Class XIth - ML Aggarwal

Limits And Deriviatives

Ex → 13.1

1.(i) $\lim_{x \to 3} (x+3)$

$\Rightarrow 3+3$

$\Rightarrow 6$ Ans

(ii) $\lim_{r \to 1} \pi r^2$

$\Rightarrow \pi . (1)^2$

$\Rightarrow \pi$ Ans

2.(i) $\underset{x \to 1}{Lt} ((2x-1)^2+5)$

$\Rightarrow ((2-1)^2+5)$

$\Rightarrow 1+5 = 6$ Ans

(ii) $\underset{x \to 1}{Lt} (x^{40} - 3x^{12} + 1)^{132}$

$\Rightarrow (1^{40} - 3.1^{12} + 1)^{132}$

$\Rightarrow (1-3+1)^{132}$

$\Rightarrow (-1)^{132} = 1$ Ans

3.(i) $\underset{x \to 0}{Lt} \frac{ax+b}{cx+1}$

$\Rightarrow \frac{0+b}{0+1}$

$\Rightarrow b$ Ans

(ii) $\underset{x \to 0}{Lt} \frac{\sqrt{1+x} + \sqrt{1-x}}{3-x}$

$\Rightarrow \frac{\sqrt{1}+\sqrt{1}}{3-0} \Rightarrow \frac{1+1}{3} \Rightarrow \frac{2}{3}$ Ans

4. (i) $\underset{x \to 0}{Lt} \frac{ax+b}{cx+d}$

$\Rightarrow \frac{0+b}{0+d} \Rightarrow \frac{b}{d}$ Ans

(ii) $\underset{x \to -1}{Lt} \frac{x^3-3x+1}{x-1}$

$\Rightarrow \frac{(-1)^3 - 3.(-1)+1}{-1-1} \Rightarrow \frac{-1+3+1}{-2}$

$\Rightarrow -\frac{3}{2}$ Ans

5.(i) $\underset{x \to 3}{Lt} \frac{x^2-9}{x-3}$ $\left[\frac{0}{0} \text{ form}\right]$

$\Rightarrow \underset{x \to 3}{Lt} \frac{x^2-3^2}{x-3}$

$\Rightarrow 2.(3)^{2-1}$

$\Rightarrow 2 \times 3$

$\Rightarrow 6$ Ans

(ii) $\lim_{x \to -1} \frac{2x^2+3x+1}{x^2-1}$ $\left[\cancel{\frac{0}{0}}\ \cancel{\text{form}}\right]$

$\Rightarrow \lim_{x \to -1} \frac{2x^2+2x+x+1}{(x-1)(x+1)}$

$\Rightarrow \lim_{x \to -1} \frac{2x(x+1)+1(x+1)}{(x-1)(x+1)}$ $\Rightarrow \lim_{x \to -1} \frac{(2x+1)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$\Rightarrow \lim_{x \to -1} \frac{2x+1}{x-1}$ $\Rightarrow \frac{2.(-1)+1}{-1 \cancel{x} - (+1)}$ $\Rightarrow \frac{-2+1}{-2}$ $\Rightarrow +\frac{1}{2}$ Ans

6.(i) $\lim_{x \to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

$\Rightarrow \lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2}$ $\Rightarrow \lim_{x \to 2} \frac{-\cancel{(x-2)}}{2x} \times \frac{1}{\cancel{x-2}}$ $\Rightarrow \lim_{x \to 2} -\frac{1}{2x}$

$\Rightarrow -\frac{1}{2 \times 2} = -\frac{1}{4}$ Ans

(ii) $\lim_{x \to 2} \frac{x^5-32}{x-2}$

$\Rightarrow \lim_{x \to 2} \frac{x^5-2^5}{x-2}$ $\Rightarrow 5.\,2^{5-1}$ $\left[\lim_{x \to 0} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$

$\Rightarrow 5 \times 2^4 \Rightarrow 5 \times 16 = 80$ Ans

7. If $f(x) = \begin{cases} x-2, & x < 0 \\ x+2, & x \geq 0' \end{cases}$ find

(i) $\lim_{x \to 1} f(x)$

$\Rightarrow \lim_{x \to 1} x+2$

$\Rightarrow \cancel{x}\,1+2 = 3$ Ans

(ii) $\lim_{x \to -1} f(x)$

$\Rightarrow \lim_{x \to -1} x-2$

$\Rightarrow -1-2 = -3$ Ans

(iii) $\lim_{x \to 0} f(x)$ (0 is non-existing on the given scale so we take 0^+ and 0^-)

$\Rightarrow$ Right Hand Limit

$\lim_{x \to 0^+} x+2$

$\Rightarrow 0+2 = 2$

$\Rightarrow$ Left Hand Limit

$\lim_{x \to 0^-} x-2$

$\Rightarrow 0-2 = -2$

$R.H.L \neq L.H.L$

so $\lim_{x \to 0} f(x)$ is not existing for $f = \begin{cases} x-2, & x < 0 \\ x+2, & x \geq 0' \end{cases}$

