Definitions

Definition 0.1. The **Regular Representation** of A is given by $\rho : A \to End(A)$, with $\rho(a)b = ab$.

Definition 0.2. A representation is called **irreducible** or **simple**, if the only subrepresentations are 0 and V.

Definition 0.3. A representation is called **semi-simple** if it is a direct sum of irreducible representations.

Definition 0.4. A non-zero representation of A is said to be **indecomposable** if it cannot be written as a direct sum of two non-zero representations.

Lemma 0.1. Let V_1 and V_2 be representations of an algebra A over a field F. Let $\phi : V_1 \to V_2$ be a non-zero morphism. Then

1. If V_1 is irreducible, ϕ is injective.
2. If V_2 is irreducible, ϕ is surjective.

Definition 0.5. Let $\mathfrak{g}$ be a vector space, and let $[,] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ be a pairing. Then $(\mathfrak{g}, [,])$ is said to be a **Lie Algebra** if it satisfies the following two properties:

1. $[,]$ is skew-symmetric bilinear.
2. $[,]$ satisfies the **Jacobi Identity**:

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0.$$

Definition 0.6. Let V be a vector space. Then, the **General Lie Algebra** of V is defined as $End(V)$ together with $[a, b] = ab - ba$. It is denoted by $\mathfrak{gl}(V)$.

Proposition 0.2. Let $g(t)$ be a differentiable family of automorphisms of an algebra A over $\mathbb{R}$ or $\mathbb{C}$, parametrized by $t \in (-\epsilon, \epsilon)$, such that $g'(0) = Id$. Then $g'(0) : A \to A$ is a derivation. Conversely, if D is a derivation, then e^{tD} is a one-parameter family of Automorphisms.

Definition 0.7. Let $\mathfrak{g}$ be a Lie algebra with basis x_i and define $[x_i, x_j] = \sum_k c_{ij}^k x_k$. Then the **Universal Enveloping Algebra** of $\mathfrak{g}$, denoted by $\mathcal{U}(\mathfrak{g})$