Robot vision - assignment 3

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task 1

task 1a

We have the transformation from world frame to camera frame given by

$$\begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \tag{1}$$

and the relationship between normalized image coordinates and points on the plane given by

$$x = X^c/Z^c = \frac{u - c_x}{f_x} = \frac{r_{11}X + r_{12}Y + t_x}{r_{31}X + r_{32}Y + t_z} \tag{2}$$

$$y = Y^c/Z^c = \frac{u - c_x}{f_x} = \frac{r_{21}X + r_{22}Y + t_y}{r_{31}X + r_{32}Y + t_z} \tag{3}$$

which gives us the relationship between normalized image coordinates (x,y) and points on the plane (X,Y). We now define the new coordinates $x = \bar{x}/\bar{z}$ and $y = \bar{y}/\bar{z}$ We can then write

$$\begin{bmatrix} \bar{x}/\bar{z} \\ \bar{y}/\bar{z} \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{r_{11}X+r_{12}Y+t_x}{r_{31}X+r_{32}Y+t_z} \\ \frac{r_{21}X+r_{22}Y+t_y}{r_{31}X+r_{32}Y+t_z} \\ 1 \end{bmatrix}$$

Multiplying both sides by $\bar{z}$ gives us the following

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} r_{11}X + r_{12}Y + t_x \\ r_{21}X + r_{22}Y + t_y \\ r_{31}X + r_{32}Y + t_z \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

which is what we wanted to prove

1b

We say the homography H is only defined up to scale because a point P represented by $\mathbf{x} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T$ can also be represented by any non-zero scalar multiple of $\mathbf{x}$ as for Hx = y, and (cH)x = cy represents the same point

2a

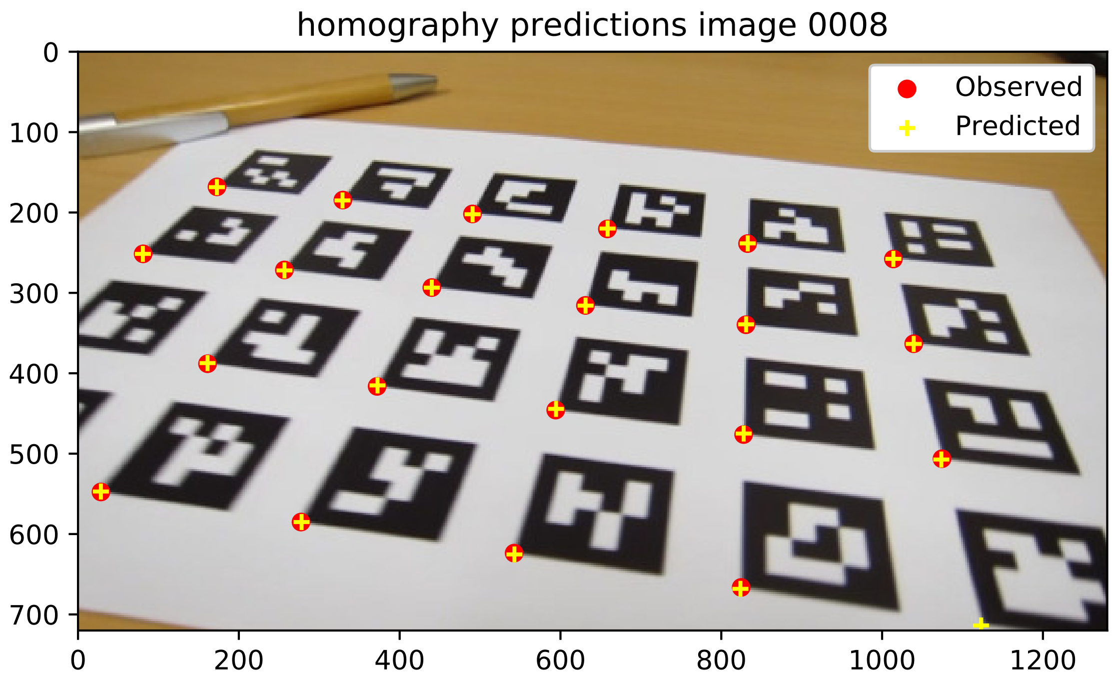

Figure 1: Result

2c

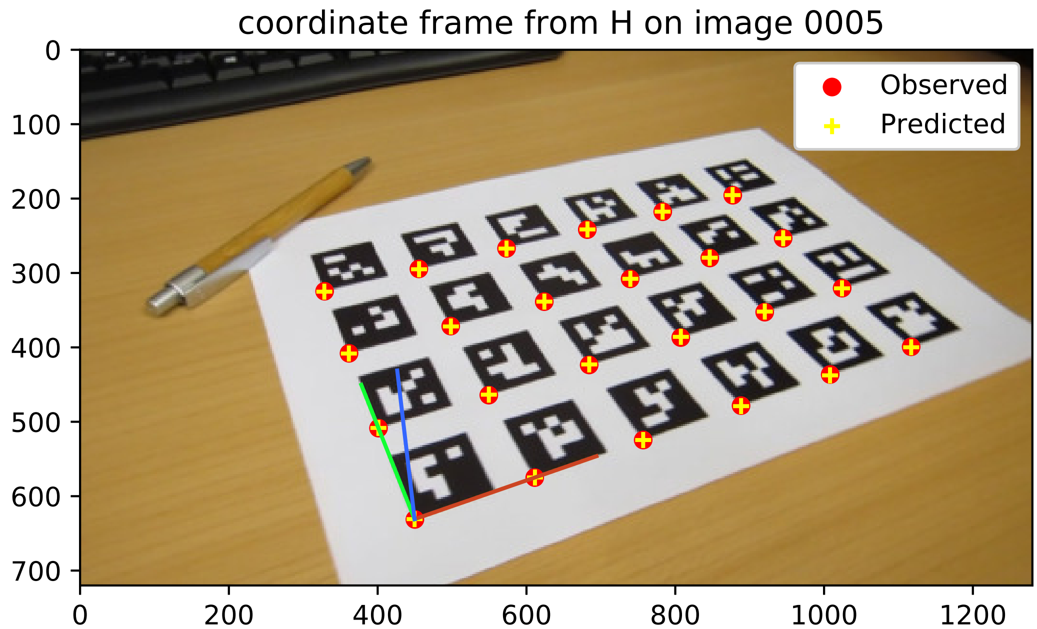

Figure 2: Result