8.(i) $\underset{x \to \frac{1}{2}}{Lt} \dfrac{4x^2-1}{2x-1}$

$\Rightarrow \underset{x \to \frac{1}{2}}{Lt} \dfrac{\cancel{(2x-1)}(2x+1)}{\cancel{(2x-1)}}$

$\Rightarrow \underset{x \to \frac{1}{2}}{Lt} 2x+1 \Rightarrow \frac{1}{\cancel{2}} \times \cancel{2}+1$

$\Rightarrow 1+1 = 2$ Ans

(ii) $\underset{x \to 1}{Lt} \dfrac{x-1}{2x^2-7x+5}$

$\Rightarrow \underset{x \to 1}{Lt} \dfrac{(x-1)}{2x^2-5x-2x+5}$

$\Rightarrow \underset{x \to 1}{Lt} \dfrac{(x-1)}{x(2x-5)-1(2x-5)}$

$\Rightarrow \underset{x \to 1}{Lt} \dfrac{\cancel{(x-1)}}{\cancel{(x-1)}(2x-5)} \Rightarrow \underset{x \to 1}{Lt} \dfrac{1}{2x-5}$

$\Rightarrow \dfrac{1}{2-5} = -\dfrac{1}{3}$ Ans

9. (i) $\underset{x \to 1}{Lt} \dfrac{x^2-3x+2}{x^2-1}$

$\Rightarrow \underset{x \to 1}{Lt} \dfrac{x^2-2x-x+2}{(x-1)(x+1)} \Rightarrow \underset{x \to 1}{Lt} \dfrac{x(x-2)-1(x-2)}{(x+1)(x-1)}$

$\Rightarrow \underset{x \to 1}{Lt} \dfrac{\cancel{(x-1)}(x-2)}{(x+1)\cancel{(x-1)}} \Rightarrow \underset{x \to 1}{Lt} \dfrac{(x-2)}{(x+1)}$

$\Rightarrow \dfrac{1-2}{1+1} = -\dfrac{1}{2}$ Ans

(ii) $\underset{x \to 3}{Lt} \dfrac{x^3-27}{x^2-9}$

$\Rightarrow \underset{x \to 3}{Lt} \dfrac{x^3-3^3}{x^2-3^2} \Rightarrow \underset{x \to 3}{Lt} \dfrac{\frac{x^3-3^3}{x-3}}{\frac{x^2-3^2}{x-3}}$

$\Rightarrow \underset{x \to 3}{Lt} \dfrac{\frac{\cancel{(x-3)}(x^2+9+3x)}{\cancel{(x-3)}}}{\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}}$

$\boxed{a^3-b^3 = (a-b)(a^2+b^2+ab)}$

$\boxed{a^2-b^2 = (a-b)(a+b)}$

$\Rightarrow \underset{x \to 3}{Lt} \dfrac{x^2+3x+9}{x+3} \Rightarrow \dfrac{9+9+9}{3+3}$

$\Rightarrow \dfrac{27}{6} = \dfrac{9}{2}$ Ans

10. (i) $\lim\limits_{x \to 2} \dfrac{x^3 - 4x^2 + 4x}{x^2 - 4}$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{x(x^2 - 4x + 4)}{x^2 - 2^2} \Rightarrow \lim\limits_{x \to 2} \dfrac{x(x^2 - 2x - 2x + 4)}{(x-2)(x+2)}$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{x[x(x-2) - 2(x-2)]}{(x-2)(x+2)} \Rightarrow \lim\limits_{x \to 2} \dfrac{x[(x-2)\cancel{(x-2)}]}{\cancel{(x-2)}(x+2)}$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{x(x-2)}{(x+2)}$ Putting Lt $\Rightarrow \dfrac{2(2-2)}{2+2}$

$\Rightarrow \dfrac{0}{4} = 0$ Ans

(ii) $\lim\limits_{x \to 2} \dfrac{x^3 - 2x^2}{x^2 - 5x + 6}$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{x^2(x-2)}{x^2 - 2x - 3x + 6} \Rightarrow \lim\limits_{x \to 2} \dfrac{x^2(x-2)}{x(x-2) - 3(x-2)}$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{x^2\cancel{(x-2)}}{(x-3)\cancel{(x-2)}} \Rightarrow \lim\limits_{x \to 2} \dfrac{x^2}{x-3}$

$\Rightarrow$ Putting Lt $\dfrac{4}{2-3} = -\dfrac{4}{1}$ Ans

11. (i) $\lim\limits_{x \to \frac{1}{2}} \left(\dfrac{8x-3}{2x-1} - \dfrac{4x^2+1}{4x^2-1} \right)$

