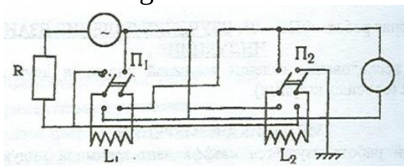

Fig.15.1

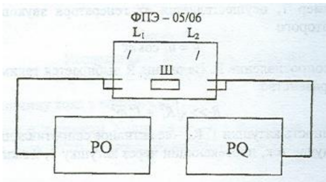

Fig. 15.2.

The cartridge is connected to acoustic generator PQ.

Voltmeter, placed on panel PQ, measures effective voltage

$$U_{eff}=U_0/\sqrt{2} \qquad (15.7)$$

Oscillograph PO is used for measurement of mutual inductance EMF amplitude.

15.4. Experimental procedure

Exercise 1

Measurement of mutual inductance coefficients $\mathbf{M_{21}}$ and $\mathbf{M_{12}}$ and study of their dependence on relative position.

1. Connect the circuit shown in fig. 15.2.
2. Preset voltage Ueff=2V and frequency $\boldsymbol{f}$ of the acoustic generator (by teacher's instructions), connect it to coil 1 (by switcher P_1), EMF of coil 2 supply to the oscillograph (by switcher P_2). Switch «V/ division» on the front panel of oscillograph PO preset to 0.02-0.05 V/division (the value of the maximal division on the display of PO).
3. Mount moveable coil 1 to the extreme position. Moving coil 1 to the opposite extreme position in intervals of 1 cm, write

into table 15.1 coordinate **Z** (the distance between the centers of the coils) and the EMF ε_{02} of mutual inductance of coil 2.

Table 15.1

$\mathbf{U_{eff}}$=2V f = … Гц						
	ε_{02}		M_{21}	ε_{01}		M_{12}
cm			H		V	H

4. Calculate M_{21} by formula (15.5). Put down the results into table (15.1).

5. Having swapped coils $\mathbf{L_1}$ and $\mathbf{L_2}$ (by switchers P_1 and P_2), repeat measurements of items 2, 3 and calculate $\mathbf{M_{12}}$.

6. Draw curves $\mathbf{M_{21}}$ and $\mathbf{M_{12}}$ against coordinate **Z** (the distance between the centers of the coils).

Exercise 2

Measurement $\mathbf{M_{21}}$ at various values of voltage of power supply.

1. Put coil 1 in the middle position relative to coil 2.

2. Preset the frequency of acoustic generator **PQ** (by teacher's instructions, e.g. 10^4 Hz)

3. Measure amplitudes of mutual inductance EMF ε_{02} at different values $\mathbf{U_{eff}}$ of coil 1 in interval 0-5 V by 0,5 V.

4. Calculate M_{21} by formula (15.5) and fill table 15.2

Table15.2

f=…Hz		$\mathbf{R}$=10^4 Ohm
U_{eff}, V	ε_{02}, V	M_{21}, Hz

Exercise 3

Measurement of $\mathbf{M_{21}}$ at various power supply frequencies.

1. Put coil 1 in the middle position relative to coil 2.

2. Preset the voltage of generator (by teacher's instruction, e.g. 2 V).

3. Measure amplitude of mutual inductance EMF ε_{02} at various frequencies of acoustic generator in the range 5 – 20 kHz (take not less then 10 values).

4. Calculate $\mathbf{M_{21}}$ by formula (15.5). Write obtained results into table 15.3.

5. Calculate absolute and relative errors for one of the calculated values of $\mathbf{M_{21}}$.

Table 15.3

$\mathbf{U_{eff}}$=…V		$\mathbf{R}=10^4$ Ohm
F, Hz	ε_{02}, V	M_{21}, H

15.5. Control questions

1. What is the value of EMF of two loops?
2. What does the mutual inductance coefficient depend on?
3. Explain $M_{21}=f(z)$ graph obtained in this work.

