Topological Data Analysis

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1. Introduction

The idea of Morse theory is that you can obtain some information about a geometric space from the functions defined on it. For example, we can have unbounded continuous functions on $\mathbb{R}$, yet this is not possible in S^1 since it is compact [**?**].

Smooth manifolds, functions between smooth manifolds, and other smooth manifold concepts are defined as in [**?**, 150]. We will usually assume functions f of manifolds to be smooth, or at least twice differentiable so that their Hessians exist.

Definition. A critical point $p_0 \in M$ is said to be **non-degenerate** for the function f if its **Hessian**

$$H_f = \tag{1}$$

$H_f(p_0)$ has non-zero determinant at p_0.

I would like to keep some visual examples on hand. To that end, consider the following surfaces:

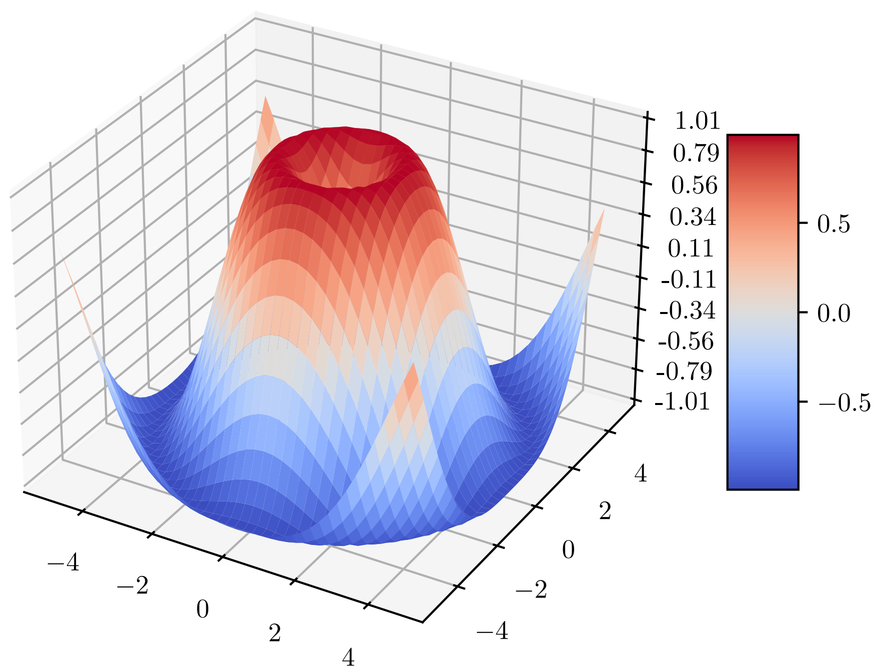

Figure 1. A surface given by the equation $Z =$

EXAMPLE.

EXAMPLE.

DEFINITION.

EXAMPLE. Let's check that the definition of a vector field on a manifold indeed generalizes the definition on $\mathbb{R}^n$.

DEFINITION. A function f is said to be **morse** if all its critical points are non-degenerate.

2. Persistent Homology

The quintessential reference for this section is [**?**].

There are a bunch of Homology groups, so it's easier to define them algebraically, and then name them by their respective chain complexes.

DEFINITION. convex hull

DEFINITION. Let X be a topological space, and $A \subseteq X$ a subspace. Then, the **relative homology groups** $H_n(X, A)$ are given by the chain complex defined by

$$C_n(X, A) = C_n(X)/C_n(A).$$

This next section is dedicated to defining the alpha complex.

DEFINITION.

DEFINITION. alpha complex.

There's a number of theorems we can prove for relative homology groups.

EXAMPLE (How to draw a persistence diagram).

2.1. Persistent Homology Morse Theory.

2.2. Extended Persistent Homology. Let $p = [p_1, \ldots, p_d] \in \mathbb{Z}^d$ and let $l = [l_1, \ldots, l_d] \in \mathbb{Z}/2\mathbb{Z}^d$

2.3. Persistent Homology. Let X be a Δ-complex.

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