$\Rightarrow \lim\limits_{x \to \frac{1}{2}} \left(\dfrac{8x-3}{2x-1} - \dfrac{4x^2+1}{(2x-1)(2x+1)} \right)$

$\Rightarrow \lim\limits_{x \to \frac{1}{2}} \left[\dfrac{(8x-3)(2x+1) - (4x^2+1)(1)}{(2x-1)(2x+1)} \right]$

$\Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{16x^2 + 8x - 6x - 3 - 4x^2 - 1}{(2x-1)(2x+1)} \Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{12x^2 + 2x - 4}{(2x+1)(2x-1)}$

$\Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{2(6x^2 + x - 2)}{(2x+1)(2x-1)} \Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{2(6x^2 + 4x - 3x - 2)}{(2x+1)(2x-1)}$

$\Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{2[2x(3x+2) - 1(3x+2)]}{(2x+1)(2x-1)} \Rightarrow \lim\limits_{x \to \frac{1}{2}} \dfrac{2[\cancel{(2x-1)}(3x+2)]}{\cancel{(2x-1)}(2x+1)}$

$\Rightarrow$ Putting Lt $\dfrac{2(3 \times \frac{1}{2} + 2)}{\cancel{2} \times \frac{1}{\cancel{2}} + 1} \Rightarrow \dfrac{3+4}{2} = \dfrac{7}{2}$ Ans

(ii) $\underset{x\to 2}{Lt}\left(\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{1}{x-2} - \frac{2(2x-3)}{x(x^2-3x+2)}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{1}{x-2} - \frac{2(2x-3)}{x(x^2-2x-x+2)}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{1}{x-2} - \frac{2(2x-3)}{x(x(x-2)-1(x-2))}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{1}{x-2} - \frac{2(2x-3)}{x[(x-2)(x-1)]}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{x(x-1) - 2(2x-3).1}{x(x-2)(x-1)}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{x^2-x-4x+6}{x(x-2)(x-1)}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{x^2-5x+6}{x(x-2)(x-1)}\right) \Rightarrow \underset{x\to 2}{Lt}\left(\frac{x^2-2x-3x+6}{x(x-2)(x-1)}\right)$

$\Rightarrow \underset{x\to 2}{Lt}\left(\frac{x(x-2)-3(x-2)}{x(x-2)(x-1)}\right) \Rightarrow \underset{x\to 2}{Lt}\frac{\cancel{(x-2)}(x-3)}{x\cancel{(x-2)}(x-1)}$

$\Rightarrow \underset{x\to 2}{Lt}\frac{x-3}{x(x-1)}$ putting Lt $\Rightarrow \frac{2-3}{2(2-1)} \Rightarrow \frac{-1}{2.1} \Rightarrow \frac{-1}{2}$ Ans

12. (i) $\underset{x\to\sqrt{2}}{Lt}\frac{x^2-2}{x^2+\sqrt{2}x-4}$

$2 = (\sqrt{2})^2$

$\sqrt{2}x = 2\sqrt{2}x - \sqrt{2}x$

$-4 = 2\sqrt{2}x \cdot (\sqrt{2}x)$

$\Rightarrow \underset{x\to\sqrt{2}}{Lt}\frac{(x-\sqrt{2})(x+\sqrt{2})}{x^2+2\sqrt{2}x-\sqrt{2}x-4}$

$\Rightarrow \underset{x\to\sqrt{2}}{Lt}\frac{(x-\sqrt{2})(x+\sqrt{2})}{x(x+2\sqrt{2})-\sqrt{2}(x+2\sqrt{2})}$

$\Rightarrow \underset{x\to\sqrt{2}}{Lt}\frac{\cancel{(x-\sqrt{2})}(x+\sqrt{2})}{\cancel{(x-\sqrt{2})}(x+2\sqrt{2})} \Rightarrow \underset{x\to\sqrt{2}}{Lt}\frac{x+\sqrt{2}}{x+2\sqrt{2}}$

$\Rightarrow$ Putting Lt $\frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+2\sqrt{2}} = \frac{2\cancel{\sqrt{2}}}{3\cancel{\sqrt{2}}} = \frac{2}{3}$ Ans

(ii) $\lim\limits_{x \to \sqrt{3}} \dfrac{x^4 - 9}{x^2 + 4\sqrt{3}x - 15}$

$4\sqrt{3}x = 5\sqrt{3}x - \sqrt{3}x$

$\Rightarrow \lim\limits_{x \to \sqrt{3}} \dfrac{(x^2 - 3)(x^2 + 3)}{x^2 + 5\sqrt{3}x - \sqrt{3}x - 15}$

