Assignment 1 - 2020-09-27

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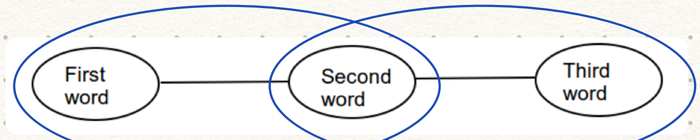
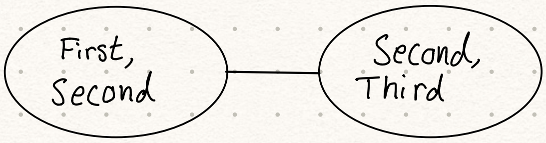
[redacted]

Assignment data



Markov model fr. Assignment

Create a clique tree from the Markov model, which result in the following clique tree:



We then have the factor function from the assignment.

ψ	Noun	Verb	Other
Noun	1	2	3
Verb	10	1	3
Other	3	5	2

Factor funktion fr. Assignment

Assignment 1.1

1. Perform calibration using Sum-Product Message Passing algorithm (it is given in the D. Koller book on the page 357, while $\delta_{1->2}$ is defined at the page 352)
   (a) Pass a message from the left clique to the right $\delta_{1->2}$: calculate β_2
   (b) Pass a message from the right clique to the left $\delta_{2->1}$: calculate β_1
   (c) Check if the resulting beliefs β_i are calibrated (if $\mu_{1,2} = \mu_{2,1}$)

   Note : you should not include $\delta_{1->2}$, while passing message $\delta_{2->1}$ back, in order not to count it twice.

1.a

$$\delta_{i\to j} = \sum_{C_i - S_{i,j}} \psi_i \cdot \prod_{k\in(Nb_i - \{j\})} \delta_{k\to i} \quad [P.\ 352]$$

Using the above method, we calculate $\delta_{1\to 2}$.

$\delta_{1\to 2} = \sum_{C_1 - S_{1,2}} \psi_1 \cdot \prod_{k\in Nb_1 - \{2\}} \delta_{k\to 1}$, As we have no incoming messages besides $\delta_{2\to 1}$ and

$S_{i,j} = C_1 \cap C_2 = Second$, we obtain

$$\delta_{1\to 2} =$$

Second	
Noun	6
Verb	14
Other	10

β_2 is then obtained through $\beta_2 = \psi_2 \cdot \delta_{1\to 2}(\text{Second}) =$

	Noun	Verb	Other
Noun	6	28	30
Verb	60	14	30
Other	18	70	20
$\sum_{C_2 - S_{2,1}} \beta_2(C_2)$	84	112	80

1.b

$$\delta_{2\to 1} = \sum_{C_2 - S_{2,1}} \psi_2 \cdot \prod_{k \in Nb_2 - \{1\}} \delta_{k\to 2} = \{\text{See 1.a for why}\} =$$

Second	
Noun	14
Verb	8
Other	8

$$\beta_1 = \psi_1 \cdot \delta_{2\to 1} =$$

	Noun	Verb	Other	$\sum_{C_1 - S_{1,2}} (\beta_1)$
Noun	14	28	42	84
Verb	80	8	24	112
Other	24	40	16	80

1.c

$$\mu_{1,2}(\text{Second}) = \mu_{2,1}(\text{Second}) =$$

Second	$\phi(\text{Second})$
Noun	84
Verb	112
Other	80

Assignment 1.2

2. Calculate the marginal distribution over the third word
   (a) Marginalize the belief $\beta(Second, Third)$ over the second word
   (b) Normalize resulting distribution

