

Assignment 3.

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Task 1

Consider the relation schema $R(A, B, C, D, E, F)$ and the following three FDs:

FD1: $\{A\} \rightarrow \{B, C\}$

FD2: $\{C\} \rightarrow \{A, D\}$

FD3: $\{D, E\} \rightarrow \{F\}$

Use the Armstrong rules to derive each of the following two FDs. In both cases, describe the derivation process step by step (i.e., which rule did you apply to which FDs).

a) $\{C\} \rightarrow \{B\}$:

SOLUTION:

FD4: $\{A\} \rightarrow \{B\}$ FD1 and Decomposition.

FD 5: $\{C\} \rightarrow \{A\}$ FD2 and Decomposition.

FD6: $\{C\} \rightarrow \{B\}$ FD4, FD5 and Transitivity.

b) $\{A, E\} \rightarrow \{F\}$:

SOLUTION:

FD4: $\{A\} \rightarrow \{C\}$ FD1 and Decomposition.

FD5: $\{C\} \rightarrow \{D\}$ FD2 and Decomposition.

FD6: $\{A\} \rightarrow \{D\}$ FD4, FD5 and Transitivity.

FD7: $\{A, E\} \rightarrow \{F\}$ FD3, FD6 and Pseudo Transitivity.

Task 2

For the aforementioned relation schema with its functional dependencies, compute the attribute closure X^+ for each of the following two sets of attributes.

a) $X = \{A\} \Rightarrow X^+ = \{A\}$ from the beginning.

FD1: $X^+ = \{A, B, C\}$ since A is a subset of $\{A\}$ and $\{B, C\}$ is not.

FD2: $X^+ = \{A, B, C, D\}$ since C is a subset of $\{A, B, C\}$ and $\{A, D\}$ is not.

FD3: $X^+ = \{A, B, C, D\}$ since $\{D, E\}$ is not a subset of $\{A, B, C, D\}$, no additions are made.

$X^+ = \{A, B, C, D\}$

b) $X = \{C, E\} \Rightarrow X^+ = \{C, E\}$ from the beginning.

FD2: $X^+ = \{A, C, D, E\}$ since $\{C\}$ is a subset of $\{C, E\}$ and $\{A, D\}$ is not.

FD1: $X^+ = \{A, B, C, D, E\}$ since $\{A\}$ is a subset of $\{A, C, D, E\}$ and $\{B, C\}$ is not.

FD3: $X^+ = \{A, B, C, D, E, F\}$ since $\{D, E\}$ is a subset of $\{A, B, C, D, E\}$ and $\{F\}$ is not.

$X^+ = \{A, B, C, D, E, F\}$

Task 3

Consider the relation schema $R(A, B, C, D, E, F)$ with the following FDs

FD1: $\{A, B\} \rightarrow \{C, D, E, F\}$

FD2: $\{E\} \rightarrow \{F\}$

FD3: $\{D\} \rightarrow \{B\}$

1. a) Determine the candidate key(s) for R.

Since A isn't present in any of the right-hand sides FD's, it should therefore be a part of every candidate-key. Neither C or F is present in the left-hand side of any FD, which then excludes them from being part of a candidate-key. Therefore, we have three possible candidate keys, $\{A, B\}$, $\{A, D\}$ and $\{A, E\}$. So we will calculate attribute closure for each one of them, to determine which of them are candidate keys.

$\{A, B\}^+ = \{A, B, C, D, E, F\}$. Since $\{A, B\}$ is a subset of $\{A, B\}$ and $\{C, D, E, F\}$ is not. This means that $\{A, B\}$ is a candidate key to R, given FD1, FD2, FD3.

$\{A, D\}^+ = \{A, B, D\}$ since $\{D\}$ is a subset of $\{A, D\}$ and $\{B\}$ is not.

$\{A, D\}^+ = \{A, B, C, D, E, F\}$. Since $\{A, B\}$ is a subset of $\{A, B, D\}$ and $\{C, D, E, F\}$ is not. This means that $\{A, D\}$ is a candidate key to R, given FD1, FD2, FD3.

$\{A, E\}^+ = \{A, E, F\}$ since $\{E\}$ is a subset of $\{A, E\}$ and $\{F\}$ is not. But from here we cannot add any more attributes to the key, which means it is not a candidate key to R, given FD1, FD2, FD3.

Candidate keys are {A, B} and {A, D}.

2. b) Note that R is not in BCNF. Which FD(s) violate the BCNF condition?

Both FD2 and FD3 violates the condition of BCNF, since none of them have super keys on the left side.

3. c) Decompose R into a set of BCNF relations, and describe the process step by step (don't forget to determine the FDs and the candidate key(s) for all of the relation schemas along the way).

R (A, B, C, D, E, F) with

FD1: {A, B} \rightarrow {C, D, E, F}	BCNF
FD2: {E} \rightarrow {F}	NOT BCNF
FD3: {D} \rightarrow {B}	NOT BCNF

Decomposing R with regards to FD2:

R1(E, F) with FD2 :{E} \rightarrow {F} and candidate key {E}.

R2(A, B, C, D, E) with FD3: {D} \rightarrow {B} and FD4: {A, B} \rightarrow {C, D, E}.

FD4 is derived from FD1 using the Decomposition rule. Candidate key {A, B}.

FD3 still violates BCNF. Decompose R2 with regards to FD3.

R2X(D, B) with FD3: {D} \rightarrow {B} and candidate key {D}.

R2Y(A, C, D, E) with FD5: {A, D} \rightarrow {C, E}.

FD5 is derived from FD3 and FD4 using the Augmentation rule, the transitivity rule and finally decomposition. Candidate key {A, D}.

The decomposition process results in three relations, R1, R2X, R2Y.

Task 4

Consider the relation schema $R(A, B, C, D, E)$ with the following FDs

FD1: $\{A, B, C\} \rightarrow \{D, E\}$

FD2: $\{B, C, D\} \rightarrow \{A, E\}$

FD3: $\{C\} \rightarrow \{D\}$

1. a) Show that R is not in BCNF.

$\{A, B, C\}^+ = \{A, B, C, D, E\}$ since $\{A, B, C\}$ is a subset of $\{A, B, C\}$ and $\{D, E\}$ is not.
 $\{B, C, D\}^+ = \{A, B, C, D, E\}$ since $\{B, C, D\}$ is a subset of $\{B, C, D\}$ and $\{A, E\}$ is not.
 $\{C\}^+ = \{C, D\}$ since $\{C\}$ is a subset of $\{C\}$ and $\{D\}$ is not.

This means that FD3 is violating BCNF, since $\{C\}$ is not a super key to R.

2. b) Decompose R into a set of BCNF relations (describe the process step by step).

Decompose R with regard of FD3.

$R_1(C, D)$ with FD3 $\{C\} \rightarrow \{D\}$ and candidate key $\{C\}$.

$R_2(A, B, C, E)$ with FD4: $\{A, B, C\} \rightarrow \{E\}$ and FD5: $\{BC\} \rightarrow \{A, E\}$. Candidate Key $\{BC\}$.

FD4 is derived from FD1 using the Decomposition Rule.

To come up with FD5 we did the following:

$\{C\} \rightarrow \{D\} \Rightarrow \{B, C, C\} \rightarrow \{B, C, D\}$ using Augmentation rule.

$\{B, C\} \rightarrow \{A, E\}$ using transitivity rule along with FD2.