$\Rightarrow \lim\limits_{x \to \sqrt{3}} \dfrac{(x - \sqrt{3})(x + \sqrt{3})(x^2 + 3)}{x(x + 5\sqrt{3}) - \sqrt{3}(x + 5\sqrt{3})}$ $\Rightarrow \lim\limits_{x \to \sqrt{3}} \dfrac{\cancel{(x - \sqrt{3})}(x + \sqrt{3})(x^2 + 3)}{\cancel{(x - \sqrt{3})}(x + 5\sqrt{3})}$

$\Rightarrow$ Putting Lt $\dfrac{(\sqrt{3} + \sqrt{3})(3 + 3)}{\sqrt{3} + 5\sqrt{3}}$ $\Rightarrow \dfrac{2\sqrt{3} \times \cancel{6}}{\cancel{6}\cancel{\sqrt{3}}} \Rightarrow 2$ Ans

$\Rightarrow 2$ Ans

13. (i) $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x} \times \dfrac{\sqrt{2 + x} + \sqrt{2}}{\sqrt{2 + x} + \sqrt{2}}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{(\sqrt{2 + x})^2 - (\sqrt{2})^2}{x[\sqrt{2 + x} + \sqrt{2}]}$ $\Rightarrow \lim\limits_{x \to 0} \dfrac{\cancel{2} + \cancel{x} - \cancel{2}}{\cancel{x}[\sqrt{2 + x} + \sqrt{2}]}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{1}{\sqrt{2 + x} + \sqrt{2}}$ putting Lt $\Rightarrow \dfrac{1}{\sqrt{2 + 0} + \sqrt{2}} = \dfrac{1}{2\sqrt{2}}$ Ans

(ii) $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 + x} - 1}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{x}{(\sqrt{1 + x} - 1)} \times \dfrac{(\sqrt{1 + x} + 1)}{(\sqrt{1 + x} - 1)}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{x(\sqrt{1 + x} + 1)}{(\sqrt{1 + x})^2 - 1^2}$ $\Rightarrow \lim\limits_{x \to 0} \dfrac{\cancel{x}(\sqrt{1 + x} + 1)}{\cancel{1} + \cancel{x} - \cancel{1}}$

$\Rightarrow \lim\limits_{x \to 0} \sqrt{1 + x} + 1$

$\Rightarrow$ Putting Lt

$\Rightarrow \sqrt{1 + 0} + 1$

$\Rightarrow \sqrt{1} + 1$

$\Rightarrow 1 + 1$

$\Rightarrow 2$ Ans

14. (i) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$

$$\Rightarrow \lim_{x \to 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} \times \frac{\sqrt{1+x^3} + \sqrt{1-x^3}}{\sqrt{1+x^3} + \sqrt{1-x^3}}$$

$$\Rightarrow \lim_{x \to 0} \frac{(\sqrt{1+x^3})^2 - (\sqrt{1+x^3})^2}{x^2\left[\sqrt{1+x^3} + \sqrt{1-x^3}\right]} \quad \Rightarrow \lim_{x \to 0} \frac{\cancel{1}+x^3 - \cancel{1}+x^3}{x^2(\sqrt{1+x^3} + \sqrt{1-x^3})}$$

$$\Rightarrow \lim_{x \to 0} \frac{2x^{\cancel{3}}}{\cancel{x^2}(\sqrt{1+x^3} + \sqrt{1-x^3})} \quad \Rightarrow \lim_{x \to 0} \frac{2x}{\sqrt{1+x^3} + \sqrt{1-x^3}}$$

Putting lt $\Rightarrow \dfrac{2 \times 0}{\sqrt{1+0} + \sqrt{1-0}} = \dfrac{0}{1+1} = \dfrac{0}{2} = 0$ Ans

(ii) $\lim\limits_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

$$\Rightarrow \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\Rightarrow \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\Rightarrow \lim_{h \to 0} \frac{\cancel{x}+h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} \quad \Rightarrow \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\Rightarrow \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$\Rightarrow$ Putting value of lt $\dfrac{1}{\sqrt{x+0} + \sqrt{x}}$

$$\Rightarrow \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \text{ Ans}$$

15. (i) $\lim_{x\to 2} \dfrac{\sqrt{3-x} - 1}{2-x}$

$\Rightarrow \lim_{x\to 2} \dfrac{\sqrt{3-x} - 1}{(2-x)} \times \dfrac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1}$

$\Rightarrow \lim_{x\to 2} \dfrac{(\sqrt{3-x})^2 - 1^2}{(2-x)(\sqrt{3-x}+1)} \quad \Rightarrow \lim_{x\to 2} \dfrac{3-x-1}{(2-x)(\sqrt{3-x}+1)}$

$\Rightarrow \lim_{x\to 2} \dfrac{\cancel{(2-x)}}{\cancel{(2-x)}(\sqrt{3-x}+1)} \quad \Rightarrow \lim_{x\to 2} \dfrac{1}{\sqrt{3-x}+1}$