15.6. References

1. Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.
2. Савельев И.В. Общий курс физики. –Алматы: «Мектеп», 1977.
3. Фриш С.Э., Тиморева А.В. Общий курс физики: -Алматы: «Мектеп», 1970.
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16. STUDY OF ELECTRIC PROPERTIES OF FERROELECTRICS

16.1. Objectives. Obtaining and studying of dielectric hysteresis in ferroelectrics, study of $\varepsilon = f(E)$ dependence.

16.2. Theory. *Dielectrics* are the substances, which practically does not carry current in usual conditions. Unlike conductors, those have no free charge carriers. Typical dielectrics are: gases, some liquids (distilled water) and solid bodies (glass, porcelain, mica).

The molecules of a dielectric are electrically neutral, they can be considered as electrical dipoles, with dipole moments

$$\vec{p} = q\,\vec{l}\,, \tag{16.1}$$

where ***q*** is net charge of all atomic nuclei in molecules, $\vec{l}$ is the vector directed from the centre of mass of molecule's electrons to the centre of mass of positive charges in atomic nuclei. Dielectric is nonpolar, if $\vec{l} = 0$ in the absence of external electric field (e.g. H_2, O_2).

In external electric field molecule's electron shell deforms, and in the result of this, molecules acquire induced dipole moment $\vec{P}_e$.

Molecules of polar dielectrics have nonzero dipole moment even in the absence of external electric field.

When dielectric is put in the external electric field, it is polarized, quantitative characteristics of this is vector $\vec{P}$, called polarization (or polarization vector).

$$\vec{P} = \frac{1}{\Delta V}\sum_{i=1}^{n}\vec{P}_{ei}\,, \tag{16.2}$$

$\vec{P}_{ei}$ - the electrical moment of a molecule, ΔV - the volume of a dielectric having *n* number of molecules. Units of polarization vector $\vec{P}$ is C/m^2. Polarization of isotropic dielectrics $\vec{P}$ is connected with electric field:

$$\vec{P} = \chi\varepsilon_0\,\vec{E}, \tag{16.3}$$

where χ is a coefficient, at first approximation independent of E, χ is called dielectric susceptibility of a substance; $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\, F/m$ is the electric constant.

Besides electric field and polarization vectors, electric displacement vector $\vec{D}$ is used for description of electric field in dielectrics.

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \tag{16.4}$$

Considering (16.3) the displacement vector can be presented as:

$$\vec{D} = \varepsilon\varepsilon_0 \vec{E}, \tag{16.5}$$

where $\varepsilon = 1 + \chi$ is a unitless quantity, called dielectric permittivity of substance. For all dielectrics $\chi \succ 0$, $\varepsilon \succ 1$.

There exists a group of dielectrics, called ferroelectrics (in Russian language –‘сегнетоэлектрик’, called after the Seignette salt $NaKC_4H_4O_6\ 4H_{20}$), having a row of interesting properties:

1. Large relative permeability ε (the Seignette salt is about 10000, titanium – a few tens of thousand)

2. Dependence of dielectric permeability on temperature.

3. Dependence of dielectric permeability on electric displacement intensity (fig. 16.1).

4. Electric hysteresis (delay), this is dependence of polarization (or displacement vector) on previous values of polarization of the ferroelectric. Then there are different values of polarization for one intensity of electric field.

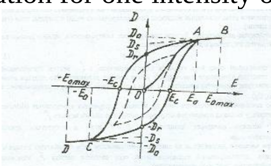

Fig.16.1

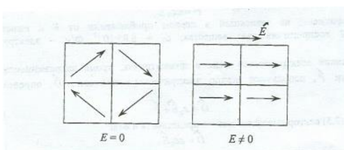

Fig.16.2

All specific features of ferroelectrics are connected with their spontaneous polarization. This can be explained that a crystal is divided into zones about 10^{-3} mm^3 in size, called domains. Domains are spontaneously polarized.

Domains realign their polarization under the influence of electric field, that results in net polarization of the crystal (fig. 16.2). Hysteresis loop is shown in fig 16.1, coordinates are ***D*** and ***E***. When ***E*** increases, displacement ***D*** changes in curve OAB, in the case when the sample was not polarized before. This curve is called principal, or initial polarization curve.