   Note : You will get the partition function as a side product of normalization

2.a

$$\beta_2 = \psi_2 \cdot \delta_{1\to 2}(\text{Second}) =$$

	Noun	Verb	Other	
Noun	6	28	30	64
Verb	60	14	30	104
Other	18	70	20	108

$$\beta_2(\text{Third}) =$$

Third	$\phi(\text{Third})$
Noun	64
Verb	104
Other	108
$\sum$	276

2.b

Normalized $\beta_2(\text{Third}) = \hat{\beta}_2 =$

Third	$\phi(\text{Third}) / \sum_{T:\text{Third}} \beta_2(T)$
Noun	0,2319
Verb	0,3768
Other	0,3913

Assignment 1.3

3. Make the following queries :

   (a) You know that the first word is noun. What does it tell you about the third word? Find condition distribution $P(Third|First = Noun)$
   Hint : The formula for unnormalized distribution from the page 369 :

$$\widetilde{P}(T = N|F = N) = \sum_S \frac{\beta_1(F = N, S) * \beta_2(S, T = N)}{\mu_{1,2}(S)}$$

where F,S,T means words : 'First, Second, Third' and N means 'Noun'

3.a

Find $P(\text{Third} \mid \text{First} = N)$.

Let's denote First, second, Third = F,S,T. and Noun, verb, other = N,V,O.

$$\widetilde{P}(T=N \mid F=N) = \sum_S \frac{\beta_1(F=N,S) \cdot \beta_2(S,T=N)}{\mu_{1,2}(S)} \qquad [1.0]$$

Let us first calculate $\delta_{1\to 2}(F=N,S) =$

S	
N	1
V	10
O	3

$\beta_2(S,T) =$

	N	V	O
N	1	20	9
V	10	10	9
O	3	50	6

$\delta_{2\to 1}(S,T) =$

S	
N	14
V	8
O	8

We then obtain $\beta_1(F=N,S) =$

	N	V	O
N	14	0	0
V	80	0	0
O	24	0	0

which results in $\mu_{1,2}(\text{Second}) =$

S	
N	14
V	80
O	24

We can then calculate $\tilde{P}(T \mid F=N,S)$ by

$$\tilde{P}(T=N \mid F=N,S) = \{\text{See } [1.0]\} = \frac{14 \cdot 1}{14} + \frac{80 \cdot 20}{80} + \frac{24 \cdot 9}{24} = 30$$

$$\tilde{P}(T=V \mid F=N,S) = 10 + 10 + 9 = 29$$

$$\tilde{P}(T=O \mid F=N,S) = 3 + 50 + 6 = 59$$

Normalization constant $Z = \sum_S \mu_{1,2}(S) = 118$

which results in

$\hat{P}(T \mid F=N,S) =$

T	
N	0,2542
V	0,2458
O	0,5000
	1,000

(b) Now you also know that the second word is verb. Update your probability that the 3rd word is "other".

Note : Do you really need to use belief in order to answer to the last query ?

3.b By also knowing the second word, we obtain the following for T=other.

$\delta_{1\to 2}(F=N, S=V) =$

S	
N	0
V	10
O	0

$\beta_2(S=V,T) =$

	N	V	O
N	0	20	0
V	0	10	0
O	0	50	0

$\delta_{2\to 1}(S=V,T) =$

S	
N	0
V	1
O	0

$\beta_1(F=N,S=V) =$

	N	V	O
N	0	0	0
V	10	0	0
O	0	0	0

$$\tilde{P}(T=O \mid F=N,S=V) = \frac{\beta_1(F=N,S=V)\cdot\beta_2(S=V,T=O)}{\mu_{1,2}(S,V)} = \frac{10\cdot 50}{10} = 50.$$

Let the normalization factor $Z = \sum_T \beta_2(S=V,T) = 80$, we then get

$\hat{P}(T=O \mid F=N,S=V) = \frac{50}{80}$.

Hint: No, we have no need for believes. Since all variables are observed but one, one only needs to look at: $\frac{\psi(S=V,T)}{\sum_T \psi(S=V,T)}$ for any given T,

For $T=O$ we get $\frac{5}{8} = \frac{50}{80} = \hat{P}(T=O \mid F=N,S=N)$