$\Rightarrow$ Putting Lt $= \dfrac{1}{\sqrt{3-2}+1} = \dfrac{1}{1+1} = \dfrac{1}{2}$ Ans

(ii) $\lim_{x\to 3} \dfrac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$

$\Rightarrow \lim_{x\to 3} \dfrac{x-3}{\sqrt{x-2} - \sqrt{4-x}} \times \dfrac{\sqrt{x-2} + \sqrt{4-x}}{\sqrt{x-2} + \sqrt{4-x}}$

$\Rightarrow \lim_{x\to 3} \dfrac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{(\sqrt{x-2})^2 - (\sqrt{4-x})^2} \quad \Rightarrow \lim_{x\to 3} \dfrac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{x-2-4+x}$

$\Rightarrow \lim_{x\to 3} \dfrac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{2x-6} \quad \Rightarrow \lim_{x\to 3} \dfrac{\cancel{(x-3)}(\sqrt{x-2}+\sqrt{4-x})}{2\cancel{(x-3)}}$

$\Rightarrow \lim_{x\to 3} \dfrac{\sqrt{x-2}+\sqrt{4-x}}{2}$ Putting Lt $\Rightarrow \dfrac{\sqrt{3-2}+\sqrt{4-3}}{2}$

$\Rightarrow \dfrac{1+1}{2} = \dfrac{\cancel{2}}{\cancel{2}} = 1$ Ans

16. (i) $\lim_{x\to 3} \dfrac{(x+1) - \sqrt{x+13}}{x-3}$

$\Rightarrow \lim_{x\to 3} \dfrac{(x+1) - \sqrt{x+13}}{x-3} \times \dfrac{(x+1) + \sqrt{x+13}}{(x+1) + \sqrt{x+13}}$

$\Rightarrow \lim_{x\to 3} \dfrac{(x+1)^2 - (\sqrt{x+13})^2}{(x-3)[(x+1)+\sqrt{x+13}]}$

$\Rightarrow \lim_{x\to 3} \dfrac{x^2+1+2x-x-13}{(x-3)[(x+1)+\sqrt{x+13}]} \quad \Rightarrow \lim_{x\to 3} \dfrac{x^2+x-12}{(x-3)[(x+1)+\sqrt{x+13}]}$

$\Rightarrow \lim_{x\to 3} \dfrac{x^2+4x-3x-12}{(x-3)[(x+1)+\sqrt{x+13}]} \quad \Rightarrow \lim_{x\to 3} \dfrac{x(x+4)-3(x+4)}{(x-3)[(x+1)+\sqrt{x+13}]}$

$$\Rightarrow \lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}[(x+1)+\sqrt{x+13}]} \Rightarrow \lim_{x\to 3} \frac{x+4}{(x+1)+\sqrt{x+13}}$$

$$\Rightarrow \text{Putting } Lt = \frac{3+4}{(3+1)+\sqrt{3+13}} = \frac{7}{4+\sqrt{16}}$$

$$\Rightarrow \frac{7}{4+4} = \frac{7}{8} \text{ Ans}$$

(ii) $$\lim_{x\to 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$$

$$\Rightarrow \lim_{x\to 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} \times \frac{\sqrt{3+x}+\sqrt{5-x}}{\sqrt{3+x}+\sqrt{5-x}}$$

$$\Rightarrow \lim_{x\to 1} \frac{(\sqrt{3+x})^2-(\sqrt{5-x})^2}{(x-1)(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$\Rightarrow \lim_{x\to 1} \frac{3+x-5+x}{(x-1)(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$\Rightarrow \lim_{x\to 1} \frac{2x-2}{(x-1)(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$\Rightarrow \lim_{x\to 1} \frac{2\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(\sqrt{3+x}+\sqrt{5-x})}$$

$$\Rightarrow \lim_{x\to 1} \frac{2}{(x+1)(\sqrt{3+x}+\sqrt{5-x})} \quad \text{Putting } Lt \Rightarrow \frac{2}{(1+1)(\sqrt{3+1}+\sqrt{5-1})}$$

$$\Rightarrow \frac{2}{2(\sqrt{4}+\sqrt{4})} \Rightarrow \frac{2}{2(2+2)} = \frac{\cancel{2}}{\cancel{2}\times 4} = \frac{1}{4} \text{ Ans}$$

17. (i) $$\lim_{x\to 2} \frac{x^8-256}{x-2}$$

$$\Rightarrow \lim_{x\to 2} \frac{x^8-2^8}{x-2}$$

$$\Rightarrow 8\times 2^{8-1}$$

$$\Rightarrow 8\times 2^7 = 8\times 128$$

$$\Rightarrow 1024 \text{ Ans}$$

(ii) $\underset{x\to 4}{Lt} \dfrac{x^{3/2} - 8}{x - 4}$ $\qquad x^{1/2} = \sqrt{x}$