If maximal value of electric field intensity is such that spontaneous polarization reaches satiation, then the resultant hysteresis loop is called loop of boundary cycle (the solid line in fig. 16.1).

Dielectric losses lead to phase shift between oscillations of current and voltage in the circuit with a capacitor, the phase shift becomes less then $\pi/2$. The difference between $\pi/2$ and the value of the phase shift is called the loss angle δ :

$$\mathrm{tg}\,\delta = \frac{1}{2\pi}\,\frac{\Delta W}{W}\,, \qquad (16.6)$$

where ΔW - the energy loss during one oscillation; W – the energy of oscillations.

Let us rewrite (16.6) in form:

$$\mathrm{tg}\,\delta = \frac{1}{2\pi}\frac{W_r}{W_0}\,, \qquad (16.7)$$

here $\mathbf{W_r}$ – energy loss of alternating electric field for dielectric hysteresis per unit volume of the ferroelectric in one time period; W_0- the maximal energy density of electric field in the crystal.

Volume density of electric field energy is

$$W=\frac{1}{2}\varepsilon\varepsilon_0 E^2\,. \qquad (16.8)$$

When intensity of electric field increases in *dE*, the volume density of energy is changed to:

$$W=\oint E dD \qquad (16.9)$$

This numerically equals to the area of hysteresis loop in *D-E* coordinates. The maximal energy density of electric field in the crystal is

$$W_0 = \frac{E_0 D_0}{2} \tag{16.10}$$

where E_0 and D_0 are the amplitudes of intensity and displacement of electric field. Substituting (16.9) and (16.10) into formula (16.7), we receive the following expression:

$$\text{tg}\,\delta = \frac{\oint EdD}{\pi E_0 D_0}.$$

16.3. Experimental equipment

Experimental unit: cartridge ФПЭ-02/07, ИП – power supply, PV – oscillograph.

The voltage U_y from the reference capacitor C_2 supplies the vertical deflection plates of the oscillograph:

$$U_y = \frac{q}{C_2}. \tag{16.11}$$

C_1 and C_2 are connected in series, so they have equal charge *q* on their plates. The magnitude of this charge *q* can be expressed through the electric displacement *D* in the studied capacitor C_1:

$$D = \sigma = \frac{q}{s}\text{, then } q = DS, \tag{16.12}$$

where σ - surface charge density on the plates of the capacitor C_1; $S = \frac{\pi d^2}{4}$ - area, *d* – diameter of the plates of capacitor C_1. Considering (16.12) the voltage is

$$U_y = \frac{S}{C_2} D. \tag{16.13}$$

Voltage U_x supplies horizontal deflection plates. This voltage is taken from resistor R_2:

$$U_x = \frac{R_2}{R_1 + R_2} U. \tag{16.14}$$

This voltage is a part of voltage U, supplying voltage divider R_1, R_2, and capacitor divider C_1 and C_2. Capacitances C_1

and C_2 satisfy condition $C_1 << C_2$. Therefore we can consider that all voltage U (accurate within $\left(\frac{C_1}{C_2}\right)$), taken from potentiometer R_3 in the capacitance divider is applied to ferroelectric capacitor C_1. Indeed, as $\frac{U_{c1}}{U_{c2}} = \frac{C_2}{C_1} \succ\succ 1$, then $U = U_{c1} + U_{c2} \approx U_{c1}$. Then, considering the electric field in capacitor C_1 to be uniform:

$$U = Eh, \tag{16.15}$$

here E – electric field intensity in the ferroelectric plate; h - the ferroelectric plate width. Considering (16.15), voltage U_x can be written as:

$$U_x = \frac{R_2}{R_1 + R_2} Eh\ . \tag{16.16}$$

Voltages U_x and U_y are measured by the oscillograph and calculated by formulae:

