Derived PHT

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In our analysis of projections of a 3-Manifold X, we would like to reconstruct our Manifold from its projections onto a direction $v \in S^2$. Each projection map is close to giving a Fiber Bundle, but the size of the fibers vary at each point. So we explore two alternatives: Sheaves and Fibrations.

Definition 0.1. A **Fiber Bundle** is a

Definition 0.2. A **Fibration** is a

Definition 0.3. A **Sheaf** is a

Abstract Sheaf Theory

A presheaf is an organizational tool that allows you to specify information of some structure locally or on open sets.

A sheaf is a presheaf that has the property that the information you have locally can glue into global information.

I will thus begin by giving an example of a presheaf that isn't a sheaf, and a presheaf that is a sheaf.

Example 1.

Example 2.

Now, it is somewhat clear from the definition of sheaf that it only depends on the notion of open covers.

Definition 0.4. Grothendieck Topology Site

As the name suggests, a Grothendieck Topology is a way to give some topological aspect to a Category. But in fact, it's not the full axioms of a topology that are passed on, rather only the information of "open covers". This is because open covers are enough to define sheaves.

Example 3. The most important example is that given by the open sets of a toplogical space X. The objects of the category will be denoted by $Op(X)$ and will consist of open subsets of X. Morphisms are inclusions.

Definition 0.5. A **presheaf** is a contravariant functor $\mathcal{F} : \mathcal{C} \to \mathcal{D}$, where $\mathcal{C}$ is a Site $\mathcal{D}$ is usually taken to be the category of sets, groups, rings, etc.

The main reason behind calling contravariant functors presheaves, is because it suggests that we want to treat that contravariant functor as a sheaf.

Definition 0.6.

Proposition 0.1. Sheafification

Example 4. constant sheaf

Example 5.

Sheaves are

Lifting Properties

Definition 1.1.

Definition 1.2.

Example 6.

Example 7.

Fibrations

Derived and Homotopy Categories

Definition 3.1.

Applications

Loop Spaces

Example 8. Let X be a PL 3-Manifold, and let $v \in S^2$. We denote the projection of X in the direction of v as X_v. Define a presheaf $\mathcal{F} : X \to \mathbb{R}^2$ by $\mathcal{F}(U) = PHT(U)$.

Magnitude Homology

Definition 5.1.

Proposition 5.1.