$\Rightarrow \underset{x\to 4}{Lt} \dfrac{x^{3/2} - 2^3}{(\sqrt{x})^2 - (2)^2} \qquad \Rightarrow \underset{x\to 4}{Lt} \dfrac{(x^{1/2})^3 - 2^3}{(\sqrt{x}-2)(\sqrt{x}+2)}$

$\Rightarrow \underset{x\to 4}{Lt} \dfrac{\cancel{(\sqrt{x}-2)}(x+4+2\sqrt{x})}{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}$ Putting Lt $\Rightarrow \dfrac{4+4+2\sqrt{4}}{\sqrt{4}+2}$

$\Rightarrow \dfrac{4+4+4}{2+2} = \dfrac{\cancel{12}\,3}{\cancel{4}} = 3$ Ans

18. (i) $\underset{x\to 2}{Lt} \dfrac{x^{10} - 1024}{x^5 - 32}$

$\Rightarrow \underset{x\to 2}{Lt} \dfrac{x^{10} - 2^{10}}{x^5 - 2^5} \qquad \Rightarrow \dfrac{\underset{x\to 2}{Lt} \dfrac{x^{10}-2^{10}}{x-2}}{\underset{x\to 2}{Lt} \dfrac{x^5-2^5}{x-2}} \qquad \Rightarrow \dfrac{10 \times 2^{10-1}}{5 \times 2^{5-1}}$

$\Rightarrow \dfrac{\cancel{10}^{\,2} \times 2^9}{\cancel{5} \times 2^4} \qquad \Rightarrow 2^6 = 64$ Ans

(ii) $\underset{x\to a}{Lt} \dfrac{x^m - a^m}{x^n - a^n}$

$\Rightarrow \underset{x\to a}{Lt} \dfrac{\dfrac{x^m - a^m}{x - a}}{\dfrac{x^n - a^n}{x - a}} \qquad \Rightarrow \dfrac{m a^{m-1}}{n a^{n-1}} \qquad \Rightarrow \dfrac{m}{n} a^{m-\cancel{1}-n+\cancel{1}}$

$\Rightarrow \dfrac{m}{n} a^{m-n}$ Ans

19. (i) $\underset{x\to 0}{Lt} \dfrac{(1+x)^n - 1}{x}$

$\Rightarrow \underset{x\to 0}{Lt} \dfrac{(1+x)^n - 1^n}{(1+x) - 1}$ adding 1 and subtracting one in denominator.

$\Rightarrow \underset{1+x\to 1}{Lt} \dfrac{(1+x)^n - 1^n}{(1+x) - 1}$ adding 1 on RHS as well as LHS of the limit

$\Rightarrow n \times 1^{n-1} = n \times 1$

$\Rightarrow n$ Ans

(ii) $\underset{x\to 0}{Lt} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

$\Rightarrow$ Let $1+x = y$

if $x = 0$; $y = 1$

so,

$\Rightarrow \underset{y\to 1}{Lt} \dfrac{\sqrt{y}-1}{\sqrt[3]{y}-1}$ $\Rightarrow \underset{y\to 1}{Lt} \dfrac{\dfrac{\sqrt{y}-1}{y-1}}{\dfrac{\sqrt[3]{y}-1}{y-1}}$

$\Rightarrow \underset{y\to 1}{Lt} \dfrac{\dfrac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}}{\dfrac{y^{\frac{1}{3}}-1^{\frac{1}{3}}}{y-1}}$

$\Rightarrow \dfrac{\frac{1}{2}\times(1)^{\frac{1}{2}-1}}{\frac{1}{3}\times(1)^{\frac{1}{3}-1}}$ $\Rightarrow \dfrac{1}{2}\times\dfrac{3}{1} = \dfrac{3}{2}$ Ans

20. If $\underset{x\to 3}{Lt} \dfrac{x^n-3^n}{x-3} = 108$ and $n \in N$, find n.

$\Rightarrow \underset{x\to 3}{Lt} \dfrac{x^n-3^n}{x-3} = 108$

$\Rightarrow n.3^{n-1} = 2\times2\times3\times3\times3$

$\Rightarrow n.3^{n-1} = 4.3^{4-1}$

$\Rightarrow n = 4$ Ans

21. If $a_1, a_2, \ldots, a_n$ are fixed real numbers and a function f is defined by $f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$, find $\underset{x\to a_1}{Lt} f(x)$.

$\Rightarrow f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$

$\Rightarrow \underset{x\to a_1}{Lt} f(a_1) = (a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)$

$= 0 \times (a_1-a_2)\ldots(a_1-a_n)$

$= 0$ Ans

For some $a \neq a_1, a_2, \ldots, a_n$, compute $\underset{x\to a}{Lt} f(x)$.