$$U_y = K_y y\,, \tag{16.17}$$

$$U_x = K_x x\,, \tag{16.18}$$

where y, x are deflections of cathode beam on the screen of the oscillograph in the axes Y and X; K_y, K_x – deflection coefficients of Y and X oscillograph channels. Considering (16.17) and (16.18), we can obtain from (16.13) and (16.16):

$$D = \frac{C_2 K_y}{S} y\,, \tag{16.19}$$

$$E = \frac{R_{11} + R_2}{R_2} \frac{K_x}{h} x \tag{16.20}$$

Moreover, from (16.15) it follows that:

$$E_0 = \frac{U_0}{h} = \frac{\sqrt{2}}{h} U\,, \tag{16.21}$$

where U-effective value of the voltage, measured by voltmeter PV.

Substituting expressions (16.19) and (16.20) into (16.7) we obtain:

$$tg\delta = \frac{1}{\pi}\frac{\oint EdD}{E_0 D_0} = \frac{1}{\pi}\frac{\oint xdy}{x_0 y_0} = \frac{1}{\pi}\frac{S_n}{x_0 y_0}, \qquad (16.22)$$

where S_n- hysteresis loop area in x, y coordinates; x_0, y_0 are the coordinates of the loop vertex.

When values of D_0 and E_0 of vertexes of several cycles are found by formulae (16.19) and (16.21), one can find values of ε by the expression:

$$\varepsilon = \frac{D_0}{\varepsilon_0 E_0} = \frac{C_2 h K_y y}{\sqrt{2}\varepsilon_0 S U}. \qquad (16.23)$$

And then study dependence $\varepsilon = f(E)$.

16.4. Experimental procedure

1. Preset handle 'Per U' on the panel of the cartridge ФПЭ-02/07 to the middle position.

2. Preset control knobs on the panels of the oscillograph to the positions, allowing observation of Lissajous figures, measurement of alternative voltage and study of dependence of two external signals.

3. Prepare power supply IP and voltmeter PV.

4. Connect circuit.

5. Check correctness of the circuit connections, then plug in the circuit to the electrical network ~220V, 50Hz, switch on toggle switch «Set» on the panels of all the gages. The oscillograph screen must show a hysteresis loop.

6. Move the hysteresis loop to the centre of the oscillograph display. Using knob '12V-120V' on the panel of the power supply, adjust such direct voltage level that the hysteresis loop is symmetrical in the display.

Exercise 1. Determination of dielectric loss tangent.

1. Obtain hysteresis loop of boundary cycle. For this purpose, turn handle 'Per U' on the panel of the cartridge to its extreme right position; adjust deflection coefficient K_y of oscillograph, so that the hysteresis loop of boundary cycle is entirely in the boundaries of the display, occupying not less then half of display (in vertical direction).

2. Measure the coordinates x_0 and y_0 of vertex of hysteresis loop. For this purpose, determine coordinates of points A and

C, $+x_0$ and $-x_0$, $+y_0$ and $-y_0$, fig. 16.1, then calculate average values of their absolute values. Write down the value of deflection coefficient K_y at changing y_0.

3. Set the hysteresis loop symmetrically to axes *Y* and *X* and draw it in cross-section paper, using coordinates of the display grid.

4. Determine the area of the hysteresis loop, using the figure in the cross-section paper.

5. Calculate $\mathbf{tg}\,\delta$ by formula (16.22).

*Exercise 2***.** Determination of residual displacement $\mathbf{D_r}$, of coercive field E_c and spontaneous satiation polarization $P_{s\,max}$.

1. Arrange boundary cycle hysteresis loop, obtained in exercise 1, item 1, symmetrically with respect to axis *Y*. Measure y_r as a half of the loop width at $x=0$. Write down value of K_y, corresponding to this measurement.

2. Align hysteresis loop symmetrically with respect to axis *X*. Measure x_c as a half of the loop width at $y=0$.

3. Prolong linear intervals of the boundary cycle loop (AB and CD in fig. 16.1) till intersection with axis *Y*, using loop picture, obtained in exercise 1, item 3. Measure value y_s as a half of the distance between the points of intersection of the extrapolated intervals with axis *Y*.

