Definition 0.1. A **universal covering** is a simply connected covering. They are unique up to homeomorphism.

Proposition 0.1. The number of sheets of a covering space $p : (\widetilde{X}, x_0) \to (X, x_0)$ with X and $\widetilde{X}$ path-connected equals the index of $p_*(\pi_1(\widetilde{X}, x_0))$ in $\pi_1(X, x_0)$.

Proposition 0.2. Let $p : (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)$ be a path-connected covering space of the path-connected, locally path-connected space X, and let $H = p_* \left(\pi_1(\widetilde{X}, \widetilde{x}_0)\right) \subset \pi_1(X, x_0)$. Then:

1. The covering space $p : \widetilde{X} \to X$ is normal (regular) if and only if H is a normal subgroup of $\pi_1(X, x_0)$.
2. The group of deck transformations $\mathrm{Deck}(\widetilde{X}/X)$ is isomorphic to the quotient $N(H)/H$, where $N(H)$ is the normalizer of H in $\pi_1(X, x_0)$.

In particular, if $\widetilde{X} \to X$ is a normal covering, then

$$\mathrm{Deck}(\widetilde{X}/X) \cong \pi_1(X, x_0)/H.$$

Hence, for the universal cover $\widetilde{X} \to X$, we have

$$\mathrm{Deck}(\widetilde{X}/X) \cong \pi_1(X, x_0).$$

Example 1. Suppose that X is a finite connected CW complex such that $\pi_1(X)$ is finite and nontrivial. Prove that the universal covering $\tilde{X}$ of X cannot be contractible.

Proof. □