$\Rightarrow \underset{x\to a}{Lt} f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$

$f(a) = (a-a_1)(a-a_2)\ldots(a-a_n)$ Ans

22. (i) If $f(x) = \begin{cases} 1+x^2, & 0 \le x \le 1 \\ 2-x, & x > 1 \end{cases}$, does $\lim_{x \to 1} f(x)$ exist?

⇒ Right ~~Left~~ Hand Limit

$\lim_{x \to 1^+} f(x) \Rightarrow \lim_{x \to 1^+} (2-x)$ ⇒ Putting Lt $= 2-1$

R·H·L $= 1$ Ans

⇒ Left ~~Right~~ Hand Limit

$\lim_{x \to 1^-} f(x) \Rightarrow \lim_{x \to 1^-} (1+x^2)$ ⇒ Putting Lt $= 1 + 1^2 = 1+1$

~~R~~·H·L $= 2$ Ans

⇒ L·H·L ≠ R·H·L Hence $\lim_{x \to 1} f(x)$ does not exist.

(ii) If $f(x) = \begin{cases} x+1, & \text{if } x > 0 \\ x-1, & \text{if } x \le 0 \end{cases}$, show that $\lim_{x \to 0} f(x)$ does not exist.

⇒ Left Hand Limit

$\Rightarrow \lim_{x \to 0^-} f(x) \Rightarrow \lim_{x \to 0^-} (x-1)$

$\Rightarrow 0-1 = -1$

⇒ Right Hand Limit

$\Rightarrow \lim_{x \to 0^+} f(x) \Rightarrow \lim_{x \to 0^+} (x+1)$

$\Rightarrow 0+1 = 1$

L·H·L ≠ R·H·L

Hence $\lim_{x \to 0} f(x)$ does not exist!

23. Let $f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ 0, & x = \frac{1}{2} \\ x-1, & \frac{1}{2} < x \le 1 \end{cases}$, show that $\lim_{x \to \frac{1}{2}} f(x)$ does not exist.

⇒ Left Hand Limit

$\Rightarrow \lim_{x \to \frac{1}{2}^-} f(x) \Rightarrow \lim_{x \to \frac{1}{2}^-} x$

$\Rightarrow \frac{1}{2} =$ L·H·L

⇒ Right Hand Limit

$\Rightarrow \lim_{x \to \frac{1}{2}^+} f(x) \Rightarrow \lim_{x \to \frac{1}{2}^+} (x-1)$

$\Rightarrow \frac{1}{2} - 1 = -\frac{1}{2} =$ R·H·L

L·H·L ≠ R·H·L

Hence, $\lim_{x \to \frac{1}{2}} f(x)$ does not exist.

24. Let f be defined by $f(x) = \begin{cases} 3x-1, & x<0 \\ 0, & x=0 \\ 2x+5, & x>0 \end{cases}$. Evaluate :-

(i) $\lim_{x\to 2} f(x)$

$\Rightarrow \lim_{x\to 2} (2x+5)$

$\Rightarrow$ Putting Lt $2\times 2+5$

$\Rightarrow 4+5 = 9$

(ii) $\lim_{x\to -3} f(x)$

$\Rightarrow \lim_{x\to -3} 3x-1$

$\Rightarrow$ Putting Lt $3\times -3 -1$

$\Rightarrow -9-1 = -10$

Does $\lim_{x\to 0} f(x)$ exist? If no, explain.

$\Rightarrow$ Left Hand Limit

$\Rightarrow \lim_{x\to 0^-} f(x) \Rightarrow \lim_{x\to 0^-} (3x-1)$

$\Rightarrow 3\times 0-1 = -1$

$\Rightarrow$ Right Hand Limit

$\Rightarrow \lim_{x\to 0^+} f(x) \Rightarrow \lim_{x\to 0^+} (2x+5)$

$\Rightarrow 2\times 0+5 = 5$

L.H.L $\neq$ R.H.L Hence $\lim_{x\to 0} f(x)$ does not exist.

25. Find k so that $\lim_{x\to 2} f(x)$ may exist where $f(x) = \begin{cases} 4x-5, & x\le 2 \\ x-k, & x>2 \end{cases}$.

$\Rightarrow$ Left Hand Limit

$\Rightarrow \lim_{x\to 2^-} f(x) \Rightarrow \lim_{x\to 2^-} (4x-5)$

$\Rightarrow$ Putting Lt $4\times 2-5$

$\Rightarrow 8-5 = 3$

$\Rightarrow$ Right Hand Limit

$\Rightarrow \lim_{x\to 2^+} f(x) \Rightarrow \lim_{x\to 2^+} (x-k)$

$\Rightarrow$ Putting Lt $2-k \Rightarrow 2-k$

$\Rightarrow \lim_{x\to 2} f(x)$ may exist so,

$3 = 2-k$

$k = 2-3$

$k = -1$ Ans

26.(i) Let f be a function defined by $f(x) = \begin{cases} \frac{|x|}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$. Does $\lim_{x\to 0} f(x)$ exist?