4. Calculate values D_r; $P_{s\,max} \approx D_s$ and E_s.

5. Estimate the accuracy of measurement of residual displacement D_r, and coercive field E_c.

Note: the values of parameters, necessary for calculations and their accuracy are written on the panel of the equipment.

*Exercise 3***.** Obtaining normal polarization curve and study of dependence $\varepsilon = f(E)$.

1. For boundary cycle hysteresis curve, obtained in exercise 1, item 1, measure the values of coordinates of cycle vertex (point B in fig. 16.1) using the method, described in the same exercise 1, item 2. Write down the value of coefficient K_y at changing y_{0max}. Determine voltage U by readings of voltmeter PV.

2. Decrease voltage U by handle ‘Per U’ on the cartridge panel and obtain boundary cycle loop, corresponding to the

minimal amplitude E_0, below which the boundary cycle disappear (i.e. the area of the loop and the coordinates of its vertexes start to change). For this purpose:

a) determine voltage U by the voltmeter PV;

b) determine x_0, y_0 and K_y by the method in exercise 1, item 2.

3. Make several particular cycles, decreasing voltage U by handle 'Per U' and changing oscillograph coefficient $K_{y|}$ for every hysteresis loop to be not less then a half of the display in vertical direction. The number of particular cycles must not be less then five for various values of coefficient K_y.

4. Write all the results of measurements by items 1-3 into table 16.1.

5. Draw normal curve of polarization in coordinates x, y.

6. Calculate values E_0 and ε for all studied cycles of repolarization.

7. Calculate uncertainty of measurement of ε.

16.5. Control questions

1. What is dielectrics polarization? What quantity is a quantitative characteristic of polarization? How is this quantity connected with electric field intensity in dielectric?

2. Describe the principal properties of ferroelectrics.

3. Draw electric scheme for obtaining hysteresis loop, explain its work.

4. Obtain the formula, by which ferroelectric dielectric permittivity is determined in this work.

16.6. Reference

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17. CHARGING AND DISCHARGING OF CAPACITOR

17.1. Objective: Familiarization with the method of study of transient processes in circuits with a capacitor. 1. Determination of circuit time constant from diagrams of charging and discharging of a capacitor. 2. Calculation of circuit time constant by nominal parameters and comparison with experimental value.

17.2. Theory. Transient processes arise when circuit goes from one steady state to another. Examples of such processes are charging and discharging of a capacitor.

Direct current rules are applicable to alternative currents as well, if change in current is not very fast. In this case instantaneous value of current (or voltage) is the same for all cross sections of the circuit. Such currents and corresponding fields are called quasi-stationary. The condition of quasi-stationary state is:

$$\tau = \ell/c << T, \qquad (17.1)$$

where **ℓ** - the length of the circuit, **c** = 3.10^8 м/с – the electromagnetic disturbance propagation speed, **T** – the period of change in current or voltage.

For example, for a circuit length $\ell = 3$ m, circuit time is $\tau = 10^{-8}$ s, the currents can be considered as quasi-stationary up to frequencies 10^6 Hz ($T=10^{-6}$ s) with inaccuracy 1%. The laws of direct current are applicable to instantaneous values for frequencies 10^6 Hz with inaccuracy 1%.

Consider the process of discharging of a capacitor. If the plates of a charged capacitor C are connected through the resistor **R**, then current goes through the resistor. Denote **I, q, U** as instantaneous values of current, charge, and voltage on the capacitor.

