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1.3 Exercises

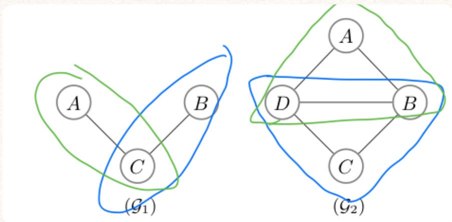
1. **Cliques in MRFs**

   (a) What is the definition of a clique in a graphical model?

   (b) What are the cliques in graph $\mathcal{G}_1$ and $\mathcal{G}_2$?

A) A set of nodes where all nodes in the set are fully connected.

b)



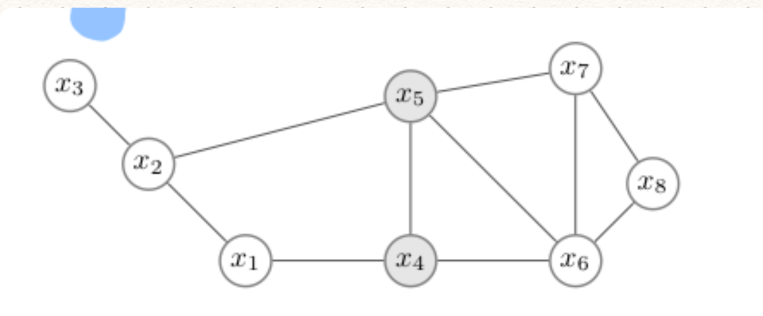
$(\mathcal{G}_1)$ $(\mathcal{G}_2)$

The above shows the max cliques.
G_2 also has pairwise cliques,
e.g $\langle D, B \rangle$.

2. **Independencies**

   (a) Which nodes (or set of nodes) are independent in graph $\mathcal{G}_3$?

   (b) What is the Markov blanket for node x_2 and node x_6 in graph $\mathcal{G}_3$?



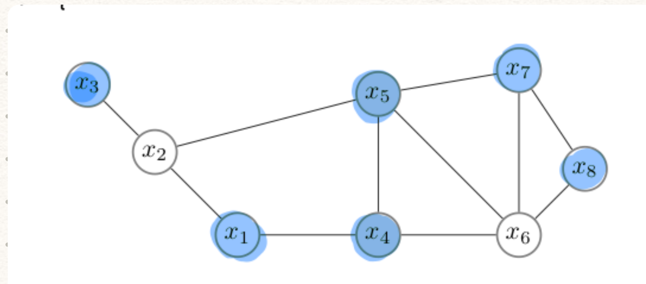
$(\mathcal{G}_3)$

a) $X_7 \perp X_2 \mid X_5$

$X_1 \perp X_6 \mid X_4$

$X_2 \perp X_6 \mid X_5, X_4$

b)



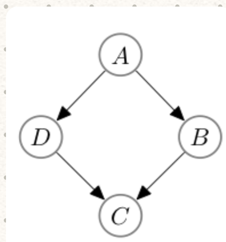
$(\mathcal{G}_3)$

3. **Graphical Representations**

   (a) Suppose that we want to define a model with a graph that satisfies the indepence conditions $A \perp C \mid \{B, D\}$, meaning A and C are independent if B and D are observed, and also $B \perp D \mid \{A, C\}$. Explain why this model cannot be represented by the directed graph $\mathcal{G}_4$?

   (b) The directed graph $\mathcal{G}_5$ satisfies $A \perp B$ when C is unobserved. Explain why can't this model be represented by an undirected graph?

1.3a)

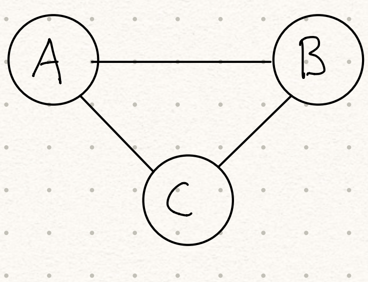


$(\mathcal{G}_4)$

$A \perp C \mid B, D$ holds
$D \perp B \mid A$ holds
$D \not\perp B \mid A, C$ as
C will let info
flow between D and
B. C is a collider.

1.3b)

To represent G_5 as
an undirected graph, we'll
need to "marry" A and B,
thus obtaining a fully
connected moral graph



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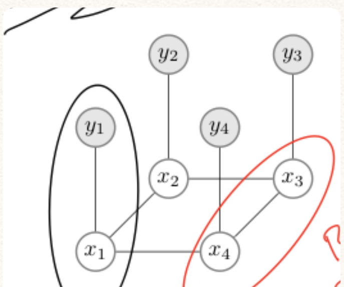
2.1 Exercises

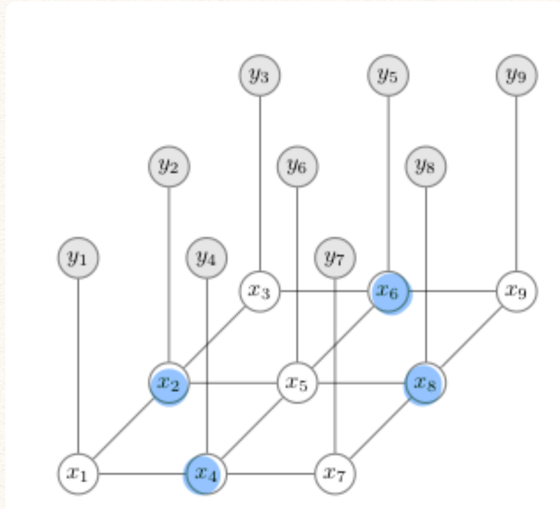
1. Explain the types of cliques that exist in the undirected graphs in Figure 4.
2. Which nodes are in the Markov Blanket of the hidden state x_5 in graph $\mathcal{G}_2$ in Figure 4?
3. Write an expression using factors over clique variables for the joint probability distribution $p(\mathbf{x}, \mathbf{y})$ of the graph $\mathcal{G}_1$ in Figure 4. Remember to define the normalizing constant Z.

1) The graphs in figure 4 only consists of pair wise cliques, i.e all neighbouring hidden values x_i, x_j and all pairs of hidden value and corresponding observed value, i.e x_i, y_i.



$(\mathcal{G}_1)$

2)



$(\mathcal{G}_2)$

X_5's Markov Blanket is the blue nodes

3)

$$P(X,Y) = \frac{e^{-\left(h \sum_{i \in [1,4]} x_i - \beta \sum_{i,j \in [1,4]} x_i x_j - \eta \sum_{i \in [1,4]} x_i y_i\right)}}{\underbrace{\sum_{i \in [1,4]} \prod_{j \in [1,4]} E(x_j, y_i)}_{\text{Sum product of all cliques}}}$$

2.3

1) MAP for ising methods leads to efficient local Computations, it is a simple algorithm that can be used but it does not guarantee that you'll find global maximum.

2.) $U(x_i, y_i)$ = Correlation between observed value y_i and hidden state y_i

$V(x_i, x_j)$ = Correlation between adjecent hidden nodes.

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3.1.2) By plotting all resulting images for varying sizes $\lambda \in [1,10]$ and $\mathcal{T} \in [1,10]$ I found that $\lambda = 2$, $\mathcal{T} = 3$ gave a good result. Around 5-6 iterations was sufficient for the energy to converge.

3.2.2) Also by visual observation, I found $\lambda = 100$ gave good results. By increasing lambda we increase the cost of having connected nodes of opposite value.

This is because if $x_i \neq x_j$, the energy function is thus increased by λ. One can see this as the edges between verticies with the same value becomes "stronger".