$\Rightarrow$ Left Hand Limit

$\Rightarrow \lim_{x\to 0^-} f(x) \Rightarrow \lim_{x\to 0^-} \frac{|x|}{x}$

$\Rightarrow \lim_{x\to 0^-} \frac{-\cancel{x}}{\cancel{x}} \Rightarrow \lim_{x\to 0^-} -1$

$\Rightarrow -1 = $ L.H.L

$\Rightarrow$ Right Hand Limit

$\Rightarrow \lim_{x\to 0^+} f(x) \Rightarrow \lim_{x\to 0^+} \frac{|x|}{x}$

$\Rightarrow \lim_{x\to 0^+} \frac{x}{x} \Rightarrow \lim_{x\to 0^+} 1$

$\Rightarrow 1 = $ R.H.L

$\Rightarrow$ L.H.L $\neq$ R.H.L

so $\lim_{x\to 0} f(x)$ does not exist.

$|x| = \begin{Bmatrix} x, & \text{if } x\ge 0 \\ -x, & \text{if } x<0 \end{Bmatrix}$

(ii) Show that $\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist. $|x-2| = \begin{cases} x-2, & \text{if } x \geq 2 \\ -(x-2), & \text{if } x < 2 \end{cases}$

$\Rightarrow$ Left Hand Limit

$\Rightarrow \lim_{x \to 2^-} \frac{-(x-2)}{(x-2)} = \lim_{x \to 2^-} -1$

$\Rightarrow -1$

$\Rightarrow$ Right Hand Limit

$\Rightarrow \lim_{x \to 2^+} \frac{x-2}{x-2} \Rightarrow \lim_{x \to 2^+} 1$

$\Rightarrow 1$

$L.H.L \neq R.H.L$

Hence $\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist.

27. If $f(x) = |x| - 5$, evaluate the following limits:

(i) $\lim_{x \to 5^+} f(x)$ [not in $\frac{0}{0}$ form]

$\Rightarrow \lim_{x \to 5^+} |x| - 5$

$\Rightarrow \lim_{x \to 5^+} x - 5$

$\Rightarrow$ Putting $\lim$ $x - 5$

$\Rightarrow 0$ Ans

~~(ii) $\lim_{x \to 5^-} f(x)$~~

~~$\Rightarrow \lim_{x \to 5^-} |x| - 5$~~

~~$\Rightarrow \lim_{x \to 5^-} -x - 5$~~

~~$\Rightarrow$ Putting $\lim$ $-5-5$~~

~~$\Rightarrow -10$ Ans~~

*Function of modulus will be normal here as it is not in the form of $\frac{0}{0}$.

(ii) $\lim_{x \to 5^-} f(x)$ [not in $\frac{0}{0}$ form]

$\Rightarrow \lim_{x \to 5^-} |x| - 5$

$\Rightarrow \lim_{x \to 5^-} x - 5$

$\Rightarrow$ Putting $\lim$ $5 - 5$

$\Rightarrow 0$ Ans

(iii) $\lim_{x \to 5} f(x)$

$\Rightarrow \lim_{x \to 5} |x| - 5$

$\Rightarrow$ Putting $\lim$ $|5| - 5$

$\Rightarrow 0$ Ans

(iv) $\lim_{x \to -5} f(x)$

$\Rightarrow \lim_{x \to -5} |x| - 5$

$\Rightarrow$ Putting $\lim$ $|-5| - 5$

$\Rightarrow 5 - 5$

$\Rightarrow 0$ Ans

28. Let $f(x) = \begin{cases} \dfrac{x - |x|}{x} & , x \neq 0 \\ -2 & , x = 0 \end{cases}$, show that $\underset{x \to 0}{Lt} f(x)$ does not exist.

$\Rightarrow$ Left Hand Limit

$\Rightarrow \underset{x \to 0^-}{Lt} f(x) \Rightarrow \underset{x \to 0^-}{Lt} \dfrac{x - |x|}{x}$

$\Rightarrow \underset{x \to 0^-}{Lt} \dfrac{x - (-x)}{x} \Rightarrow \underset{x \to 0^-}{Lt} \dfrac{2\cancel{x}}{\cancel{x}}$

$\Rightarrow$ Putting Lt 2

$\Rightarrow$ Right Hand Limit

$\Rightarrow \underset{x \to 0^+}{Lt} f(x) \Rightarrow \underset{x \to 0^+}{Lt} \dfrac{x - |x|}{x}$

$\Rightarrow \underset{x \to 0^+}{Lt} \dfrac{x - x}{x} \Rightarrow \underset{x \to 0^+}{Lt} \dfrac{0}{x}$

$\Rightarrow$ Putting Lt 0

L.H.L $\neq$ R.H.L

Hence $\underset{x \to 0}{Lt} f(x)$ does not exist.

~ END ~