Consider current as positive, when it charges from positively charged plate to the negatively charged one (fig. 17.1a), then I= **-** dq/dt. The Ohm’s law for resistor R: U = IR. Using U = q/C, we obtain a first order differential equation:

$$dq/dt + q/(RC) = 0. \qquad (17.2)$$

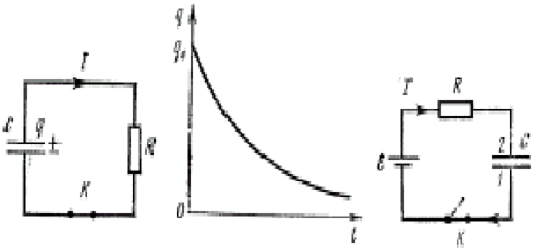

Рис. 17.1

The solution for this equation is:

$$q = q_0 e^{-t/\tau}, \qquad (17.2)$$

where q_0 – the initial charge on the capacitor, τ - the circuit time constant:

$$\tau = RC. \qquad (17.3)$$

Change of current against time can be found from (5.2):

$$I = -dq/dt = I_0 e^{-t/\tau}, \qquad (17.4)$$

Where $I_0 = q_0/\tau$ – current at t=0.

The q(t) diagram is shown in picture 17.1(b). The same dependence has current I(t).

Now let us study the process of charging of the capacitor. The circuit consists of in-series connected EMF source, resistor **R**, capacitor **C** and key K, it is shown in fig 17.1 (c). Key K is open at t<0, then at t=0 the key is closed, and charging current goes in the circuit. The growing charge on the capacitor inhibits current flow, gradually decreasing it.

We consider the current in the circuit as positive towards the positively charged plate: I = dq/dt. Now we apply the Ohm law to the inhomogeneous part of circuit 1-ε-R-2, where R equals the net resistance of the part, including internal resistance of the source: $IR = \varphi_1 - \varphi_2 + \varepsilon$. Substituting I = dq/dt and $\varphi_1 - \varphi_2 = -q/C$, we get $dq/dt = [\varepsilon - q/C]/R$. After separation of variables we get $Rdq/(\varepsilon - q/C) = dt$. Integration, considering the initial condition (q = 0 at t = 0) gives the equation $RC \ln (1-q/\varepsilon C) = -t$, from this equation we get

$$q = q_m (1 - e^{-t/\tau}), \qquad (17.5)$$

where $q_m = \varepsilon C$ – the limiting value on the capacitor (at $t \to \infty$), $\tau = RC$. The relation for current is:

$$I = I_0 e^{-t/\tau}, \qquad (17.6)$$

where $I_0 = \varepsilon / R$. These relations are in fig. 17.2.

17.3. Experimental procedure

1. Connect the scheme on the jack panel. Connect adjustable power supply of rectangular positive impulses with U_m = 10 V, f = 200 … 250 Hz to input terminals of the scheme. The connected gauges A1 and V0 are corresponding pairs of jacks on the connector.

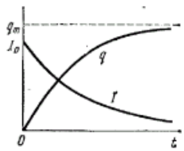

Fig. 17.2

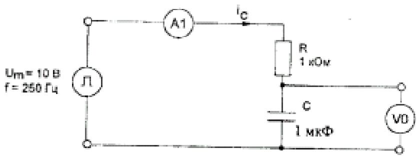

Fig.17.3

2. Connect computer, click two times on the label “VP_Physics” on the desktop, unit “ Pribory I ” appears. Click on the “Menu” of the opened window “ Pribory II ” choose “Oscillograph”.

3. Adjust the virtual oscillograph, so there is a picture of processes of charging and discharging of capacitor.

4. Draw oscillograms of current and voltage (in different colours) in the grid picture.

5. Designate curves and processes in the picture, draw one or two tangents to the curves of current or voltage and determine circuit time constant by the tangents.

6. Calculate circuit time constant by nominal parameters R and C, written on miniblocks (fig. 17.3) and compare it with the experimental value.

17.4. Control questions

1. What processes are called transient?

2. Give definition of quasi-stationary currents. What laws such currents obey? Specify the condition of quasi-stationary state.

3. What is the circuit time constant or relaxation time?

4. Derive differential equations for charging and discharging of the capacitor.

5. Solve differential equations for charging and discharging of the capacitor and draw q, I, U diagrams.

17.5. Reference

1.Fishbane P.M., Gasiorowicz S.G., Thornton S.T. Physics for Scientists and Engineers with Modern Physics. 3rd ed., Pearson Prentice Hall, New Jersey, USA, 2005.

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